TDFD-based Measurement of Analog-to-Digital Converter Nonlinearity

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Analog-to-Digital Converters (ADCs) are central to modern audio, video, and RF systems, with some modern converters providing 23-plus "significant" bits. With such systems, performance is usually limited by audible distortion. Consequently linearity tests, analogous to distortion measurements, are adapted to test performance. The Total Difference-Frequency Distortion (TDFD) test is a modern standard for distortion measurement in the audio field, and its uniquely-powerful advantages may be transferred to the realm of ADC testing. Building on existing work, we describe and demonstrate a straightforward TDFD test for ADC systems. Measurements of example ADC systems are presented to demonstrate the method. A simple relationship between distortion and effective bits is derived, as a measure of performance, and testing guidelines are given.

0 Introduction

Simple engineering specifications for a typical ADC include its resolution, which usually means the width of the digital word provided by the electronics. However, there is no guarantee that this figure is an honest representation of the converter's performance [1]: at least, a figure of linearity is also required. A noise figure may also be relevant, although many modern applications involve oversampling, averaging, Fourier post-processing, or some other mechanism effective against random noise.

A full specification of ADC performance involves such measures as differential non-linearity, missing codes, integral linearity, aperture jitter, noise, and bandwidth, gain and offset errors [2]. For modern, high-resolution ADCs, these specifications typically reduce to a hard noise floor of -110dB and distortion of the order of 80–90dB. Fielder has presented an objective technique suitable for evaluating the audibility of nonlinearity produced by digital converters. In [1] it is shown that noise and adjacent signal masking effects can often ameliorate nonlinearities, but that "high-level tests show that seemingly small discontinuities can be quite audible under the right circumstances". In the absence of an exact requirement for inaudibility of distortion, a tight overall specification must be sought.

The measurement of this distortion is a problem for two reasons. Firstly, the distortion introduced by the ADC is typically much less than the harmonic distortion present in the output of a good signal generator used as a test source. Secondly, the thermal noise present in all such systems may be far in excess of the distortion, necessitating extensive signal averaging.

In this paper we investigate the use of intermodulation distortion as a measure of ADC linearity, using the standard audio Total Difference-Frequency Distortion (TDFD) test. In this, two input test tones are applied and the resulting intermodulation products identified. This has the advantage of being relatively immune to the harmonic content of the input test signals, it being easier to produce a test signal of low intermodulation distortion content than it is to produce one of low harmonic distortion content.

We then perform a discrete Fourier transform (DFT) on the data sampled by the ADC to identify the intermodulation terms. From this we can gauge the ADC's linearity, expressed as "effective bits".

In Section 1 we review existing work on the measurement of the linearity of ADCs, while Section 2 is a discussion of the TDFD test as a measure of system linearity. Section 3 develops the theory of expressing linearity in terms of effective bits and of the identification of distortion terms using the DFT. In Section 4 we discuss the design of such tests while in Section 5 we present a simple but effective method for the verification of such TDFD tests. Finally, in Section 6 we present some example measurements of high-resolution ADCs and draw conclusions in Section 7.

1 ADC Linearity Tests

The favoured method for expeditious ADC testing emerged in the early 1980s. It consisted of sampling a sinewave, and examining the converted digital output. Integral nonlinearity is assessed by looking for harmonics by means of discrete Fourier analysis—effectively performing a THD test, albeit with the receiver half of the test set implemented in software [2, 3, 4]. Additionally, improved sinewave fitting algorithms can be developed for computational efficiency and immunity from noise and sampling coherence [5].

An alternative to harmonic analysis is that of histogram or code-density techniques [6, 7]. In this, a statistically-significant number of samples of a known input signal are taken and the frequency of code occurrence plotted as a function of code. The shape of this histogram plot then represents the probability density function of the input waveform. Linearity can be assessed by deviation of this from that expected for the known input waveform.

The limitation of the above techniques is that harmonics in the input signal will be measured as arising from the ADC itself. They are thus limited to the testing of low-resolution ADCs, having distortion much greater than the harmonic distortion (THD) of the test signal source.

More recently, intermodulation tests, modelled on the usual two-tone test for radio receivers, have been used [8]. Intermodulation (IMD) testing confers one main advantage over harmonic testing, namely immunity from confusion resulting from the presence of any harmonic components on the test tones. This is particularly important for sources at higher frequencies. This advantage transfers identically to the case of ADC testing. As will be discussed in later Sections, it is easier to produce a multi-tone test signal of low IMD than it is to produce a single-tone test signal of low THD.

