FORWARD by Thomas Kabele, Ph.D. FSA
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Around 1985 the Equitable Society of London gave me a coy of the 1757 handwritten manuscript by James Dodson, Fellow of the Royal Society. The 1757 Manuscript, First Lectures on Insurance, defined the whole life policy, the policy reserve, and net level premium and also included some asset share projections. Dodson developed the whole life policy after he applied for insurance from the Amicable, but was turned down because he was too old. The Amicable had "uniage" premiums but had a maximum issue age of about 45

A typed version of the Dodson manuscript was submitted in 1989 to the Centennial celebration of the American Actuarial Association, one of the predecessor bodies to the Society of Actuaries. A copy was also given to the Institute of Actuaries library. A few years ago a copy of the handwritten manuscript was published.

Dodson computed premiums using pounds, shilling and pence ( 20 shillings to the pound and 12 pence to the shilling. We have also shown the decimal equivalent. We have not changed the spelling of the following words (except the Microsoft Word "quick correct" changed some spellings):

Ballance (p18)
Corperation (p5).
Altho (p 9)
Totall (p60)
Servents (p44)
Agoe (p45)
Endeavour (p54)

This paper is dedicated to the Equitable Society of London, founded posthumously after James Dodson's death in 1762.

Material in [] was added by Thomas G. Kabele, who typed this manuscript. In some cases we have added rows and columns to the various charts. We have also added section numbers and titles.

| Section | Manuscript |  |
| :--- | :--- | :--- |
| 1 Bills of Mortality | $1-11$ |  |
| 2. Sample Premium Calculation | $11-16$ |  |
| 3A. Fund with Greatest Deaths, 10 year term | $16-22$ |  |
| 3B. Fund with Greatest Death \& Public Funds | $22-27$ |  |
| 3C. Fund with 1731-50 Deaths and Public Funds | $27-35$ |  |
| 3D. Funds with Mean Mortality | $35-38$ |  |
| 4A. Whole Life Projection | $38-41$ |  |
| 4B. Accumulation of Individual Premiums | $41-42$ |  |
| 4C. Amicable Society | $42-45$ |  |
| 4D. Calls \& Initial funds | $46-62$ |  |
| 5. Table of Decrements | $56-63$ | $63-64$ |
| 6. Contingent Insurance |  |  |

Dodson discusses some interesting concepts. On page 5 he discusses the 1665 London Plague, page 2 he discusses mutual company Guaranty Capital ("Public funds") on page 37 he deals with assumption reinsurance. Dodson does several projections of cash flow.

## ======

Forward by the Equitable Society of London.
The First Lecture on Insurances by James Dodson was written in 1756 to demonstrate that [whole] life assurance would be practicable. The treatise gives examples of show a life assurance society would work. (this [manuscript] appears to be a copy made by his son, James Dodson, in 1764)

## [p 1 ms ] <br> James Dodson, F.R.S. FIRST LECTURE ON INSURANCES. 1756

## [1. BILLS OF MORTALITY 1602-1750 ]

Before the method of calculating the value of the premium which ought to be paid for the insurance of a life can be explained, it will be necessary to consider the construction and use of the Bills of Mortality. That London was seldom clear of the plague in the preceeding century seems to be the reason why the keeping an account of the number of persons dying annually was first instituted; for from the year 1602, when the first Bill of Mortality was made, to the year 1663 , both inclusive, there is no other distinction made among persons whose death was registered than whether they died of the plague or some other disease. It is so long since this Metropolis has had that misfortune that many readers may think this hardly credible, but it is melancholy truth, for by the bills it appears that except in the years 1629,1635 , and 1679 , in every year from 1602 to 1679 both inclusive, one person or more died of the plague in London, but some years great numbers from 1602 to 1611 both inclusive, died of that distemper, upwards of 51000 in the year 1625; 35417 in the year 1630; 1317 from the year 1636 to the year 1643, both inclusive; 29190 in the year 1665; 60596 and in the following years 1998 and the whole number of such deaths from 1602 to 1679 both inclusive, being 78 years, was 188571 which one year with another, amounts to near 2418 yearly.

From the year 1664 to the year 1727, both inclusive, the Bills of Mortality distinguished the sex of the deceased, but no attempt toward distinguishing the age was made till 1728 , since which time the bills inform us how many have died yearly.

