

Notes can be downloaded from:

www.geo.ucalgary.ca/~wu/TUDeft/index.htm

Ice Age, Climate & Ice Model

- Ice Age & Astronomical Theory
- Other causes of Paleoclimate Change
- Climate in the last 20,000 years
- Introduction to Glaciology
- Constructing ICE Model for GIA
- GIA feedback on Ice Inception

Kettle lakes



Drumlin



Striations in bedrock



Erratic boulder



At the height of the last Ice Age 20,000 years ago: ~2,000 meters ice covered Canada, northern U.S., Europe, Asia, Antarctica.
Total Ice Mass $\sim 3 \times 10^{19}$ kg, sea level fell by ~120 meters

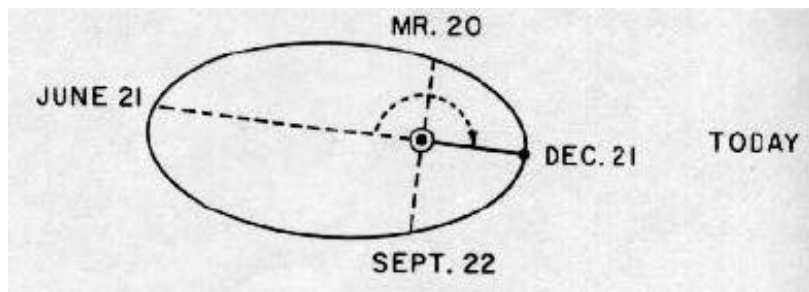


Louis Agassiz

Theory of Ice Age:

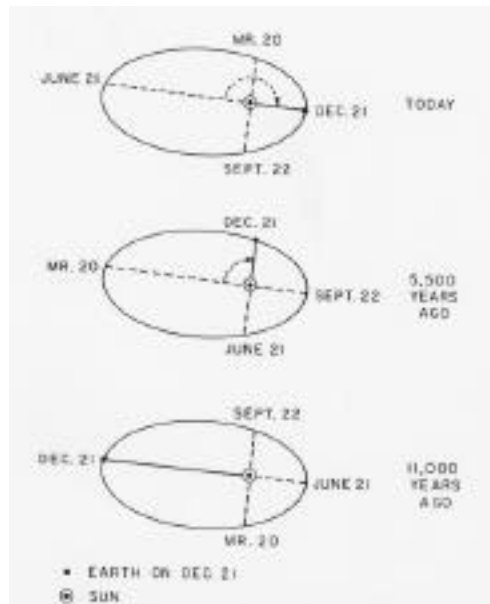
- 1787 Bernard Kuhn: erratic boulders in Swiss Jura is evidence of ancient glaciation.
- 1794 James Hutton visited the Jura and arrived at the same conclusion.
- 1824 Jens Esmark found evidence of extensive glaciation in Norway.
- 1832 Reinhard Bernhardt argued that a polar ice cap covered Northern Europe reaching as far south as central Germany.
- 1833 Charles Lyell argued that the huge erratics were transported by boulder-laden icebergs and ice rafts of the great flood.
- 1837 Louis Agassiz, based on observations of Venetz and de Charpentier, argued that erratics were evidence of past glaciation and an ancient Ice Age.

Birth of Astronomical Theory



In the Northern Hemisphere, spring and summer contain 168 hours (7 days) of daylight hours more than fall and winter. In the Southern Hemisphere, this situation is completely reversed.

Joseph Adhémar (1842) argued that because the Southern Hemisphere has more hours of darkness each year than daylight, that hemisphere must be growing colder.



Adhémar's Theory:

The equinoxes precess with a period of about 22,000 years.

Whichever hemisphere had a longer winter would experience an ice age.

Thus, every 11,000 years, an ice age would occur alternately in one hemisphere and then in the other.

Alexander Von Humbolt's (1852) opposition to Adhémar's Theory:

- 1) It is not the number of hours of daylight and darkness that affects the temperature of the hemispheres, but the calories of energy from the sun that is received each year.
- 2) As shown by d'Alembert, any decrease in solar heating during the season when the earth is farther from the sun, is exactly balanced by an increase during the opposite season, when the earth is closer to the sun.

Thus, the total solar energy received by one hemisphere during the year is always the same as that received by the other hemisphere.



James Croll

Leverrier (1843): gravitational pull of the planets causes the **orbital eccentricity** and the **tilt** of the rotation axis to change.

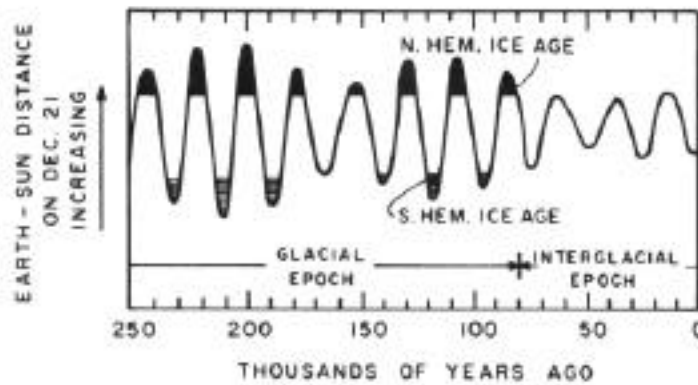
James Croll (1867) :

- 1) **Eccentricity varies period-ically between 1%(more circular) to 6% (more elliptical).** High-eccentricity ~100,000 years ago. Low eccentricity afterwards.
- 2) **Intensity of radiation received by the earth during each season is strongly affected by changes in eccentricity.**

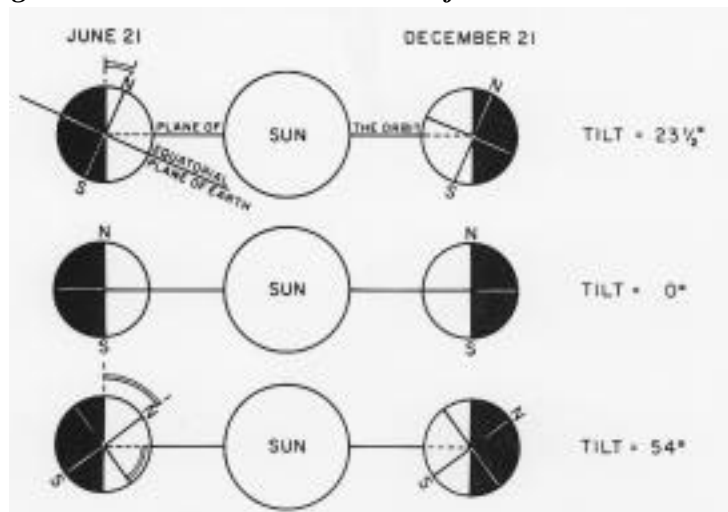
Croll reasoned that:

- 1) Decrease in the amount of solar radiation received during the winter favors the accumulation of snow. “Albedo feedback” will result in an additional loss of heat by reflecting more sunlight back into space.
- 2) If winter occurs when the earth is close to the sun, winter will be warmer than usual. On the other hand, if winter occurs when the sun is far from the sun, temperatures are colder than usual.
- 3) If the polar area of one hemisphere becomes colder, the stronger will be the trade winds in that hemisphere, but the warm equatorial currents in the ocean will be forced to shift towards the other hemisphere, so that even more heat is lost.
- 4) If the orbit were circular, the precession of the equinoxes would have no effect at all on climate because each season would occur at the same distance from the sun.

- 5) Combining both Precessional cycle and variation in Eccentricity, Croll predicted that an Ice Age (a markedly elongate orbit and a winter solstice that occurs far from the sun) occurred about 250,000 years ago and ended about 80,000 years ago.



Croll (1875) : Tilt of the earth's axis also varies about 3 deg (22 - 25 deg.) An Ice Age would be more likely to occur *when the axis is closer to vertical, for the polar regions receive a smaller amount of heat.*



Objections to Croll's theory:

- 1) the last glacial period ended not 80,000 years ago but around 6,000 to 10,000 years ago.
- 2) southern hemisphere glaciers may actually be in phase with that in the northern hemisphere and ice ages did not occur alternately in one hemisphere and then in the other one.
- 3) meteorologists found that the variations in solar heating described by Croll were too small to have any noticeable effect on climate.

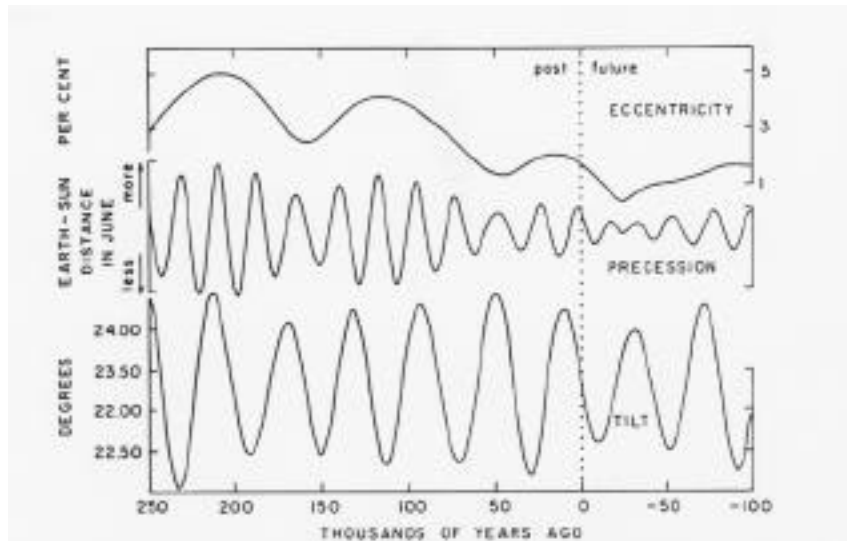
Ludwig Pilgrim (1904) calculated the combined effects of eccentricity of the orbit, the tilt of the axis of rotation and the precession of the equinoxes in the last 1 million years.



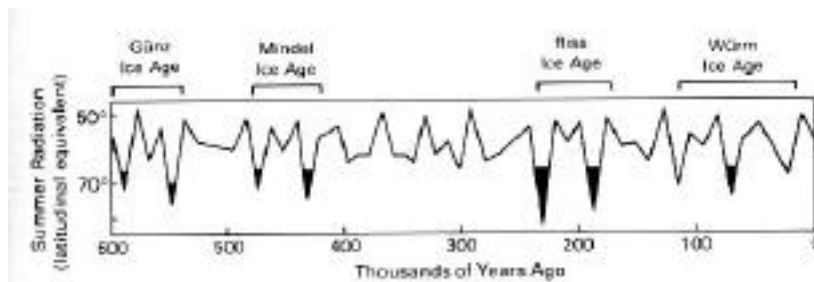
Milutin Milankovitch

Milankovitch used Pilgrim's results to compute the geographic and seasonal distribution of sunlight for the past 1 million years. He showed that the effect of the tilt angle on climate is more important than Croll had suggested.

Milankovitch 's Theory of Ice Age

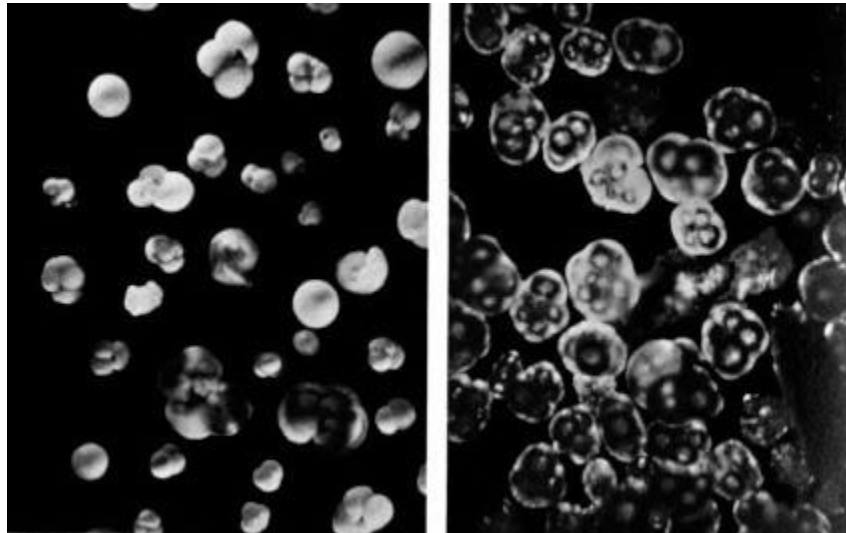
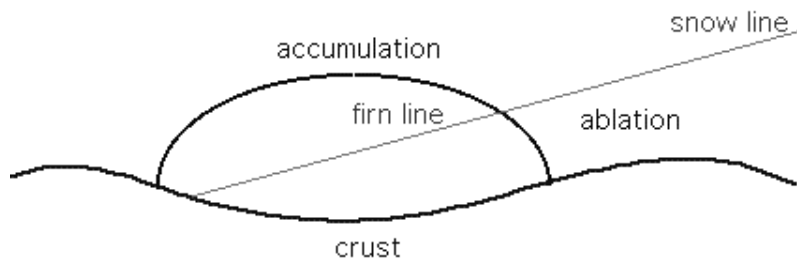


Milankovitch (in discussion with Koppen & Wegener), found that annual snow budget is most affected during the summer when modern glaciers melt. *Any decrease in the intensity of summer sunlight would inhibit melting, making the annual snow budget positive, and lead to glacial expansion.*

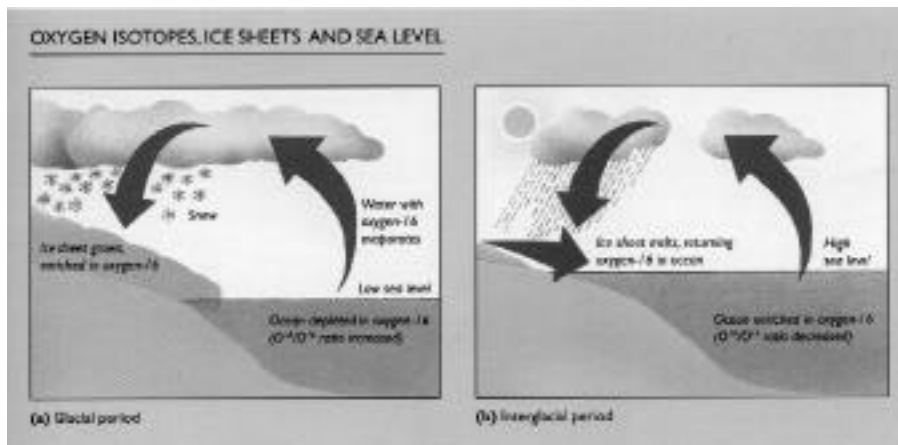


Snowline – the elevation above which there is at least some snow all year round.