A further advantage arises in the case of sophisticated ADC subsystems employing DSP calibration schemes. Such schemes are generally necessary in highly-linear ADCs offering 20 or more effective bits. An improvement in linearity-correction of 10dB can be achieved with a multi-tone signal over linearity-correction with a single input tone [9].

An example of an ADC employing DSP calibration is given in [10]. This employs a 2-pass architecture to achieve a hard (high order) distortion limit of -110dB relative to full peak output, or -110dBr. Statistical correlation of injected digital noise is utilized to derive correction factors for both the internal DAC and residue gain amplifier. Fig.(5) shows the effectiveness of this calibration. Although the calibration algorithm can theoretically eliminate hard distortion, unavoidable coupling of the digital output to the input as spurious noise will degrade performance. The addition of a second test tone, more representative of a real-world application, provides sufficient statistical dither to smooth this coupled noise.

The original two-tone IMD test, used as a basis of the tests in [8], was developed for testing narrow-band RF systems. It calls for two test tones at

$$\begin{aligned} \omega_1 &= \omega_0 + \delta \text{ and} \\ \omega_2 &= \omega_0 - \delta \ . \end{aligned} \tag{1}$$

From this one obtains information about odd-order IMD components at tones of

$$\begin{aligned} 2\omega_1 - \omega_2 &= \omega_0 + 3\delta \text{ and} \\ 2\omega_2 - \omega_1 &= \omega_0 - 3\delta \ . \end{aligned} \tag{2}$$

Typical test tones might be 100.005MHz and 99.995MHz, leading to IMD components at 100.015MHz and 99.985MHz when testing a receiver tuned to a nominal centre frequency of 100.00MHz.

The authors of [8] also search for even-order tones, since the ADC bandwidth is relatively wide. They limit their mathematics to 3^{rd} order for simplicity, and derive an estimation of ADC linearity, as effective bits, in terms of measured IMD and THD. The inclusion of IMD reduces the influence of source tone THD on the measure of total distortion, but still limits testing to ADCs having IMD much greater than the THD of the test signals.

It should be further noted that the test signals used in the numerical example in [8] were obtained using sources with particularly low susceptibility to intermodulation, while the converter performance was of the order of 10 effective bits, so the test system easily exceeded the performance of the Device Under Test (DUT). This situation is unrealistic, particularly in the case of modern audio and video converters.

2 The TDFD Linearity Test

The TDFD standard for intermodulation distortion measurement stipulates different conditions from those of the two-tone test, and is explicit about components contributing to the nonlinearity measure [11]. This standard can also be transferred to the testing of ADC converters.

TDFD may be viewed as employing the wider channel bandwidth of an audio system (exceeding 1, and typically taken as 3 decades) by using widely-spaced stimulus tones. Signals of

$$\begin{aligned} \omega_1 &= 2\omega_0 \text{ and} \\ \omega_2 &= 3\omega_0 - \delta \end{aligned} \qquad (3)$$

are added and passed through the device under test, where the magnitudes of the stimulus signals and the components at

$$\begin{aligned} \omega_2 - \omega_1 &= \omega_0 - \delta \text{ and} \\ 2\omega_1 - \omega_2 &= \omega_0 + \delta \end{aligned} \qquad (4)$$

are examined to yield the TDFD figure. Typically, $\omega_0 = 4$kHz and $\delta \approx 50$Hz, giving signals of interest at 8kHz, 11.95kHz, 3.95kHz and 4.05kHz. The definition of TDFD does not need the separate magnitudes of the two tones near $\omega_0 = 4$kHz, which may be measured together. It should also be noted that the TDFD test takes into account only the two simplest IMD components, although any higher-order ones will appear adjacent, as will be seen.

The ADC transfer function can be represented as a polynomial in the input voltage, to be followed by ideal quantisation. For a 3^{rd} order system;

$$v_{out} = a_0 + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 \ . \qquad (5)$$

