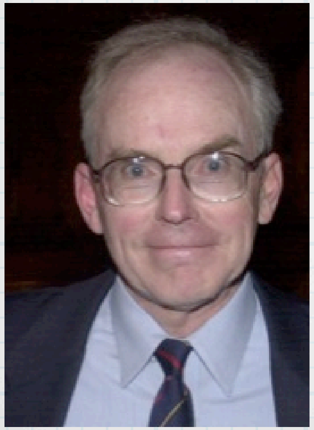
The Third Claude E. Shannon Memorial Lecture
April 29, 2005

UCSD

Are There Turbo-Codes on Mars?



Robert J. McEliece
California Institute of Technology

“The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point.”

To solve this problem, Shannon created a branch of applied mathematics which is today called Information Theory...

Information Theory
1101

Entropy

Entropy *H(X)* measures our uncertainty about the event X.

$$H(X) = -\sum_{x} p(x) \log p(x)$$

Relative entropy *H(X|Y)* measures our uncertainty about X after Y is observed

$$H(X|Y) = -\sum_{x,y} p(x,y) \log p(x|y)$$

Mutual Information

Mutual Information *I(X;Y)* measures the amount of information the event Y provides about the event X

$$I(X;Y) \quad = \quad H(X) - H(X|Y)$$

Channel Capacity

The capacity C of a channel is the highest possible rate (in bits per second) at which reliable communication over the channel is possible

$$C = \max_X I(X;Y)$$

Compressibility

The Compressibility Function $R(\delta)$ is the minimum number of bits per second required to communicate the source output with "distortion" δ .

$$R(\delta) = \min_{Y:|X-Y|\leq\delta} I(X;Y).$$

Shannon’s Equations

$$H(X) = -\sum p(x) \log p(x)$$

$$I(X;Y) = H(X) - H(X|Y)$$

$$C = \max_X I(X;Y)$$

$$R(\delta) = \min_Y I(X;Y)$$

Dr. Shannon's Prescription for Excellent Communications

Channel Coding (Error Correction)

Source Coding (Data Compression)

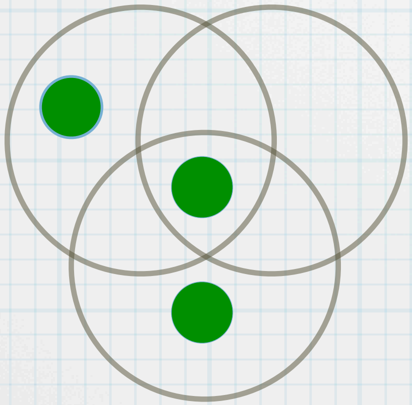
Summary

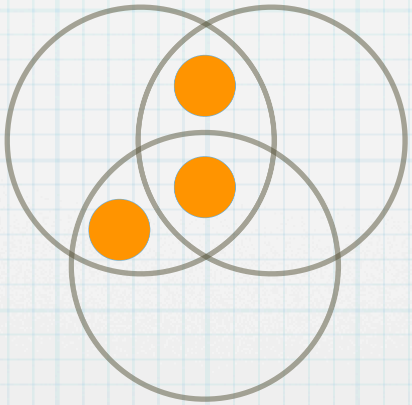
- Of the 35 patterns of three erasures:

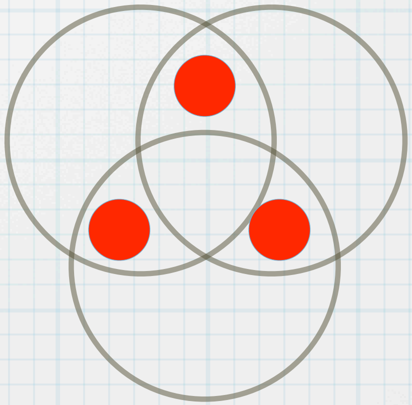
25 are correctable with the simple algorithm

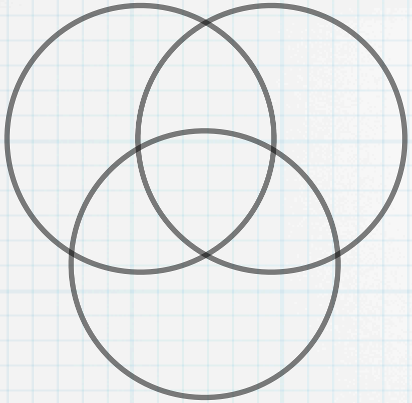
3 more are correctable with the complex algorithm

7 are uncorrectable (codewords)

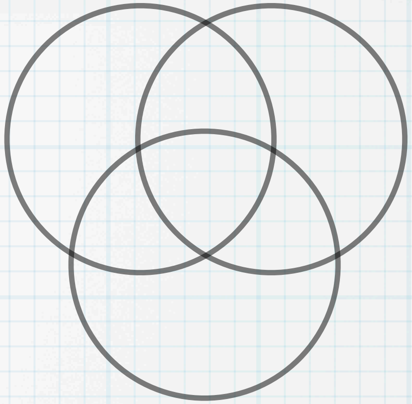








In a Hamming Code of Length n:



Theorem 1. *The number of erasure patterns of weight 3 is* $\sim \frac{1}{6}n^3$.

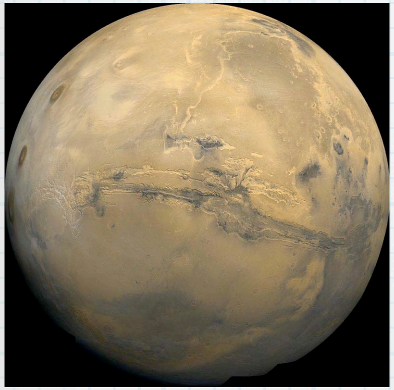
Theorem 2. *The number of [easily] correctable erasure patterns of weight 3 is* $\sim \frac{1}{6}n^{\log_2 5} = \frac{1}{6}n^{2.322}$.

Theorem 3. *The number of uncorrectable erasure patterns of weight 3 is* $\sim \frac{1}{6}n^2$.

Visit To A Small Red Planet

“The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point.”

Mars



Point A

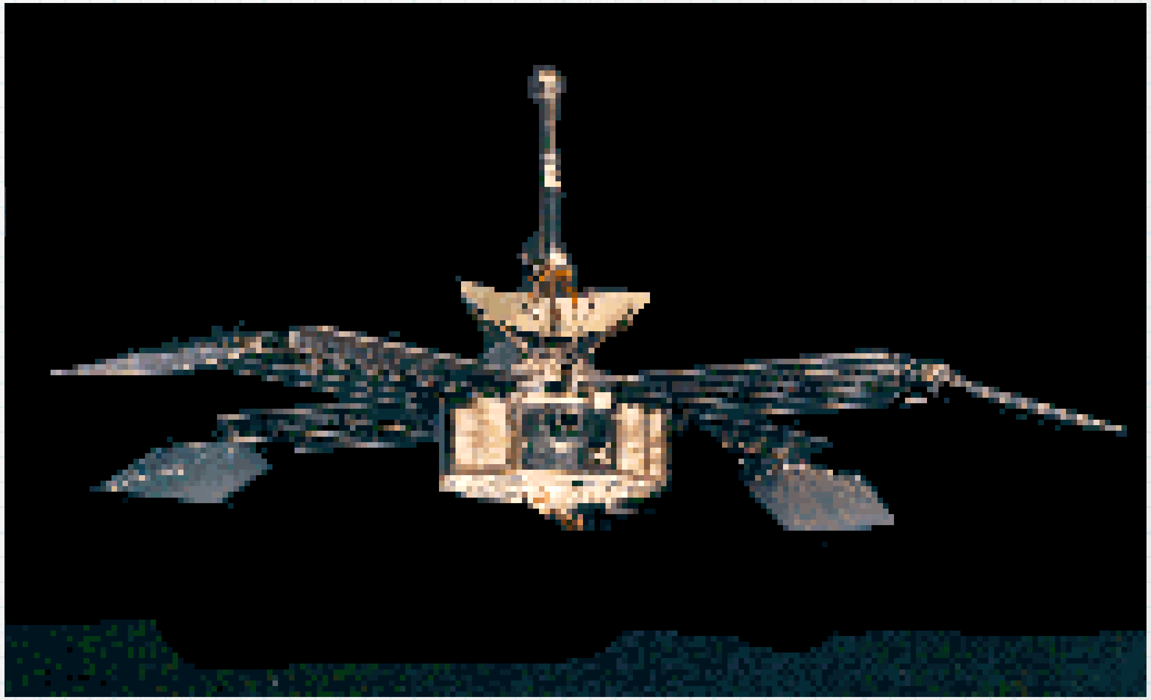
Rate: R bits per second

Earth

Point B

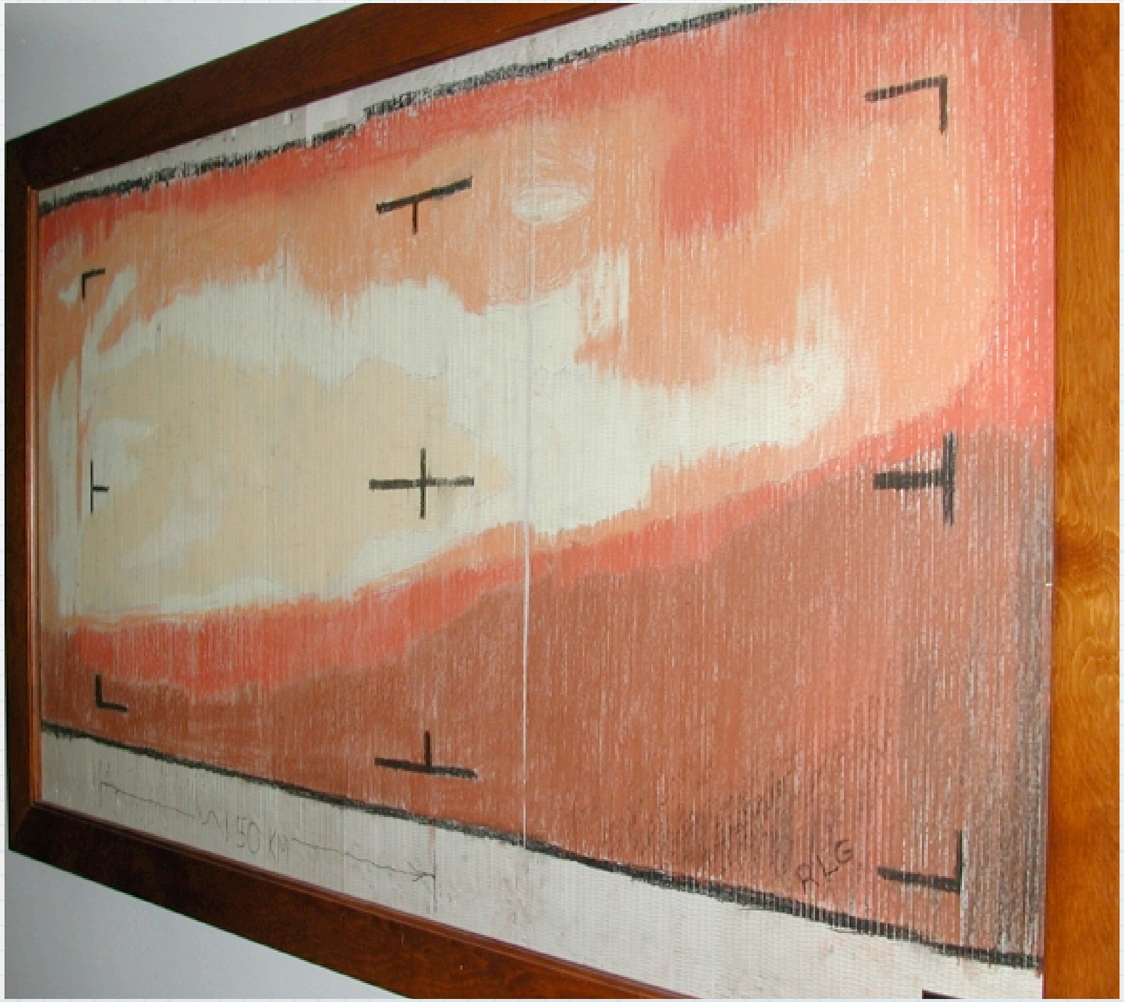
Example: Mariner 4 (1965)

- F = 2.3 GHz (S-band)
- BPSK modulation
- R = 8.33 bits per second
- No Error Correction
- No Data Compression



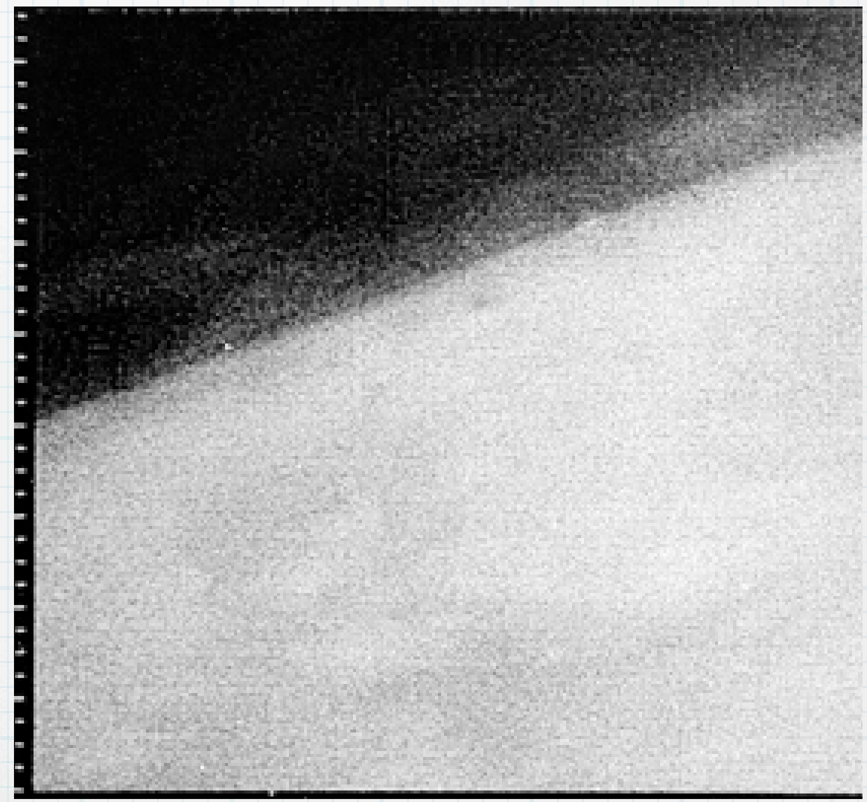
This is our baseline system.

The First Close-Up of Mars!

