Filter Circuits

ECEN2260

R.W. Erickson

The ideal low-pass filter

  Not physically realizable

Practical low-pass filters

  Parameters and properties

Real poles

Butterworth filter

Chebyschev filter

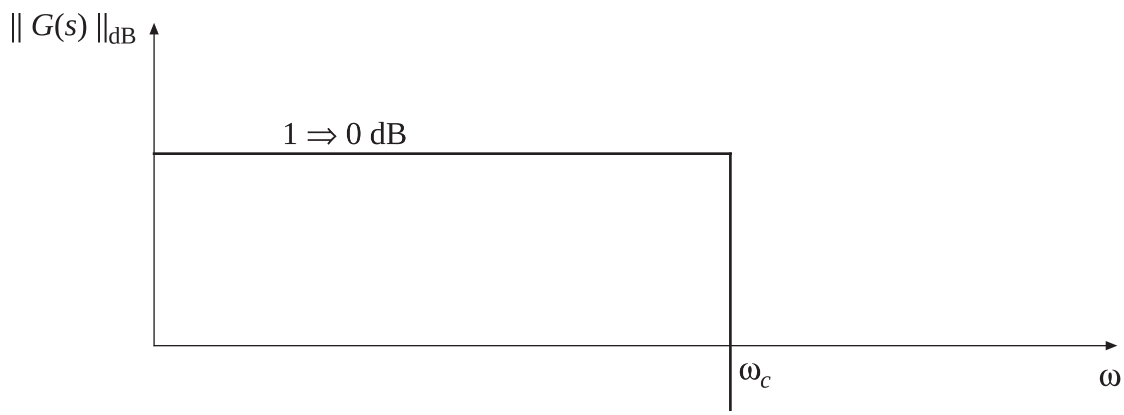
Bessel filter

Comparison of filter responses

Additional points

The “ideal” low-pass filter

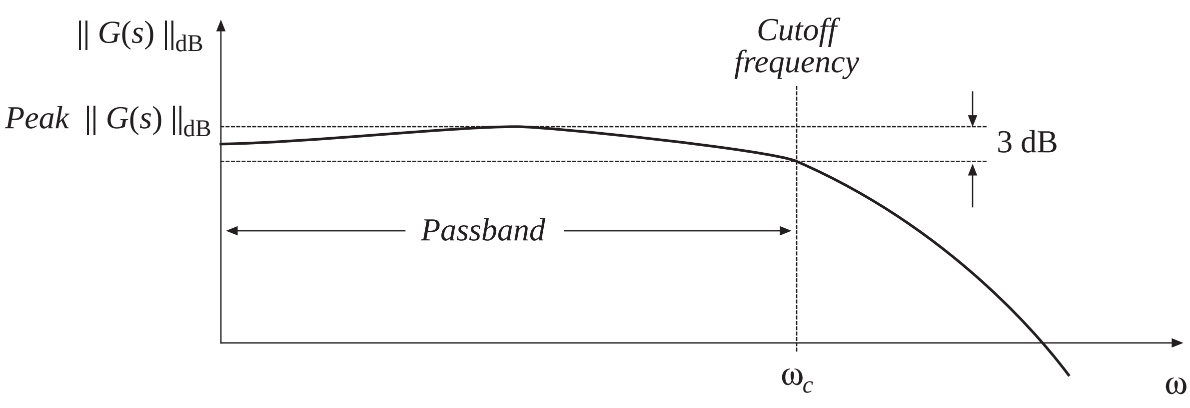
Frequency response



Using the Fourier transform, it can be shown that this transfer function leads to a nonphysical transient response. The output signal must change before the input changes, and the response is not causal. Hence, it is physically impossible to construct a filter having this frequency response.

A practical low-pass filter

Frequency response



Inside the passband, the magnitude response remains within 3 dB of the maximum value.

At the cutoff frequency, the magnitude is 3 dB below the maximum.

At frequencies greater than the cutoff frequency, the magnitude “rolls off”: the filter attenuates high-frequency sinusoids.