For two sinusoidal inputs of

$$\begin{aligned} v_1 &= A_1 cos\omega_1 t \text{ and} \\ v_2 &= A_2 cos\omega_2 t \ , \end{aligned} \qquad (6)$$

this yields an output voltage to be sampled of

$$\begin{aligned} v_{out} = a_0 &+ \frac{a_2 A_1^2}{2} + \frac{a_2 A_2^2}{2} \\ &+ (a_1 A_1 + \frac{3}{4} a_3 A_1^3 + \frac{3}{2} a_3 A_1 A_2^2) cos\omega_1 t \\ &+ (a_1 A_2 + \frac{3}{4} a_3 A_2^3 + \frac{3}{2} a_3 A_1^2 A_2) cos\omega_2 t \\ &+ \frac{a_2 A_1^2}{2} cos2\omega_1 t + \frac{a_2 A_2^2}{2} cos2\omega_2 t \\ &+ \frac{a_3 A_1^3}{4} cos3\omega_1 t + \frac{a_3 A_2^3}{4} cos3\omega_2 t \\ &+ a_2 A_1 A_2 cos(\omega_2 \pm \omega_1) t \\ &+ \frac{3}{4} a_3 A_1^2 A_2 cos(2\omega_1 \pm \omega_2) t \\ &+ \frac{3}{4} a_3 A_1 A_2^2 cos(2\omega_2 \pm \omega_1) t \ . \end{aligned} \qquad (7)$$

Note that this contains a DC component, linear components in ω_1 & ω_2, second and third harmonics, 2^{nd} order IMD terms of $\omega_2 \pm \omega_1$ and 3^{rd} order IMD terms of $2\omega_1 \pm \omega_2$ and $2\omega_2 \pm \omega_1$. The DC term is hereafter ignored, but can readily be obtained as the average of all samples.

The IMD terms of interest in the TDFD test are either side of ω_0. These are at

$$\begin{aligned} \omega_{IM2} &= \omega_2 - \omega_1 = \omega_0 - \delta \text{ and} \\ \omega_{IM3} &= 2\omega_1 - \omega_2 = \omega_0 + \delta \ , \end{aligned} \qquad (8)$$

i.e.,

$$\begin{aligned} IM2 &= a_2 A_1 A_2 cos(\omega_2 - \omega_1) t \text{ and} \\ IM3 &= \frac{3}{4} a_3 A_1^2 A_2 cos(2\omega_1 - \omega_2) t \ . \end{aligned} \qquad (9)$$

For a 3^{rd} order system, measurement of these two components then yields a measure of the distortion of the system. Additionally, the amplitudes of the input test tones are normally set to be equal, *i.e.*, $A_1 = A_2$, and at half the full-scale range of the ADC.

It is straightforward if tedious to show that, if the system is represented by a polynomial with non-negligible coefficients associated with the terms of order greater than three, then those higher terms will contribute to the terms at ω_{IM2} and ω_{IM3}. This guarantees that the final TDFD figure picks up all orders of nonlinearity. Further, the higher-order terms will generate signals that cluster about ω_0 but which are spaced away from ω_0 by various multiples of δ. As seen in the example of Fig.(5) below, a look at the spectrum in the vicinity of ω_0 gives an immediate impression of the order of the nonlinearity present. This may be useful if some idea of the order of the nonlinearity is desired. For example, in [1], it is noted that higher order contributions can be of greater concern in the case of audio signals.

The spacing of stimulus and response tones specified for TDFD allows the use of simple filters in both the signal generator ("transmitter") part, and the signal analyser ("receiver") part, of a distortion test system. The engineering advantages of the spacing, together with the well-defined nature of the TDFD method, make it very appealing as a method for ADC converter testing.

3 Effective Bits

In this Section we follow the development of the expression of ADC linearity in terms of effective bits.

The error waveform arising from the quantisation of an ADC can be thought of as a real signal added to the input of an infinite-precision quantiser along with the signal. For high-level signals, sample-to-sample error will be statistically independent and uniformly distributed over a range of the quantising interval Δ [12]. The rms quantisation noise can then be approximated by integrating the probability density of the error signal;

$$v_n = \left[\frac{\Delta^2}{12}\right]^{1/2} \quad . \tag{10}$$

For a 2's complement ADC of B bits, the largest signal that can be represented is given by

$$v_{rms,max} = \frac{2^{B-1}\Delta}{k\sqrt{2}} = \frac{2^B}{k2\sqrt{2}}\Delta \ , \tag{11}$$

where k is the number of equal-amplitude tones in the input.

The signal-to-noise power ratio is then given by

$$\begin{aligned} SNR_k &= 10\log_{10}\left[\frac{k \times v_{rms,max}^2}{v_n^2}\right] \\ &= 10\log_{10}\left[2^{2B} \times \frac{3}{2k}\right] \\ &= 6.02B + 10\log_{10}\left[\frac{3}{2k}\right] \ . \end{aligned} \tag{12}$$

Note that the term $3/2k$ involves both the quantisation noise and a loss of input signal power with multiple input tones. For a single input tone, $k = 1$, and

$$SNR_1 = 6.02B + 1.76 \ . \tag{13}$$

For input amplitudes of less than the maximum, k represents the ratio of the single-tone amplitude to that of the actual test tones.