## [p 2 ms ]

| Under two years of age | $40 \& 50$ |
| :--- | :--- |
| Between $2 \& 5$ | $50 \& 60$ |
| $5 \& 10$ | $60 \& 70$ |
| $10 \& 20$ | $80 \& 90$ |
| $20 \& 30$ | $90 \& 100$ |
| $30 \& 40$ | $100 \&$ upwards. |

It is much to be wished that some further distinctions were to be made in the bills, specimens of which \& of their use may be met with in pamphlets called observations on the past growth \& present rate of the city of London by Mr. Corbis Morris, \& in the 47th Vol of the Philos. Transactions page 333 by the author. But since neither of the plans above mentioned have been put in execution, we must content ourselves, at present, with making the best use we can of those that are extant. The above mentioned Mr. Morris has oblidged the
public with a table of the Bills of Mortality during each of the periods above mentioned till the year 1750 his treatise bearing date 1751 by which it appears that in the time during which the ages have been distinguished viz in 23 years beginning 1728 and ending 1750--there have died as follows.

| Between | [Deaths over 23 years] | [Deaths per year] |
| :--- | :--- | :--- |
| under 2 years | 218,810 | 9513 |
| $2 \& 5$ | 51,343 | 2232 |
| $5 \& 10$ | 21,300 | 926 |
| $10 \& 20$ | 18,701 | 813 |
| $20 \& 30$ | 47,322 | 2057 |
| $30 \& 40$ | 57,495 | 2500 |
| $40 \& 50$ | 59,208 | 2574 |
| $50 \& 60$ | 47,691 | 2074 |
| $60 \& 70$ | 37,609 | 1653 |
| $70 \& 80$ | 26,991 | 1174 |
| $80 \& 90$ | 13,764 | 598 |
| $90 \& 100$ | 2,313 | 101 |
| $100 \&$ upwards | 200 | 9 |
| Total 0-50 | 474,179 |  |
| Total 51-up | 120,568 |  |
| Total | 602,747 |  |

## [p.3, ms.]

By accommodating a prosess of the like kind to each period of time, the following table was constructed which nearly shows at one view the number of persons living at every age from the time of birth to the age of 108 years according to the suppositions above recited. The first column contains the age. The second the number alive at that age and the third, the number of persons that die before they attain that age which is one year older. (p.3)

| Age | Living | D | Age | Living | D | Ave | Living | D | Age | Living | D |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Born | 26206 | 4757 | 30 | 10665 | 250 | 60 | 3518 | 164 | 90 | 109 | 10 |
| 1 | 21449 | 4756 | 31 | 10415 | 250 | 61 | 3354 | 164 | 91 | 99 | 10 |
| 2 | 16693 | 744 | 32 | 10165 | 250 | 62 | 3190 | 194 | 92 | 89 | 10 |
| 3 | 15949 | 744 | 33 | 9915 | 250 | 63 | 3026 | 164 | 93 | 79 | 10 |
| 4 | 15205 | 744 | 34 | 9965 | 270 | 64 | 2862 | 164 | 94 | 69 | 10 |
| 5 | 14461 | 186 | 35 | 9415 | 250 | 65 | 2698 | 163 | 95 | 59 | 10 |
| 6 | 14275 | 185 | 36 | 9165 | 250 | 66 | 2535 | 163 | 97 | 39 | 10 |
| 7 | 14090 | 185 | 37 | 8915 | 250 | 67 | 2372 | 163 | 97 | 39 | 10 |
| 8 | 13805 | 185 | 38 | 8665 | 250 | 68 | 2209 | 163 | 98 | 29 | 10 |