Milankovitch formulated a mathematical relationship between summer radiation and the altitude of the snowline and determined how much increase in snow cover would result from any given change in summer radiation.



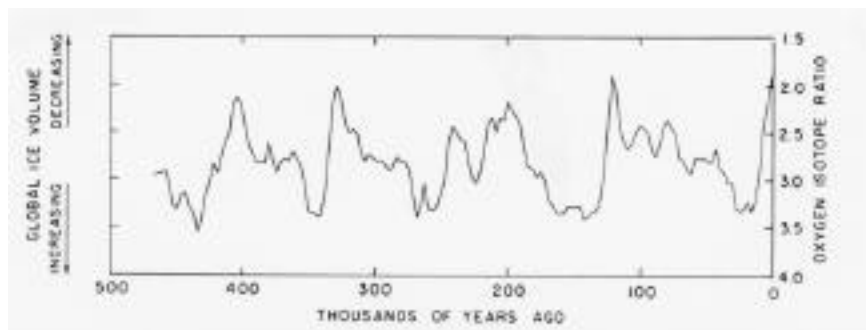
The shell of these tiny sea animals gives a complete record of variation in O^{16} isotope in the sea (or ice volume fluctuation) during the past millions of years.



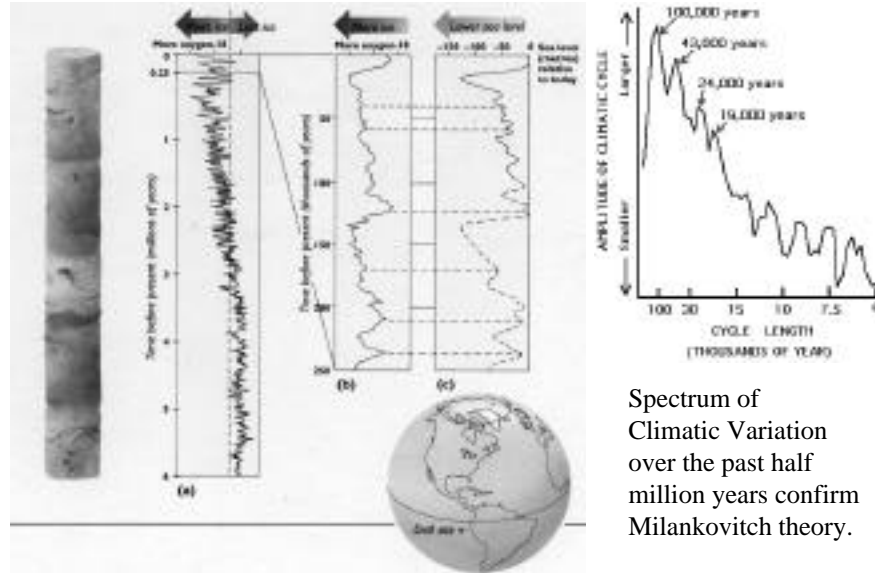
The waxing and waning of large ice sheets on land can affect the ratio of the light O^{16} and heavy O^{18} isotopes in seawater. During the glacial period, the water in the oceans is depleted in O^{16} . During an interglacial period, the ice sheets melt, raising the sea level and enriching the oceans again with O^{16} .

Confirmation of Milankovitch Theory:

Oxygen isotope data from deep-sea cores confirm the existence of the astronomical cycles with periods of 100, 43, 24 & 19 ka (According to Imbrie 1974, these correspond to eccentricity, tilt and 2 precession peaks respectively.)

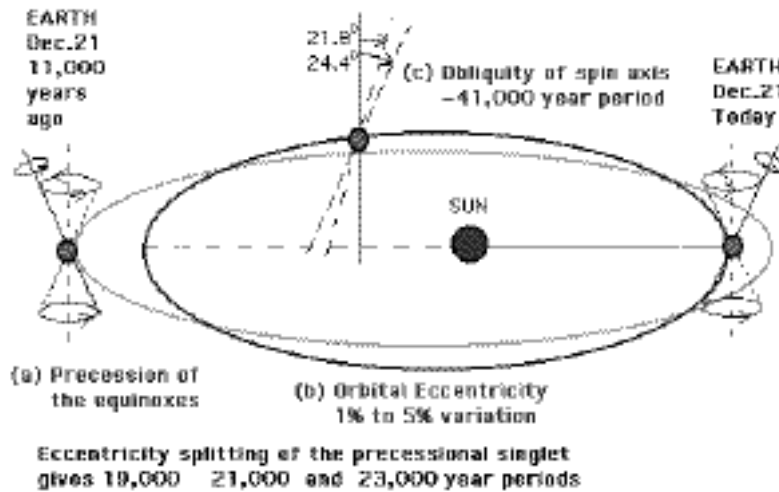


Oxygen Isotope data from Deep Sea Cores reveal past climate/ice volume fluctuations

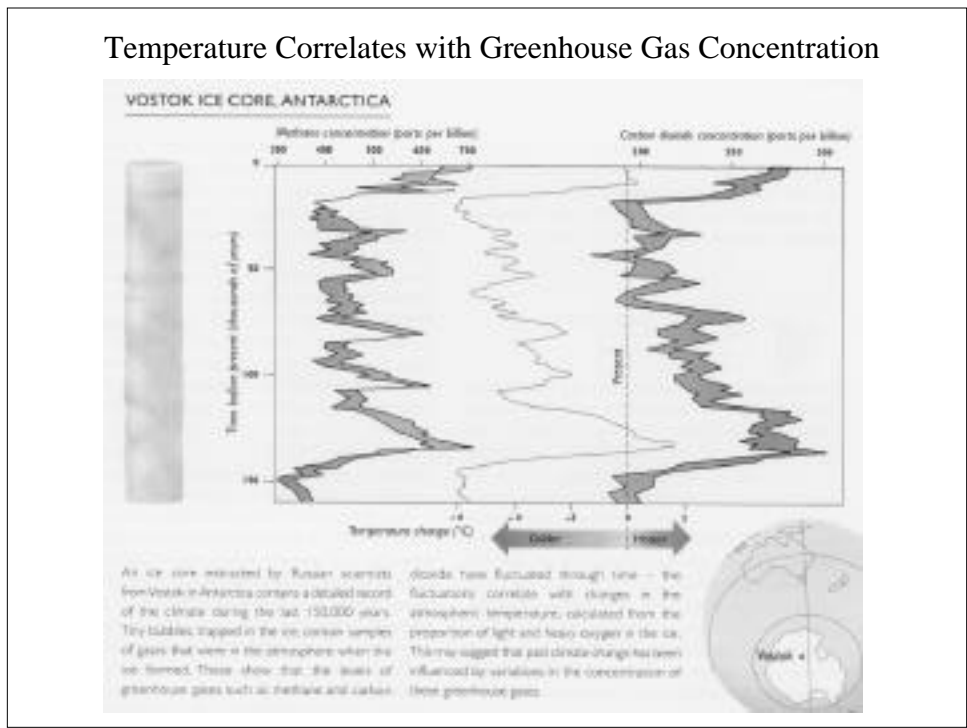


Spectrum of Climatic Variation over the past half million years confirm Milankovitch theory.

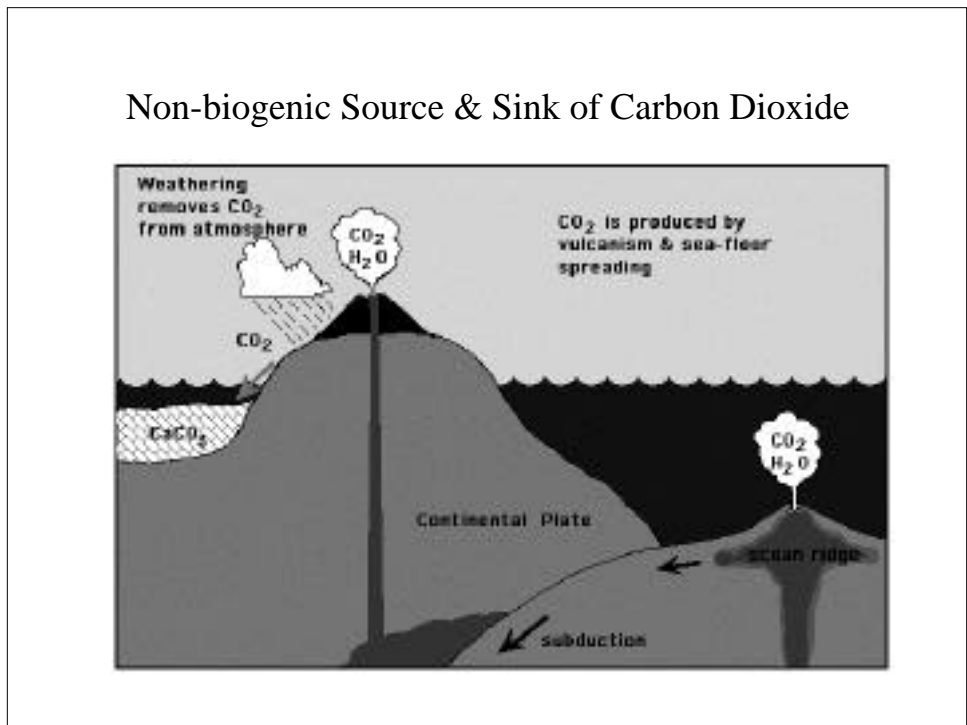
Milankovitch's Orbital Theory

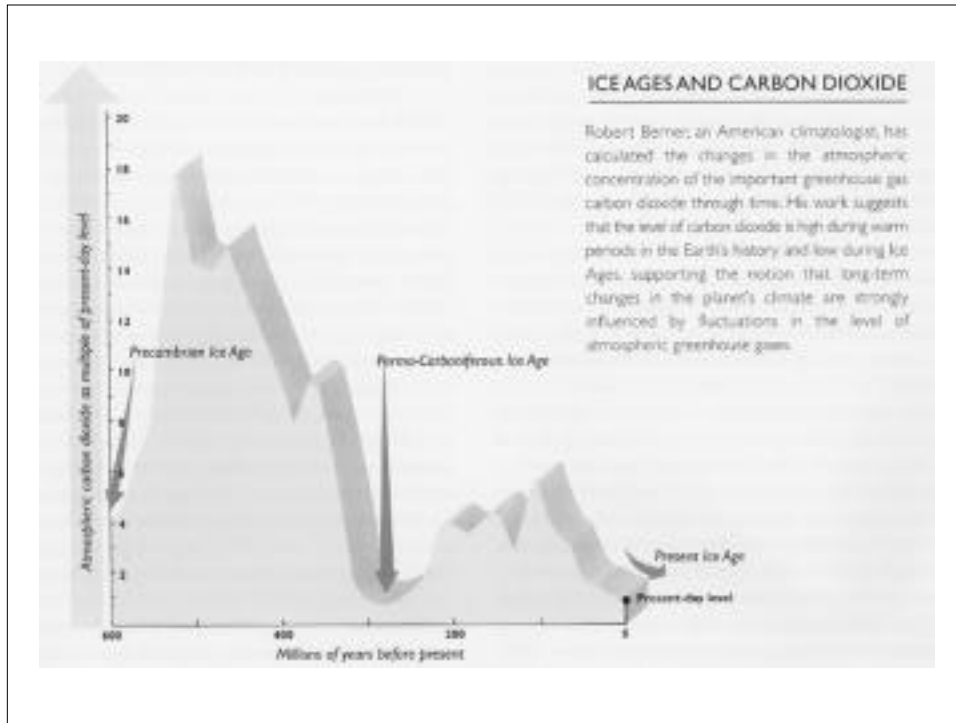


Temperature Correlates with Greenhouse Gas Concentration



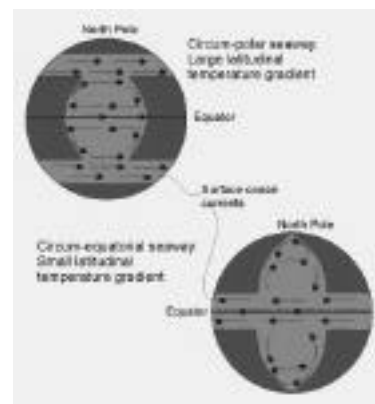
Non-biogenic Source & Sink of Carbon Dioxide