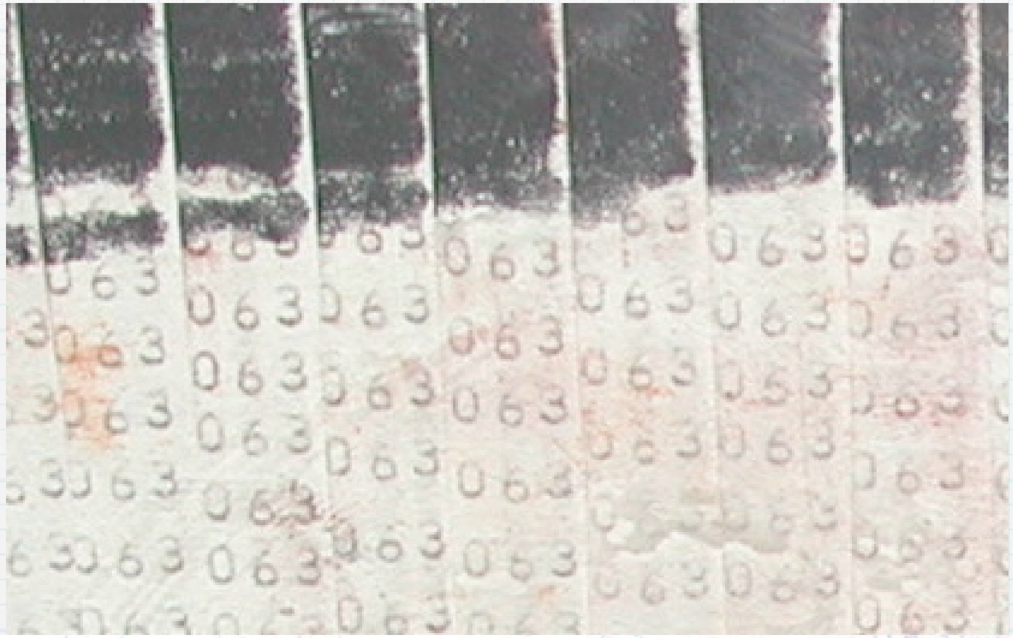


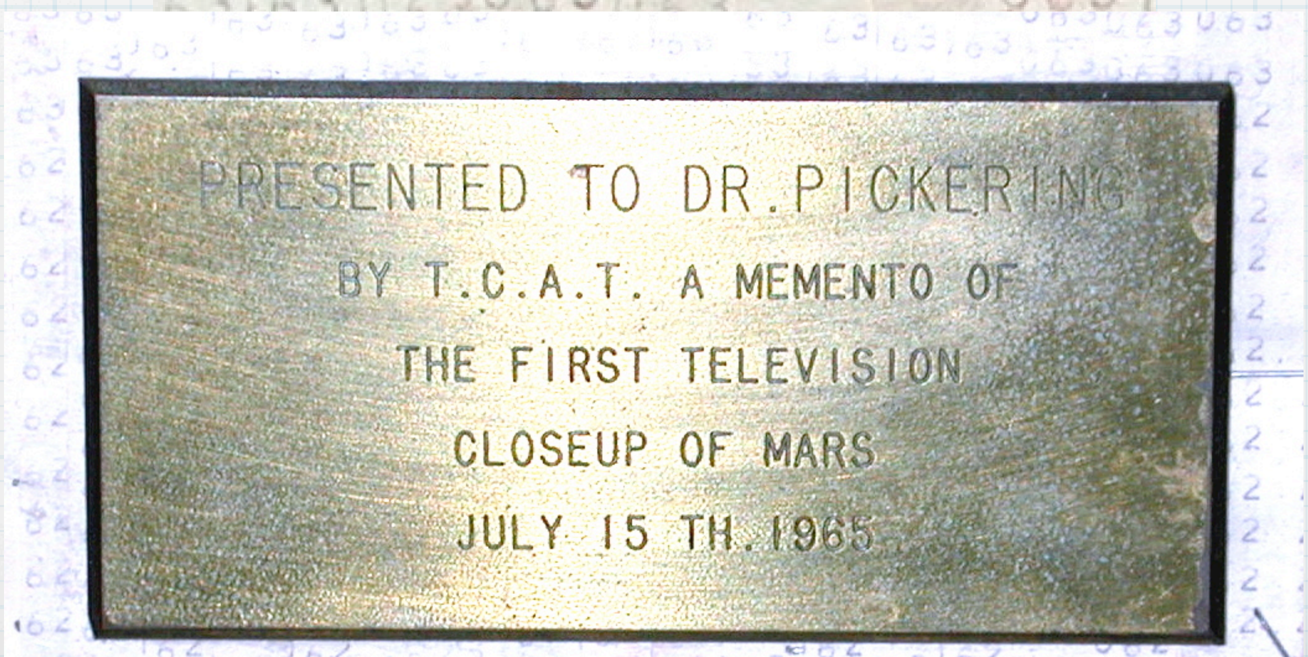
Mariner 4

Before and After

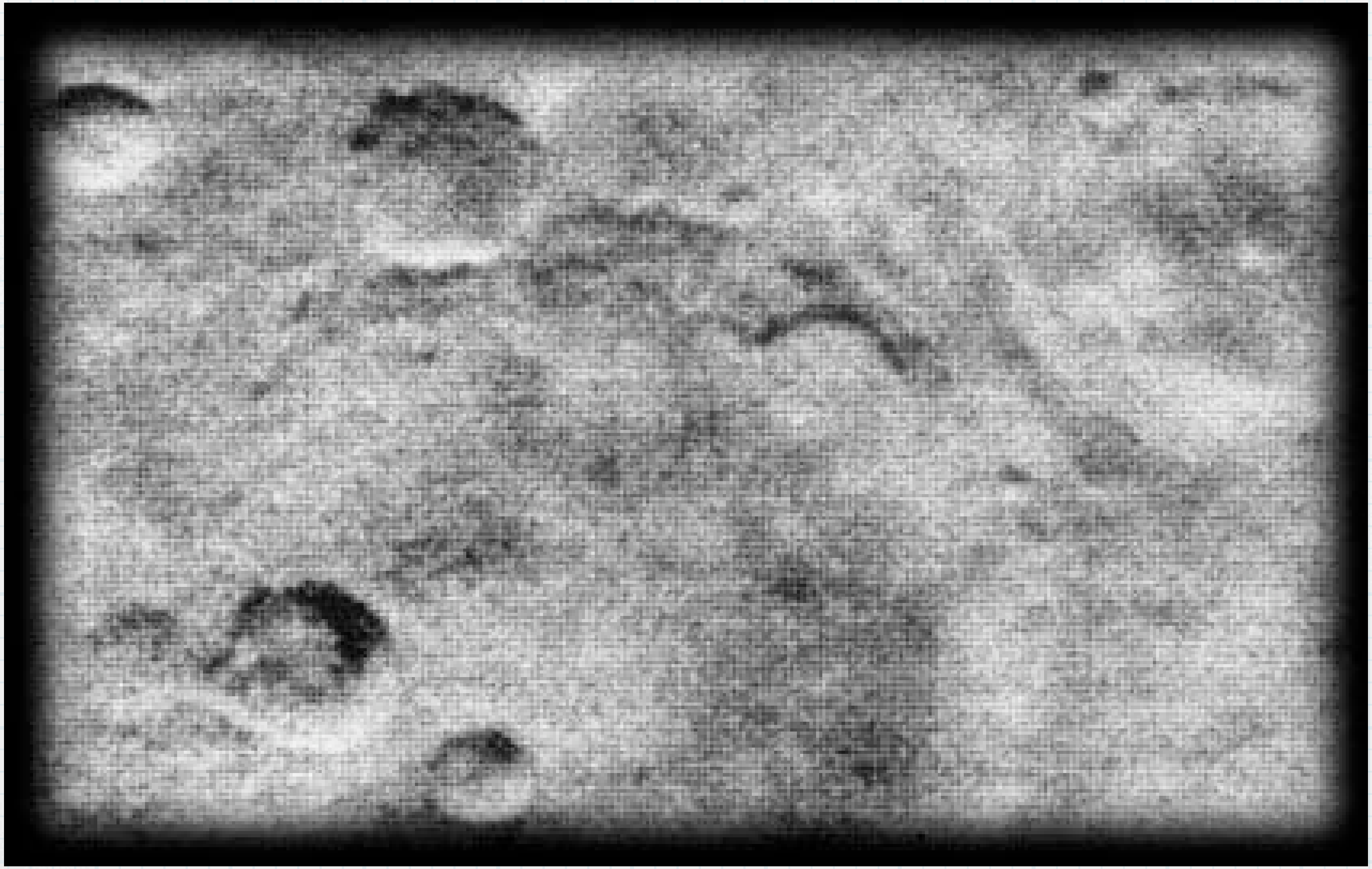


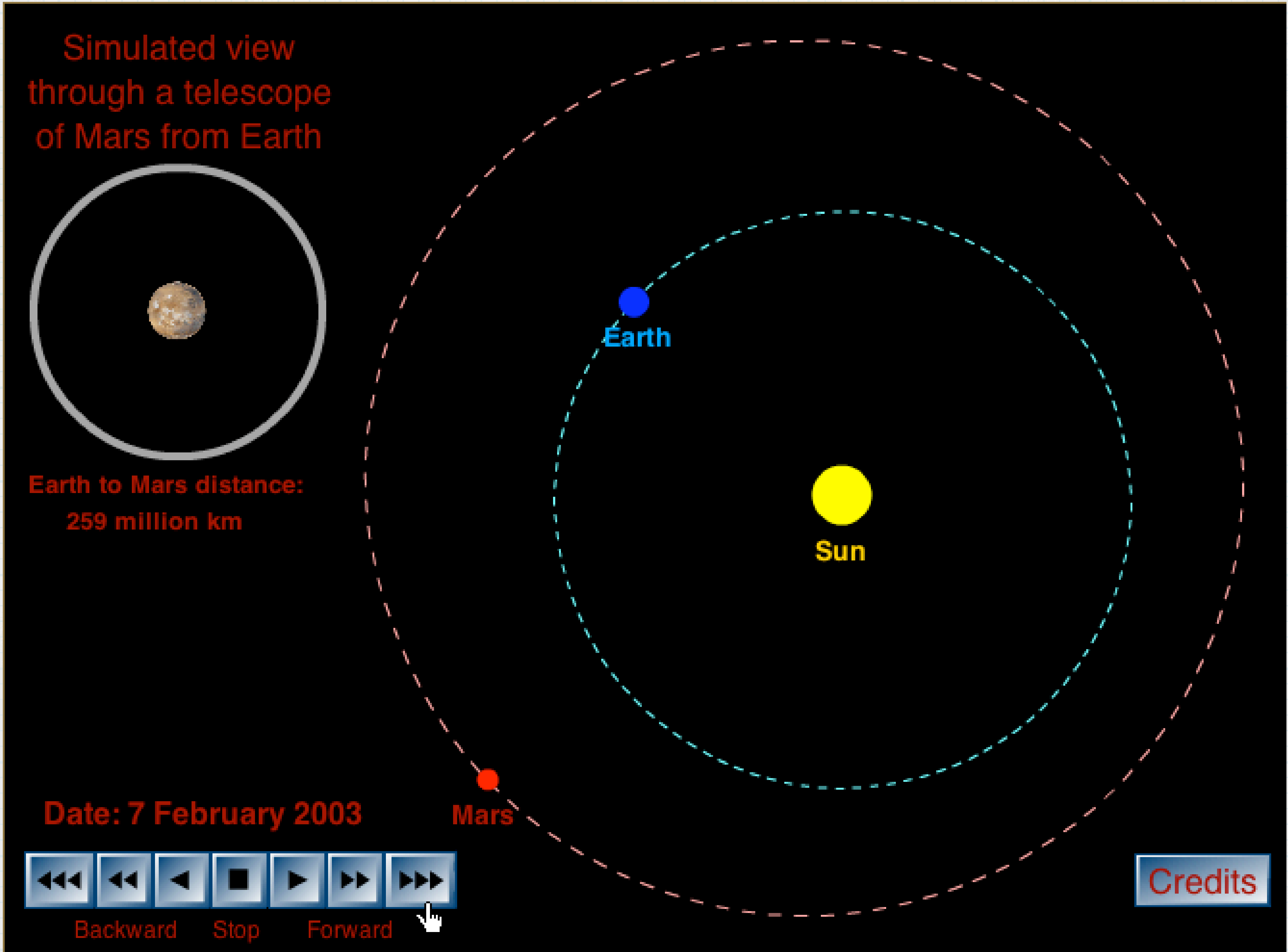
A Memento of Mariner 4



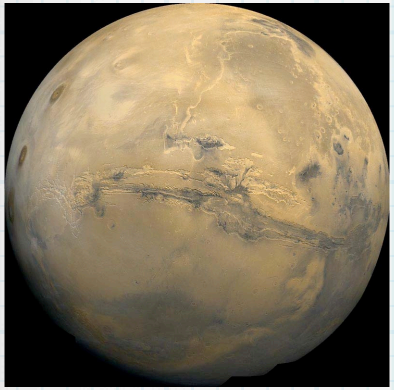


Another Mariner 4 Picture




Simulated view
through a telescope
of Mars from Earth
Earth to Mars distance:
259 million km
Earth
Sun
Mars
Date: 7 February 2003
Backward
Stop
Forward
Credits