Some properties of interest:

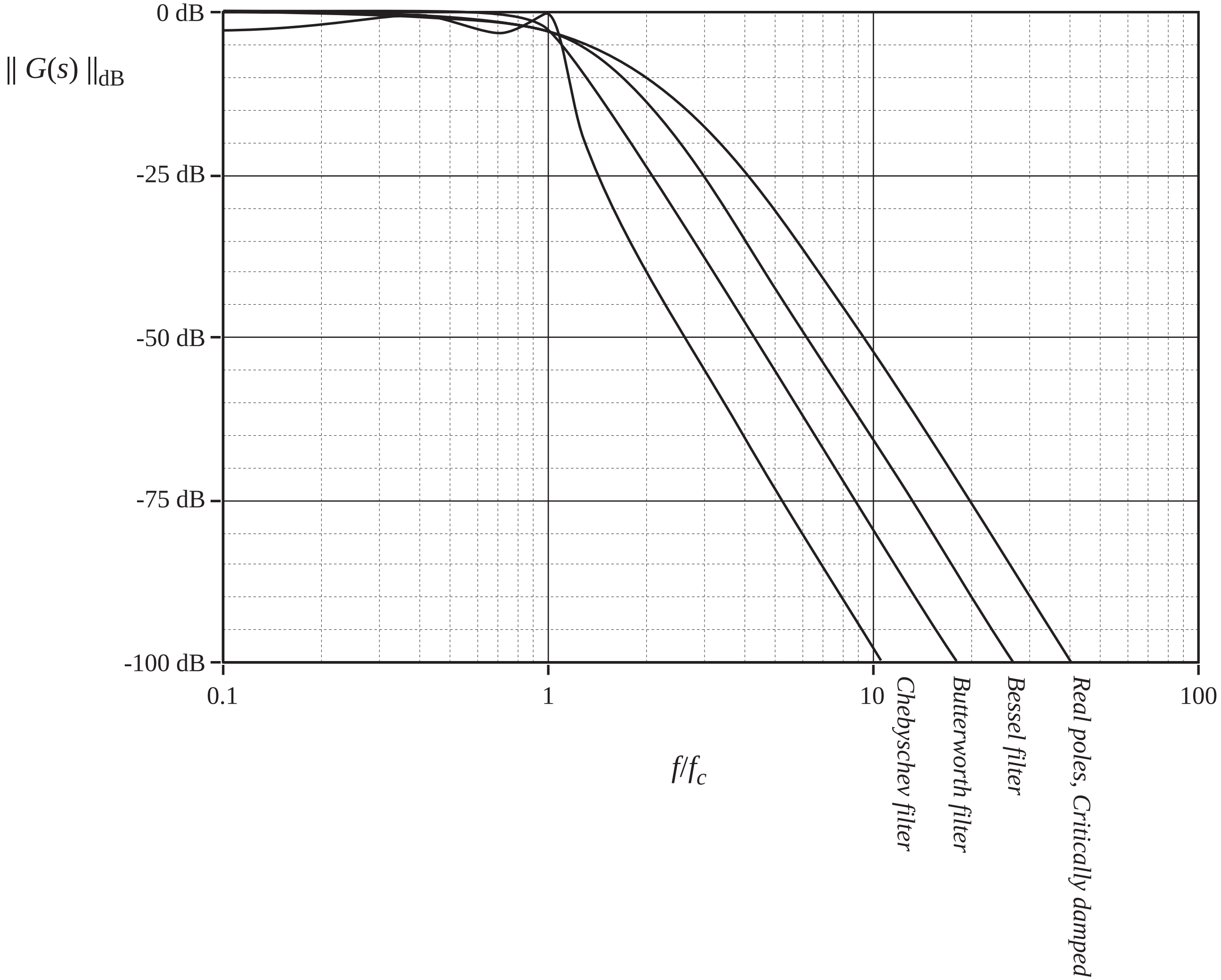
Flatness within passband

Fast rolloff

Amount of overshoot and ringing in step response

Phase response: constant time delay

Comparison of Fourth-Order Filter Responses



Real poles, critical damping

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_0}\right)^n}$$

$$\left\| G(j\omega) \right\| = \frac{1}{\left(1 + \left(\frac{\omega}{\omega_0}\right)^2\right)^{\frac{n}{2}}}$$

Note that the -3 dB point occurs at $\omega_c = \omega_0$ only for $n = 1$. When $n > 1$, the -3 dB point ω_c is given by the solution of