The conventional notion of "effective bits" is to model the actual measured ADC SNR as arising simply from quantisation noise. Thus, for an ADC of measured signal-to-noise power ratio of SNR_k, effective bits is given as

$$B_{\text{eff}} = \frac{SNR_k - 10\log_{10}(3/2k)}{6.02} \ . \tag{14}$$

High-resolution, wide-bandwidth ADCs have random analog noise added to the input signal prior to quantisation that is very much greater than the quantisation interval Δ. This noise ensures the validity of the quantising noise model [10]. The expected output error value can be solved by integrating the product of the quantising error and the noise distribution. Such integration of Gaussian distributions is normally avoided, but can be computed numerically to give a peak expected output error as

$$E_p = \frac{e^{-2(\pi\sigma)^2}}{\pi} \ , \tag{15}$$

where σ, the standard deviation of the Gaussian noise, and E_p are both relative to Δ.

For a typical high-resolution ADC having noise of -70dBr and $>$ 16 bits resolution, the exponential form of this error means that quantisation noise is not significant.

The error in the instantaneous sample of a signal will be dominated by the added thermal noise. To identify small distortion components in a discrete Fourier transform (DFT) of the sampled waveform, a large number of samples are required. For N samples, noise of σ and a noise-equivalent bandwidth E_B of the DFT window function, the signal-to-noise of the DFT records is given by [13]

$$SNR \approx 10\log_{10}\left[\frac{N}{\sigma^2 E_B}\right] \ . \qquad (16)$$

For a noise figure of -70dBr and a Blackman-harris window, for which $E_B = 2$, this gives

$$SNR \approx 10\log_{10} N + 67.0 \ . \qquad (17)$$

If it is required to see down to -100dBr, then we require $N = 2000$ samples. Successive doublings of the number of samples give improvements of 3dB. With sufficient samples, distortion components can be indentified above thermal noise, assuming they are above the hard noise floor arising from other, correlated, noise sources. By measuring the intermodulation distortion components we approximate the signal-to-noise ratio and then, from Eq.(14), calculate effective (linear) bits.

Assuming a 3^{rd} order system and two equal-amplitude input signals, from Eq.(7) we note two 2^{nd} order harmonic terms, two 3^{rd} order harmonic terms, two 2^{nd} order intermodulation terms and four 3^{rd} order intermodulation terms. Distortion power is then given by the rms sum of these;

$$\begin{aligned} D^2 &= \sum IMD^2 + \sum HD^2 \\ &= \frac{5}{4}a_2^2 A^4 + \frac{19}{16}a_3^2 A^6 \\ &= \frac{5}{2} \times IM2_{rms}^2 + \frac{38}{9} \times IM3_{rms}^2 \ . \qquad (18) \end{aligned}$$

Here we have expressed the distortion power in terms of the intermodulation components, using Eq.(7) and (9), as these are larger than the harmonic components and less contaminated by input signal components. Then, by measuring the intermodulation distortion components, the resulting signal-to-noise ratio can be related to effective bits by applying it to the quantisation noise model of Eq.(12) and (14);

$$B_{\text{eff}} = \frac{SNR_2 + 1.25}{6.02} \ . \qquad (19)$$

Note that Eq.(18) does not represent the total distortion power observable in the sampled waveform. This waveform will also contain harmonics present in the input test tones and distortion terms arising from these. Eq.(18) does, however, represent the distortion power that would be present if the input consisted of two pure tones. This, of course, assumes a 3^{rd} order system modelled by Eq.(5), with other noise very much lower than the intermodulation distortion.

If the ADC requires a higher-order model to accurately represent its non-linearity, Eq.(5), (7) and (18) can be expanded to include these higher-order distortion components. Here we have restricted ourselves to 3^{rd} order, for simplicity. Although with successive-approximation converters this is not a good assumption, the contribution of higher-order terms to the measured 2^{nd} and 3^{rd} order intermodulation products ensures there inclusion.