Mars



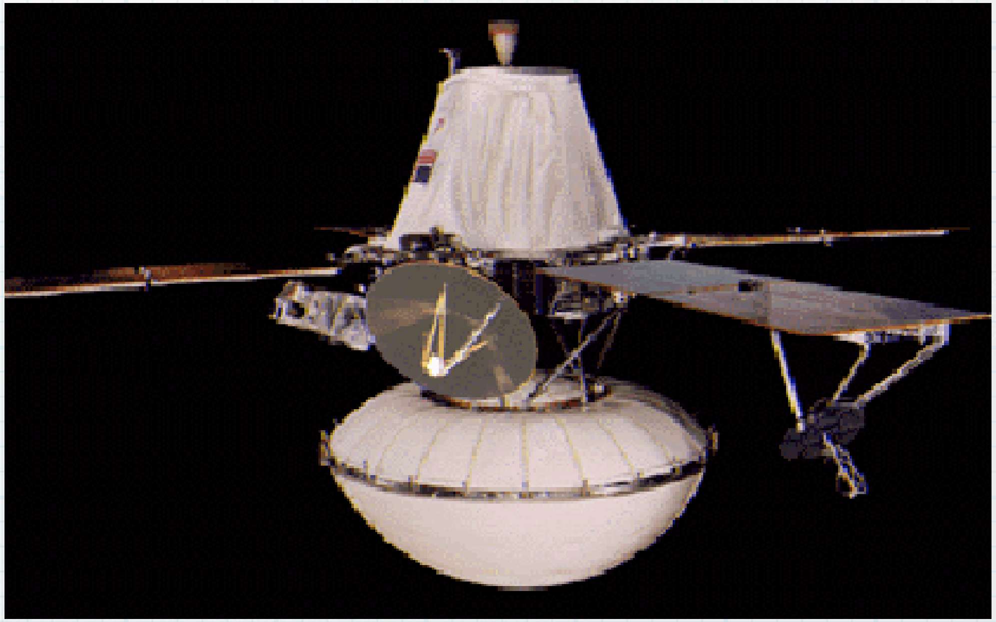
Earth

Normalized Rate R*

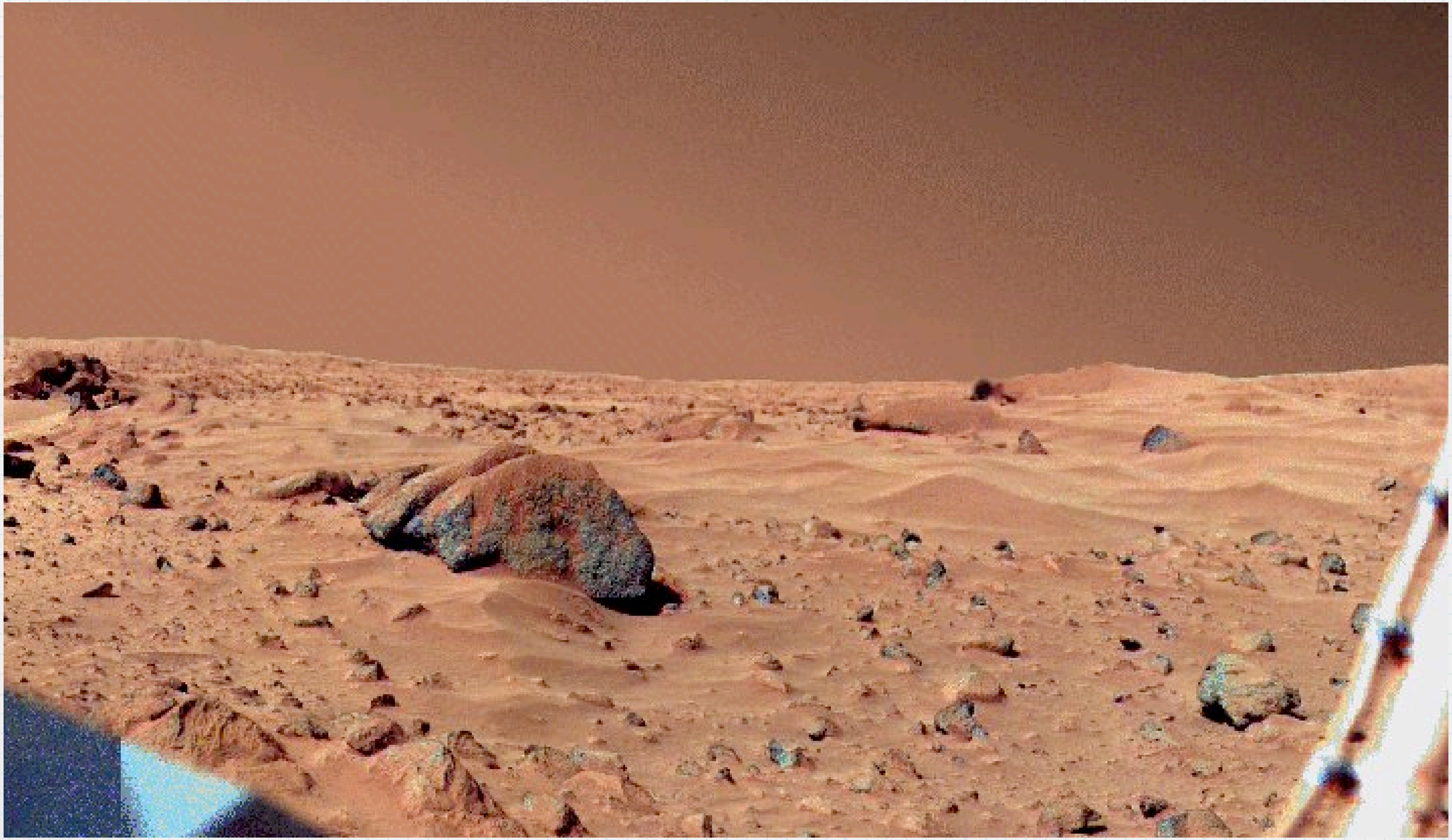
We normalize the data rate R to R*, the rate in image-bits/sec to account for the distance to Mars and a few other factors.

Viking Mars Orbiters/ Landers (1976)

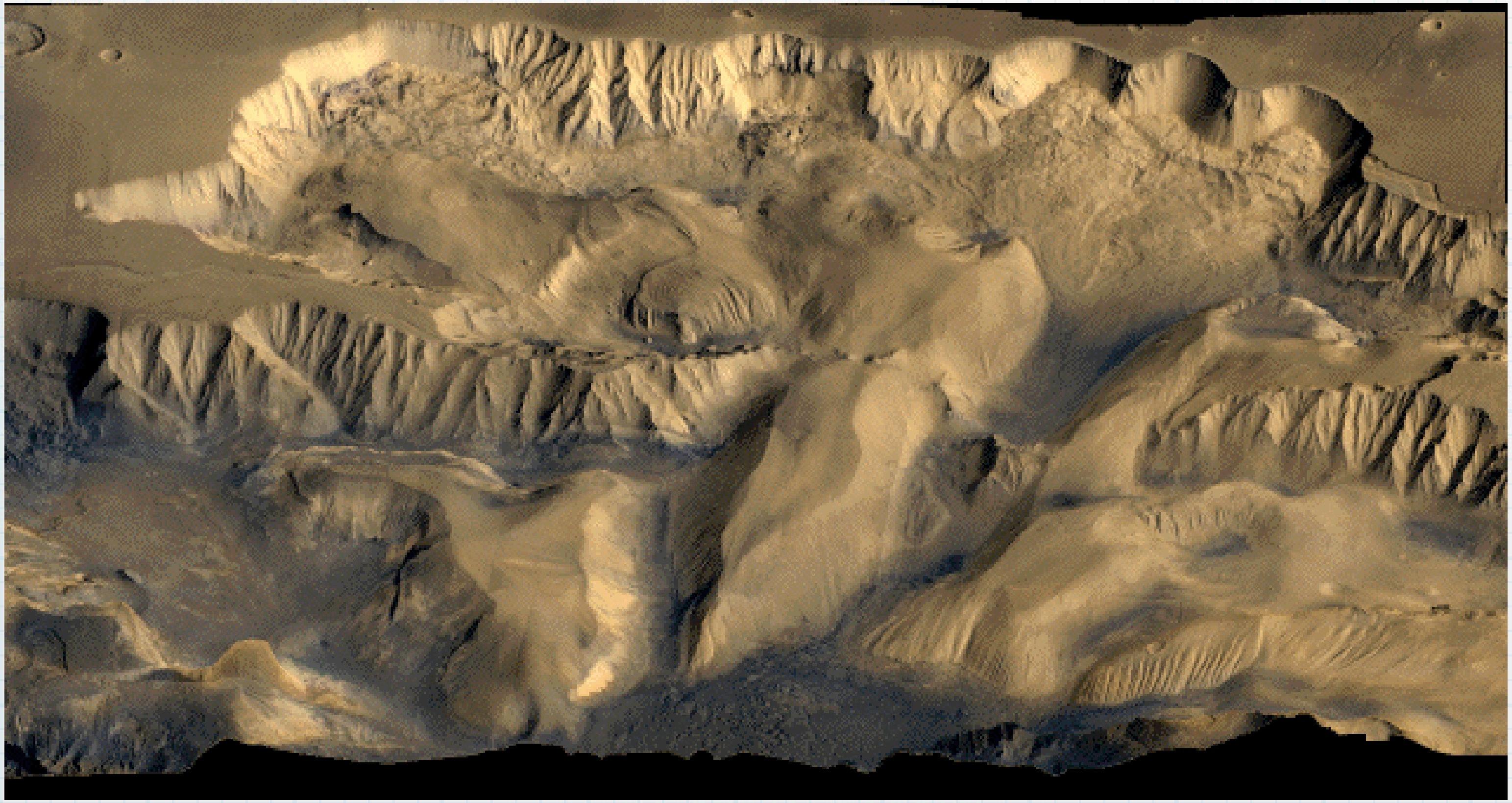
- F = 2.3 GHz (S-band)
- BPSK Modulation
- R* = 3K ibps
- (32,6) Biorthogonal Code
- No compression



Viking I Landscape



The Great Equatorial Canyon

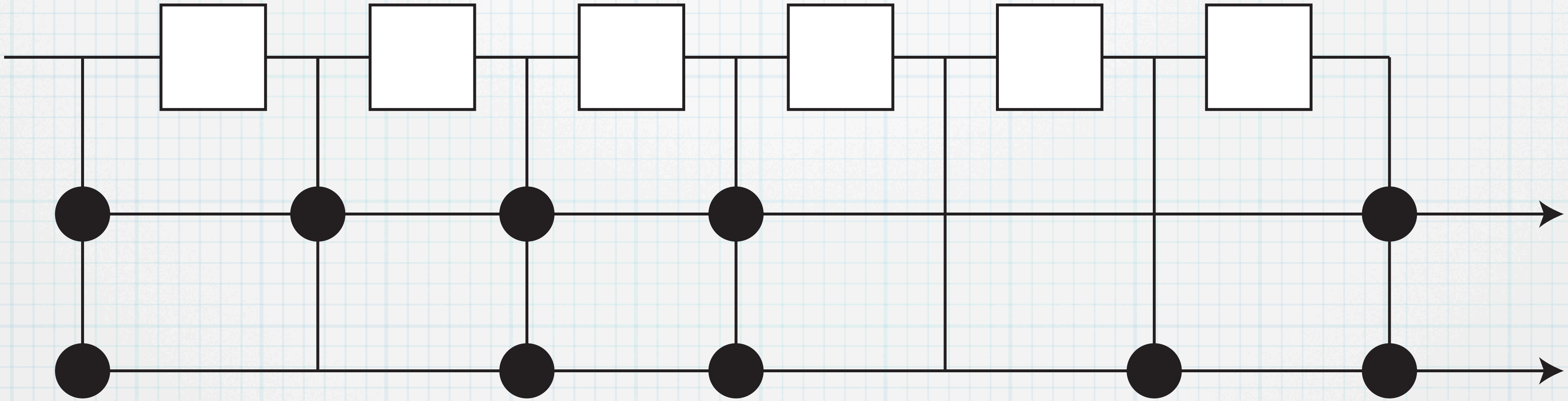


A 20-Year Gap and Then:

Mars Global Surveyor (1997)

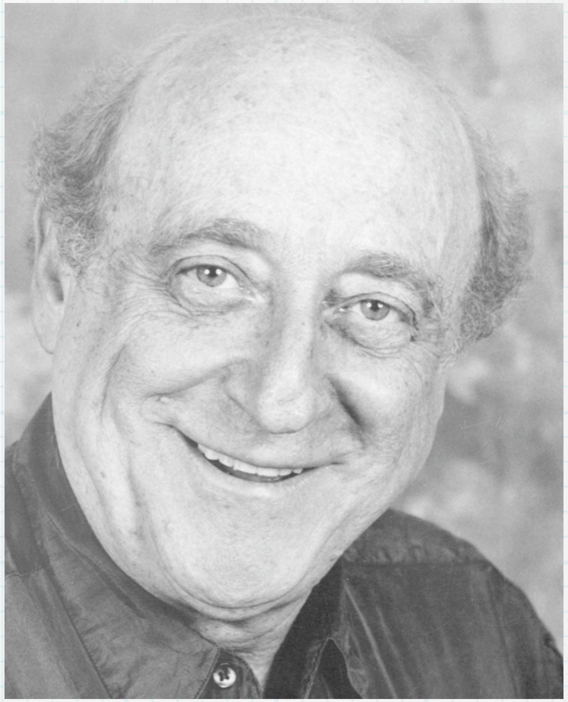
- F = 8.4 GHz (X - band)
- BPSK Modulation
- R^* = 128K ibps
- (7, 1/2) CC + (255,223) RS
- 2:1 lossless Rice compression