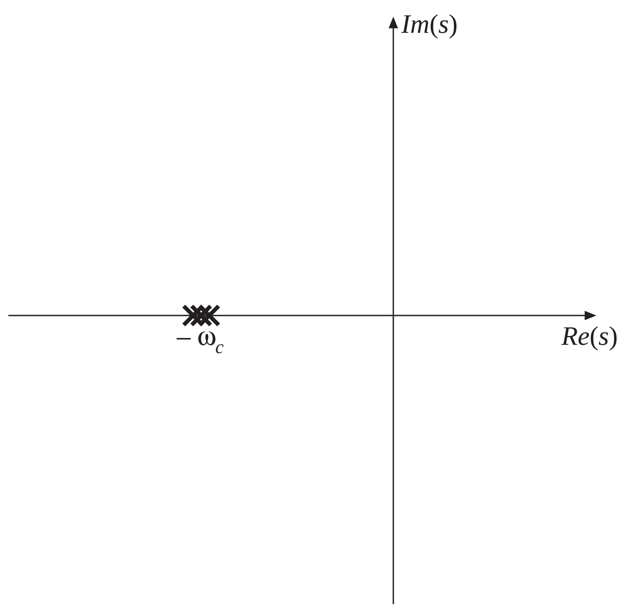
$$\frac{1}{\sqrt{2}} = \frac{1}{\left(1 + \left(\frac{\omega_c}{\omega_0}\right)^2\right)^{\frac{n}{2}}}$$

Solve for ω_c (cutoff frequency) as a function of the corner frequency:

$$\omega_c = \omega_0 \sqrt{2^{\frac{1}{n}} - 1}$$

When $n > 1$, ω_c can be significantly less than ω_0. For example, for $n = 4$, $\omega_c = 0.435\omega_0$. This leads to a very gradual rolloff characteristic.

Obtaining a steeper rolloff requires use of complex poles.



Butterworth filter

$G(s) = \dfrac{1}{B(s)}$ $\qquad\qquad$ $B(s)$ = *Butterworth polynomial* (*below*)

$B(s)$ is a polynomial containing complex roots evenly spaced in the left half–plane around a circle of radius ω_c (for general formula, see Thomas and Rosa, 4th Edition, p. 691).

n	***B*(*s*)**
1	$\left(1 + \frac{s}{\omega_c}\right)$
2	$\left(1 + \sqrt{2}\,\frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)$
3	$\left(1 + \frac{s}{\omega_c}\right)\left(1 + \frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)$
4	$\left(1 + 0.7654\,\frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)\left(1 + 1.848\,\frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)$

see Thomas and Rosa, *4th Edition*, *Table* 14–1 *on p*. 694

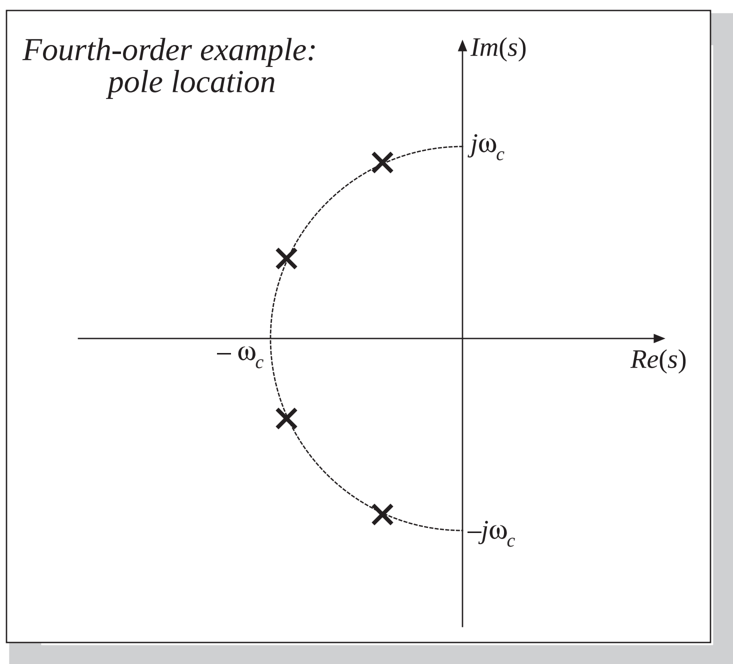
Properties of the Butterworth filter

Maximally flat: in the order *n* Butterworth filter, the first *n* derivatives of the magnitude response are equal to zero at $\omega = 0$.

