The Perfect Basketball Shot

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May 6, 2004

A basketball was shot several times at a rim 10 feet above the ground. It was found that air resistance and spin played little role in the trajectory of the ball and that the ball moved as a point particle projectile. Two shots were also compared, a missed shot as well as a made shot, and it was found that no single factor is responsible for determining a good shot, but it was the right combination of both the angle and the velocity, provided the horizontal and vertical distances remain constant.

INTRODUCTION AND THEORY

In basketball the art of an overhand throw in basketball is called a “shot”; however, there are many types of shots: hook shot, jump shot etc. The shot analyzed in this experiment is the two-hand push shot, which can vary with the shooter. The two handed push shot gives the ball two different motions: the motion of the center of mass that the ball exhibits, and the rotation of the ball about its center of mass. In this experiment this rotational motion is treated separately at first. The perfect shot is a shot that does not hit any part of the rim as it goes through.

In order to analyze the projectile motion of the ball, it is first important to break down the parabolic motion into two basic components, the horizontal motion and the vertical motion. If the ball experiences no force in the horizontal (or x direction), then the acceleration is zero in that direction and the ball moves with a constant velocity. The horizontal velocity of the ball is the x component of the initial velocity u_x(equation 1). The only component of velocity that varies in a parabolic motion is the vertical velocity because it experiences an acceleration due to gravity. When the acceleration in the x direction is zero,

$$s_x = u_x t \quad \text{where} \quad u_x = u_o \cos\theta \quad (1)$$

where, s_x is the range (or horizontal distance) at time t, u is the ball’s velocity , θ is the angle of projection, u_o is the initial velocity and t is the time elapsed from when the object was released.

For the vertical distance, the acceleration is assumed to be that of gravity.

$$s_y = u_{oy} t - \frac{gt^2}{2}, \qquad (2) \qquad u_{oy} = u_o \sin\theta,$$

It is possible to write an equation in terms of s_y and s_x. Using $\frac{s_x}{u_x} = t$, from equation 1 into equation 2.

$$s_y = \tan\theta - \frac{g s_x^{\ 2}}{2u^2 \cos^2\theta} \qquad (3)$$

These three equations (1, 2 and 3) basically dictate the parabolic motion of a particle through a drag free medium and with only gravity acting on it. In this experiment, however there are external forces acting on the particle moving (in this case the smooth basketball). These external forces are air resistance and the force due to the spinning motion of the ball.

The error needed to make a shot is the maximum distance from the center of mass of the ball to the center of the hoop for the shot to be perfect[5].

$$\frac{P}{2} = \frac{D_r}{2} - \frac{D_b}{2\sin\phi} = \text{Error needed to make a shot}$$

<u>Spin Factor</u>

The Spin factor of the ball is important simply because how fast the ball spins could determine whether there are any additional forces that are acting on the ball apart from the gravitational force. It is these additional forces caused by the rapid rotation of the ball that would allow the path of ball to be altered, as seen by a bend in the path of a baseball when a curve ball is thrown or a curl in a cross or free-kick when a soccer ball is kicked. The equation that represents the lift brought about by the spinning of the ball in flight is[2]:

$$F_L = \frac{1}{2} C_L \rho v^2 A, \qquad (4)$$

where F_L = Lift force
v= velocity of the ball
A = cross-sectional area of the ball = 0.045 m^2
C_L= Co-efficient of Lift (related to spin factor)
ρ = density of medium =1.29kg/m^3

Air Resistance

Air Resistance is the force brought about by the ball moving through the viscous air, it opposes the direction of motion of the ball and should affect the motion of the ball by lowering the speed of the ball.

In this experiment the effects of drag can be analyzed through the horizontal motion of the ball. If there is no acceleration in the horizontal direction, the constant velocity should not decrease[12].

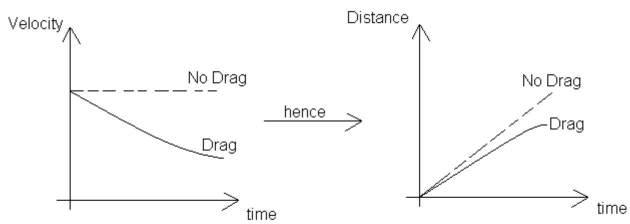

Figure 1: Shows the effects of drag on a velocity-time and distance-time graph.