Other Factors for Climatic Change:

- Biological pump feedback
- Thermohaline circulation
- Circum-polar vs Circum-equatorial Ocean Currents
- Dust particles from volcanic eruptions
- Presence of supercontinents near the pole



Principle sources of Proxy data for Palaeoclimate Reconstructions (1)

- Glaciological (Ice Cores)
Oxygen isotopes,
Physical properties,
Trace element & microparticle concentrations
- Geological
Marine & Terrestrial Sediments
Sedimentary Rocks
- Biological
- Historical

- ## Principle sources of Proxy data for Palaeoclimate Reconstructions (2)
- Geological
 - A. Sediments
 - 1. Marine (ocean sediment cores)
 - i) Organic sediments (planktonic & benthic fossils)
Oxygen isotopes, Faunal & floral abundances, Morphological variations
 - ii) Inorganic sediments
Mineralogical composition & surface texture,
Distribution of terrigenous material, Ice-rafted debris, Geochemistry
 - 2. Terrestrial
Periglacial features, Glacial deposits & erosional features,
Glacio-eustatic features (shorelines), Aeolian deposits (sand dunes)
Lacustrine deposits/varves (lakes)
 - B. Sedimentary Rocks
Facies analysis, Fossil/microfossil analysis, Mineral analysis,
Isotope geochemistry

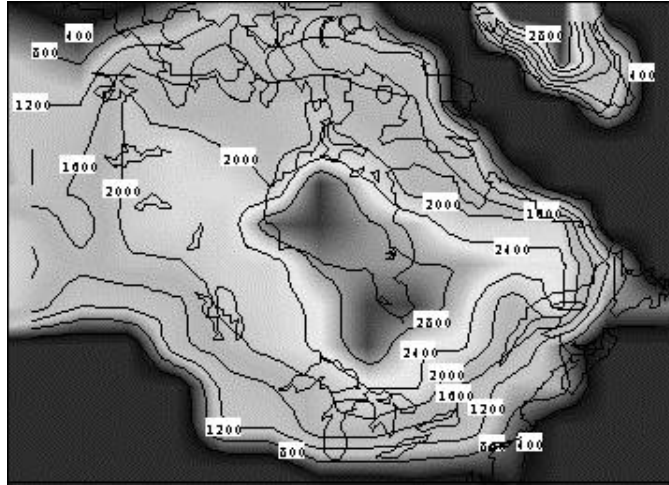
Principle sources of Proxy data for Palaeoclimate Reconstructions (3)

- Biological
 - Tree rings (width, density, isotope analysis)
 - Pollen (species, abundances)
 - Insects
- Historical
 - Meteorological records
 - Parameteorological records (environmental indicators)
 - Phenological records (biological indicators)

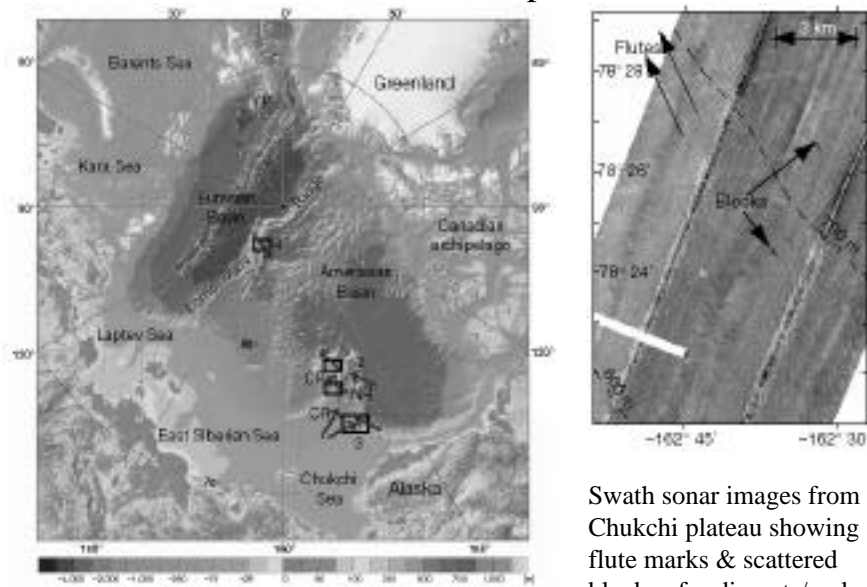
Climate at last Glacial Maximum:

- Ice & Snow Cover
 - ✓ Ice thickness ~3 km in Laurentide, slightly thinner in Fennoscandia. Antarctica ~ 500 m greater than present. Total ice is equivalent to ~120 m drop in mean sea level.
 - ✓ Ice covered Arctic Ocean? Tibetan Plateau?
 - ✓ Atlantic ocean south of 45°N remained free of sea ice during the winter due to migration of polar front (c.f. today it is around 78°N)
 - ✓ Snowline depression of ~1000 m, temperature depression of 5-6 °C at elevations > 2000 m. (pollen data)
 - ✓ Southern Hemisphere glaciation synchronous with that of the Northern Hemisphere.
 - ✓ Large changes in Antarctic sea ice cover. At 18,000 BP, sea ice was approximately twice (?) the present area of Antarctic ice cover.

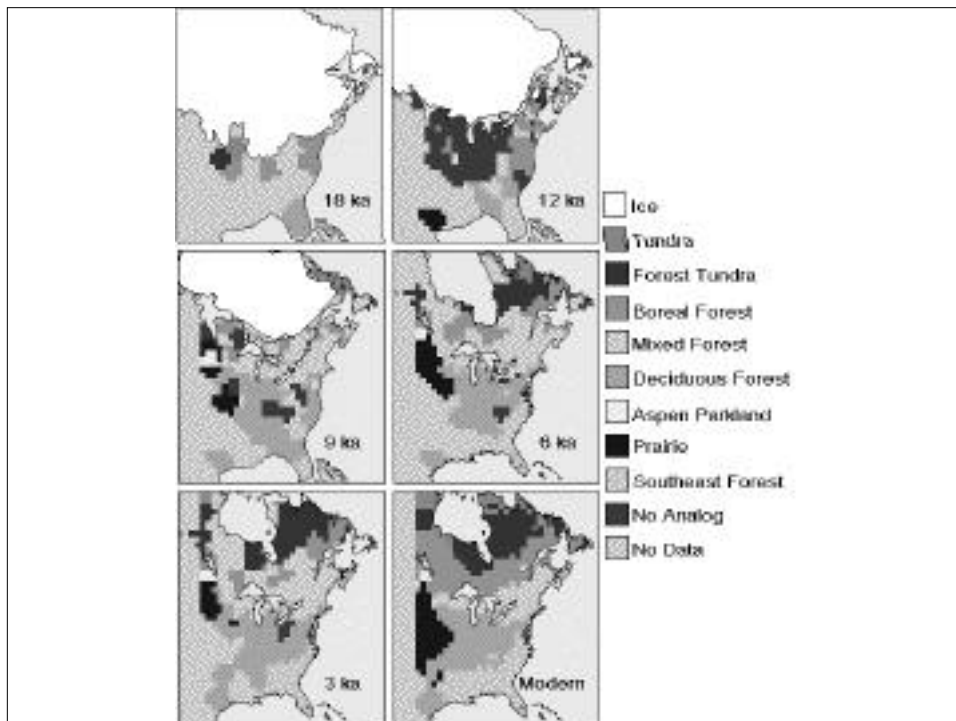
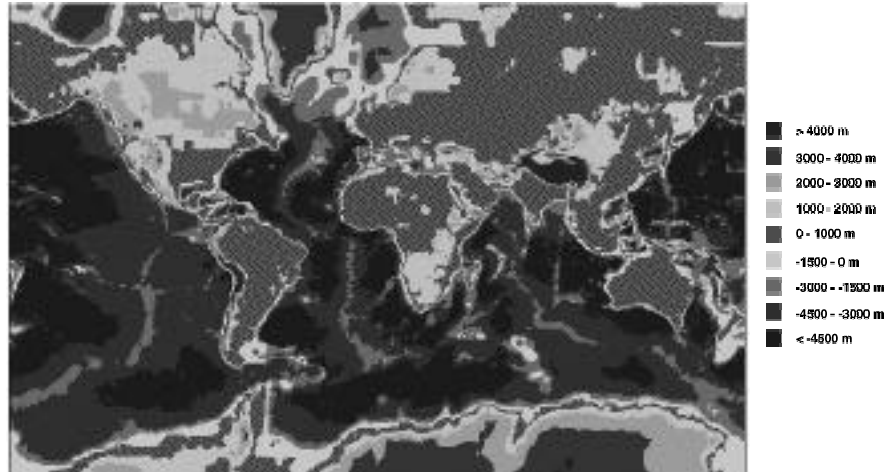
Ice thickness at Glacial Maximum



Submarine SCICEX Expedition 1999



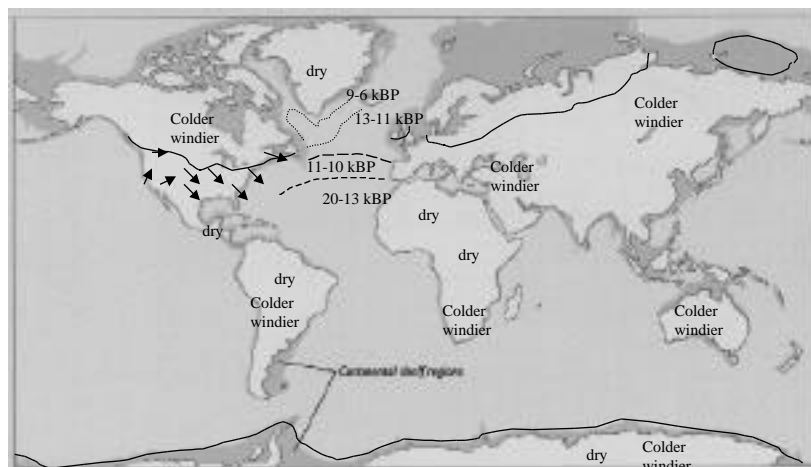
Paleo-Topography at 18 ka BP



Climate at last Glacial Maximum:

- Temperature Changes on Land
 - ✓ High-latitude: annual temperature generally decreased ~ 10°C ; winter T decrease by ~15-20° C (due to polar wind?)
 - ✓ Mid-latitude: generally decrease ~ 5-8° C
 - ✓ Tropics: generally decrease ~ 4-5° C
- Sea Surface Temperature (SST) Changes
 - ✓ SST decrease by ~ 6-10° C in regions affected by oceanic polar fronts; ~ 6° C along eastern boundary currents.
 - ✓ Tropical oceans (within 40° latitude) decrease by ~ 1-2° C
 - ✓ Globally averaged decrease in SST ~ 1.6° C (CLIMAP 81)

Position of North Atlantic polar front & Surface wind direction from eolian features



Climate at last Glacial Maximum:

- Precipitation Changes
 - ✓ Generally drier during the last ice age. Greenland & Antarctic ice cores suggest 50% decrease in polar regions. Increased aridity is consistent with an increase in atmospheric dust found in ice cores & wind blown eolian sediments in Atlantic deep sea cores.
 - ✓ Tundra extended southward from the ice margins and spruce-pine boreal forest existed south of ~34°N.
 - ✓ Mid-latitude areas affected by equatorially displaced westerlies were moist.
 - ✓ Tropical lowlands were drier, lakes in tropical Africa & Central America were very low (250-500 m below present in E. Africa). Sand dunes expanded in sub-Sahara & Central America. Amazon rain forest may have reduced to a few “refugia”.

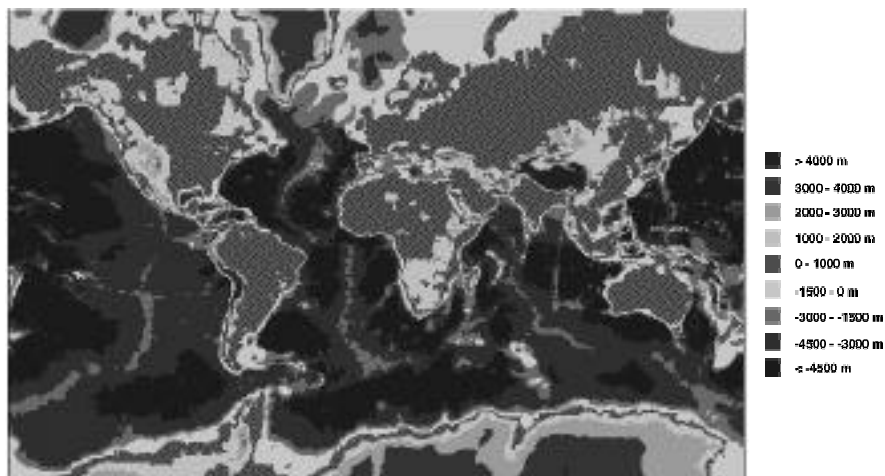
Climate at last Glacial Maximum:

- Atmospheric Circulation Changes
 - ✓ In North America, wind direction changes from present south-westerlies to ice age north-westerlies. Wind speed increased ~20-50% or more. The advection of very cold air by the north-westerlies might significantly affect evaporation rates in the Gulf Stream.
 - ✓ Upwelling indices & windblown (eolian) material in deep-sea cores suggest ~20% increase in speed for the North Pacific westerlies, ~30% increase for the North Pacific trades, ~50% increase for the North Atlantic trades, ~30-50% increase for the South Pacific trades.
 - ✓ Chloride concentration in ice cores also indicates 50-80% (or 5-8 m/sec) increase in wind speed of the North Atlantic and Southern Ocean westerlies.
 - ✓ Increased wind speed might strongly affect ocean circulation & sea ice formation.

