Background for the Energy Cost Calculator

The energy cost calculator is a simple tool for estimating constant and current dollar level annual costs of energy. The following briefly reviews some of the essential concepts concerning the time value of money and the major components of an engineering economic analysis including fixed and variable costs and the effects of inflation or deflation. Further information on the subject can be found in any engineering economics text. The calculator employs what is known as a revenue requirements approach to determine the energy revenue ($/kWh) required to earn the desired rate of return. The notes that follow give some background to the equations used by the calculator and discuss an example for a generic biomass power plant. The values included here show as defaults when you link to the calculator from the main web page. You can replace the defaults with your own values. The calculator is a tool for making simple estimates only. Please send any comments or questions to bmjenkins@ucdavis.edu.

Cash flow equivalence:

The investment of money is intended to generate additional money ("return"). For example, money placed in a bank account ("loaned" to the bank) is expected to earn interest. If the interest earned is reinvested (for example, left in the bank account), then interest is expected to be earned on the interest already earned. This is known as compounding. Under compounding, cash flows of various types can be found to be equivalent. A future sum of money resulting from the investment of a present sum of money is to be greater than the present sum (exclusive of inflation) due to the effects of interest. A future sum, *F* ($) can be found from the present sum, *P* ($), known as the present worth or present value, given an interest rate, *i* (period^{-1}), and a number of compounding periods, *n*, as

$$F = P(1+i)^n \qquad [1]$$

The factor $(1+i)^n$ is called the compound amount factor, *F/P*.

The present worth of a future sum of money can be found from the inverse relationship as

$$P = F(1+i)^{-n} \qquad [2]$$

where $(1+i)^{-n}$ is called the present worth factor, *P/F*.

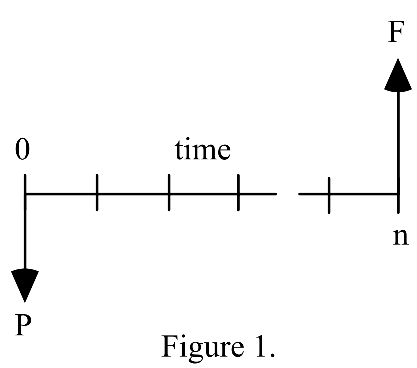

Figure 1.

A cash flow diagram of the above relationships appears as in Figure 1. By convention, an arrow pointing downwards represents an investment, an arrow pointing upwards represents a return. Note that for future amounts, an end of period convention is utilized, that is, formulas [1] and [2] apply to present worth at time "zero" with future amounts after the end of n periods.

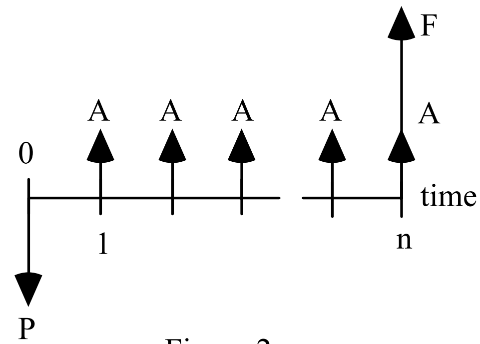

Figure 2.

Another equivalence can be found between a present worth or a future worth and a uniform series of cash flows, designated A (for annuity, although the periods can be other than years). The cash flow diagram appears as in Figure 2. This is the typical case of the monthly payment on a house or car loan (the figure shows the bank's perspective). The uniform series of cash flows begins at the end of period 1 in the following equations.

The equivalence between A and P is determined as

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \qquad [3]$$

where the term in brackets is known as the capital recovery factor, *A/P*, or sometimes, *CRF*.

For A and F, the equivalence is

$$A = F\left[\frac{i}{(1+i)^n - 1}\right] \qquad [4]$$

and the term in brackets is known as the sinking fund factor, *A/F*.

Note that $A/P = A/F + i$. Also note that if $n = \infty$, then $A/P = i$.

A number of other factors can be derived as well based on arithmetic and geometric gradients.