EXPERIMENT AND METHOD

The Canon ZR10 Digital Camcorder with a 15 frame per second shutter speed, was placed in such a manner so as to achieve the best perspective of the ball's motion. The camera should be far enough from the ball so as to record the full trajectory of the ball; however, it must be close enough to be able to record the spin of the ball. The camera was placed 50 inches off the floor, the horizontal distance of the camera from the trajectory of the ball was measured to be 32ft 2inches, +/- 6 inches.

Initial images were made of a 4 meter rule (two, 2 meter sticks taped together), placed vertically at the point the shooter stood when the balls were thrown. The shooter then stood at the free throw line, that was a horizontal 14.5ft from the center of the rim, the shooter was then filmed taking a number of shots. The movie was then downloaded from the camcorder onto the computer using QuickTime version 6.5. After reviewing the tape, only the perfectly "made" shots were selected, (along with one missed shot) and these shots were finally downloaded onto Video Point Version 2.5 to be analyzed further. First the scale was set, since the number of pixels per meter is to remain constant throughout the experiment (100.5 pixels per meter in this experiment). The data was analyzed frame by frame by clicking at the center of the ball giving the ball's position in x-pixels and y-pixels. These values when divided by 100.5 pixels per meter gave the value of the range and height in meters.

As a check to see whether the camera gave the correct perspective, the pixel co ordinate from the shooter's foot was compared to the pixel co-ordinate of the rim. The height of the pixels was then converted to meters, the difference gave the height of the rim. The height of the rim was found to be 3.005m +/-0.013m which is close to the regulation height of 10 feet, 3.05m). Hence this verified that the perspective caught by the camera was valid to 2%.

RESULTS

All the shots taken in this experiment were taken in an indoor gym. Four of the five shots analyzed were perfect shots, however one was a bad shot.

Since the ball rotates slowly in this experiment, and the velocity keeps varying, the lift force is not constant. However, since the rotationary velocity of the ball is so low, (the ball was estimated to rotate about 1.25 times during a flight time of approximately 1 second), as compared to velocity of the ball, the basketball would have a ratio of rotational velocity to velocity of the ball close to that of $\frac{\pi D\omega}{u} = 1.3$, which gives a relatively low co-efficient of lift value of C_L= 0.3 Ns/kg[11].

Using equation 4, the lift force was calculated to be 0.14 N, which is 2% of the weight, and is too small to significantly affect the motion of the ball over such a small time interval.

The results of the data taken were compiled into two distinct graphs, the height (s_y) versus time graph (Figure 2) and the range (s_x) versus time graph (Figure 3). In order to show that the motion of the ball mimicked the trajectory of a body exhibiting projectile motion, the co-efficient of t^2 in the height as a function time equation, ($y = at^2 + bt + c$) should equal 4.9m/s^2. The projectile motion in figure 4 starts from approximately 2.2 –2.4 m off the ground, because the path of the ball traced was from the point the ball left the tips of shooter's fingers to the point of the ball's entrance into the hoop.

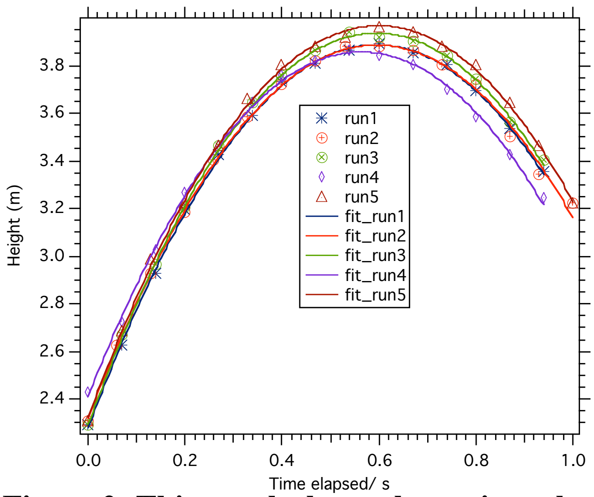

Figure 2: This graph shows the various shots taken plotted in a height versus time graph, the outline for run 4 is a missed shot, all other shots were made.