“Voyager” (7, 1/2) Convolutional Encoder



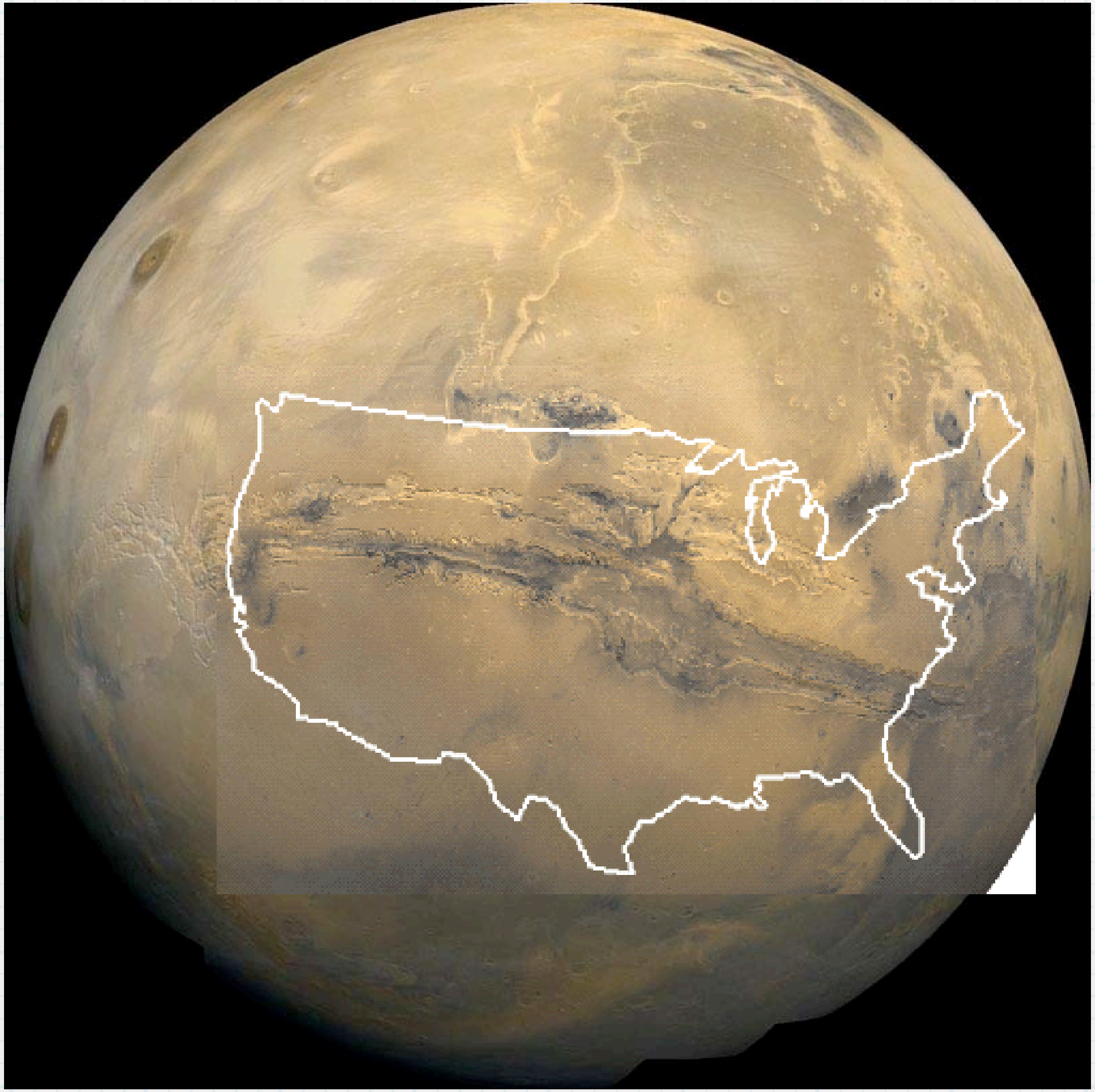
Reed-Solomon Codes

Mr. Reed



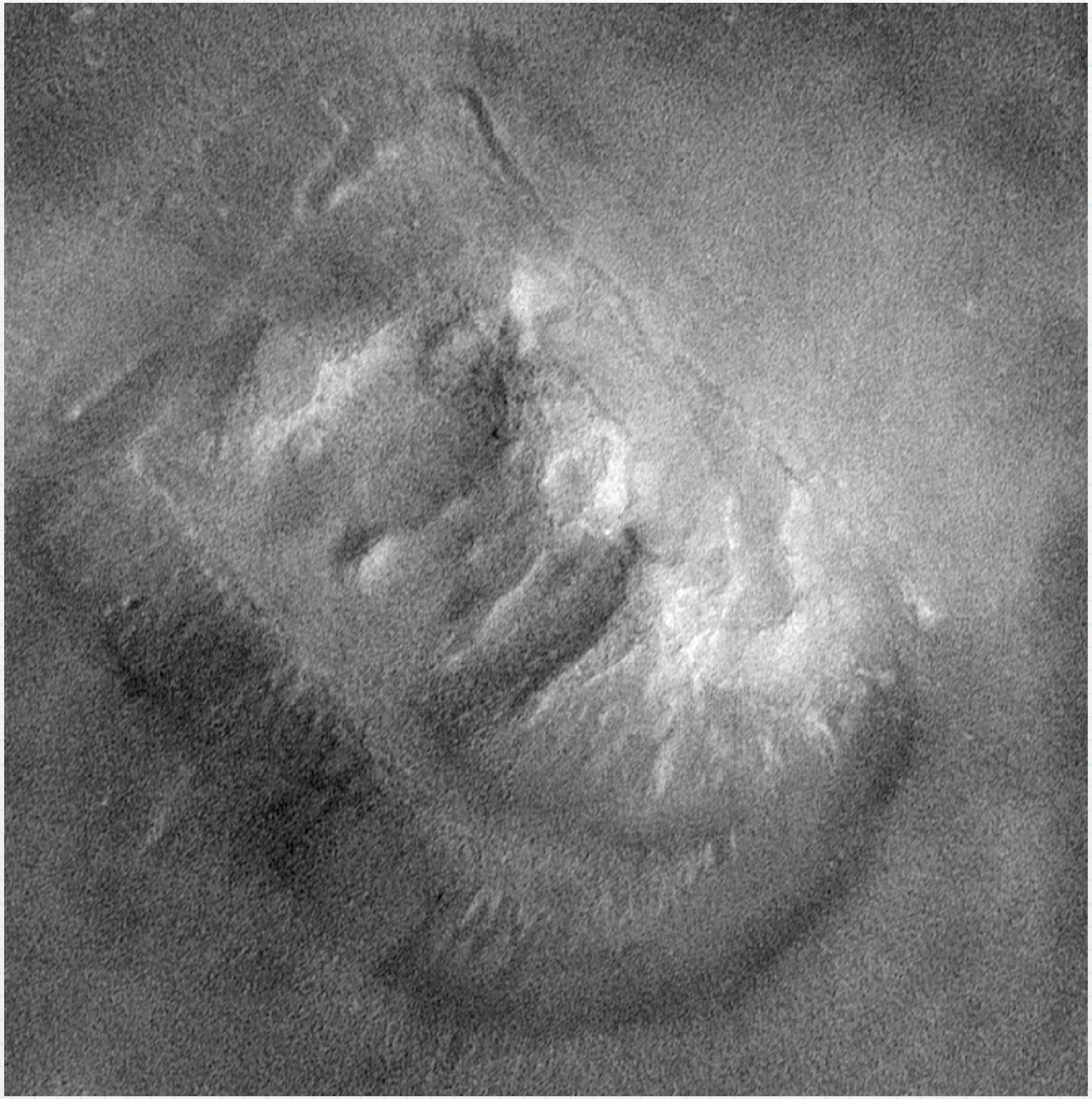
Mr. Solomon

MGS



MGS

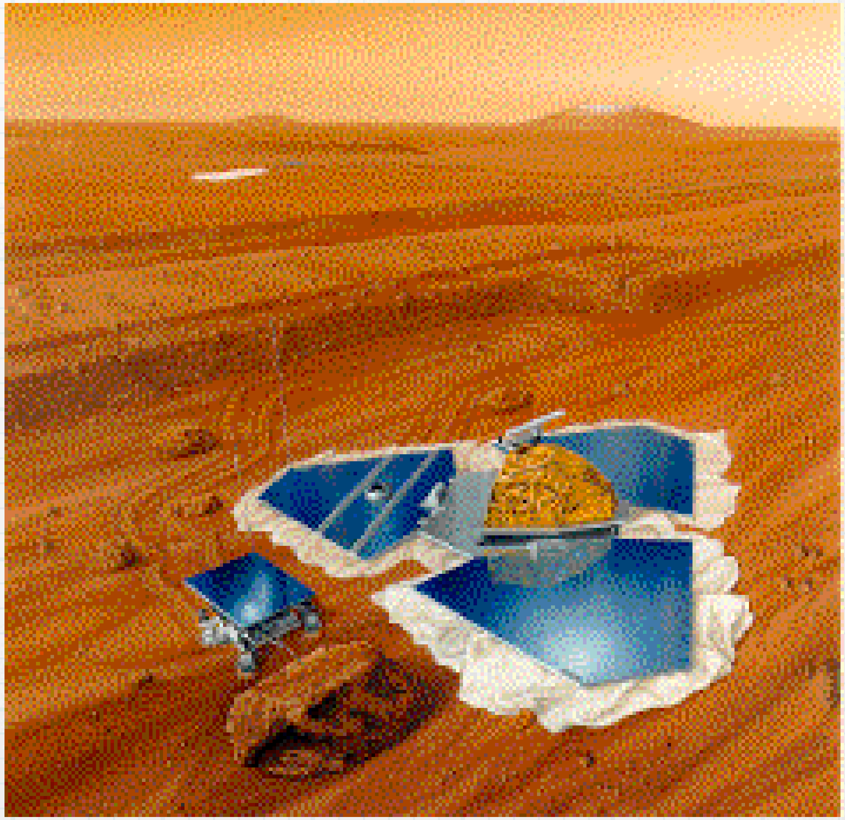
The "Face" on Mars (Cydonia)



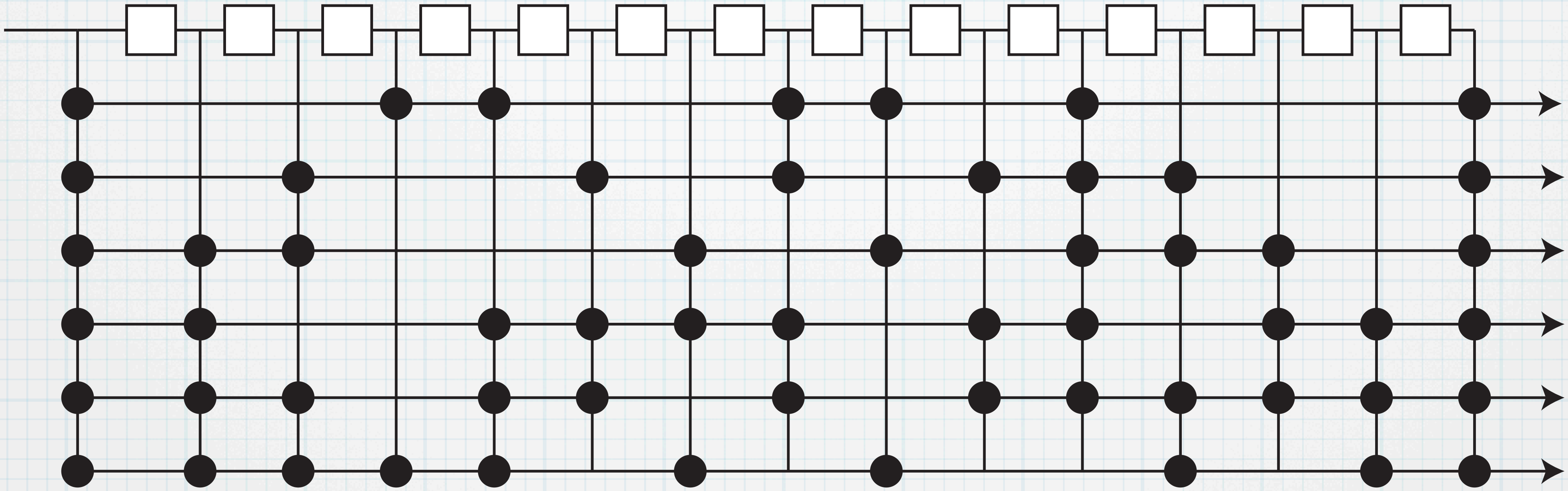
Earth and Moon from MGS

Mars Pathfinder (1997)

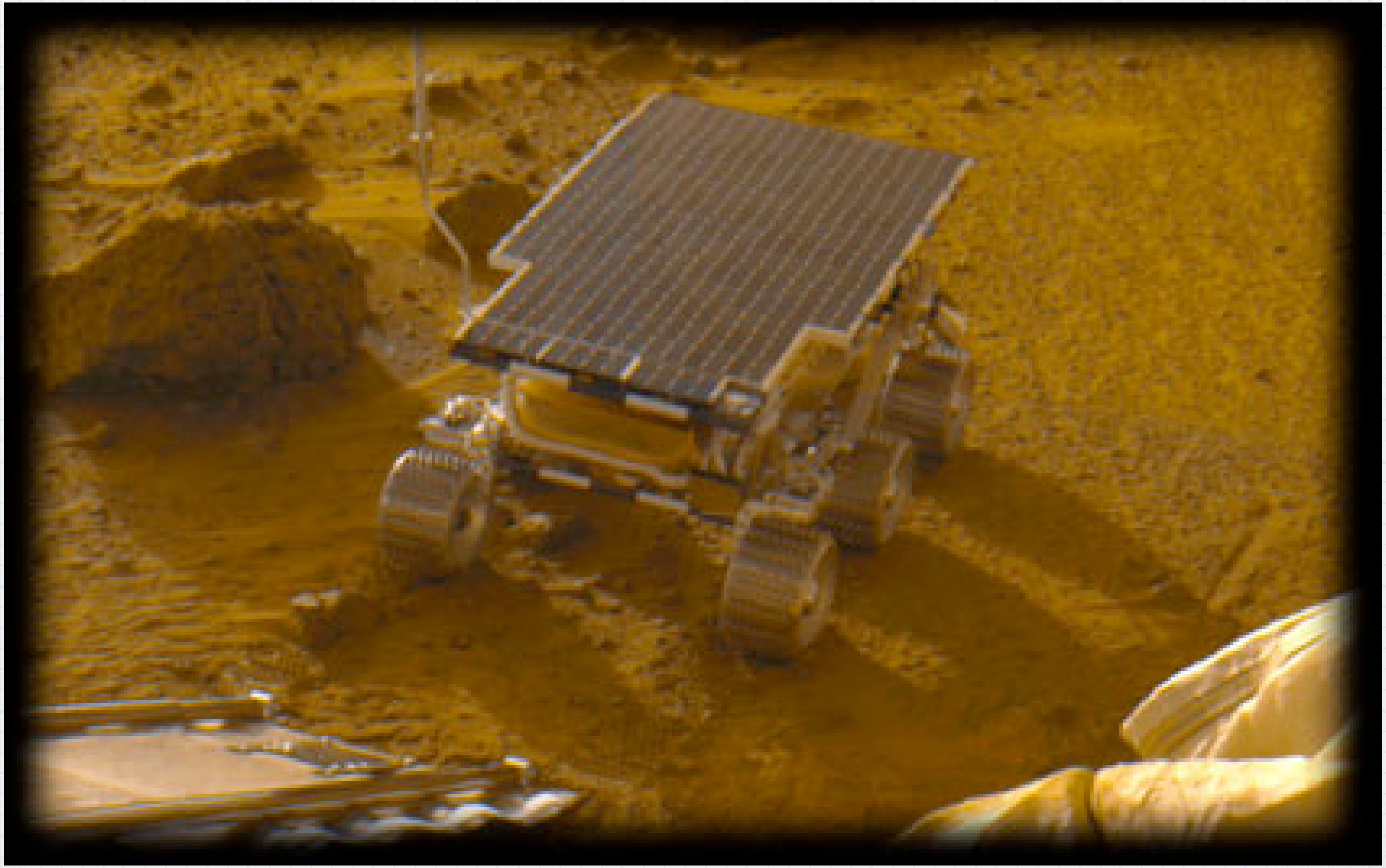
- F = 8.4 GHz (X-Band)
- BPSK Modulation
- R* = 8K ibps
- (15, 1/6) CC + (255,223) RS
- 6:1 lossy JPEG compression