The interest rate can be expressed in one of several ways. The *nominal* interest rate, r, is the annual interest rate not including compounding, expressed as

$$r = i\,m \qquad [5]$$

where m is the number of compounding periods per year. The effective interest rate is the interest rate per year including compounding, expressed as

$$i_e = (1+i)^m - 1 \qquad [6]$$

where i is the interest rate per interest period $(= r/m.)$, as before. Under continuous compounding, $m = \infty$, and $i_e = e^r - 1$, where e is the base of the natural logarithms.

Depreciation:

Depreciation accounts for the loss in value of assets over time, due to wear-out, obsolescence, or economic management. Accounting for depreciation is necessary to ensure that replacement assets can be purchased at the end of the useful life of the existing asset. Depreciation is also an important tax deduction. A number of depreciation methods may be employed. For tax purposes the tax code normally specifies the method to be used. The three most common methods for calculating depreciation are straight line, sum of year's digits, and declining balance. The latter two methods are said to be "accelerated" relative to straight line. Accelerating the depreciation allowance is good for tax purposes if the business is profitable because it reduces the amount of tax early in the analysis and increases present worth.

<u>Straight line depreciation</u>

The depreciation allowance to be taken in each year by the straight line method can be found as

$$D_n = \frac{B - S}{l} \qquad [7]$$

where B = *Book value at the beginning of the year*
S = *Salvage value of asset*
l = *remaining useful life at the beginning of the year*

The straight line method in this form will depreciate the asset to its salvage value. The book value is the value of the asset, equal to the first cost less accumulated depreciation. Commonly, the value D_n is the same in each year and computed as $(P - S)/N$ where

P = first cost (or purchase cost) of the asset
N = total useful or economic life of the asset

Sum of the year's digits depreciation

The sum-of-the-year's digits depreciation allowance taken in each year is

$$D_n = \frac{l}{SOYD}(P - S) \qquad [8]$$

$$SOYD = sum\ of\ year's\ digits = 1 + 2 + 3 + ... + N = \frac{N}{2}(N+1)$$

Note that this method will also depreciate to the salvage value.

Declining balance depreciation

The declining balance depreciation allowance taken in each year is

$$D_n = \frac{\phi}{N} B \qquad [9]$$

where ϕ = *decline rate*. Typically, double (or 200%) declining balance is used, in which case, $\phi = 2$. The book value at the end of n years, (the beginning of $n+1$ years), is $P(1 - \phi/N)^n$, so that

$$D_n = \frac{\phi}{N} P\left(1 - \frac{\phi}{N}\right)^{n-1} \qquad [10]$$

Only by coincidence will declining balance depreciate to the salvage value. At some point, the depreciation allowance by declining balance may fall below that by straight line or SOYD based on the remaining book value and life. The depreciation method can be switched at that point to maintain the highest depreciation allowance in each year.

Tax depreciation procedures will normally specify the schedule of depreciation, including how to handle salvage, although they are usually based on one or more of the above methods (e.g., double declining balance switching to straight line, possibly with restrictions on the amount of depreciation allowed in the first year). The federal modified accelerated cost recovery system (MACRS) uses depreciation schedules as specified in the tax code.

Taxes:

Taxes are fees assessed on income, generally determined as

$$T = (t)(I_t) \qquad [11]$$

where T = taxes ($)
t = tax rate (--)
I_t = taxable income ($)

and taxable income is gross income adjusted by deductions,

$$I_t = I_g - E - D_t - D_i \qquad [12]$$

I_g = gross income ($)
E = expenses ($)
D_t = depreciation for tax purposes ($)
D_i = interest on debt ($)

For federal purposes, state tax is a deduction, but federal tax is not normally considered a deduction for state tax purposes. A combined tax rate, t, can be computed as

$$t = t_F\,(1 - t_S) + t_S \qquad [13]$$

t_F = federal tax rate (--)
t_S = state tax rate (--)

Escalation and Inflation:

Inflation results in an increase in the cost of a good or service over time, and is generally modeled in a manner similar to interest rate. Under inflation (or deflation), a so-called "real" interest rate can be determined as

$$i' = \frac{i-f}{1+f} = \frac{1+i}{1+f} - 1 \qquad [14]$$

where i' is the real interest rate (accounting for the effect of inflation), i is the apparent (or quoted) interest rate, as above, and f is the inflation rate (period^{-1}). Note that the apparent interest rate is

$$i = i' + f + i'f = (1+i')(1+f) - 1 \qquad [15]$$

Under inflation, a future amount of money, F, will not have the same purchasing power as if there had been no inflation. Expressed in the same value units as when the investment was made, that is, in year zero units, the year zero adjusted future value is

$$F_o = \frac{F}{(1+f)^n} \qquad [16]$$

where n is the number of periods into the future. The present worth of F_o is then

$$P = \frac{F_o}{(1+i')^n} = \frac{F}{(1+i')^n(1+f)^n} = \frac{F}{(1+i)^n} \qquad [17]$$

and P is related to F in the same manner as before (compare equation [2])

<u>Current and Constant Dollar Analysis, Level Annual Cost</u>

The consideration of inflation in an economic analysis leads to the concepts of current and constant dollar analysis. A current dollar analysis includes the effect of inflation, while a constant dollar analysis attempts to adjust for the effect of inflation so that economic values may be compared on an equivalent basis (e.g., comparisons of the cost of alternative fuel resources in the future to the present cost of existing resources, exclusive of general inflation in the economy). Consider, for example, that the cost of fuel, C, over time is expected to escalate at the apparent rate, e, as

$$C_n = C_o(1+e)^n \qquad [18]$$

where C_o is the present cost of fuel.

The present worth of C_n is

$$P_n = C_o \frac{(1+e)^n}{(1+i)^n} = C_o k^n \qquad [19]$$

in which $k = (1+e)/(1+i)$. The total present worth of the fuel cost over the entire analysis time is

$$P = C_o \sum_{n=1}^{N} k^n \qquad [20]$$

Factoring k in [20],

$$\frac{P}{kC_o} = 1 + \sum_{n=1}^{N-1} k^n \qquad [21]$$

which together with [20] yields

$$P = kC_o \frac{(1-k^N)}{(1-k)} \qquad [22]$$

The level annual cost, or *LAC*, is sometimes found to be a useful measure for comparing fuels and and other forms of energy. The current dollar *LAC* is found simply as

$$LAC = (P)(A/P) = kC_o \frac{(1-k^N)}{(1-k)} \frac{i(1+i)^N}{(1+i)^N - 1} \qquad [23]$$

The constant dollar *LAC* adjusts for inflation by using the "real" values of the escalation and interest rates in equation [23]. In the constant dollar case, k is computed as

$$k' = \frac{(1+e')}{(1+i')} \qquad [24]$$

where

$$e' = \frac{(1+e)}{(1+f)} - 1 \qquad [25]$$

$$i' = \frac{(1+i)}{(1+f)} - 1 \qquad [14]$$

The value of k' can be inserted in equation 22 in place of k. However, from equations [24], [25], and [14], the value of k' can be seen to be identical to k, as required by the invariance of the present worth (P) in equation 22. The essential difference between the current and constant dollar analyses is the use of the "real" interest and escalation rates in the constant dollar analysis. More complex economic analyses may be solved for current and constant dollar results in the same manner.

Evaluation of Alternatives:

Alternative energy technologies or strategies may be compared economically using any of a number of different rational methods, all of which are equivalent and yield the same decision if employed properly. Table 1 presents the major methods for evaluating alternatives.

Other techniques, such as simple payback analysis, are not generally performed in a rational manner and so do not yield proper economic decisions. Payback is useful when the rapid recovery of capital investment is required, however, it will not necessarily select the alternative that maximizes the amount of money made over the economic life (more precisely, that maximizes net present worth or NPW).

Net Present Worth, Level Cash Flow, and Benefit/Cost analysis assume an interest rate (or discount rate). Rate of return analysis computes the interest rate earned on investment (equal to that rate which makes NPW=0).

Table 1. Common rational economic accounting methods for comparing alternatives.