Table 1 shows the missed shot. Unlike most of the shots, it has the lowest set of angles and the lowest velocity. Such small values only affirm the notion that the shot missed by falling short of the rim.

Table 1 compare the different vector values (where s_{ox} is the horizontal postion at time t = 0, s_{oy} is the vertical distance at time t=0, u_o is the velocity at t=0 or the velocity ball was projected with) determined by analyzing the missed shot (run4) and the made shots. The values are extremely similar. However, it is evident that that the ball in the made shots were an average of over $53^0 = \theta$ as compared to about 52^0 for the missed shot and with a larger velocity of at least 6.5 m/s, for all the made shots as compared to 6.43 m/s +/-0.70m/s for the missed shot, this means that the made shots reached a higher height, and were carried further than run 4. It is evident that run3 and run5 had the largest angles, (an average of over 54^0 for each shot) but they also had the largest velocities and hence, a more "arcing" shot.)

The values of the acceleration due to gravity determined from the two graphs (s vs t and s_y vs s_x) were low: approximately 9.10 m/s^2, as compared to the accepted value of 9.8 m/s^2.

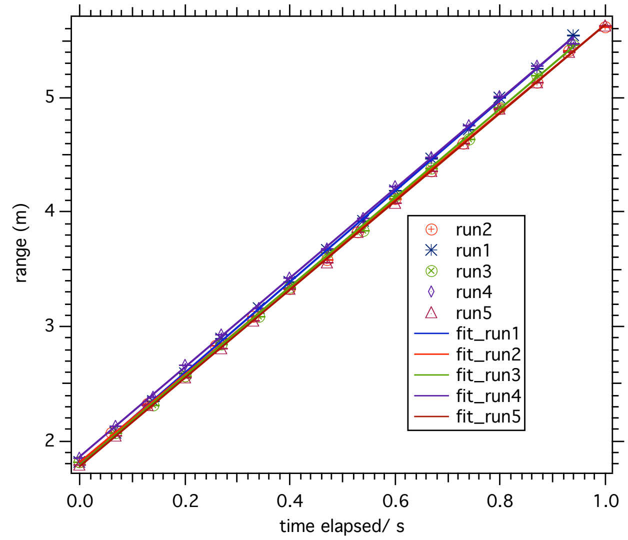

Figure 3: This graph shows the various shots taken plotted in a range versus time graph

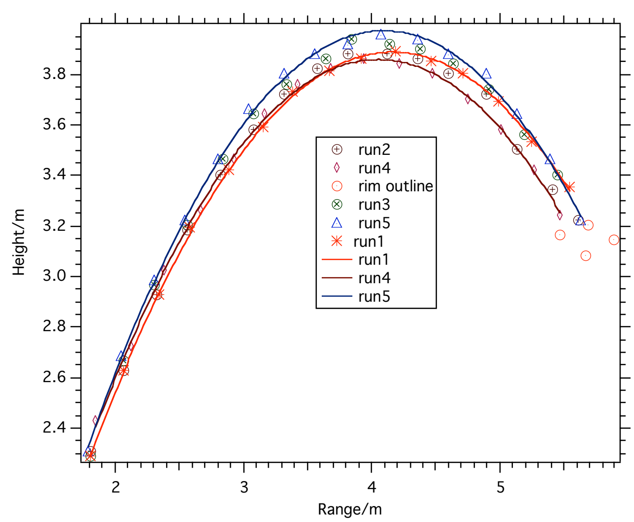

Figure 4: This graph shows the various shots taken plotted in a height versus range graph

The results of all the shots analyzed in a height against range are shown in figure 4. The missed shot (run4) can be seen to fall a little short of the target and ends up hitting the front iron of the rim. As Table 3 shows, the missed shot has a slightly larger error needed to make the shot and has a fairly large angle of entrance as it approaches the rim, which is even larger than runs 1, 3 and 5. The projected velocity for the missed shot was too low, this made the shot "short" as it hit the front iron of the rim. A perfect shot can only be made with a right range of combinations of angle and velocity, a large velocity would cause the ball to overshoot, a small velocity would cause the ball to come up short (as seen in shot 4). Similarly a large angle would cause the ball to come up short whilst a low angle may cause the ball to be overshot.