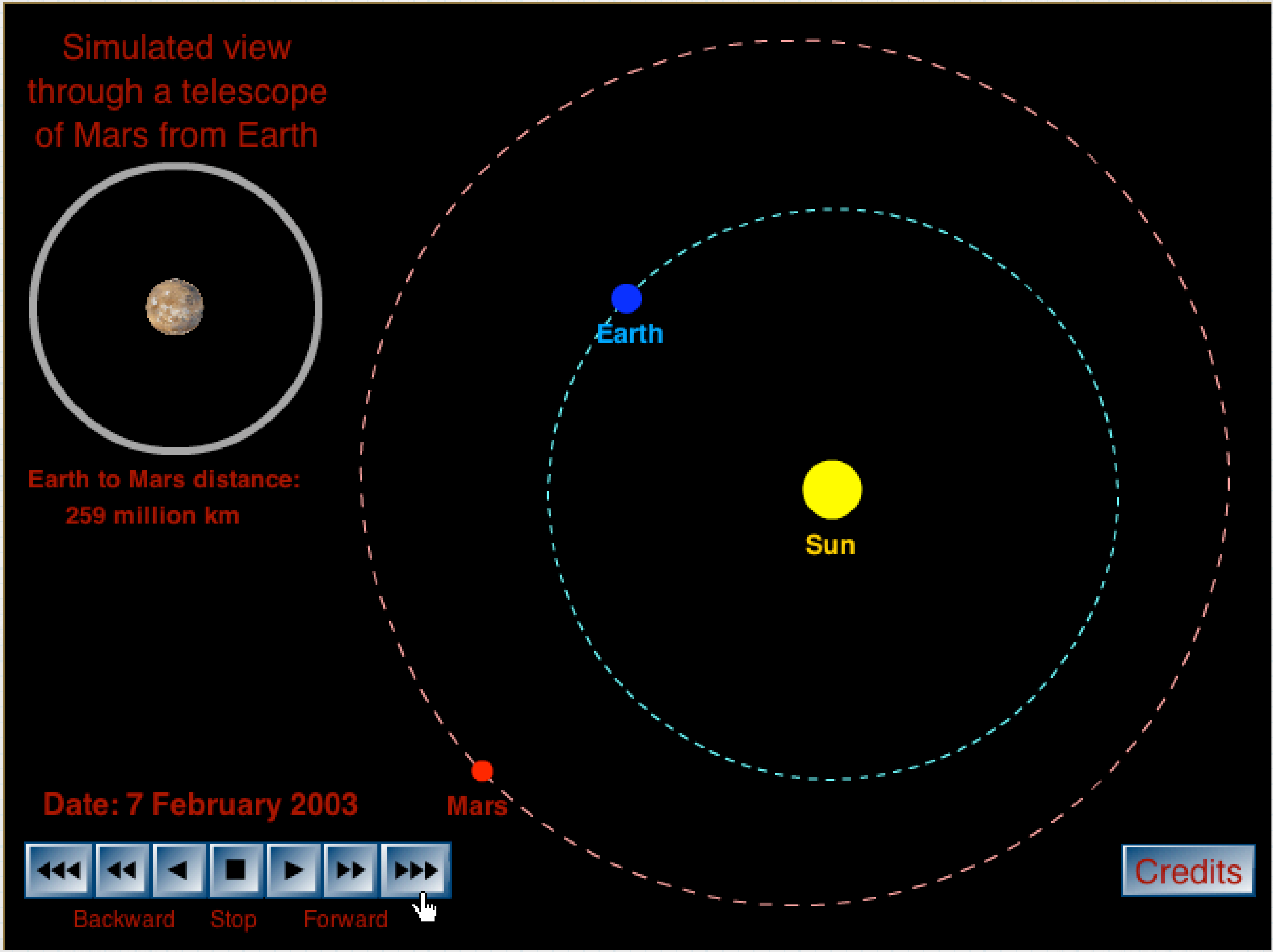


“Galileo” (15, 1/6) Convolutional Encoder



“Sojourner”




Simulated view
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Earth to Mars distance:
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Forward
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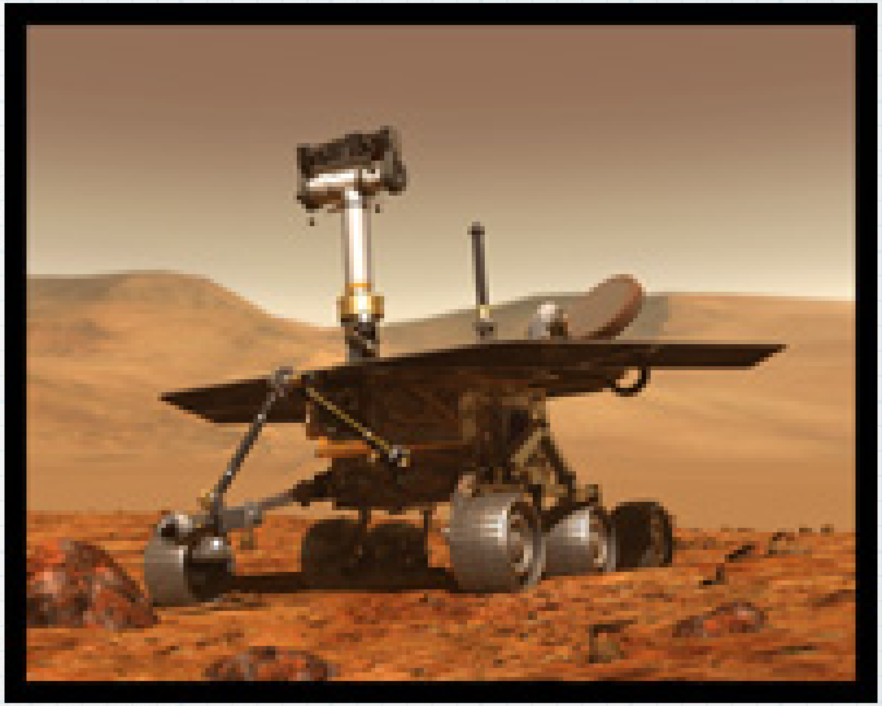
Mars Exploration Rovers (2004)

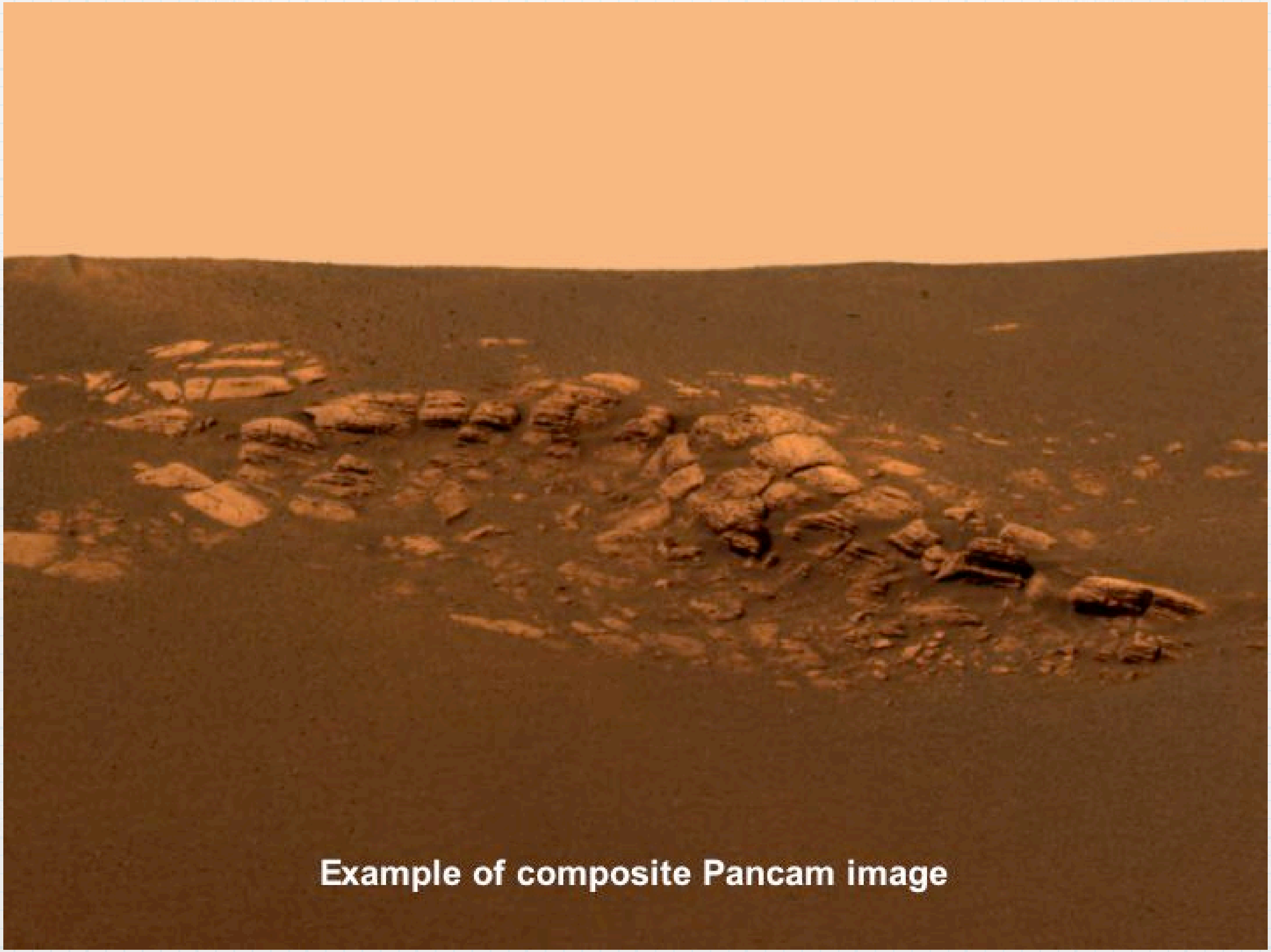
- F = 8.4 GHz (X -Band)
- BPSK Modulation
- R* = 168K ibps
- (15, 1/6) CC + (255,223) RS
- 12:1 lossy “ICER” compression



Leaving the Lander

The "Columbia Hills" (Spirit)

Eagle Crater (Opportunity)

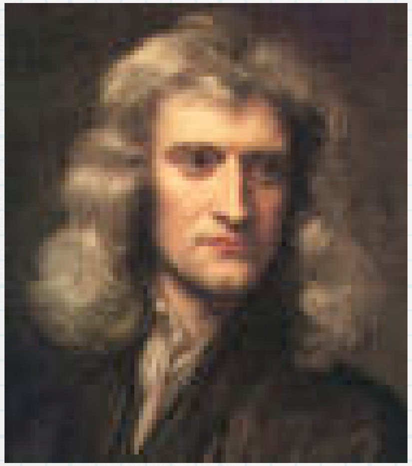


Progress, 1965-2004

- **1965 (Mariner 4): R*= 8.33 ibps**
- **2004 (MER): R*= 168K ibps**
- **This is a 20000-fold increase, or 4.3 orders of magnitude (43 dB).**

Clash of the Titans

Newton vs. Shannon



- **Newton (Physics)**
  - **Aperture**
  - **Frequency**
  - **Power**

- **Shannon (Mathematics)**
  - **Error-Correction**
  - **Data Compression**

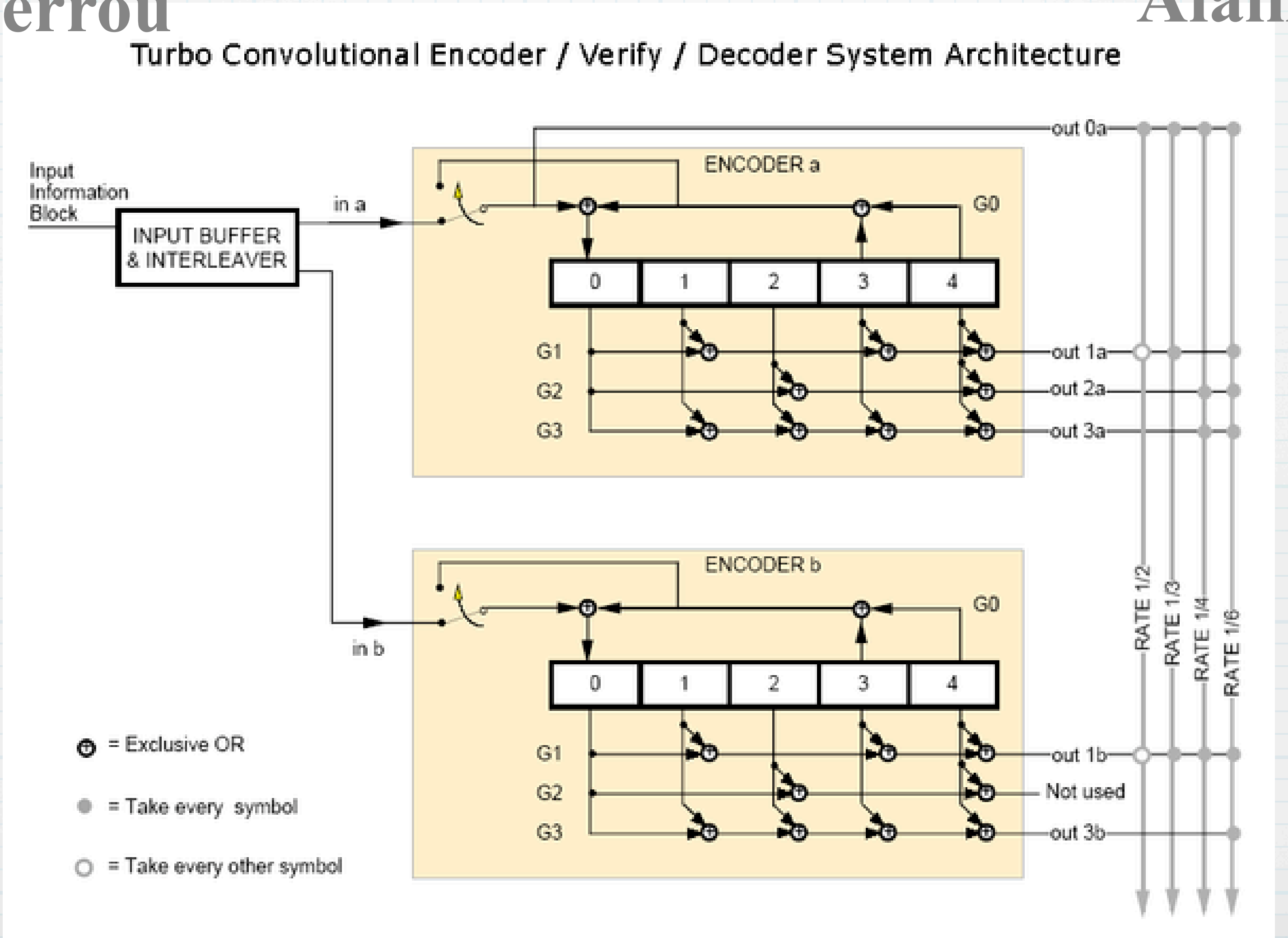
4.3 Orders of Magnitude Improvement in Image Bit Rate, 1965-2004