Method	Objective
Present Worth	Compute net present worth (present worth of benefits less present worth of costs) of alternatives. Maximize Net Present Worth (NPW)
Uniform (Level) Cash Flow	Compute net level (typically annual) cash flow (level benefits less level costs) of alternatives. Maximize Net Level Benefits (Minimize Net Level Costs)
Rate of Return (ROR)	Find $i = ROR$ such that NPW=0 For feasible alternatives with ROR ≥ Minimum attractive rate of return (MARR), compute incremental ROR (ΔROR) between alternatives: If ΔROR≥MARR, choose higher cost alternative. If ΔROR<MARR, choose lower cost alternative.
Benefit-Cost Ratio (B/C)	Find feasible alternatives with ratio of present worth of benefits (PWB) to present worth of costs (PWC) greater than unity, i.e., PWB/PWC ≥ 1. Compute incremental B/C (ΔB/C) between feasible alternatives. If ΔB/C≥1, choose higher cost alternative. If ΔB/C<1, choose lower cost alternative.

Revenue Requirements Method

An approach similar to the Level Cash Flow method is the Revenue Requirements method. A revenue requirements analysis attempts to determine the necessary energy price to yield the desired rate of return. The revenue requirements fall generally into four categories:

Capital repayment
Return on investment
Expenses
Taxes

Capital repayment recovers the capital cost of the project over the economic life. The return represents the interest earned on the investment over the life.

Because the revenue requirements method specifies the rate of return to be earned, taxes, which are part of the revenue requirement, are computed in a special way, rather than directly as if the revenues were known already.

By equation [11], the tax payment in each year of the analysis is found by:

$$T = (t)(I_t) \qquad [11]$$

and the taxable income, I_t, is

$$I_t = I_g - E - D_t - D_i \qquad [12]$$

The revenue requirements, I_g, are the sum of the four items above,

$$I_g = C_r + I_r + T + E \qquad [26]$$

where C_r = capital (principal) repayment ($)
I_r = return on investment ($)

The taxes are

$$\begin{aligned} T &= t(C_r + I_r + T + E - D_t - E - D_i) \qquad [27] \\ &= t(C_r + I_r + T - D_t - D_i) \end{aligned}$$

which is solved for T as

$$T = \frac{t}{1-t}(C_r - D_t + I_r - D_i) \qquad [28]$$

The term $I_r - D_i$, the difference between the total return on the unrecovered investment and the interest on debt, is simply the return on equity. Defining the debt ratio, r_D (--), as the fraction of the cost of the project covered by debt (as opposed to equity), and C as the unrecovered investment to date, the total return at rate of return, i, is

$$I_r = iC \tag{29}$$

while the interest or return on debt is

$$D_i = i_D r_D C \tag{30}$$

where i_D is the interest rate on debt. Then

$$\frac{D_i}{I_r} = \frac{i_D r_D}{i} \tag{31}$$

and equation [28] becomes

$$T = \frac{t}{1-t}\left[C_r - D_t + I_r\left(1 - \frac{i_D r_D}{i}\right)\right] \tag{32}$$

Example Cost Calculation:

An example of the revenue requirements methodology is given below. Table 2 includes definitions of the primary terms used in the analysis. The example analyzes the current and constant dollar revenue requirements for electricity generated by a biomass fueled power plant. The basic assumptions of the analysis and some intermediate calculations are listed in Table 3.