Climate at last Glacial Maximum:

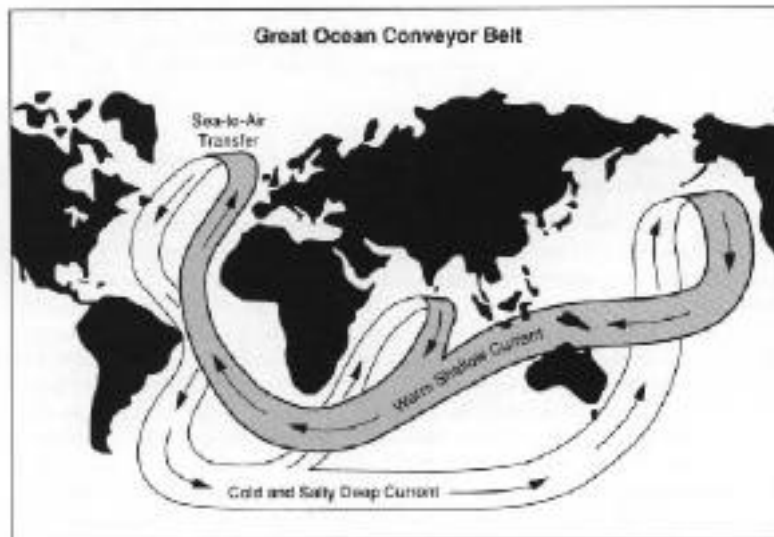
- Atmospheric Composition
 - ✓ High dust level
 - ✓ CO₂ concentration (ice cores) decreased to ~200 ppm (i.e. ~80 ppm less than the pre-industrial value)
 - ✓ Methane concentration (ice cores) also decreased to ~400 ppm from ~700 ppm
- Deep Ocean Changes
 - ✓ Cd/Ca ratios of benthic (bottom dwelling) forams suggest decrease in production rate for North Atlantic Deep Water (NADW) by 1/3 to 1/2.
 - ✓ Antarctic Bottom Water rates are variable.
 - ✓ No decrease in North Pacific deep water formation?
 - ✓ Reduced deep-ocean overturn ? temperature by ~1-2°C ?

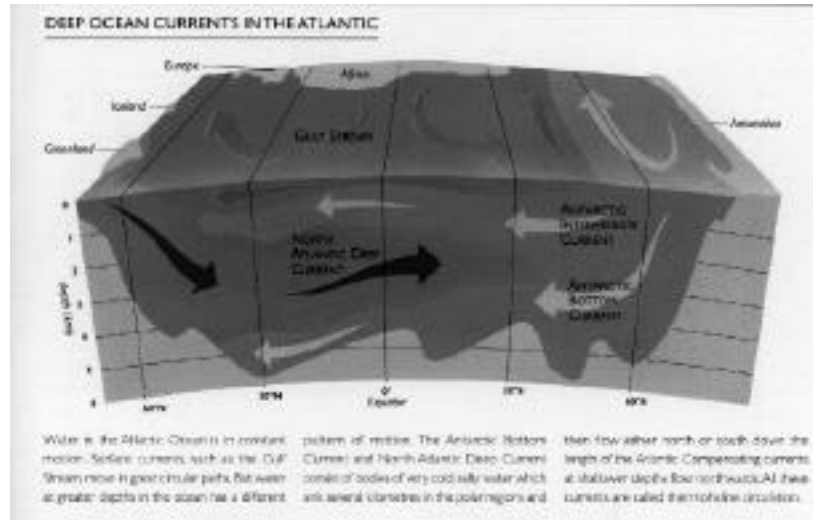
Paleo-Topography at 12 ka BP



Temporal Structure of Deglaciation:

- Abrupt warming ~14-13 kBP
 - ✓ Fossil beetles from England suggest summer T°C at 12 kBP was as warm as today. Alpine glacial retreat indicate warming from Alaska to Chile.
 - ✓ Antarctica melting started 16-17 kBP? SST in the Southern Ocean reached present level by 13 kBP.
- Younger Dryas Cooling ~11-10 kBP
 - ✓ North Atlantic polar front readvanced southward to ~50°N
 - ✓ Cooling in circum-subpolar North Atlantic Basin, Caribbean basin. Rapid change in Ethiopian lake levels.
 - ✓ Outflow of melted ice water decreased NADW production & thermohaline circulation, thus brought cooling.
- 2nd stage warming ~9-8 kBP
 - ✓ Final outflow of Laurentide ice from Hudson Strait & Hudson Bay



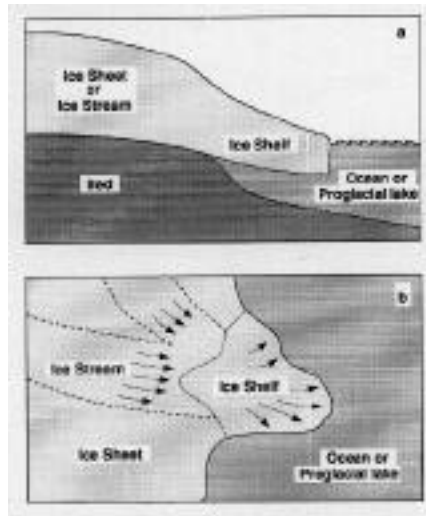


Holocene:

- Early Holocene ~9-4 kBP
 - ✓ ~2°C warmer than the last 4.5 kBP.
 - ✓ Latitudinal displacement of vegetation zones and vertical displacement of vegetation & mountain glaciers in North America, western Europe, New Guinea
 - ✓ A pulse of Holocene warmth in northern Canada ~ 9 kBP
 - ✓ North American Great Plains: 20% decrease in precipitation while monsoon belt became more moist.

- Late Holocene ~4 kBP to present
 - ✓ Cooler and drier

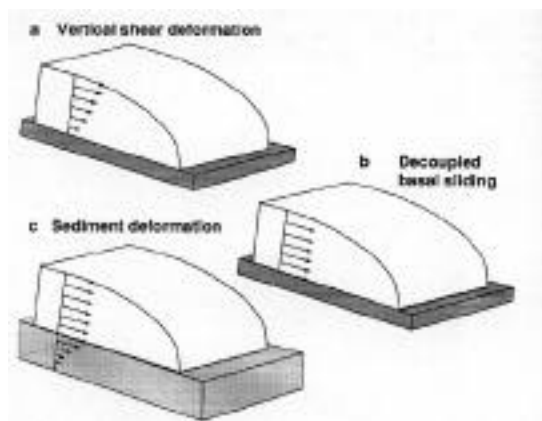
Dynamical Elements of Ice Sheets



- Sheet Ice - grounded & well-coupled with the bed, has steep margins
- Ice Streams - fast flowing currents within an ice sheet, little basal shear
- Ice Shelves - floating tongues of ice which spread out over proglacial lakes or continental shelves. Ice shelf flow is generally rapid (no basal shear). Calving into the water is a significant means of ablation.

Mechanisms of Ice Flow & Ice Elements

- Vertical shear deformation under gravity (sheet ice)
- Decoupled sliding at ice-bed interface (ice streams, ice shelves) warm based
- Basal motion via deformation of subglacial sediments (ice streams) warm based

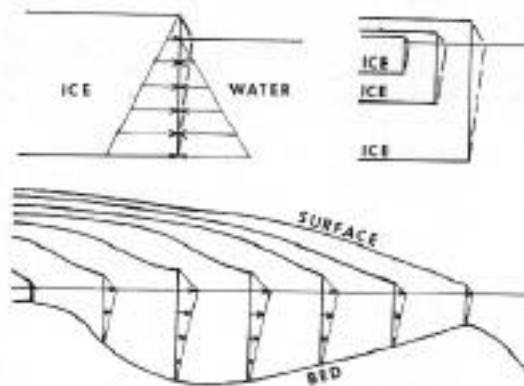


Introduction to Ice Sheet Physics

- Mass Balance (annual accumulation vs ablation) - controlled by winter accumulation & length & strength of summer ablation. Ablation can be from surface melt (solar energy), basal melting (geothermal heat), calving.
- Ice dynamics - temperature dependent viscous creep. Viscosity dependent on temperature which is affected by frictional heat from sliding, strain energy from internal deformation & heat advection.
- Basal flow condition - depends on the ice thermal regime. Sliding & sediment deformation due to warm based.

Pulling force in Shelf Ice

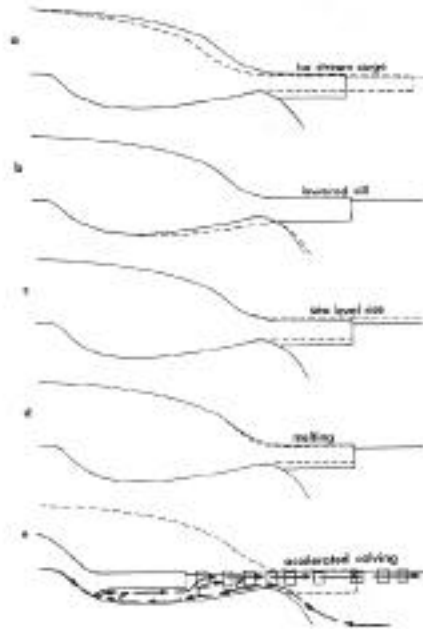
Due to difference between lithostatic & hydrostatic forces in ice and water. The pulling force increases with the square of floating ice thickness. As the grounding line retreats, the pulling force increases on a downslope bed, but increases on an upsloping bed.



Mechanism triggering collapse of Shelf Ice:

Irreversible collapse occurs when the grounding line migrates over its basal sill as a result of

- (a) ice thinning during an ice-stream surge,
- (b) lowering of sill by glacio-isostatic depression of the bed,
- (c) rising sea levels that lifts the ice shelf,
- (d) climatic warming that melts upper or lower surfaces of ice shelf,
- (e) accelerated calving and evacuation of ice bergs by water currents



Construction of Ice Model:

3 types of data are needed:

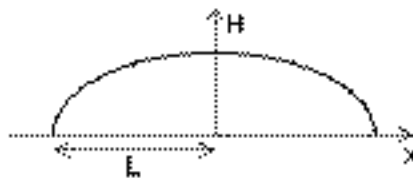
- 1) Ice margin (map & date terminal moraines give isochrone maps)
- 2) Ice profile (from study of ice dynamics)
- 3) World sea level changes



Isochrone & Flow direction Map of Fennoscandia

Theoretical Steady State Ice Profile:

Neglecting effects of topography, isostatic adjustment, imperfect plasticity, calving, surges or stagnation:



$$H = H_0 \left(1 - x/L \right)^{1/2}$$

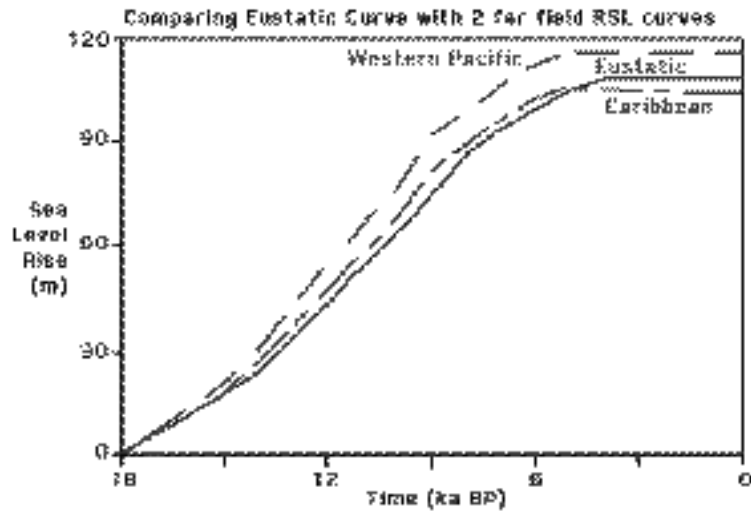
Maximum thickness $H_0 = (2 \tau_0 / \rho g)^{1/2}$

ρ = density of ice

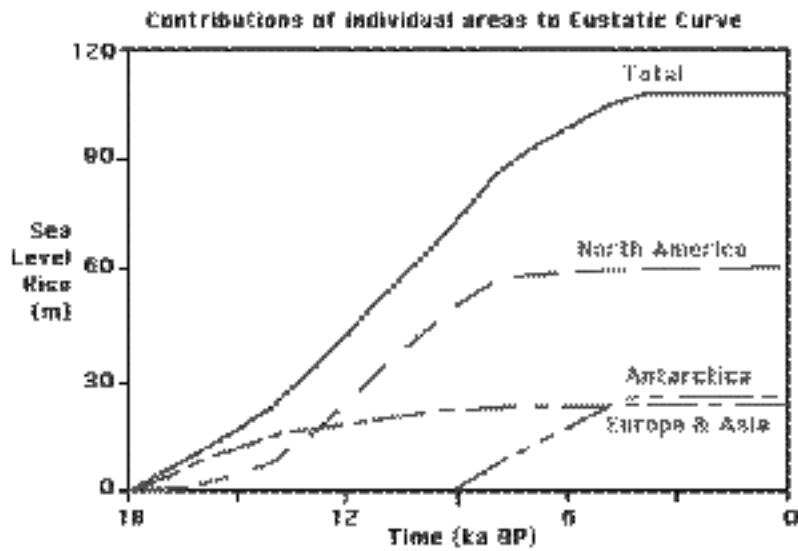
τ_0 depends on the base temperature & pressure/velocity

Thus, ice thickness depends on the lateral extent of the ice sheet.
Ice volume can be estimated from the area (isochrone data).