A Look at the Future

"Turbo Codes" (1993)

Claude Berrou

Alain Glavieux



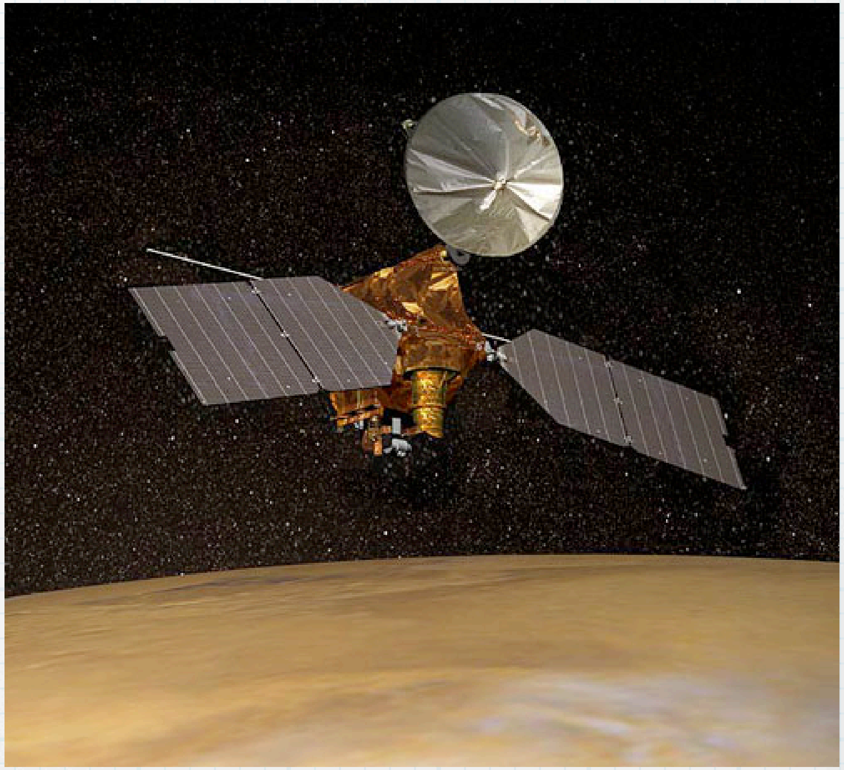
Newton Fights Back with More Aperature

Green Bank 100m Antenna

Array of 12m Antennas

Mars Reconnaissance Orbiter (2006)

- F = 32 GHz (Ka - Band)
- QPSK Modulation
- R* = 6M ibps
- (8920, 1/6)CCSDS turbo code
- 2:1 lossless compression 



Was It Worth the Effort?

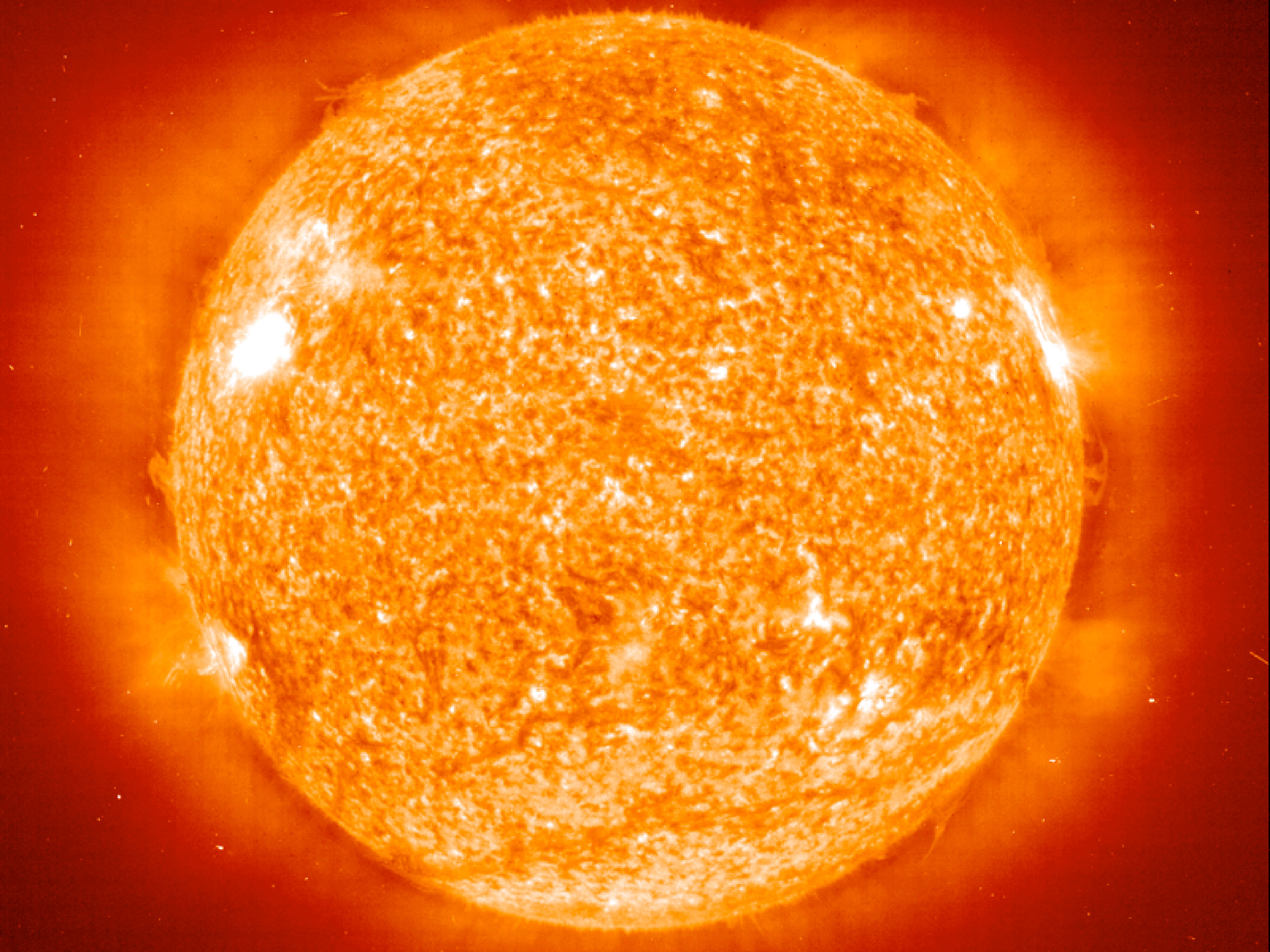
“Frequently the messages have meaning”

A Tour of the Solar System.

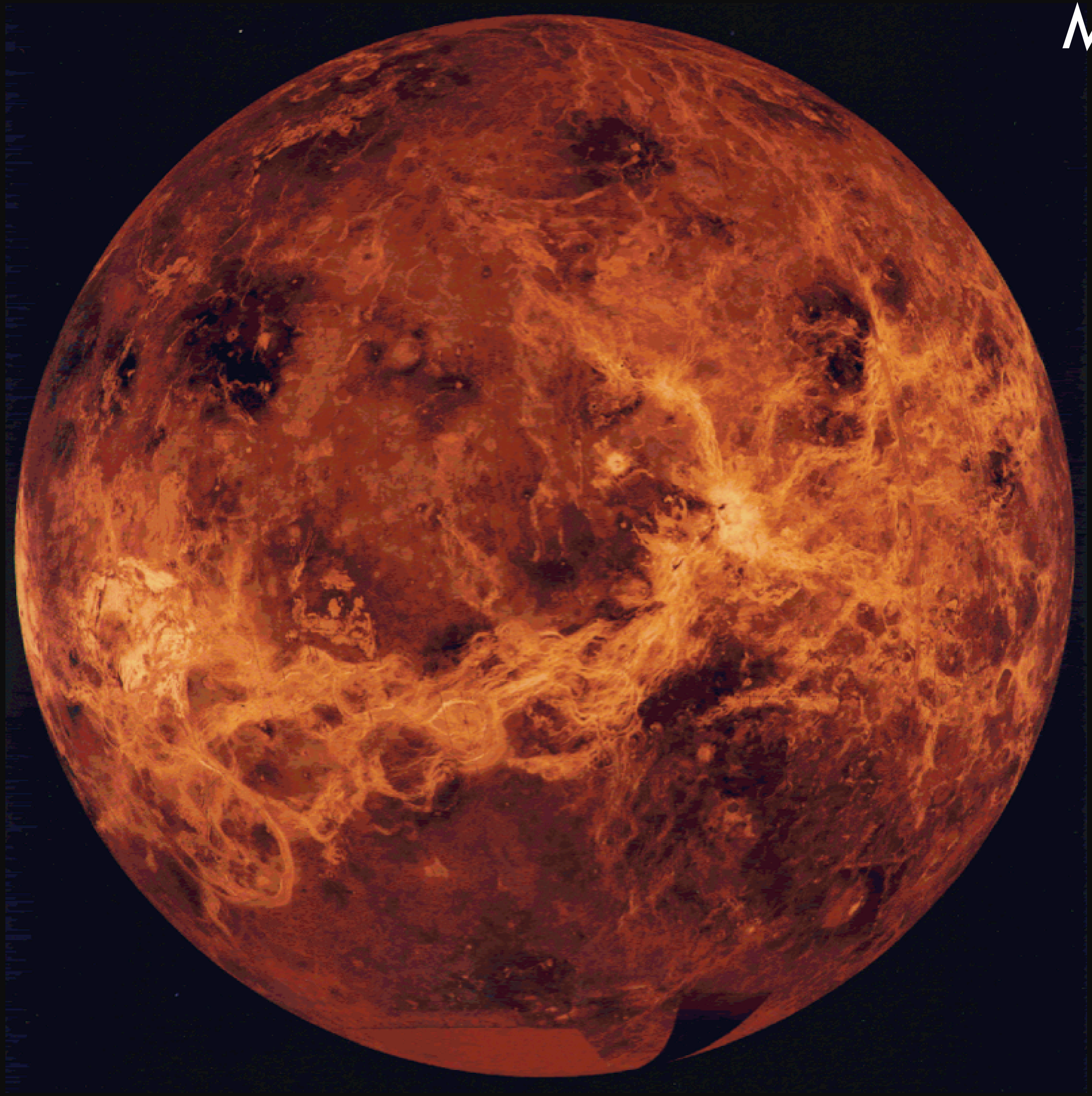
A Tour of the Solar System.