The power plant of the example is assumed to convert biomass with a net electrical capacity of 25 MW_e and a capacity factor of 88%. The cost of the plant is covered by both debt and equity funds, with 75% as debt (75% borrowed from the bank, 25% equity supplied from the owner). The rate of return on equity is assumed to be 15%. The debt interest rate is 12%. As part of the financing plan, the owner is required to place an amount equal to one year of debt repayment in a savings account in case technical problems cause a suspension of plant operations and revenue is not generated. This "debt reserve" account earns simple interest, which is available to the plant as revenue. The plant also receives a payment for "capacity," that is, a payment made to the facility for guaranteeing to supply power, not just energy. This payment and the interest on the debt reserve reduce the amount of revenue required to cover the cost of generating the electrical energy. A MACRS depreciation schedule based on double declining balance switching to straight line is used for tax purposes (a 5 year method that extends over 6 years due to a half-year convention in the first year). Negative taxes are permitted under the assumption that the owner company is large, and negative taxes merely reduce the tax burden of the company as a whole. If this is not the case, then negative taxes would not be allowed. Fuel costs and expenses are assumed to escalate at 5% annually, with inflation also running at 5%. Taxes can only be paid in current dollars. Sample calculations are listed in Table 4 with results shown for the first year. Current and constant dollar level revenue requirements are given in Table 5. Note that the current dollar level cost of power is \$0.0924 kWh^{-1} over the 20 year period. With the effect of general inflation removed, the cost is \$0.0641 kWh^{-1}, which may be compared with existing costs of alternative sources in the same base year.

<u>Sensitivity Analysis</u>

Sensitivity analysis is commonly employed to understand how the economic feasibility varies as changes are made to the input assumptions. Figure 3 displays the sensitivity of the energy revenue requirements for the example as fuel cost, capital cost, availability, MARR, and thermal efficiency are varied independently, all other parameters constant. In this way, an assessment can be made as to how sensitive the results are to any parameter of the analysis, and where attention should be focused to improve the economic performance. For example, Figure 3 shows that the solution is particularly sensitive to reductions in capacity factor (availability) and efficiency, and increases in capital costs.

Table 2. Nomenclature for revenue requirements example.

		units
M_p	plant capacity or size	variable, e.g., kW
n	life of plant (or duration of analysis)	y
h	annual operating hours	h y^{-1}
η	thermal efficiency	--
Q_f	fuel heating value	kJ kg^{-1}
Cost:		
C_p	capital cost of plant (total)	$
K_p	capital cost per unit energy output	e.g., $ kW^{-1}
C_f	fuel cost	$/physical unit, e.g., $ t^{-1}
C_r	non-fuel operating costs	$/unit output, e.g., $ kWh^{-1}
R_d	debt reserve in account	$
Tax:		
t_F	federal tax rate	--
t_S	state tax rate	--
t	combined tax rate	--
*Inco*me:		
I_c	non-energy income	variable
i_{ra}	interest rate earned on reserve accounts	--
*Esca*lation/Inflation:		
f	general economic inflation rate	--
e_f	escalation rate on fuel cost	--
e_c	escalation rate on other expenses	--
*Finan*cial:		
i_e	rate of return on equity capital	--
i_D	interest rate on debt	--
r_D	debt ratio	--
D_n	depreciation allowance in year n	--
P	present value of money	$
A/P	capital recovery factor	--
LAC	level annual cost	$

Table 3. Assumptions and computed results for the revenue requirements example.

Capital Cost ($/kW)	K_p	2,800
Net Plant Capacity (kW)	M_p	25,000
Capacity Factor (%)	$h/8760$	88
Thermal Efficiency (%)	100η	20
Fuel Heating Value (kJ/kg) or	Q_f	18,608
Fuel Heating Value (Btu/lb)		8,000
Fuel Consumption Rate (metric tons/h)--*computed*	$m_f = 3.6M_p/(\eta Q_f)$	*24*
Fuel Consumption Rate (short tons/h)--*computed*		*26.5*
Fuel Cost ($/short ton)	C_f	20
EXPENSES		
Labor Cost ($/kWh)		0.01038
Maintenance Cost ($/kWh)		0.00778
Insurance/Property Tax ($/kWh)		0.00726
Utilities ($/kWh)		0.00104
Ash Disposal ($/kWh)		0.00052
Management/Administration ($/kWh)		0.00104
Other Expenses ($/kWh)		0.00208
Total Expenses ($/kWh)--*computed*	C_r	*0.03010*
TAXES		
Federal Tax Rate (%)	$100t_F$	34
State Tax Rate (%)	$100t_S$	9.6
Combined Tax Rate (%)--*computed*	$100t$	*40.34*
INCOME other than energy		
Capacity Payment ($/kW-y)	I'_c	166
Interest Rate on Debt Reserve (%)	$100i_{ra}$	7
ESCALATION/INFLATION		
General Inflation (%)	$100f$	5
Escalation--Fuel (%)	$100e_f$	5
Escalation--Other (%)	$100e_c$	5