Validity of Eustatic Sea Level Curve



Eustatic Sea Level Curve

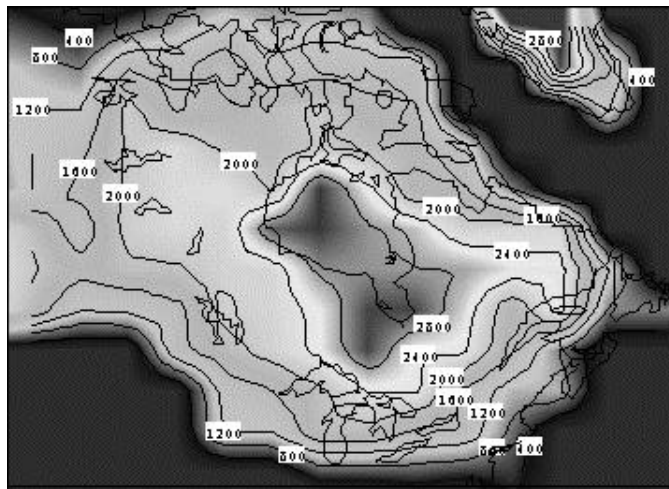


Construction of ICE1 :

For every glacier and at every time step:

- 1) Ice margin (isochrone maps) or lateral dimension
- 2) Ice profile (from ice dynamics) - thus estimate ice volume from isochrone maps and assumed basal shear σ .
- 3) World sea level changes - constrains total ice volume (and thus σ).

Ice thickness at Glacial Maximum

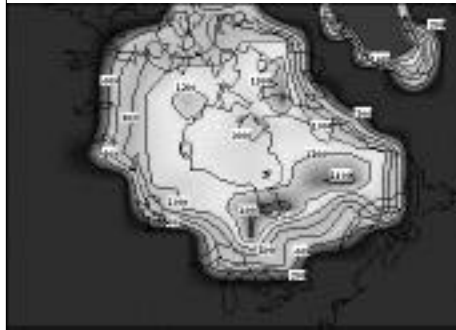




18 ka BP



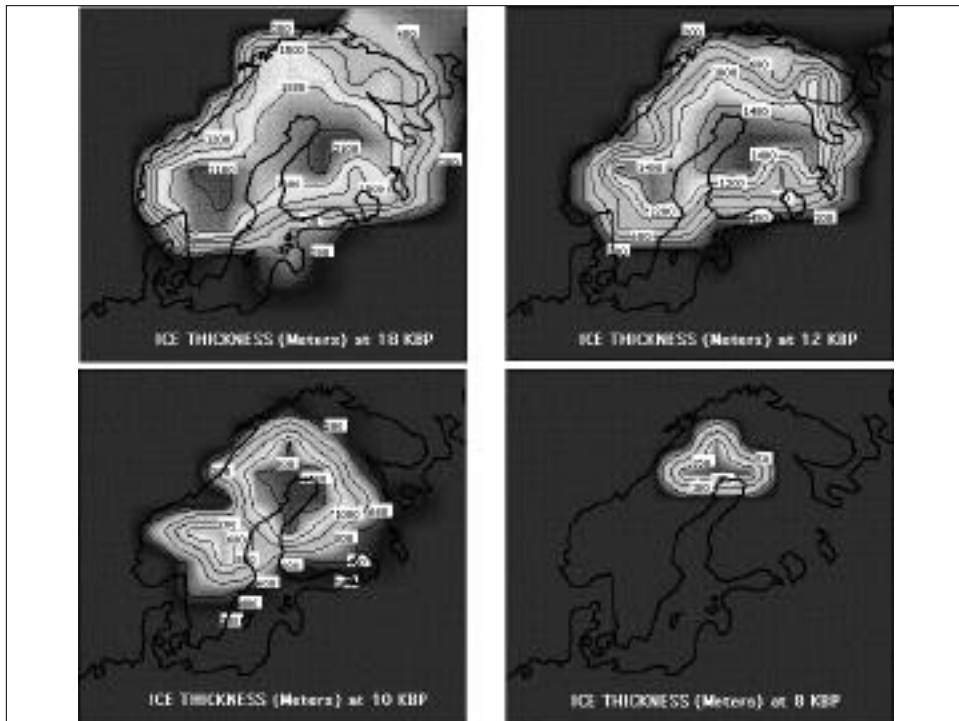
12 ka BP



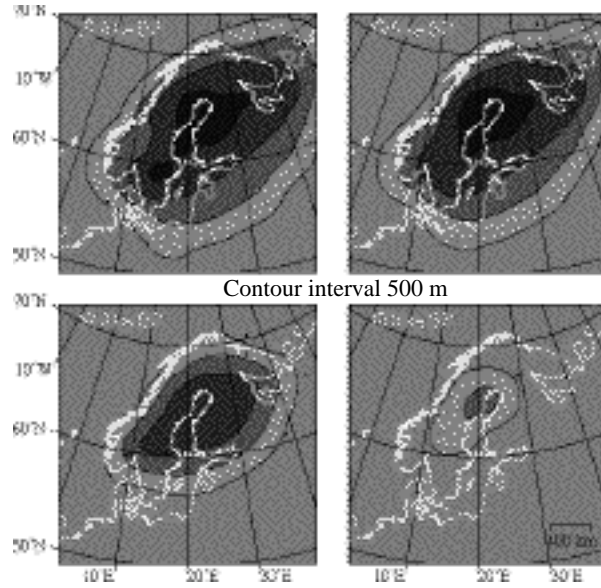
10 ka BP



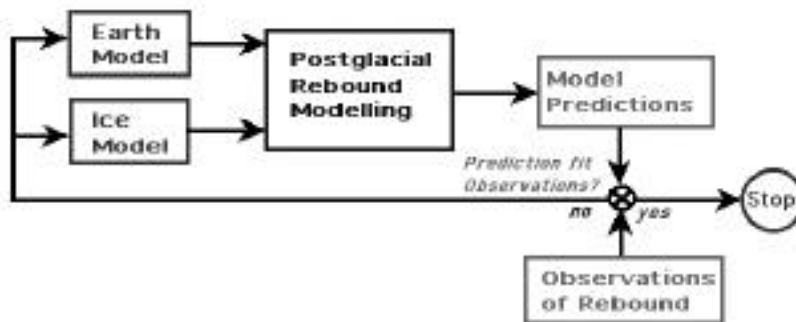
8 ka BP



Ice deglaciation history of Lambeck et al. (1998)



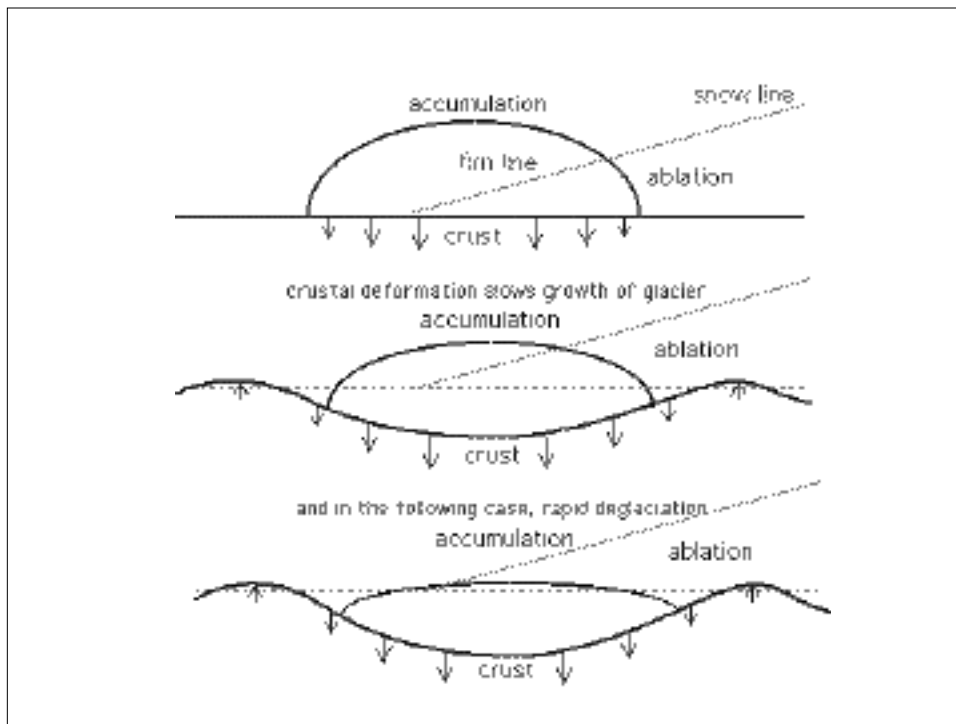
Forward Modelling



Reconstructing Quaternary Ice Sheets

- CLIMAP Reconstruction at Last Glacial Maximum -assumed steady-state ice sheets, no capability for simulating rapid changes, simplistic treatment of surface & basal mass balance
- Reconstruction using GIA observations - give ice thickness that are much thinner than that from CLIMAP
- Ice sheets that grew and collapsed quickly cannot be detected / recorded by the sea level data, thus the 'true' thickness probably lie between that estimated from GIA and steady-state estimates.





Motivation

- Bedrock deformation affects the growth/retreat of ice sheets, thus must be included in Ice models
- Many ice models use a simple local compensation mechanism:

$$-\frac{h^B}{t} = \frac{h^B - h_0^B}{\tau} + \frac{\rho^I H^I}{\rho^B \tau}$$

- So far, full models of Glacial Isostatic Adjustment (GIA) can only handle linear rheology in the mantle
- Rock physics tells us that mantle rheology may be nonlinear and recent analysis of Sea Level data around Laurentia (Wu 1999,2002) indicates that the lower mantle may be nonlinear.

Purpose

- Compute ice sheet growth and decay for linear and nonlinear (power-law) mantles with the new Finite-Element-Iterative Technique
- Compare and study how the different bedrock isostatic adjustment mechanisms (local compensation, linear and nonlinear rheology) affect ice sheet growth and decay

Outline

- Description of Ice Model
- GIA Bedrock Compensation Model
- Results:
 - a) Fennoscandia size ice sheets
 - b) Laurentia size ice sheets
- Summary of results

Ice Models:

- Axially symmetric
- Standard glaciological constitutive relationships to describe ice sheet dynamics and mass balance (Glen's flow law with $n = 3$)
- Ice is prohibited from sliding over the bed, all of the ice flux is associated with internal shear deformation
- All experiments begin with no initial ice and a flat bed
- Ice sheet mass balance is specified using an annual degree-day model to predict the amount of precipitation (cf. Marshall et al., 2000)

Ice Model: (continue)

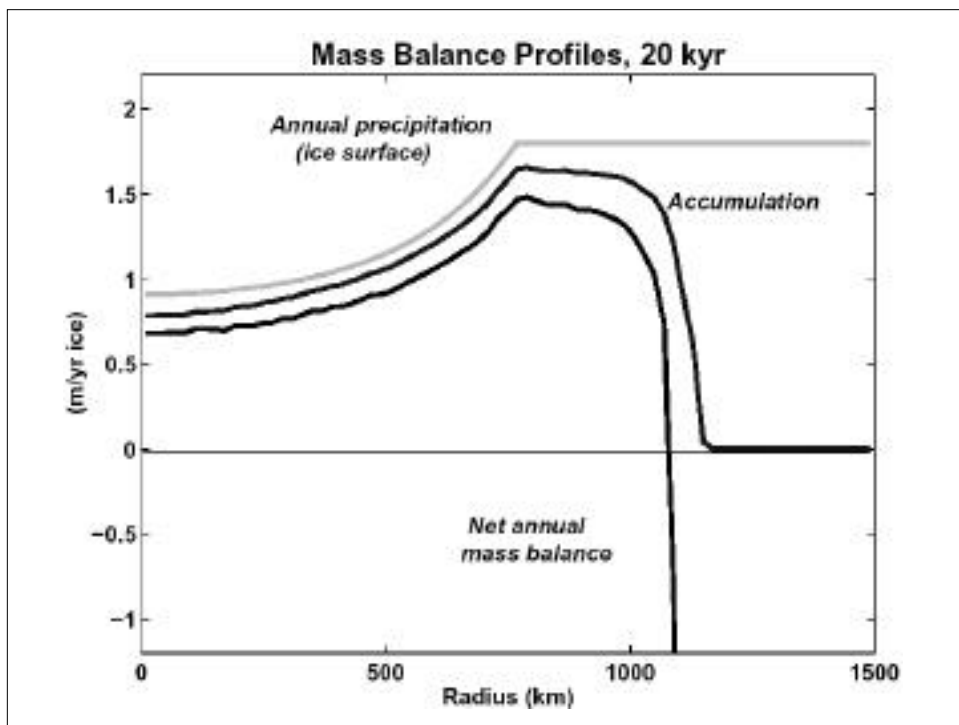
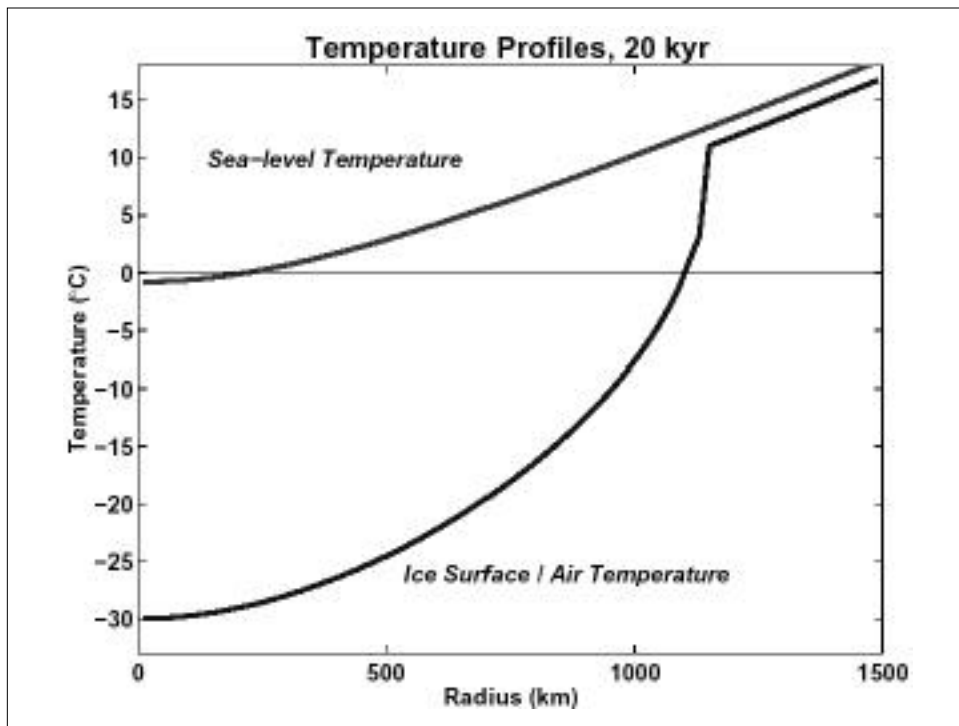
- Climate inputs :
 - (i) total annual precipitation, P_{SEA} , and
 - (ii) mean annual temperature, T_{SEA} .