Distortion components are extracted from the sampled waveform using the DFT, using the methods outlined in [14]. If one is confident of the applicability of the model, the absence of significant spurious components and the level of the input test tones are known, then only two spectral lines need be computed; $IM2@\omega = \omega_0 - \delta$ and $IM3@\omega = \omega_0 + \delta$. These are calculated as;

$$\begin{aligned} R_i &= \frac{1}{N}\sum_{n=1}^{N} W_n D_n cos\left[\frac{2\pi k(n-1)}{N}\right] \ , \\ I_i &= \frac{1}{N}\sum_{n=1}^{N} W_n D_n sin\left[\frac{2\pi k(n-1)}{N}\right] \ , \text{ and} \\ mag_i &= \sqrt{R_i^2 + I_i^2} \ . \qquad (20) \end{aligned}$$

Here, R_i & I_i are the real and imaginary components of the i^{th} spectral line, N is the number of data samples, W_n is the window function and D_n the time-function data point. Sample size N is chosen according to Eq.(17). This will normally be quite large. The approach taken will be to compute the contribution of each data point to all required spectral lines, rather than the contribution of all data points to each required spectral line. The difference is how many passes through the data set are required. Thus only one run through the large data set is required for the small number of spectral lines and large memory arrays are not required.

An alternative to the DFT in estimating distortion component signal power is digital filtering [15].

Throughout we have ignored the internal architecture of the ADC and the sampling frequency f_s. Depending on f_s, several of the higher IMD and HD terms will appear at aliased frequencies, including the input test tone harmonics. This need not be a problem, provided they do not land on the $IM2$ and $IM3$ components of interest. If, however, the ADC employs over-sampling and post-filtering then some of the higher-order distortion terms might be removed. For example, for an audio-frequency ADC, using the test tones of Section 2, two $IM3$ and three HD terms are lost. Eq.(18) then needs to be modified accordingly.

With these assumptions, Eq.(19) represents a measure of the linearity of the ADC. This measure is insensitive to the harmonic content of the input test tones and is obtained by simple measurement of the 2^{nd} and 3^{rd} order intermodulation products. It requires only that the IMD contained in the input test tones is significantly less than that of the ADC. This is discussed in following Sections.

4 Design and Calibration

Any test setup will have some residual noise and nonlinear components, fixing its dynamic range. We address the problem of generating a high-quality test signal source, and subsequently determining the system resolution limit.

In both THD and IMD tests, a suitable (analog) test signal source is required. The improved immunity to harmonics of test tones makes generation of suitable signals less demanding in the IMD case. Nevertheless, experience shows that the purity of the source soon becomes the limit on system measurement capability. Since the receiver part of the test system is necessarily carried out digitally in the case of testing ADCs, this can be achieved with almost arbitrary precision. Hence the position of the source as the limiting component is even more firmly fixed when the test is applied to ADC testing.

Generation of a test signal reduces in essence to summing two test signals of reasonable quality in a fashion that minimises their interference with each other. Consider the alternatives shown in Fig.(1). In the simple summation scheme, (A), signal from each generator gives rise to current injected into the other. This current is free to intermodulate in the output electronics, giving

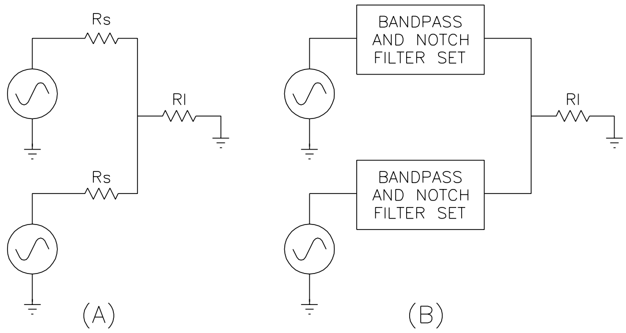

Figure 1: Schemes for combining tones for IMD testing: (A) simple addition, and (B) including filtering.

rise to residual IMD components. This can be controlled with careful attention to the design of the sources, as was done in [16], but such an approach is complex, and impossible at high frequency. Also, for convenience it is desirable to use off-the-shelf signal generators, whose intermodulation susceptability is then fixed.

Using widely-spaced stimulus signals, as in a TDFD rather than a two-tone regime, permits the scheme of Fig.(1B) to be employed. The spacing of the test tones makes for relatively uncomplicated, passive, linear, filters to pass one test tone and reject the other [17, 18]. By this means the dynamic range obtained using ordinary signal generators is greatly extended. For example, resistive summing of high quality RF generators can give intermodulation residuals 75dB below fundamental (dBc) near maximum level, whereas two filters each comprised of a few passive components will give in excess of 100dBc even with modest generators. Above a couple of MHz an LC filter is suitable, and at audio frequencies a twin-T or similar RC filter is adequate.