| 9 | 13720 | 185 | 39 | 8415 | 250 | 69 | 2046 | 163 | 99 | 19 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 10 | 13535 | 81 | 40 | 8165 | 258 | 70 | 1883 | 118 | 100 | 9 | 1 |
| 11 | 13454 | 82 | 41 | 7907 | 258 | 71 | 1765 | 118 | 101 | 8 | 1 |
| 12 | 13372 | 81 | 42 | 7649 | 258 | 72 | 1647 | 117 | 102 | 7 | 1 |
| 13 | 13291 | 81 | 43 | 7391 | 258 | 73 | 1530 | 117 | 103 | 6 | 1 |
| 14 | 13210 | 82 | 44 | 7133 | 257 | 74 | 1413 | 118 | 104 | 5 | 1 |
| 15 | 13128 | 81 | 45 | 6876 | 257 | 75 | 1295 | 117 | 105 | 4 | 1 |
| 16 | 13047 | 81 | 46 | 6619 | 257 | 76 | 1178 | 117 | 106 | 3 | 1 |
| 17 | 12966 | 82 | 47 | 6362 | 257 | 77 | 1061 | 118 | 107 | 2 | 1 |
| 18 | 12884 | 81 | 48 | 6105 | 257 | 78 | 943 | 117 | 108 | 1 | 1 |
| 19 | 12803 | 81 | 49 | 5848 | 257 | 79 | 826 | 117 | 109 | 0 | 0 |
| 20 | 12722 | 206 | 50 | 5591 | 207 | 80 | 709 | 60 |  |  |  |
| 21 | 12516 | 205 | 51 | 5384 | 208 | 81 | 649 | 60 |  |  |  |
| 22 | 12311 | 206 | 52 | 5176 | 207 | 82 | 589 | 60 |  |  |  |
| 23 | 12105 | 206 | 53 | 4969 | 207 | 83 | 529 | 60 |  |  |  |
| 24 | 11899 | 205 | 54 | 4762 | 208 | 84 | 469 | 60 |  |  |  |
| 25 | 11694 | 206 | 55 | 4554 | 207 | 85 | 409 | 60 |  |  |  |
| 26 | 11488 | 206 | 56 | 4347 | 207 | 86 | 349 | 60 |  |  |  |
| 27 | 11282 | 205 | 57 | 4140 | 208 | 87 | 259 | 60 |  |  |  |
| 28 | 11077 | 206 | 58 | 3932 | 207 | 88 | 229 | 60 |  |  |  |
| 29 | 10871 | 206 | 59 | 3725 | 207 | 86 | 169 | 60 |  |  |  |

## [P.4, ms.]

From the foregoing tables may be collected the numbers of persons of all ages, that is the whole number of persons residing in London for since the third column contains the number of persons which die yearly out of a certain number alive at the beginning of the year which stands in the second and since the sum of the whole column is the number of persons that died in one year therefore the sum of the whole second column will be the number of persons the beginning of the year and consequently about one person in 26 dies annually taking the inhabitants of London all through but it may be worth while to be more particular in the following table.

A table of the Bills of Mortality from the year 1728 to 1750 in inclusive is 23 years in which time there have died as follows:

| [Age] | [Living] | [Mean Deaths] | [Greatest Deaths] | [Deaths our <br> Calculation $]$ |
| :--- | :--- | :--- | :--- | :--- |
| Under 2 years | $218810^{*}$ | 9513 | 10438 |  |
| Between 2 \& 5 | 51343 | 2232 | 2318 |  |
| $5 \& 10$ | 21300 | 926 | 1072 |  |


| $10 \& 20$ | 18701 | 813 | 1048 |  |
| :--- | :--- | :--- | :--- | :--- |
| $20 \& 30$ | 47322 | 2058 | 2316 | 2057 |
| $30 \& 40$ | 57495 | 2500 | 3476 |  |
| $40 \& 50$ | 59208 | 2574 | 3731 | 2073 |
| $50 \& 60$ | 47691 | 2074 | 2351 |  |
| $60 \& 70$ | 37609 | 1635 | 1977 |  |
| $70 \& 80$ | 26991 | 1174 | 540 |  |
| $80 \& 90$ | 13764 | 598 | 754 |  |
| $90 \& 100$ | 2313 | 101 | 150 |  |
| $100 \&$ upwards | 200 | 9 | 14 |  |
| [Total] | 602747 | 26207 | 32169 |  |

* Which being favorably divided the second column will give the number that have died on a mean in any one of the said periods of life.

Whereof between 10 \& 70 there have died 268026 [see page 2] which will upon a mean be about 11653 yearly we have therefore by the help of these Bills not only found out how many persons may be expected to die yearly upon a

## [p5 ms]

mean but also the number of such in each of the periods of life. By examining the table of the Bills of Mortality above quoted it will appear that the greatest number of persons that have died in any one year since the great plague year in 1665 is 32169 , which happened in the year 1741 which number exceeds the said mean number 26207 by 5962 which excess amounts to something less than $1 / 4$ of the mean numbers and exceeds $1 / 5$ thereof.

Now first if the calculations of the premiums of insurance of the lives of those persons who become members of the corperation and are to bare a proportion of the loss if such happen are made for all lives indiscriminately in proportion to the said mean number of deaths. Secondly if the persons who shall desire to be insured without being liable to such a call should be rated in proportion to the greatest number of deaths that have happened within our knowledge as I think they ought, then the latter ought to pay near $1 / 4$ part in the premium more than the former, that is if the premium for insuring the life of one of the former, be, 4 , then that for the life of one of the latter should be made 5 pounds.