$$T_s(\lambda, \theta, t) = T_{sea}(\lambda, \theta, t) + \beta h_s(\lambda, \theta, t)$$

with lapse rate $= -0.0075^\circ\text{C}/\text{m}$. Similarly for precipitation.

- Elevation feedback on precipitation rate: above a threshold elevation $= 1200$ m, precipitation rates decline as:

$$P_s = P_{sea} \exp[-\gamma (h - h_t)]$$



Definition of Strain Rate: $\dot{\epsilon}_{ik} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right)$

Momentum Balance: $\frac{\partial \sigma_{ik}}{\partial x_k} = -\rho^I g_k$

Glen's Flow Law for Ice: $\dot{\epsilon}_{ik} = B(T^I) \left\| \sigma'_{ik} \right\|^{(n-1)/2}$

where $B(T^I)$ is the ice stiffness coefficient

deviatoric stress tensor: $\sigma'_{ik} = \sigma_{ik} - \sigma_{rr} \delta_{ik} = \sigma_{ik} - p^I \delta_{ik}$

glaciostatic (internal) ice pressure: p^I

second invariant tensor : $I_2 = \frac{1}{2} \sigma'_{ik} \sigma'_{ki}$

Mass Balance:

$$-\frac{\partial H}{\partial t} + \frac{\partial (\bar{v}_j H)}{\partial x_j} = \dot{b}$$

where Ice thickness: $H(\lambda, \theta, t) = h^I(\lambda, \theta, t) - h^B(\lambda, \theta, t)$

(ice surface height minus bed elevation)

\dot{b} = ice equivalent accumulation minus ablation

\bar{v}_j = vertically averaged horizontal velocity field

Combining Momentum Balance & Glenn's flow law, gives
Horizontal Ice Velocities:

$$u(z) = u(h^B) - 2(\rho^I g)^n \parallel j h^I \parallel^{n-1} \frac{1}{R_E \sin \theta} \frac{h^I}{\lambda} \frac{h^I}{h^B} B(T^I) (h^I - z)^n dz$$

$$v(z) = v(h^B) - 2(\rho^I g)^n \parallel j h^I \parallel^{n-1} \frac{1}{R_E \sin \theta} \frac{h^I}{\theta} \frac{h^I}{h^B} B(T^I) (h^I - z)^n dz$$

where ice stiffness coefficient $B(T^I) = EB_0 \exp \frac{-Q}{RT^I}$

Energy Balance:

$$\frac{T^I}{t} = -v_k \frac{T^I}{x_k} + \kappa^I(T^I) \frac{2T^I}{z^2} + \frac{1}{\rho^I c^I(T^I)} \frac{k^I}{T} \frac{T^I}{z} + \frac{d}{\rho^I c^I}$$

where strain heating:

$$d(T^I) = 2B(T^I) \left\{ \rho^I g \parallel j h^I \parallel (h^I - z) \right\}^{n+1}$$

Isostasy Models:

Case 1: No Isostasy (i.e. no crustal deformation).

Case 2: Local Compensation only, with 2 ka relaxation time.

Case 3: Full GIA model, where the earth contains a 100 km thick elastic lithosphere overlying a stratified linear mantle. The viscosity of the upper mantle is 1×10^{21} Pa-s and the viscosity of the lower mantle below 670 km is 2×10^{21} Pa-s

Case 4: Full GIA model just as in case 3, except that rheology is nonlinear throughout the mantle with stress exponent $n=3$ with creep parameter $A=3 \times 10^{-35}$ Pa⁻³s⁻¹.

Assumptions:

- i) Steady state creep and
- ii) no interaction between rebound stress and tectonic stress (Karato 1998).

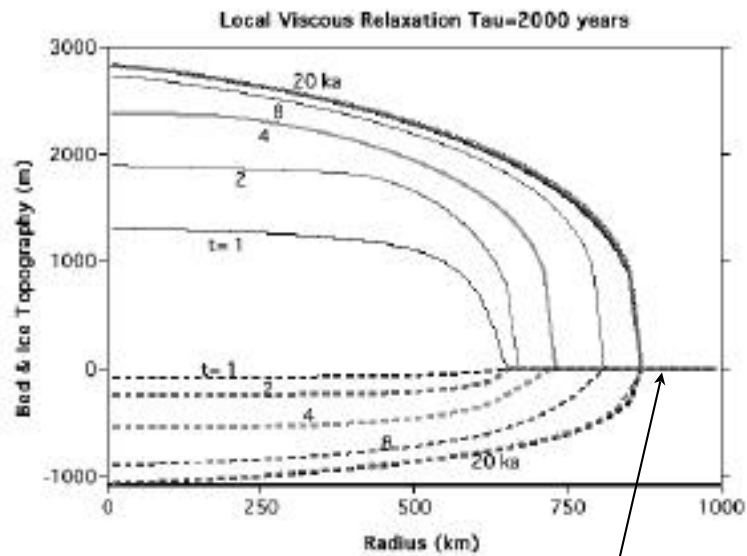
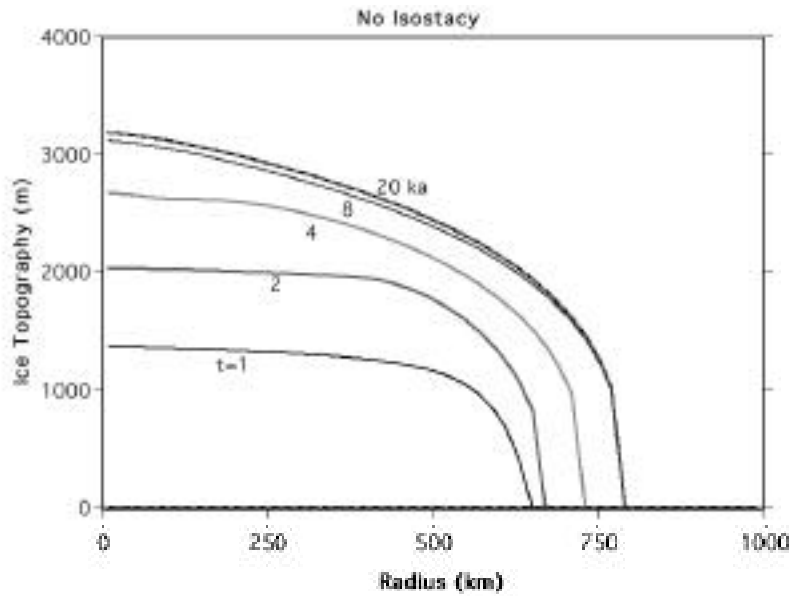
The Finite-Element-Iterative Technique

The loading at the surface of the earth is provided by the ice thickness generated by the above ice model and crustal deformation is calculated by the finite element method for case 3 & 4.

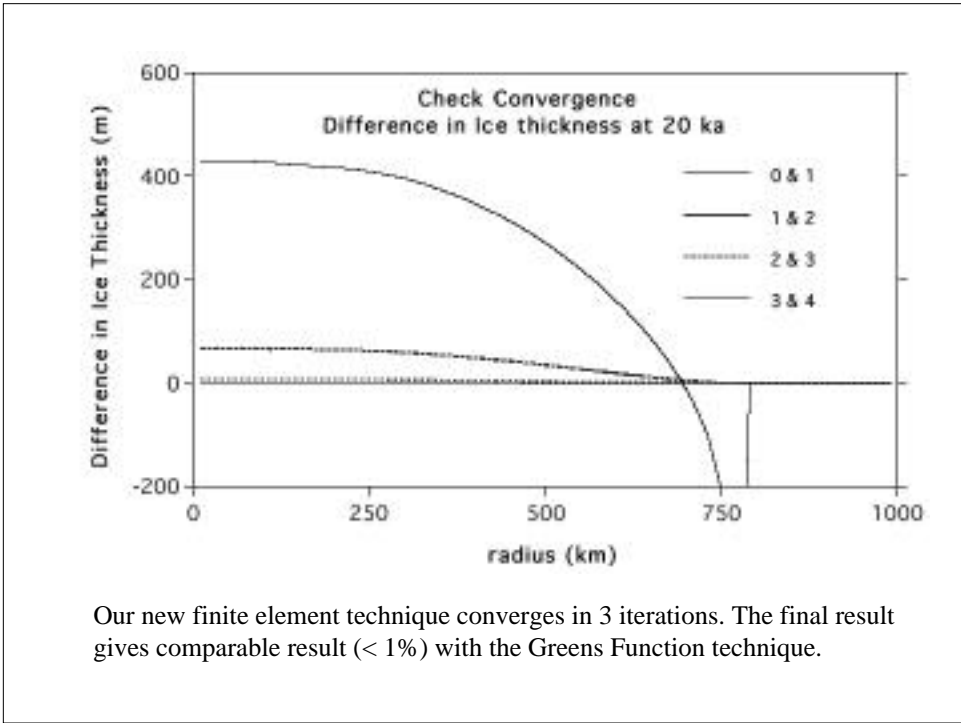
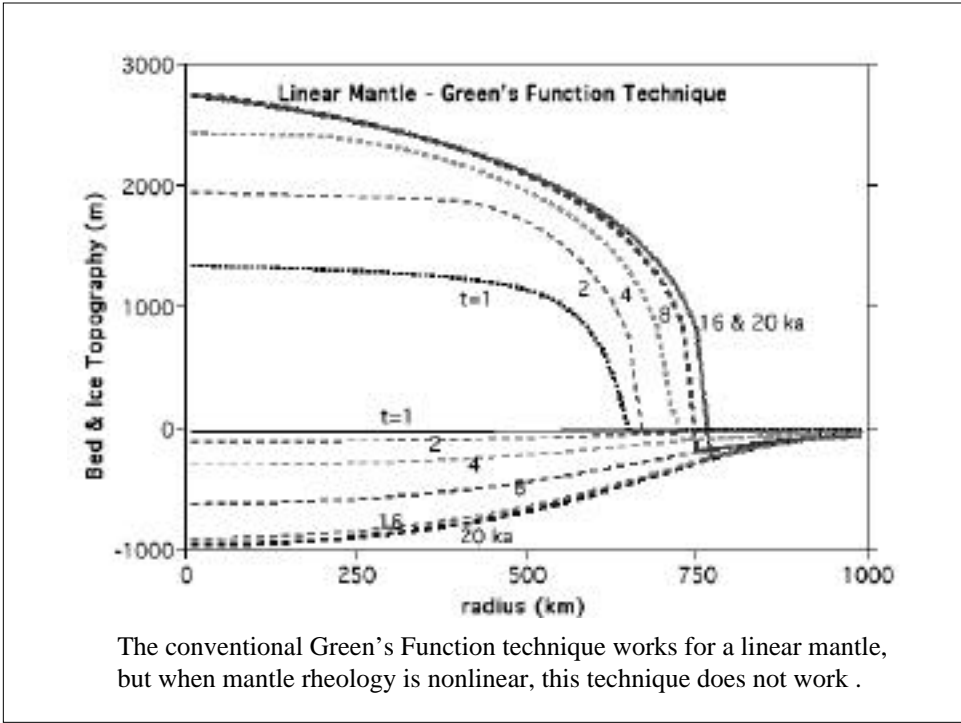
Crustal deformation is then provided as an input to the ice model.

This process is iterated until solution converges (generally <3 iterations).