Determination of system dynamic range is straightforward in the case of an all-analog measurement: the source is simply connected directly to the receiver circuitry, and the residual distortion read off. Unfortunately, this is not possible for an ADC test. The receiver is fashioned in software, and cannot be connected to the signal source without going through a conversion to digital format. If an ADC is not available with performance known to be superior to the converter to be tested, there would seem to be no way of determining whether the nonlinearity found in the post-processing came from the source, or the

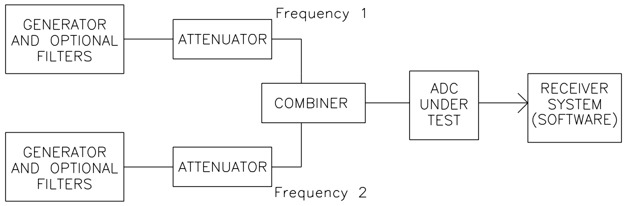

Figure 2: Block diagram of the measurement setup suitable to measure and verify ADC linearity.

ADC under test.

It may be possible to investigate the signal in the analog domain. However, most spectrum analysers have a *linear* dynamic range of less than 80dB, many as little as 65dB, equivalent to about 13 bits at most. We are not aware of any with greater than 100dB, or just over 16 bits. In addition, they are expensive. A scheme of preceding a spectrum analyser with passive filters, such as used in [16] and [17], involves complicated custom design, and does not eliminate the need for a quality spectrum analyser (although there are some commercial units available which include such filters).

Fortunately, it is possible to determine by simple experiment the source of distortion residuals. If, by following the procedure described in the next Section, the residuals are identified as arising in the converter subsystem, then the distortion reading specifies the ADC converter's linearity. Otherwise, an upper bound is set on its performance—the measurement system residual—and the generator causing it is found.

5 Test Verification

Consider the arrangement shown in Fig.(2). Two attenuators are installed in addition to the basic requirements for a TDFD test, one in series with each generator, so that the amplitudes of each test component may be varied independently. The attenuators are assumed to be matched in both directions and so act reciprocally.

Initially the attenuators are set for a modest attenuation, and the generators adjusted to provide a summed signal with two test tones of equal amplitude, and with the desired peak-to-peak level. The receiver reads the amplitudes of the components at ω_1, ω_2, $\omega_0 - \delta$, and $\omega_0 + \delta$. Next the attenuators are adjusted to increase one test tone, say ω_1, by a small factor, say ϵ, and decrease the other test tone, ω_2, by the same factor (A typical value of ϵ might be 3dB, *i.e.*, $\epsilon = \sqrt{2}$). From Eq.(9), the amplitude of the term $IM2@\omega = \omega_0 - \delta$ is given by

$$|IM2| = a_2 A_1 A_2 \ . \tag{21}$$

With this increase in one test tone and corresponding decrease in the other test tone, the amplitude of $IM2$ would then be

$$|IM2| = a_2(A_1 \times \epsilon)(\frac{A_2}{\epsilon}) = a_2 A_1 A_2 \ . \tag{22}$$

Thus, if the intermodulation product arises in the ADC (i.e., downstream of the combiner), the difference product, $IM2$, may be expected to remain constant in amplitude. (The third-order product, $\omega_0 + \delta$ in this example, may be expected to increase or decrease in magnitude by the factor ϵ, depending upon whether ω_1 or ω_2 was increased.) However, if the intermodulation is arising in one of the generators by virtue of signal from the other generator passing through both attenuators and the combiner, the difference (second-order) intermodulation component will have to pass through one of the attenuators after generation. Since the attenuators were adjusted so that their total attenuation remained unchanged, the insurgent test tone at the guilty generator did not change in amplitude, but any IMD products had to pass through one attenuator en-route to the ADC, and that attenuator was changed. Ergo the level read at the receiver after the perturbation will change, by $\pm\epsilon\ dB$, betraying the flaw in the measurement system. The sign of ϵ identifies the guilty generator.

A simple verification experiment is then available: Small perturbations in the relative amplitudes of the two test tones should not affect the difference component, provided their sum remains constant. If this is not so, the nonlinearity is not arising in the converter.