Faster roll-off than Bessel filter or real poles

Step response exhibits overshoot and ringing

A popular filter characteristic

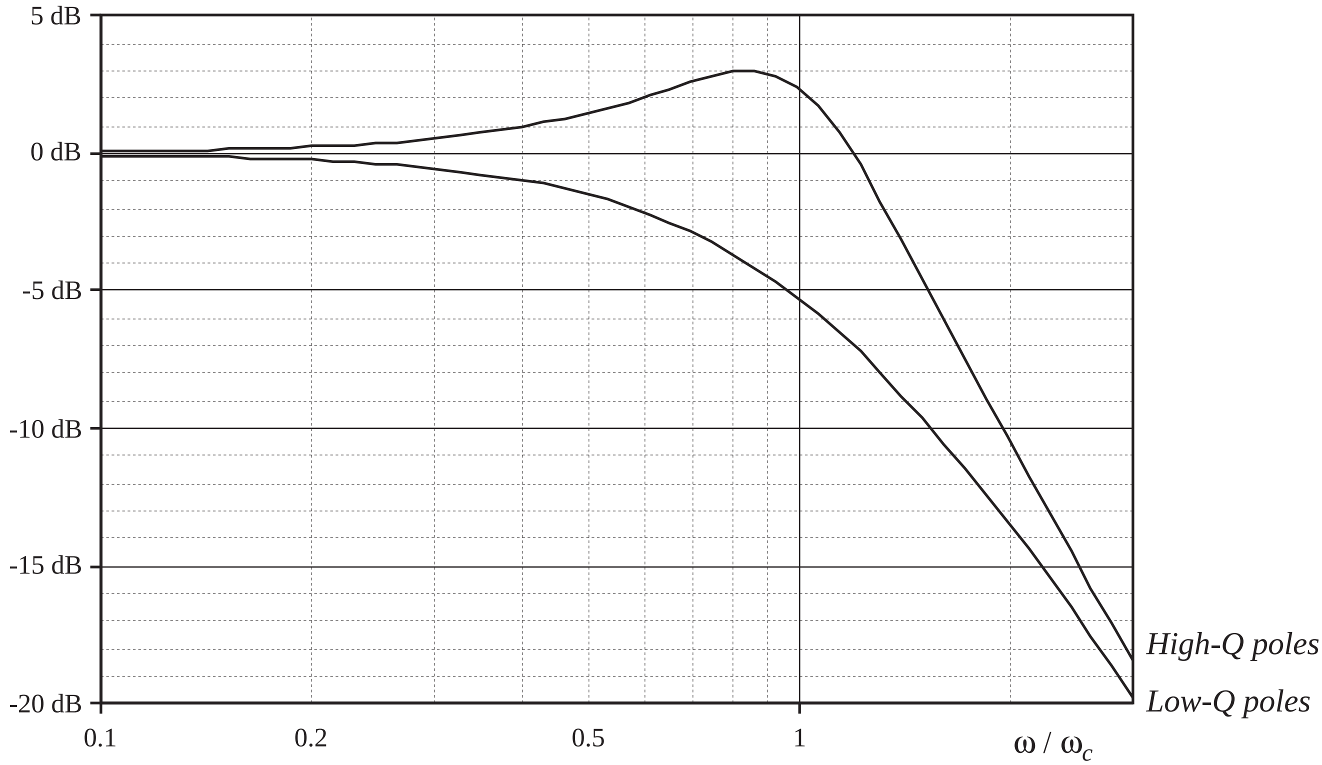


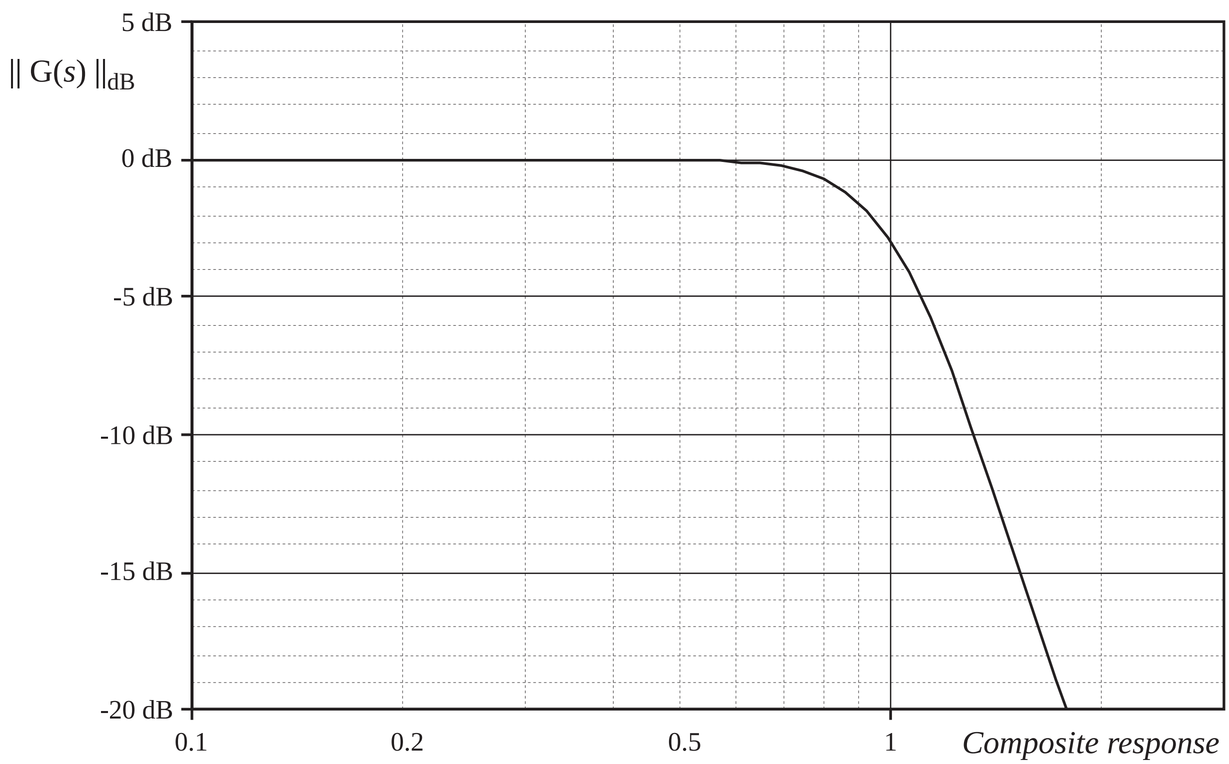
Fourth-Order Butterworth Filter: How the Complex-Conjugate Pole Pairs Combine