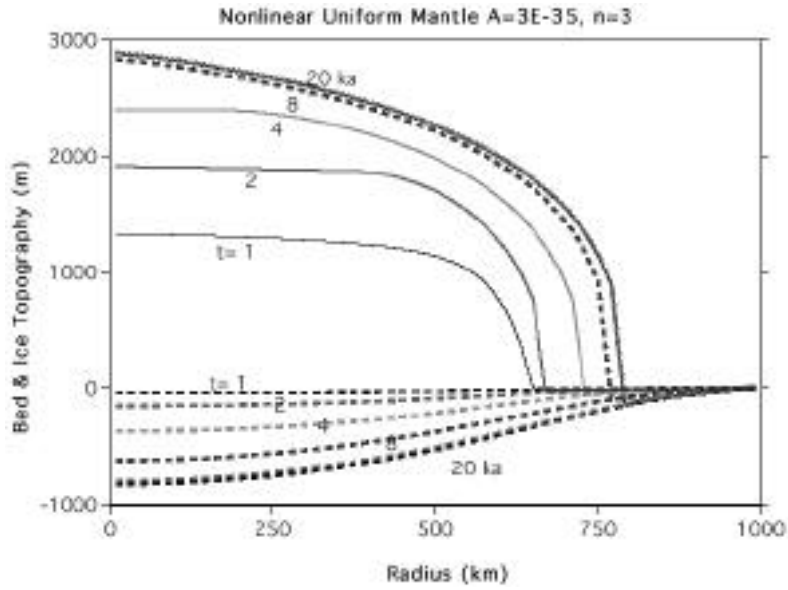
Predict how ice sheet grows in time when there is no bed compensation.



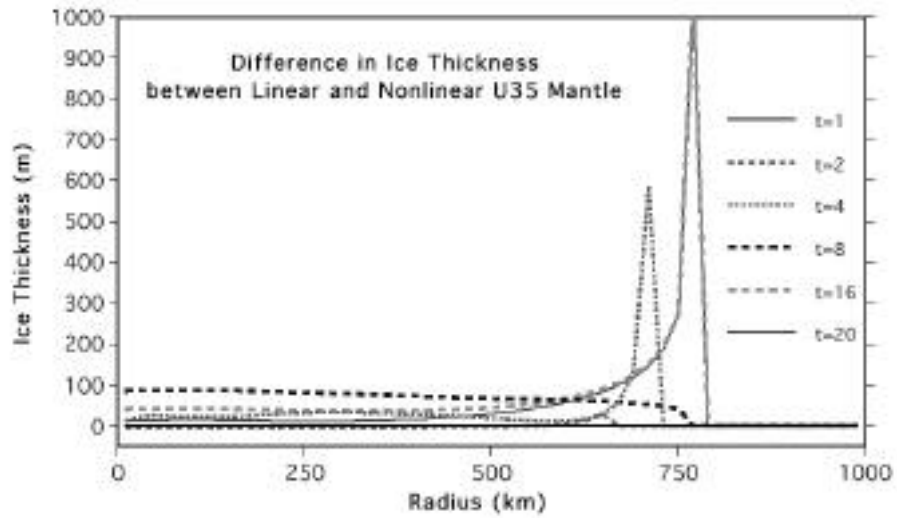
For local compensation, there is no bed deformation outside the ice sheet.



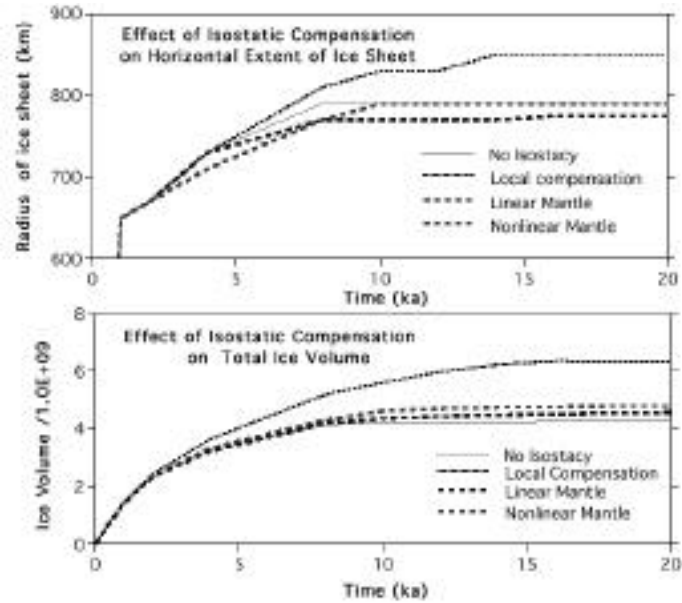
Predict how ice sheet grows in time when mantle rheology is nonlinear.



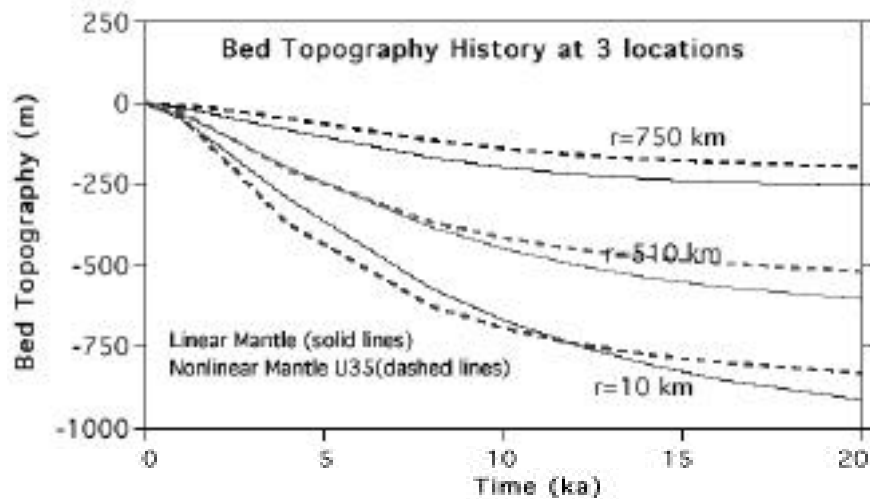
Comparing Ice Thickness predicted by linear and nonlinear mantles.



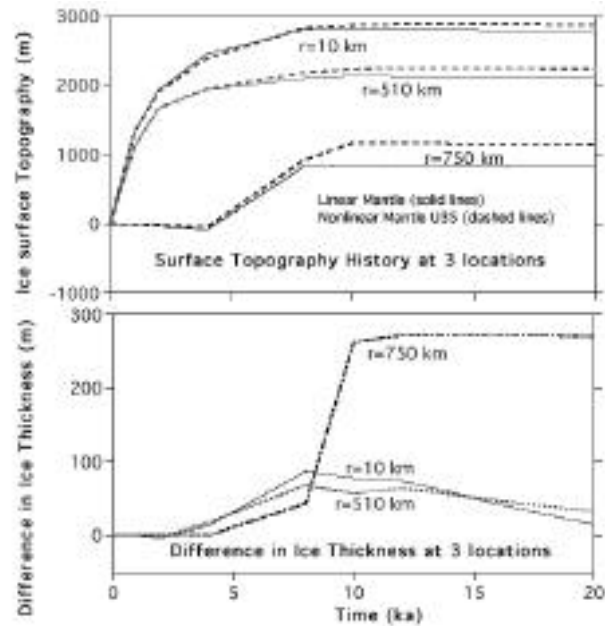
Comparing the temporal variation of the horizontal extent (radius) and the volume of the ice sheet due to the different bed compensation mechanisms.



How does nonlinear rheology affect the motion of the bedrock?



How does nonlinear mantle affect ice surface topography & ice thickness?

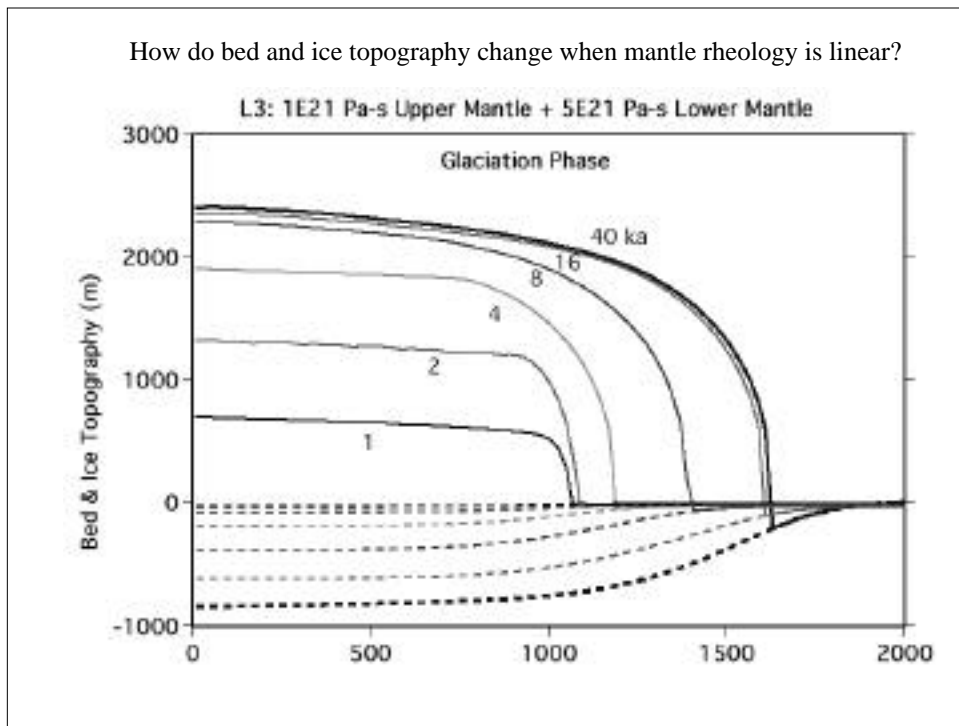


So far, only the glaciation phase of Fennoscandia size ice sheets are considered. Below, we consider ice sheets with size comparable to the Laurentide ice sheet. Both glaciation and deglaciation phases are considered.

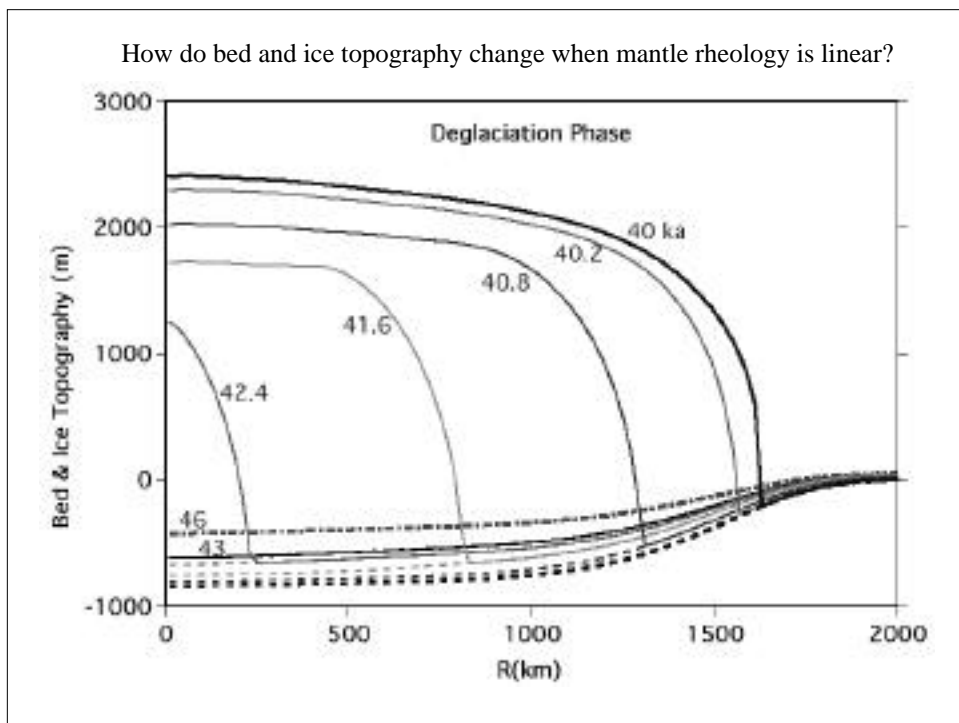
For Laurentia size ice sheets, the appropriate relaxation time for local compensation is 4000 years rather than 2000 year.

Relative Sea Level data around Laurentia shows that rheology in the upper mantle is linear (10^{21} Pa-s) but that in the lower mantle (below 670 km depth) may be nonlinear ($A=3 \times 10^{-35} \text{ Pa}^{-3} \text{ s}^{-1}$, $n=3$) or linear with high viscosity ($5 \times 10^{21} \text{ Pa-s}$). Both of these models will be considered below.