A double-check is available via similar logic applied to the third-order TDFD component. It can be shown that we could perform further checking by reversing the frequencies provided by each generator, reversing the changes introduced at the attenuators, etc. In practice the disturbances

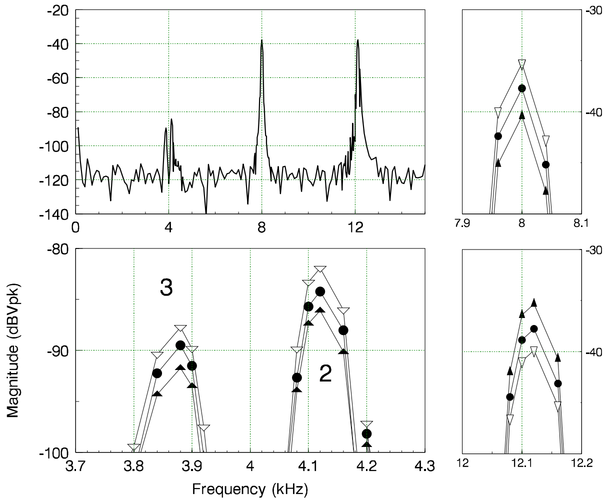

Figure 3: Three superimposed sets of data for the digitiser being measured using two different, inexpensive function generators. The upper-left graph shows the overall spectrum, the others expand the peaks of the four TDFD tones.

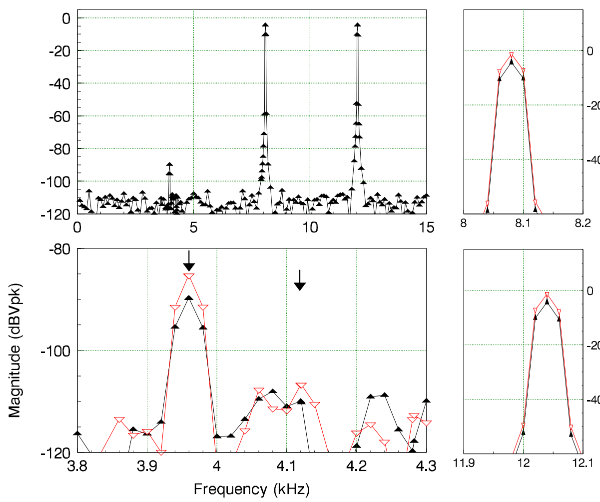

Figure 4: Spectral results of a TDFD test carried out on a digitiser using the signal source of [16]. Two sets of peaks correspond to the test carried out at two slightly different levels. The two arrows in the lower-left plot identify the TDFD IMD components.

in the odd-order component may be hard to follow with sufficient accuracy to be reliable. This is because it is frequently lower than the second, and closer to the inevitable noise floor.

One assumption has been implicit in our logic, that the distortion is arising in one system component only. This clearly might not be the case if identical sources were to be used. However, we may artificially establish this condition by adding an additional "padding" attenuator in series with one of the generators, so that its electronics are isolated by a different return-path loss from that of the other. This can be achieved, of course, by simple adjustment of the same attenuators as already required. It is also assumed that the second-order term has been "simply generated", i.e., that the polynomial transfer function of the source of nonlinearity is dominated by low order terms. While less likely in complex ADC subsystems, this is not an unreasonable assumption for the simple analog electronics of most signal generators.

6 Example Measurements

As a first example, a digitiser of the type described in [9, 10] is subjected to two test tones from inexpensive function generators. These have their outputs simply connected together, so that summing of the two tones is done in their output impedances. The results are shown in Fig.(3). A TDFD SNR of 45dB is returned, giving a linearity of $B_{\rm eff} = 10.8$, whereas we believe it to be far more linear. Subsequently, the perturbation test of Section 5 is carried out, and the data for the two alternatives also appear in Fig.(3). Note that the 2^{nd} order term rises and falls with the 8KHz $2\omega_0$ term, whereas if this distortion originates in the ADC then from Eq.(22) it would be expected to remain constant (the traces identified with the • symbols correspond to the original TDFD test). This behaviour betrays the 8KHz generator as the source of the non-linearity.

Fig.(4) shows the same digitiser tested using the signal source described in [16], in which careful attention to the combining network is given. A TDFD SNR of 83dB is found from the first trace ($\bigtriangledown$), giving a linearity of $B_{\rm eff} = 14.1$. This is typical for this digitiser at this level of input (the 100dBr performance potential of this digitiser requires both multiple tones and that these be at or below a level of -30dBc). The second set of traces in Fig.(4) indicate what happens when the test tones are both dropped by 3dB. The 2^{nd} order TDFD tone drops by 6dB, as expected from Eq.(9). This confirms that the non-linearity

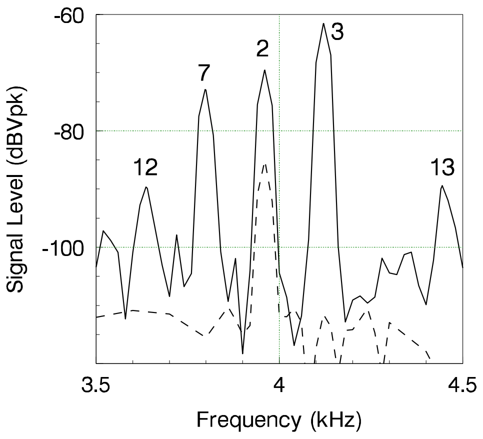

Figure 5: Spectrum of TDFD IMD components returned by the same digitiser as used in Fig.(4) under the same conditions except that the digital linearising system has been switched off. The dotted trace is a repeat of the data with the linearising system operating normally. The numbers indicate order of the IMD component, although only the 2 and 3 components may be important.

arises in the ADC rather than the source.