$$G(s) = \frac{1}{\left(1 + 0.7654\left(\frac{s}{\omega_c}\right) + \left(\frac{s}{\omega_c}\right)^2\right)\left(1 + 1.848\left(\frac{s}{\omega_c}\right) + \left(\frac{s}{\omega_c}\right)^2\right)}$$

High-Q poles $Q = 1.307$

Low-Q poles $Q = 0.5412$





Chebyschev filter

$G(s)$ contains complex poles spaced in the left half–plane around an ellipse inscribed inside the Butterworth circle. The poles are "stagger–tuned": they occur at different frequencies (see T&R, p. 696).

n	$G(s)$
1	$\dfrac{1}{\left(1+\dfrac{s}{\omega_c}\right)}$
2	$\dfrac{1}{\sqrt{2}}\dfrac{1}{\left(1+0.7654\left(\dfrac{s}{0.8409\,\omega_c}\right)+\left(\dfrac{s}{0.8409\,\omega_c}\right)^2\right)}$
3	$\dfrac{1}{\left(1+\left(\dfrac{s}{0.2980\,\omega_c}\right)\right)\left(1+0.3254\left(\dfrac{s}{0.9159\,\omega_c}\right)+\left(\dfrac{s}{0.9159\,\omega_c}\right)^2\right)}$
4	$\dfrac{1}{\sqrt{2}}\dfrac{1}{\left(1+0.1789\left(\dfrac{s}{0.9502\,\omega_c}\right)+\left(\dfrac{s}{0.9502\,\omega_c}\right)^2\right)\left(1+0.9276\left(\dfrac{s}{0.4425\,\omega_c}\right)+\left(\dfrac{s}{0.4425\,\omega_c}\right)^2\right)}$

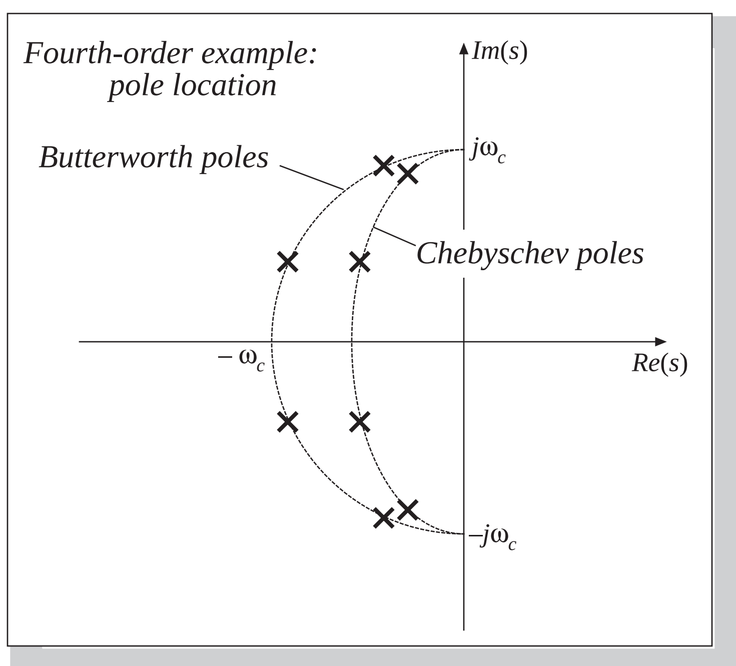
see Thomas and Rosa, fourth edition, Table 14– 2 *on p.* 698

Properties of the Chebyschev filter

Equal ripple in the passband: each complex pole pair causes a hump in the magnitude, varying from –3 dB to 0 dB.