FINANCE		
Debt ratio (%)	$100r_D$	75
Equity ratio (%)	$100(1-r_D)$	25
Interest Rate on Debt (%)	$100i_D$	12.00
Economic life (y)	N	20
Cost of equity (ROR, %)	$100i_e$	15.00
Cost of Money (%)--*computed*	$100i = 100(r_Di_D + (1-r_D)i_e)$	*12.75*
Total Cost of Plant ($)--*computed*	$C_p = K_pM_p$	70,000,000
Total Equity Cost ($)--*computed*	$(1-r_D)C_p$	17,500,000
Total Debt Cost ($)--*computed*	r_DC_p	52,500,000
Capital Recovery Factor (Equity) --*computed*	$(A/P)_e$	0.1598
Capital Recovery Factor (Debt) --*computed*	$(A/P)_D$	0.1339
Capital Recovery Factor (Total) --*computed*	(A/P)	0.1402
Annual Equity Recovery ($/y) --*computed*	$(1-r_D)C_p(A/P)_e$	2,795,826
Annual Debt Payment ($/y) --*computed*	$r_DC_p(A/P)_D$	7,028,636
Debt Reserve ($)	R_d	7,028,636
Annual Debt Reserve Interest ($/y) --*computed*	R_di_{ra}	492,005
Annual Capacity Payment ($/y) --*computed*	I'_cM_p	4,150,000
Tax depreciation schedule (ACRS):		
Year 1	D'_1	0.2000
Year 2	D'_2	0.3200
Year 3	D'_3	0.1920
Year 4	D'_4	0.1152
Year 5	D'_5	0.1152
Year 6	D'_6	0.0576
Total tax depreciation--*computed*	$\Sigma D'_n$	1.0000
Annual Production (kWh) --*computed*	hM_p	192,720,000
Annual Hours--*computed*	h	7,709

Table 4. Sample calculations for year 1.

Year	n	1
Equity Recovery	$ER_n = (1-r_D)C_p(A/P)_e$	2,795,826
Equity Interest	$EI_n = i_e C_{e,n-1}$	2,625,000
Equity Principal Paid	$EP_n = (1-r_D)C_p(A/P)_e - i_e C_{e,n-1}$	170,826
Unrecovered equity[1]	$C_{e,n} = C_{e,n-1} - EP_n$	17,329,174
Debt Recovery	$DR_n = r_D C_p(A/P)_D$	7,028,636
Debt Interest[2]	$DI_n = i_D C_{D,n-1}$	6,300,000
Debt Principal Paid	$DP_n = r_D C_p(A/P)_D - i_D C_{D,n-1}$	728,636
Unrecovered Debt	$C_{D,n} = C_{D,n-1} - DP_n$	51,771,364
Fuel Cost	$F_n = hm_f C_f(1+e_f)^{n-1}$	4,110,628
Non-fuel Expenses	$NF_n = M_p h C_r(1+e_c)^{n-1}$	5,800,872
Debt Reserve	$R_d = DR_1$ *in first year only*	7,028,636
Depreciation (tax)	$D_n = C_p D'_n$	14,000,000
Capacity Income	$I_c = I'_c M_p$	4,150,000
Interest on Debt Reserve	$I_{ra} = R_d i_{ra}$	492,005
Taxes[3]	$T_n\ (t/(1-t))(ER_n + DP_n - D_n)$	-7,082,015
Energy Revenue Required[4]	$RR_n = (ER+DR+F+NF+T+R_d-I_c-I_{ra})_n$	15,040,579
Present Worth (time 0)	$RR_n(1+i)^{-n}$	13,339,759

[1] $C_{e,0} = (1-r_D)C_p$. [2] $C_{D,0} = r_D C_p$ [3] Capital repayment = $EP_n + DP_n$, return on investment = $EI_n + DI_n$ [4] revenue required to cover the cost of energy is reduced by revenue received as capacity payments and interest on the debt reserve.