	graph	u_{ox}/ (m/s)	u_{oy}/(m/s)	u_o/(m/s)	θ	g/(m/s^2)
run1	s vs t	3.97+/-0.01	5.41+/-0.04	6.71+/-0.19	53.7^0+/-2.3^0	9.10+/-0.08
	s vs s	-	-	-	53.5^0+/-0.2^0	9.14+/-0.26
run2	s vs t	3.81+/-0.02	5.27+/-0.08	6.50+/-0.19	54.1^0+/-2.0^0	9.10+/-0.08
	s_y vs s_x	-	-		51.3^0+/-0.2^0	9.00+/-0.26
run3	s vs t	3.89+/-0.01	5.52+/-0.04	6.80+/-0.19	53.7^0+/-1.9^0	9.28+/-0.08
	s_y vs s_x	-	-		54.7^0+/-0.4^0	9.09+/-0.26
run4	s vs t	3.90+/-0.02	5.12+/-0.04	6.43+/-0.70	52.8^0+/-8.5^0	9.28+/-0.08
(missed shot)	$s_y$vs s_x	-	-		52.0^0+/-0.2^0	9.13+/-0.26
run5	s vs t	3.86+/-0.01	5.49+/-0.04	6.71+/-0.19	53.7^0+/-1.9^0	9.10+/-0.08
	s_y vs s_x	-	-	-	54.8^0+/-0.2^0	9.23+/-0.26

Table 1: This table shows the values determined from all the runs.

Horizontal **Vertical**

	intercept	slope	a	b	c
Run 1	1.80 +/-0.01	3.97 +/-0.01	-4.55+/-0.04	5.41+/-0.04	2.28+/-0.01
Run 2	1.82 +/-0.01	3.81 +/-0.02	-4.42+/-0.08	5.27+/-0.08	2.32+/-0.02
Run 3	1.79 +/-0.01	3.89 +/-0.02	-4.64+/-0.05	5.52+/-0.05	2.29+/-0.01
Run 4	1.86 +/-0.01	3.90 +/-0.02	-4.64+/-0.06	5.12+/-0.06	2.41+/-0.01
Run 5	1.78 +/-0.01	3.86 +/-0.01	-4.58+/-0.05	5.49+/-0.05	2.32+/-0.01

Table 2: Shows the intercept and the slope determined from figure 3, as well as the co-efficients of the equation that defines figure 2.

	Angle of entrance	Range	Time of flight	Margin of error (P/2)	Comments
Run1	38.6^0 +/- 1.7^0	3.731m +/- 0.003m	0.94s	0.04m	Made it
Run2	42.0^0 +/- 2.1^0	3.471m +/- 0.003m	0.94s	0.05m	Made it
Run3	37.8^0 +/- 1.7^0	3.642m +/- 0.003m	0.94s	0.04m	Made it
Run4	39.9^0 +/- 2.1^0	3.601m +/- 0.003m	0.94s	0.05m	Missed it
Run5	38.5^0 +/- 1.7^0	3.602m +/- 0.003m	0.94s	0.04m	Made it

Table 3: This estimates the different values of both shots, and compares them, it also shows that accuracy is vital, the right velocity is crucial, because even with a good angle, as seen with run4, it does not guarantee a made shot.

The angle ϕ (angle of entrance) was determined by finding the tan inverse of the gradient at the last point closest to the rim.

CONCLUSION

The following conclusions could be reached by analyzing the shots taken:
1) The ball shot in an indoor gym displays a trajectory that is equivalent to that of a point projectile, where air resistance and the spin factor of the ball have no measurable effect on the motion of the ball.
2) The shots that are made or missed do not depend on one given factor, but a set of factors. There is no one projection angle that would yield a perfect shot, however it is the combination of both the initial launch velocity, the angle of projection, the diameter of the ball as well as the rim (if they are varied). Each factor is more or less dependant on each other to get a perfect shot.

An improvement in this experiment could come by analyzing more shots with a camcorder that has a higher frequency and better resolution. Another improvement could come analyzing the technique of actually shooting the ball, and seeing how that affects the trajectory of the ball, as well as a shooter shooting at different distances and analyzing how the shot changes, and whether air resistance plays a larger role, especially as the shots get longer.

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