A table of the greatest and mean numbers of death for the ages between $10 \& 70$ according to the foregoing Bills of Mortality.

| [Age] | Greatest Number | Mean <br> Number | the difference is |  |
| :--- | :--- | :--- | :--- | :--- |
| Between $10 \& 20$ | 1048 | 813 | 235 | more than $1 / 4$ of mean |
| $20 \& 30$ | 2816 | 2058 | 758 | more than $1 / 3$ |
| $30 \& 40$ | 3476 | 2500 | 976 | more than $1 / 3$ |
| $40 \& 50$ | 3731 | 2574 | 1157 | not much short of 2 |
| $50 \& 60$ | 2851 | 2074 | 777 | more than $1 / 3$ |
| $60 \& 70$ | 1977 | 1635 | 342 | not quite $1 / 4$ |
| [Total] | 15899 | 11654 | $[4245]$ | [more than $1 / 3]$ |

here the difference

## [P.6, ms.]

between 15899 the greatest, and 11654 the mean number is 4245 which is considerably above $1 / 3$ of the said mean number.

From whence it appears that in almost all of the ages proposed to be insured by this corperation the greatest number of death has exceeded the number of the by above a fourth part of the said mean number and consequently that the premiums calculated upon the greatest numbers will exceed them calculated upon the mean numbers by more than a fourth part of those found from the mean numbers.

First then since 9 persons die yearly upon a mean at the age of 100 or upwards therefore at any time there will be 9 persons alive of the age of 100 . Secondly there will be always 110 persons alive of the age of 90 in London, for there die 101 persons between $90 \& 100$, \& 9 persons out of 100 , which could not be unless there was 110 alive at the age of 90 .

There will be 708 persons alive aged 80 for between $80 \& 90$ there die 598 and 110 alive to be 90 .
And by arguing in the like manner it will appear that there will be $1882,3517,5591,8165,10665,12723$, $13536,14462 \& 16694 \ldots$ persons alive of the respective ages of $70,60,50,40,30,20,10,5, \& 2$ years. And lastly the numbers of persons born yearly upon a mean will be 26207 for there die under two years old 9513 persons and 16694 are alive at that age which could not be unless 9513 \& 16694 persons were born. Let us now suppose that the number dying yearly during each of the above periods of age are equal for instance since the die 2500 persons in the 10 years between $30 \& 40$ let us suppose that 250 of them die in each year then the number of persons alive in each particular age may be

## [P.7, ms.]

obtained in the following manner:
From 10,665 persons alive at age 30
Take 250 the number dying yearly
Remains 10,415 the number alive at age 31
Subtract 250 the number dying year
10,165 the number alive at age 32
And so on, at 40 will be living 8165 persons hence many be deducted the following conclusions. Viz that $1 / 5$ part of the persons who are living under 2 years of age do not survive that period.

1. That is to say one person dies out of 5 and consequently if lives so young were to be insured the annual premium of insurance for $£ 100$ ought to be $£ 20$ because 5 such Premiums would just pay one Death.
2. That the numbers dying between $2 \& 5$ are less than $1 / 20$ part of the numbers living and consequently $£ 100$ may be insured on such a life for an annual premium of $£ 5$ for 20 such premiums will pay one death.
3. That the number dying between 3 and 10 years are less than $1 / 75$ \{p. 73 of the numbers living whence $£ 1.6 .8$. [1.333] will during that period be a sufficient annual premium for insuring $£ 100$ for 75 such premiums will pay one death.
4. That the dead between 10 and 20 years are not the $1 / 160$ part of the living and therefore if this part of the Bills could be relied on $£ 0.12 .6[0.625]$ would be a sufficient annual premium during this Period of insuring , 100 but concerning this want of correctness in the London Bills arriving from great numbers of strangers who flock to London during this and some following periods more will be said thereafter.
5. That the number of persons dying between $20 \& 30$ years are not $1 / 50$ part

## [p.8, ms.]

of those alive whence the annual premium of insurance for $£ 100$ during this period may be $£ 2$.
6. That the number dying between $20 \& 30$ years are about $1 / 38$ part of the living and the annual premium may be about £2.13.0 [2.650].
7. Between $40 \& 50-1 / 27$ part of the living the annual premium $£ 3.14 .6$ [3.725].
8. Between 50 \& $60-1 / 22$ the annual premium $£ 4.11 .0$ [4.550]
9. Between 60 \& $70-1 / 16--£ 6.5 .0 \quad$ [6.250]
10. Between $70 \& 80-1 / 11-$ - £9.2.0 [9.100]

In like manner the deaths in the succeeding periods will be about $1 / 7$ th, $1 / 6$ th, $\& 1 / 5$ th of the living and consequently the premiums about $£ 14.6 .0$ [14.300], $£ 16.13 .4$ [16.667], $£ 20$.