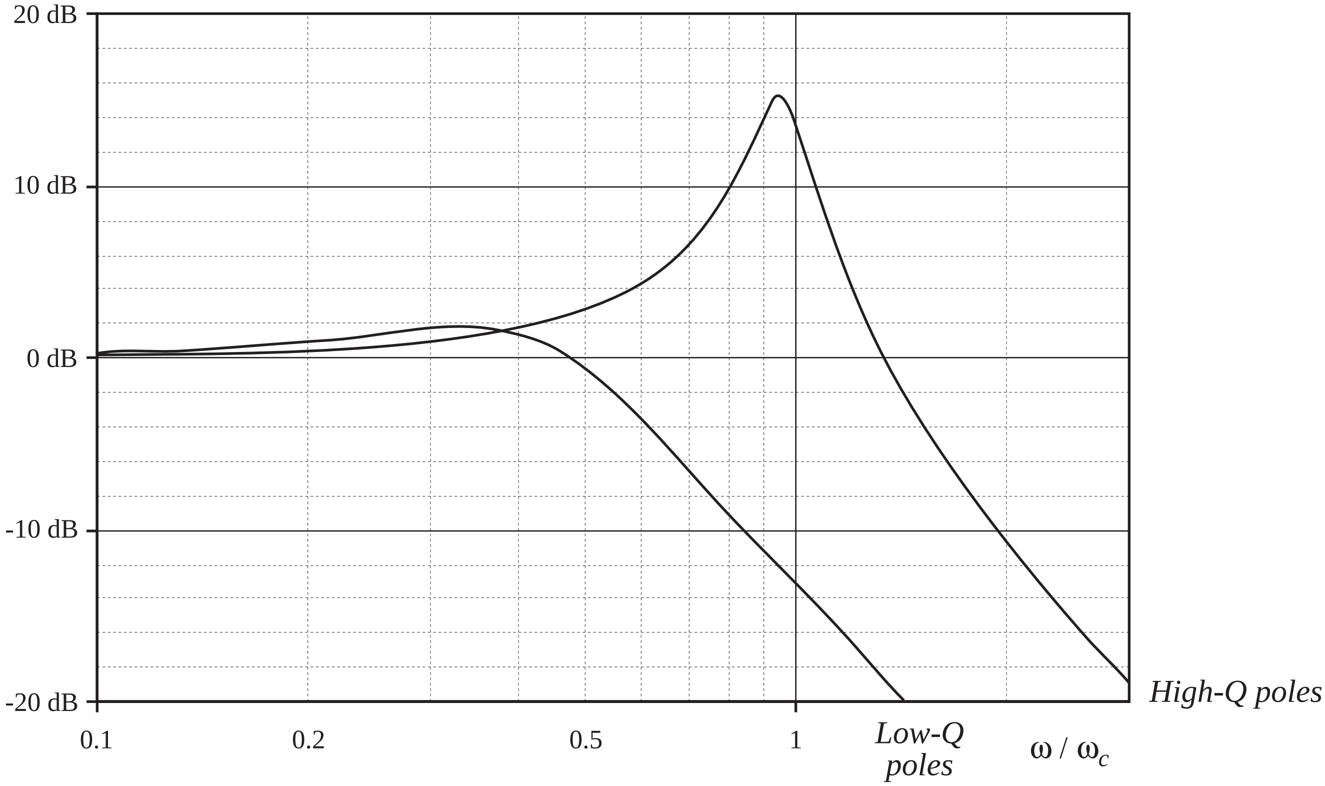
Faster roll-off than Butterworth, Bessel, or real poles