Table 5. Current and constant dollar level annual costs.

Total Present Worth ($)	$P = \Sigma P_n$	127,013,294
Capital Recovery Factor (current)	$(A/P = (i(1+i)^n)/((1+i)^n-1)$	0.1402
Level Energy Revenue Requirements ($)	$P(A/P)$	17,809,771
LAC per unit production ($ kWh^{-1})	$P(A/P)/(M_p h)$	0.0924
Real Cost of Money	$i' = (1+i)/(1+f)-1$	0.0738
Capital Recovery Factor (constant)	$(A/P)' = (i'(1+i')^n)/((1+i')^n-1)$	0.0972
Constant Level Revenue Requirements ($)	$P(A/P)'$	12,346,433
LAC in constant dollars ($ kWh^{-1})	$P(A/P)'/(M_p h)$	0.0641

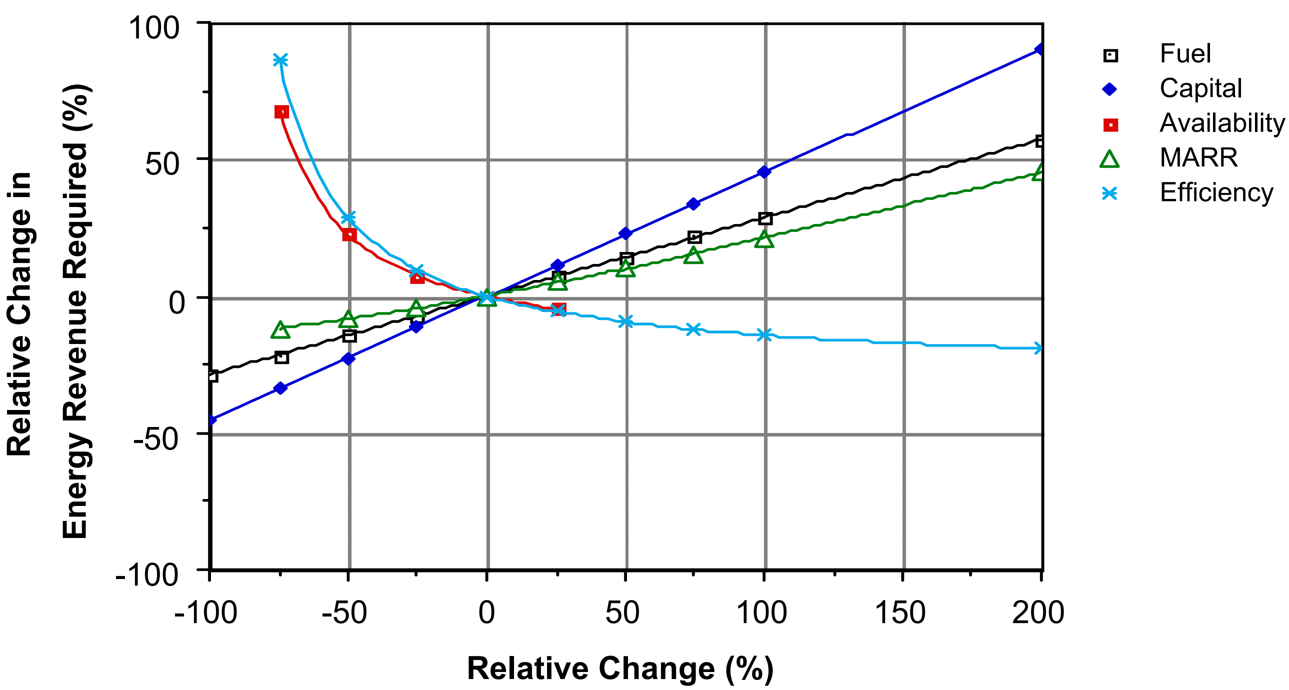

Figure 3. Sensitivity analysis.