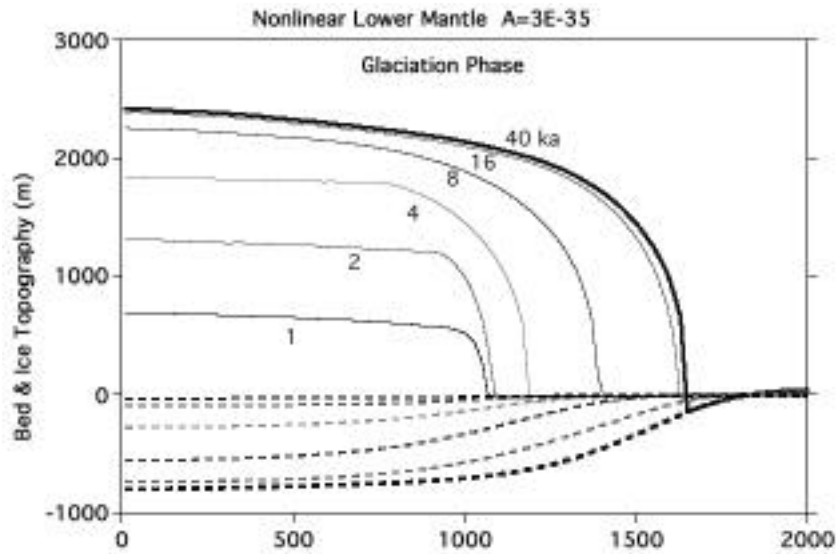
How do bed and ice topography change when mantle rheology is linear?



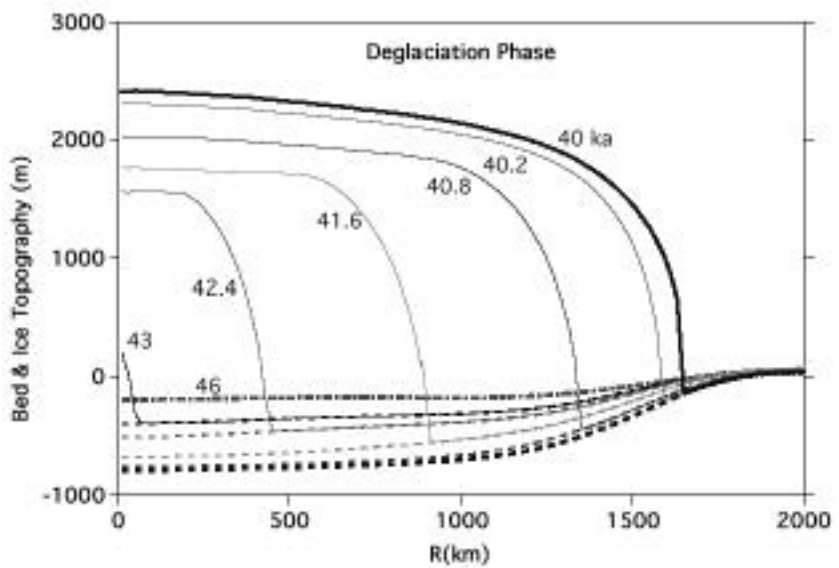
How do bed and ice topography change when mantle rheology is linear?

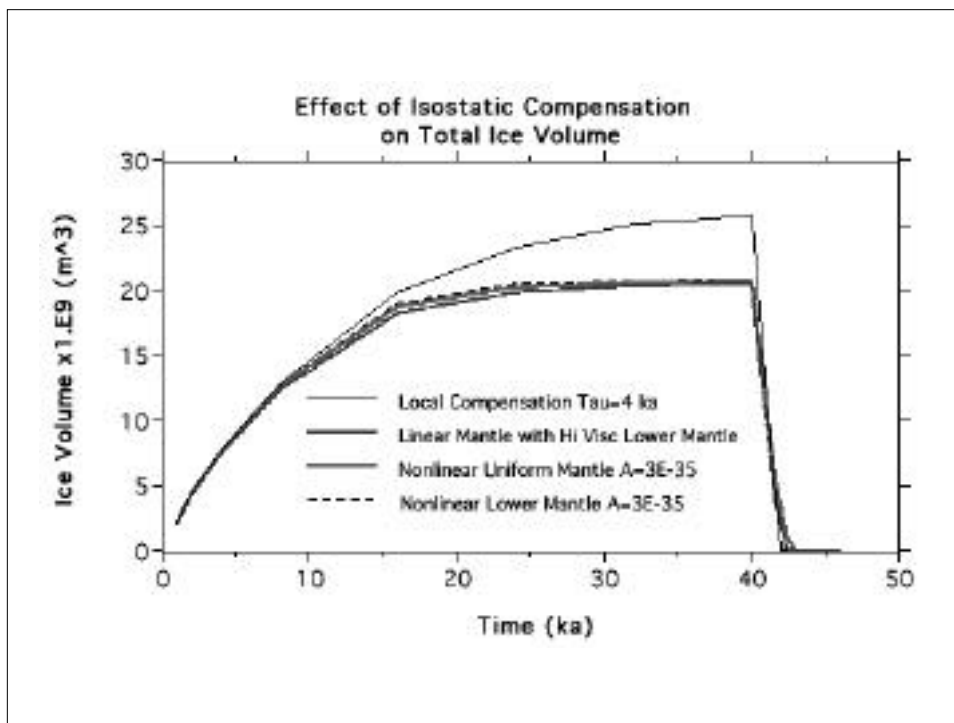
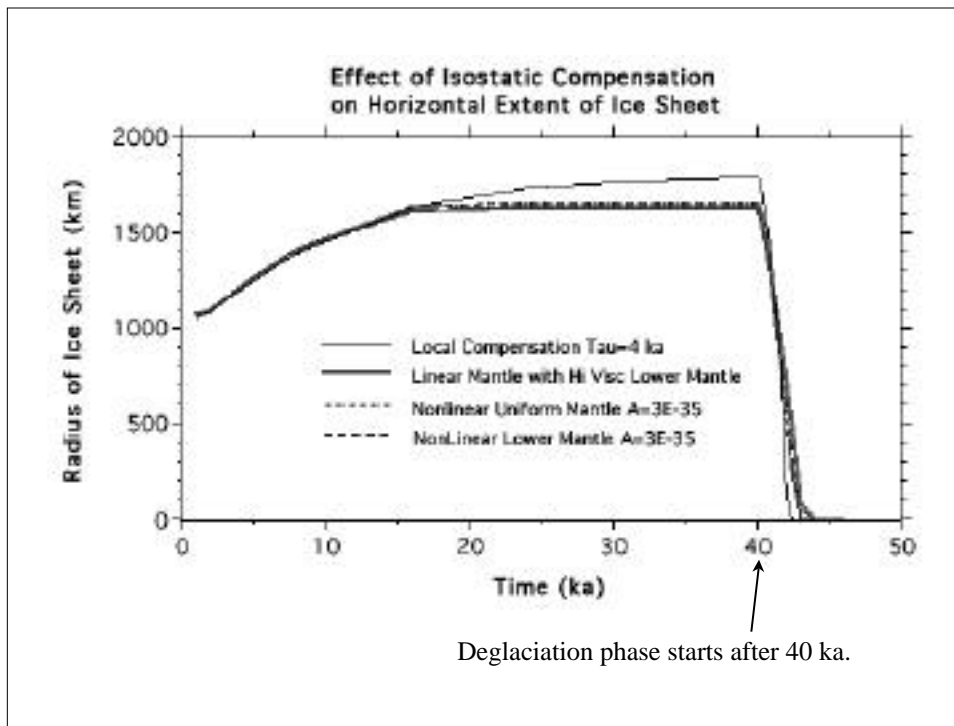


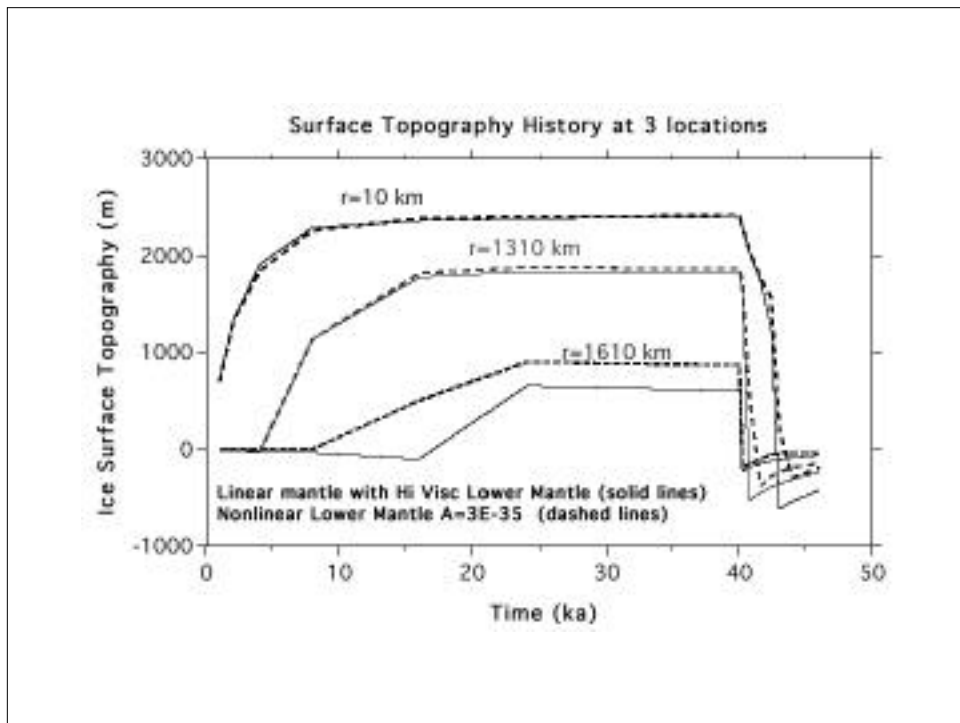
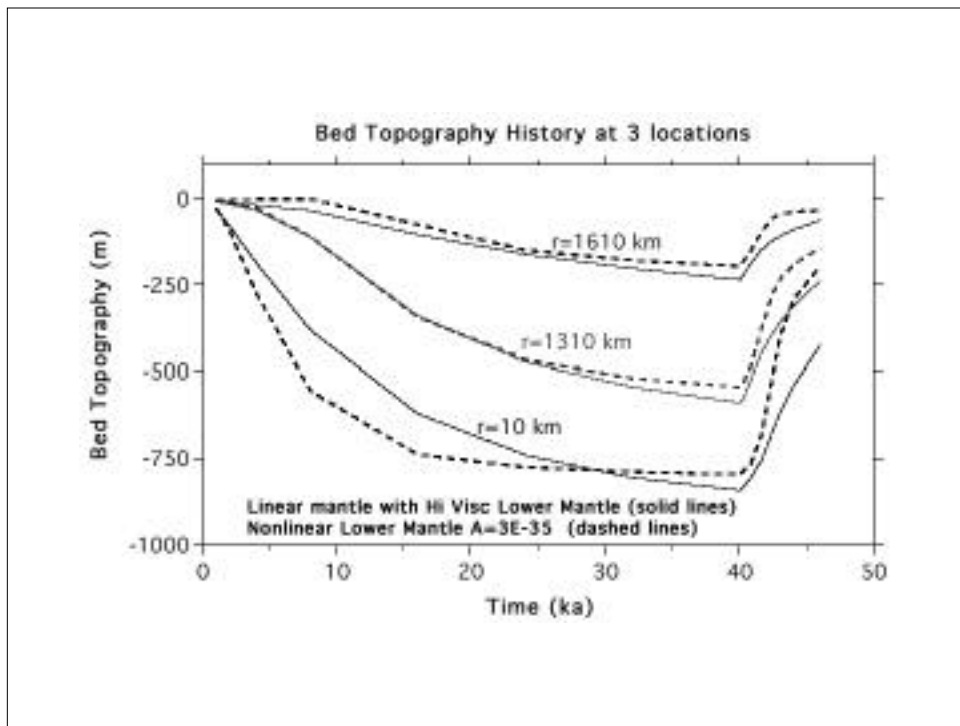
How do bed and ice topography change when mantle rheology is nonlinear?

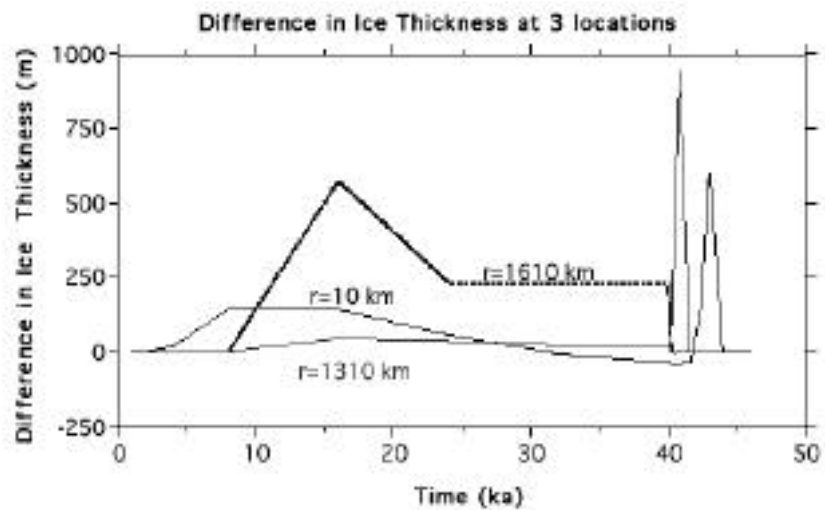


How do bed and ice topography change when mantle rheology is nonlinear?









Largest difference in ice thickness occurs during the deglaciation phase or near the edge of the ice sheet during the glaciation phase.

Summary:

- 1) Isostatic adjustment of the bedrock affects ice sheet inception.
- 2) Local compensation over predicts the horizontal extent (radius) of the ice sheet and the total ice volume.
- 3) Nonlinear mantle models give larger ice sheets (both in horizontal extent and volume) than linear rheology