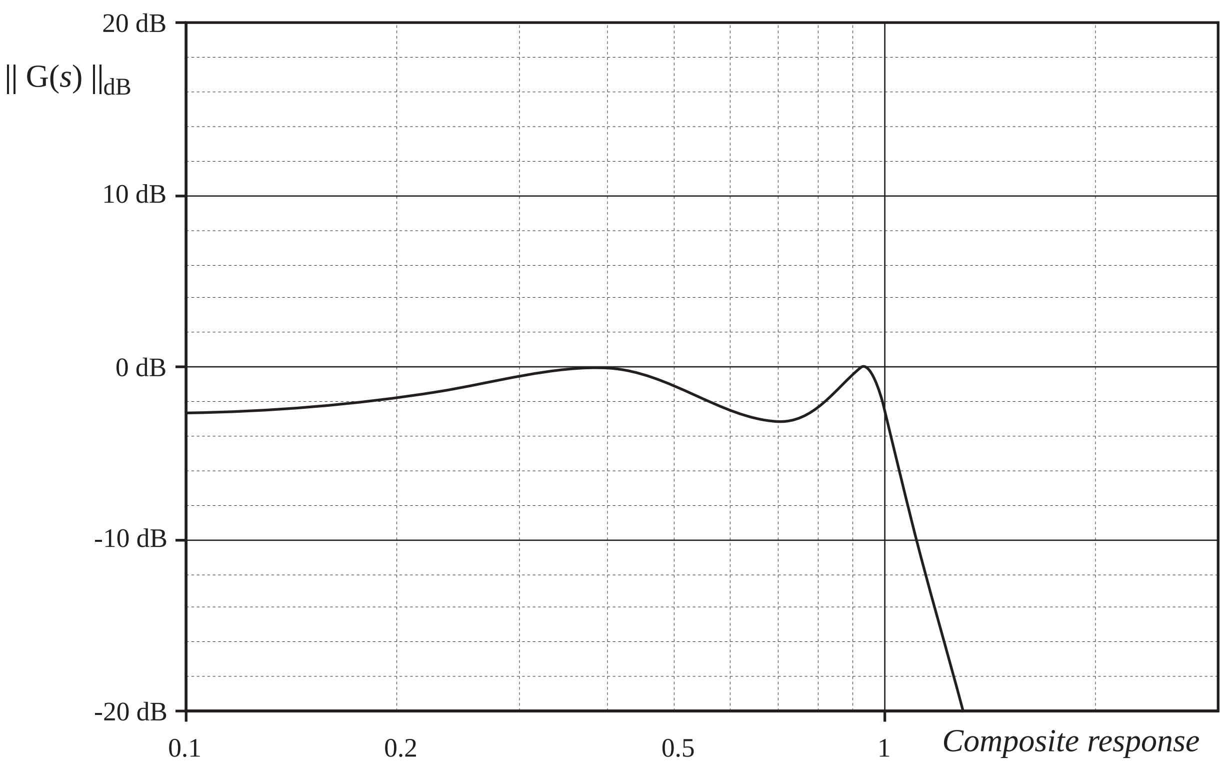
Step response exhibits overshoot and ringing



Fourth-Order Chebyschev Filter:
How the Complex-Conjugate Pole Pairs Combine

$$G(s) = \frac{1}{\sqrt{2}} \frac{1}{\underbrace{\left(1 + 0.1789\left(\frac{s}{0.9502\omega_c}\right) + \left(\frac{s}{0.9502\omega_c}\right)^2\right)}_{\substack{\textit{High-Q poles} \\ Q\,=\,5.60,\ \omega_0\,=\,0.9502\,\omega_c}}\underbrace{\left(1 + 0.9276\left(\frac{s}{0.4425\omega_c}\right) + \left(\frac{s}{0.4425\omega_c}\right)^2\right)}_{\substack{\textit{Low-Q poles} \\ Q\,=\,1.08,\ \omega_0\,=\,0.4425\,\omega_c}}}$$





Bessel filter

$$G(s) = \frac{K_0}{Be_n(\frac{s}{\omega_0})} \qquad Be_n(s) = Bessel\ polynomial\ (below)$$

K_0 is chosen such that the dc gain is 1.
ω_0 is chosen such that the desired cutoff frequency ω_c is obtained.
$Be_n(s)$ is a Bessel polynomial generated as follows:

$$Be_0(s) = 1$$

$$Be_1(s) = 1 + s$$

$$Be_n(s) = \left(2n - 1\right)Be_{n-1}(s) + s^2 Be_{n-2}(s)$$

n	***Be$_n$(s)***	***K$_0$***
1	$1 + s$	1
2	$3 + 3s + s^2$	3
3	$15 + 15s + 6s^2 + s^3$	15
4	$105 + 105s + 45s^2 + 10s^3 + s^4$	105