Ludwig van Beethoven, Moonlight Sonata
Daniel Barenboim, pianist

UCSD
April 29, 2005



Mercury
Mariner 10
1974

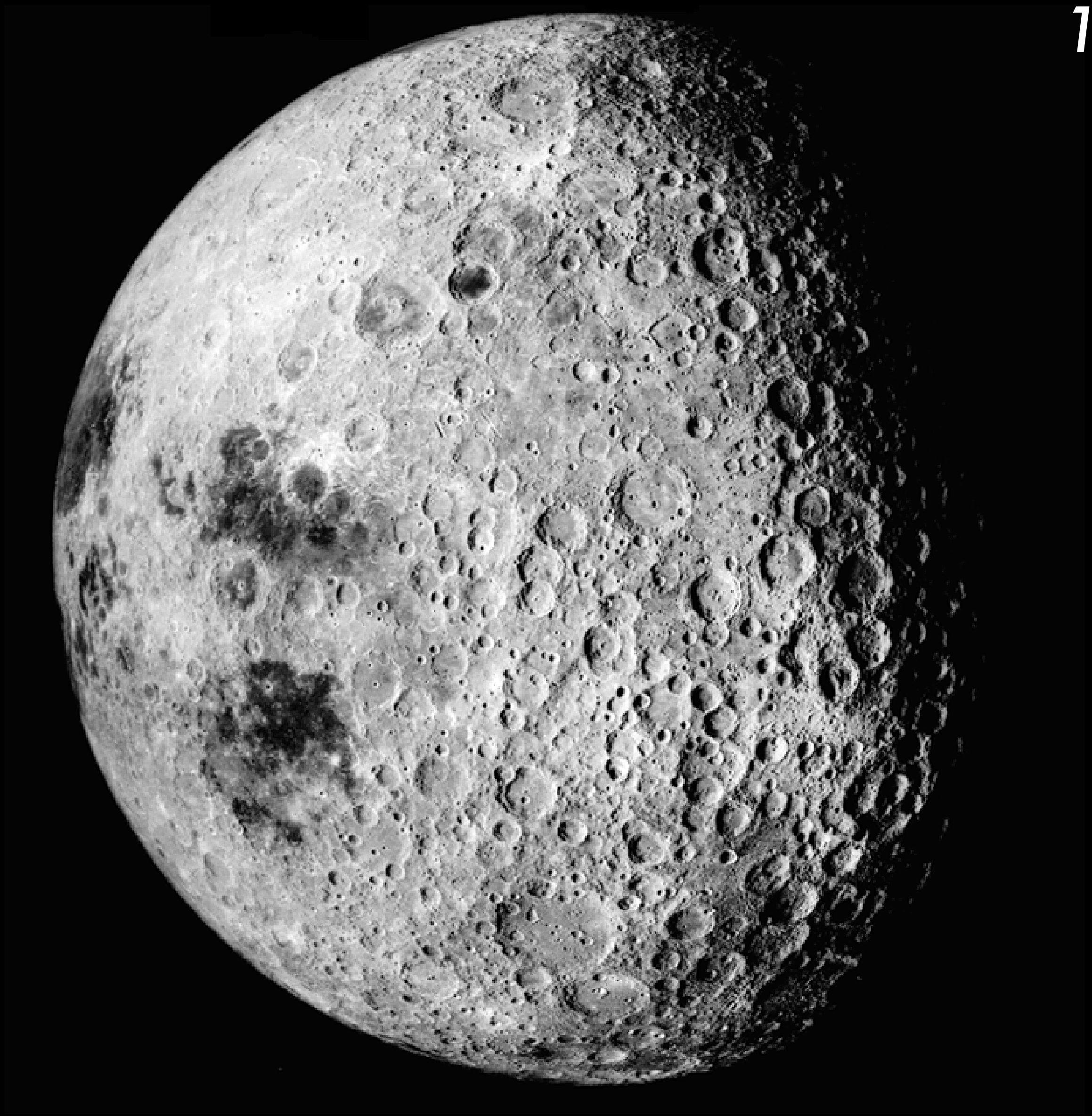


Venus

Magellan

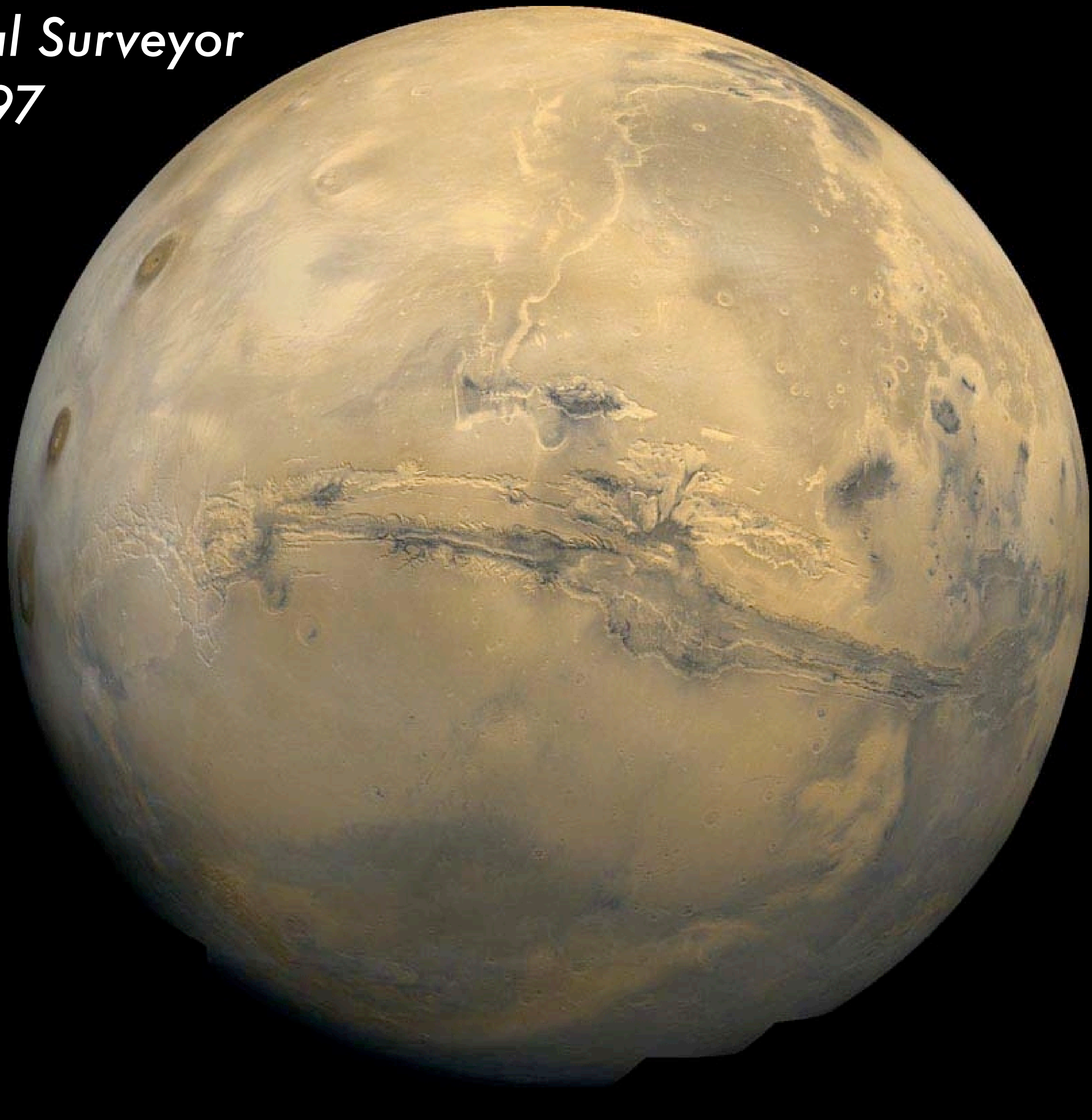
1990

The Far Side of the Moon
Apollo 16
1972



Mars

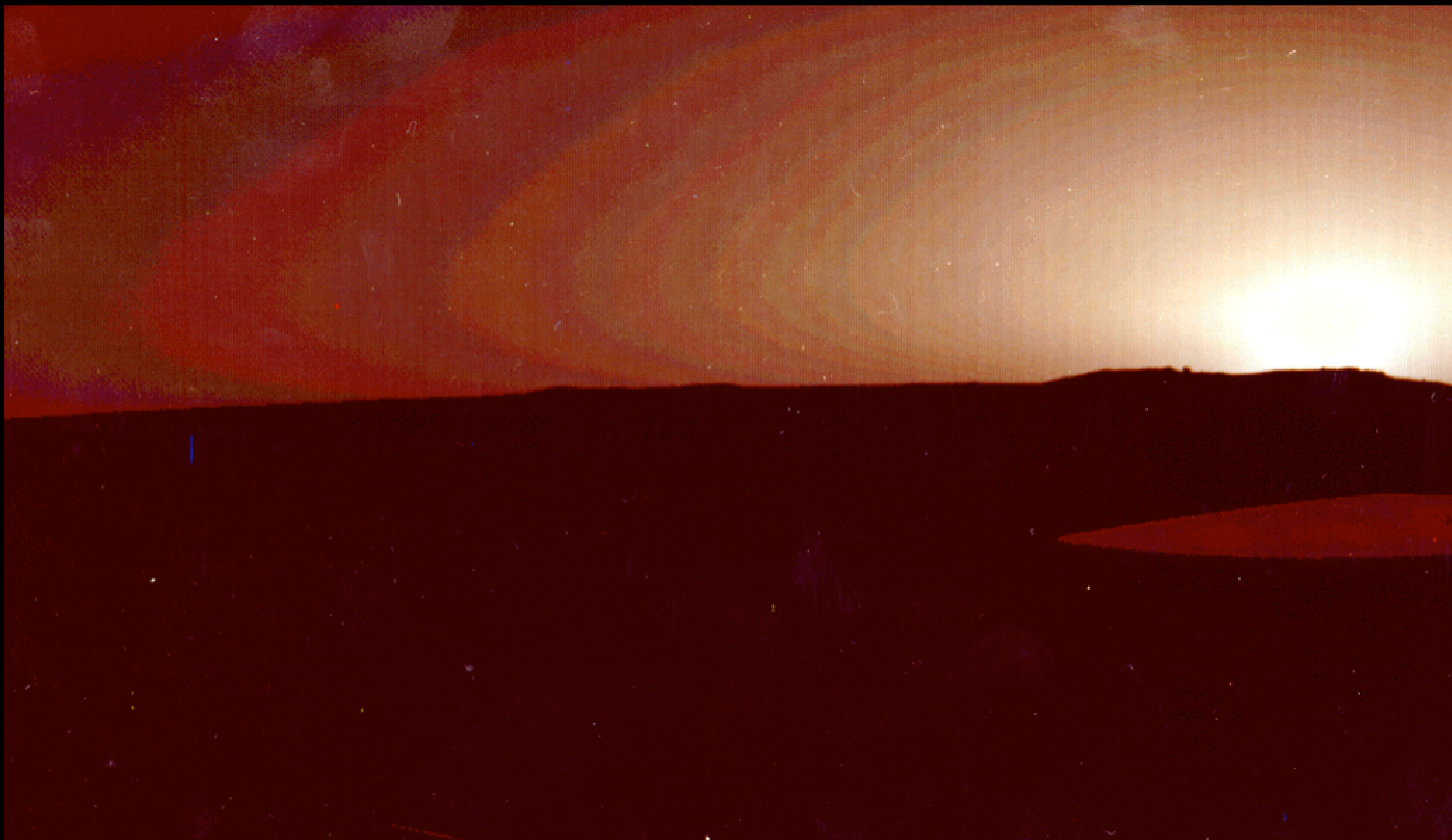
Mars Global Surveyor
1997



Sunset on Mars

Viking Lander

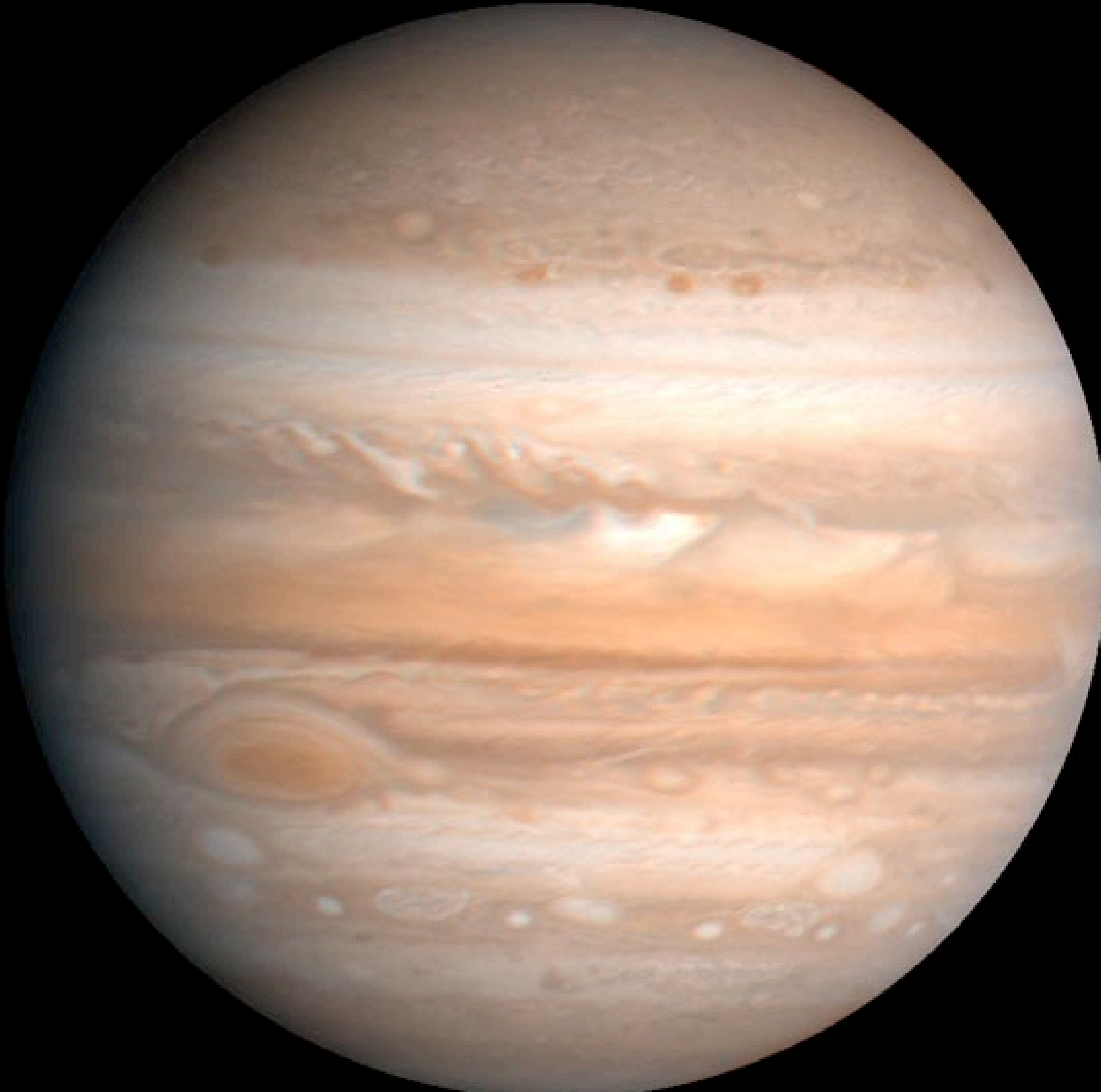
1976



The Asteroid Gaspra

Galileo

1991

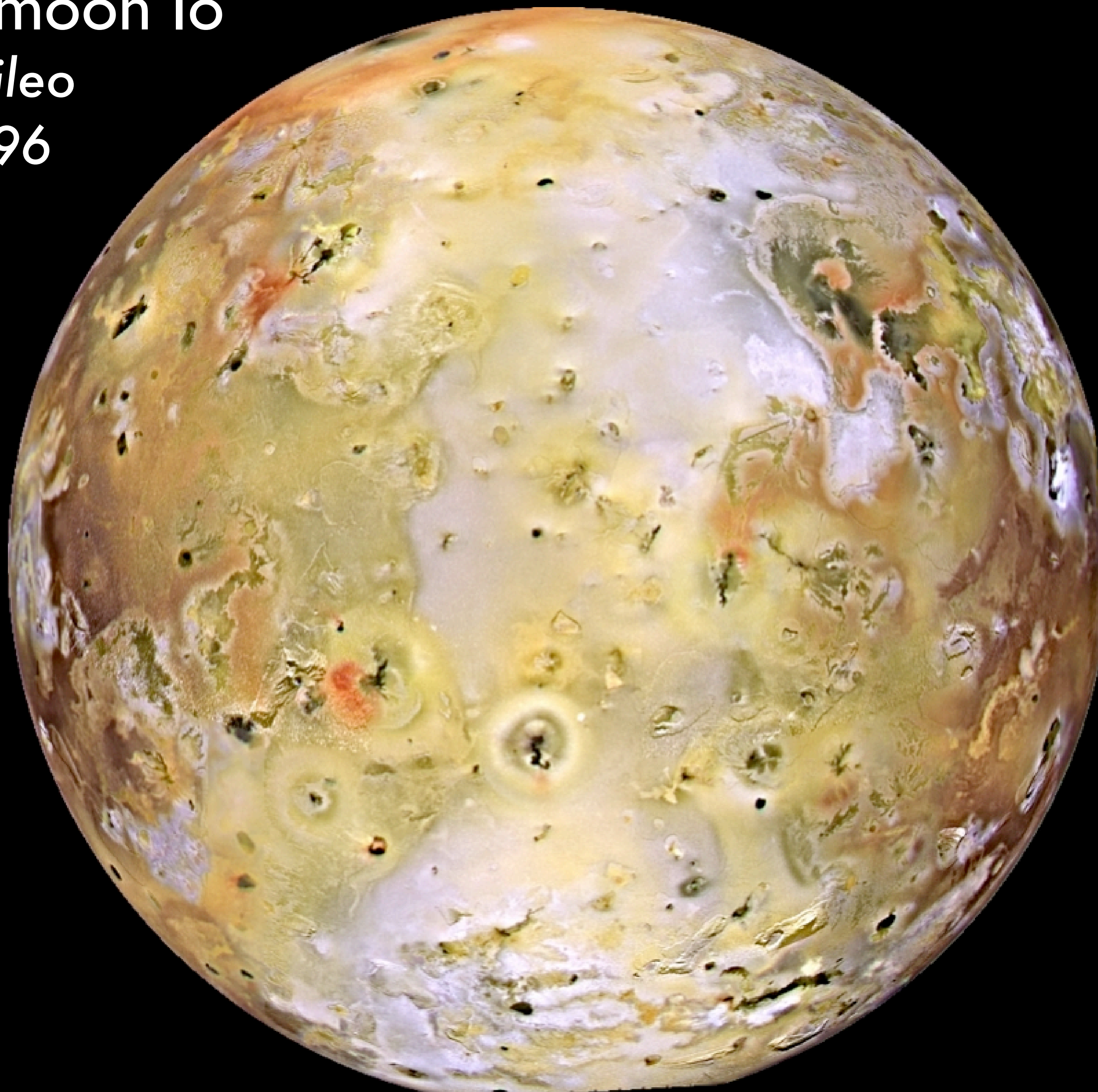


Jupiter
Voyager 1
1979

Jupiter's moon Io

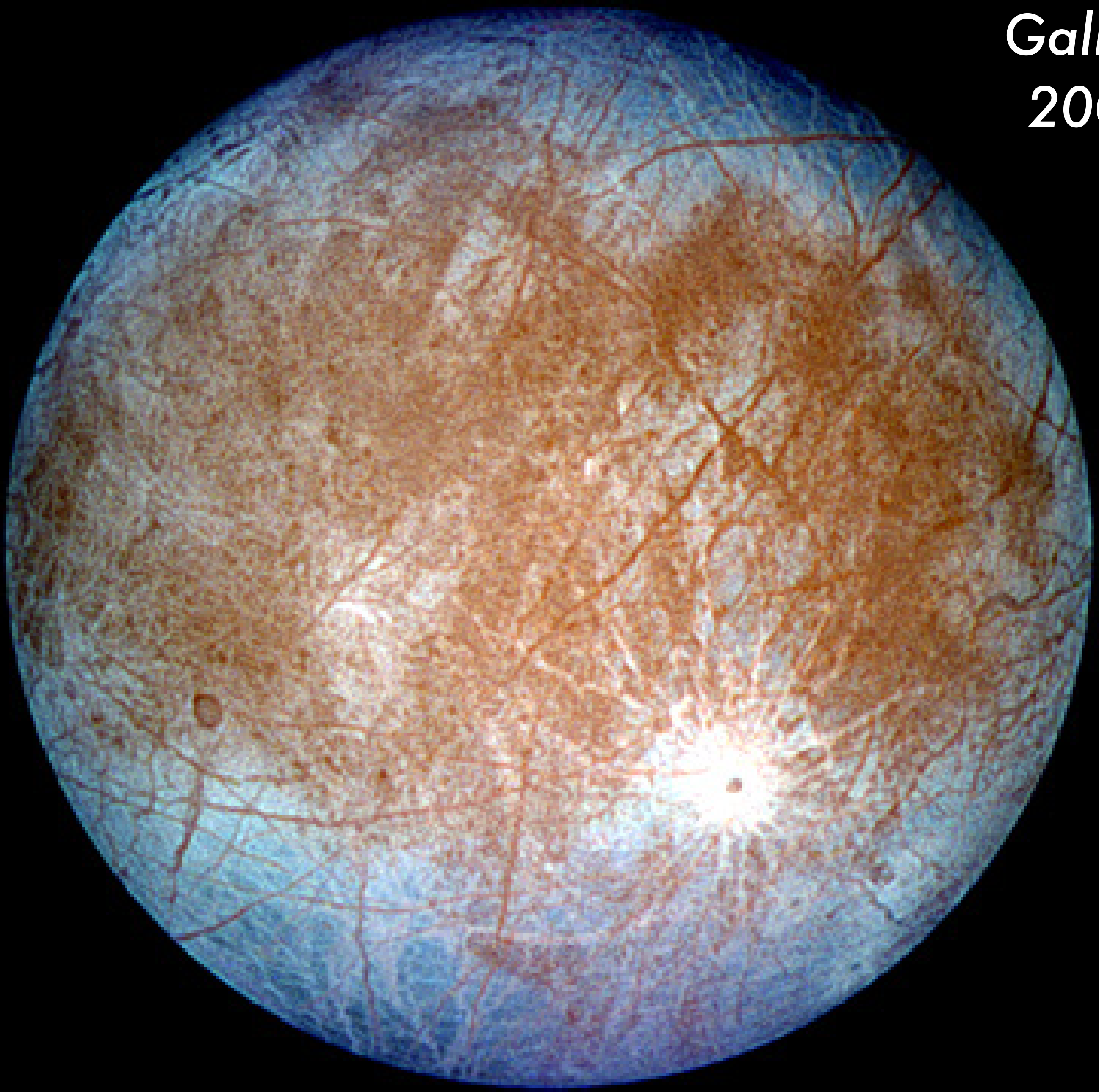