Fig.(5) shows the same test performed with the digitiser's autocalibration feature disabled. The overall level of intermodulation distortion has greatly increased, indicating the effectiveness of the autocalibration scheme. A plethora of IMD terms appear (the orders of some appear next to their respective peaks in the Figure). Clearly, a higher-order model would be required for Eq.(5) to model this distortion.

The next two examples are for high-performance audio-frequency ADCs. The first ADC, shown in Fig.(6), employs over-sampling and post-filtering. The TDFD spectrum of interest is then free of aliased harmonics. Input tones are at $-9.8\mathrm{dBV}_{pk}$. The $IM2$ and $IM3$ terms are clearly shown and a noise floor of -140dB is obtained from a 4-second data record. A TDFD SNR of 88.1dB is obtained, giving a linearity of $B_{\mathrm{eff}} = 15.2$.

A second example ADC, in Fig.(7), shows slightly higher noise floor for a smaller data set of 1 second. Over-sampling and post-filtering are not employed, giving many aliased harmonics in

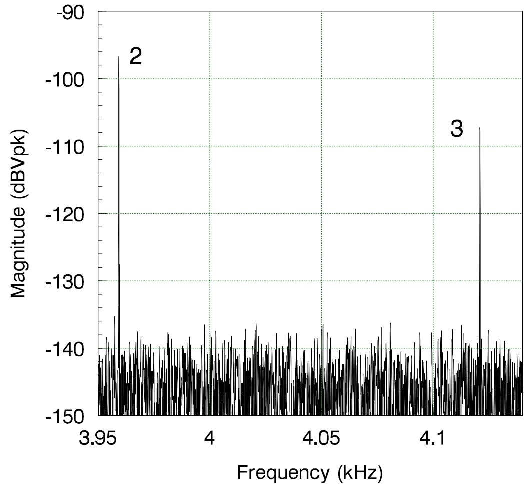

Figure 6: A high-performance audio-frequency ADC employing over-sampling and post-filtering. The 2^{nd} and 3^{rd} IMD terms are clearly indentified.

the TDFD spectrum, in particular a large component at 4134Hz. This does not affect the TDFD measurement. Input tones are at $-19.3\mathrm{dBV}_{pk}$, giving a much lower distortion for this level of input. The $IM2$ term is prominent at $-120\mathrm{dBV}_{pk}$ whereas the $IM3$ term is at the noise floor. A TDFD SNR of $\approx 102dB$ is obtained, giving a linearity of $B_{\mathrm{eff}} = 18.2$.

7 Conclusion

Convincing arguments have been given for using two-tone tests to characterise the linearity of ADC converters in preference to single tone methods. Adaption of the existing TDFD approach to ADC tests confers further advantage. We have shown how it may be used confidently to investigate converter performance.

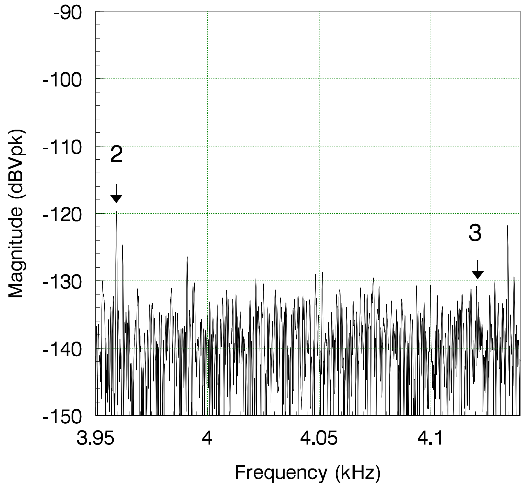

Figure 7: A second high-performance audio-frequency ADC. This ADC does not employ oversampling and post-filtering. Many aliased input harmonics are visible. The 2^{nd} IMD term is clearly indentified, although the 3^{rd} IMD term is at the noise floor.

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