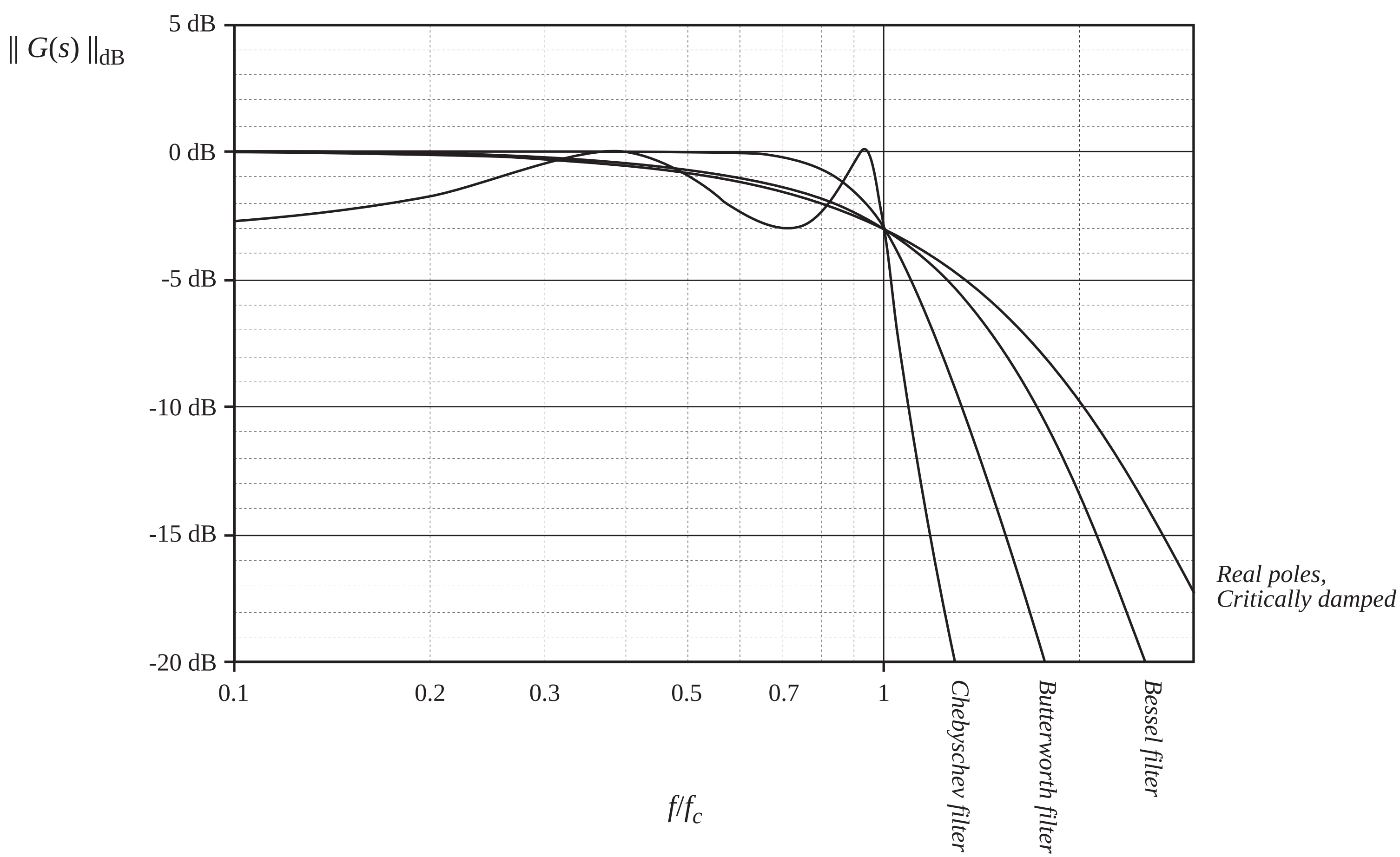
Properties of the Bessel filter

Good phase response: filter exhibits a nearly constant time delay to frequencies within the passband

Gradual roll-off, but still faster than real poles

Step response exhibits negligible overshoot and ringing

Comparison of Fourth-Order Filter Responses



A few additional points

To obtain a high-pass characteristic

Use inverted poles: replace (s / ω_c) with (ω_c /s).

To obtain a bandpass characteristic

Cascade low-pass and high-pass characteristics.

How to realize a circuit having a given filter characteristic:

Given a circuit, solve for its analytical transfer function.

Given the desired filter transfer function, find numerical values for the coefficients of the denominator polynomial.

Equate the coefficients of the denominator polynomial in the desired filter transfer function to the corresponding coefficients in the circuit analytical transfer function. Hence, select element values.

Several op-amp circuits that produce complex poles

see Thomas and Rosa, Section 14-3

Biquad circuit: *see* supplementary course notes on block diagrams

Bibliography

Thomas and Rosa, *The Analysis and Design of Linear Circuits*, fourth edition, Prentice Hall, 2004.

Kuo, *Network Analysis and Synthesis*, Wiley, 1966.

Weinberg, *Network Analysis and Synthesis*, McGraw-Hill, 1962.