Galileo

1996




Io and Jupiter
Cassini
2004

Jupiter's moon Europa
Galileo
2000



Jupiter's moon Callisto

Galileo

2001

Saturn

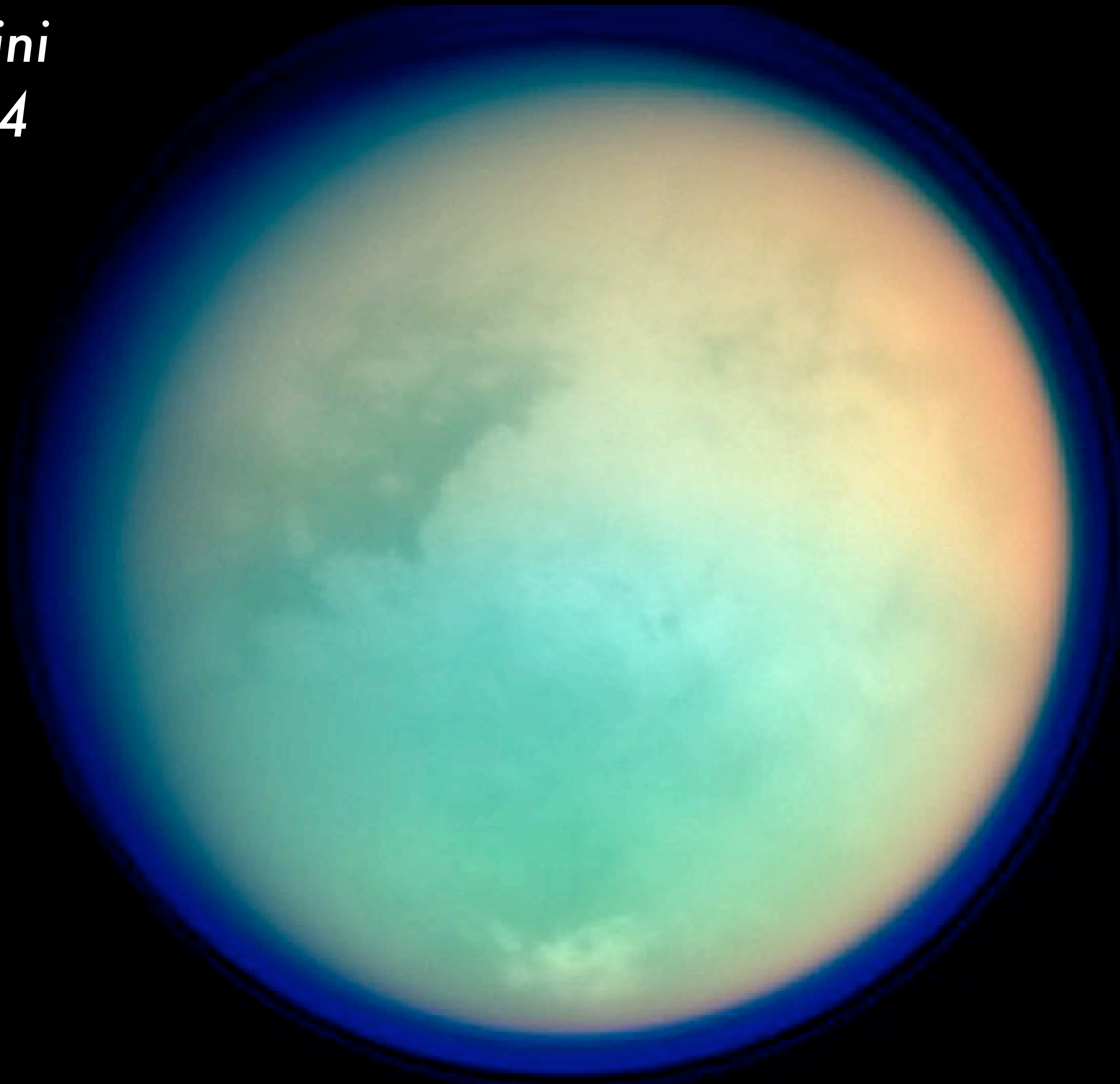
Cassini

2004

Saturn’s moon Titan

Cassini

2004



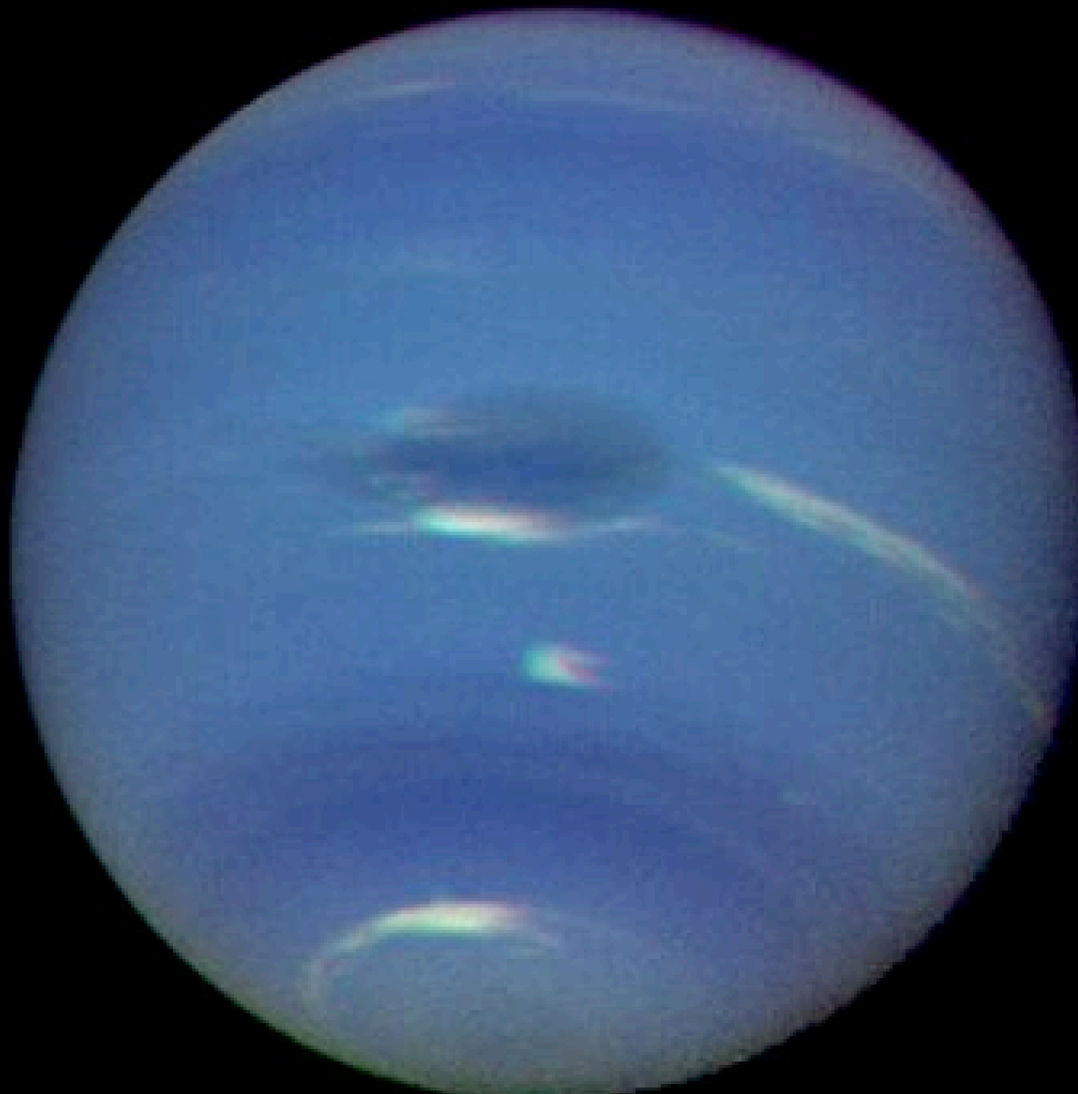
Saturn's moon Phoebe

Cassini

2005

Uranus
Voyager 2
1986

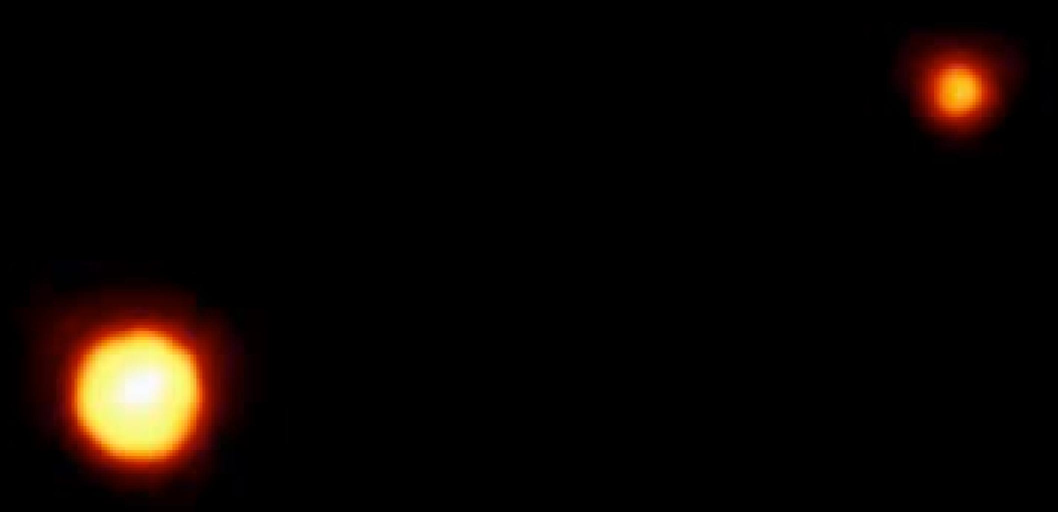
Neptune
Voyager 2
1989



Pluto and its moon Charon

Hubble Space Telescope

1994



We shall not cease from exploration
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time.
-T. S. Eliot, Little Gidding.