Lives according to the Foregoing calculation may be insured for:

| 10 to 20 | $£ 0.12 .6$ |
| :--- | :--- |
| 20 to 30 | 2 |
| 30 to 40 | 2.13 |
| 40 to 50 | 3.14 .6 |
| 50 to 60 | 4.11 |
| 60 to 70 | 6.5 |
| 70 to 80 | 9.2 |

[The following chart summarizes the chart on page 3, but Dodson might have made some minor arithmetic errors. Dodson's interval $10 \& 20$ was really 10 to 19.]

| Age | Persons Living | Deaths | $[$ Approx. Ratio <br> Living / Deaths] |
| :--- | :--- | :--- | :--- |
| Under 2 yrs | 47,655 | 9513 | $[5]$ |
| 2 \& 5 | 47,847 | 2232 | $[20]$ |
| 5 \& 10 | 70,451 | 926 | $[75]$ |
| $\mathbf{0}$ \& 19 | $\mathbf{1 6 5 , 9 5 3}$ | $\mathbf{1 2 6 7 1}$ | $[13]$ |
|  |  |  |  |
| $10 \& 19$ | 131,690 | 813 | $[162]$ |
| 20 \& 29 | 117,965 | 2058 | $[58]$ |
| 30 \& 39 | 95400 | 2500 | $[38]$ |
| 40 \& 49 | 70,061 | 2574 | $[27]$ |
| 50 \& 59 | 46,580 | 2074 | $[22]$ |
| 60 \& 69 | 26,820 | 1635 | $[16]$ |
| $\mathbf{1 0 ~ \& ~ 6 9}$ | 488,516 | 11654 | $[42]$ |
|  |  |  |  |
| $70 \& 80$ | 13,541 | 1174 | $[11]$ |
| $80 \& 90$ | 4,390 | 600 | $[7]$ |
| $90 \& 100$ | 600 | 100 | $[6]$ |
| Upwards of 100 | 45 | 9 | $[5]$ |
| Total | $\mathbf{6 7 3 , 0 8 5}$ | $\mathbf{2 6 , 2 0 8}$ | $[42]$ |

From whence it appears that the deaths during the above period in which the corperation proposes to insure are about 42 part of the living in that time and consequently that if the premiums were to be established without

## [p9 ms]

regard to age as is done by the Amicable Society then the Insurance of $£ 100$ for one year might cost about $£ 2.80$ but if the greatest number of deaths 15,899 be considered instead of 11,652 the mean number they will appear to take $1 / 31$ of the Living so the Premium $£ 3.11 .6$ [3.225] which confirms the former conclusion.

Now as it appears on those small Periods the Premiums are very different from each other, that is to say altho
upon the whole time between 10 and 70 it was found that $£ 2.8 .0$ [2.400] was the Premium for insuring one Age with another yet in the Particular Periods between $10 \& 20$ the premium is but $12 / 6$ in that between 60 and 70 it is $£ 6.5 .0$ [6.250] so if any of those Periods be broken into particular years the Premium for those years will also be different, as may be seen by the following Table

| Age | Living | Out of which die |
| :--- | :--- | :--- |
| 40 | 8165 | 258 |
| 41 | 7907 | 258 |
| 42 | 7649 | 258 |
| 43 | 7391 | 258 |
| 44 | 7133 | 257 |
| 45 | 6876 | 257 |
| 46 | 6619 | 257 |
| 47 | 6362 | 257 |
| 48 | 6105 | 257 |
| 49 | 5849 | 257 |
| Total |  | 2,576 |

Now since out of 8165 Persons living at the age of 40,258 die before they arrive at the age of 41 , therefore if all their lives were insured for $£ 100$ there would be 258 times $£ 100$ to be paid on account of the Deaths, whence $£ 25,800$ must he received in Premiums of the 8165 Persons whose Lives are insured and each person must pay the one 8165 -the part of $£ 25,800$ that is about $£ 3.3 .21 / 2$ [3.160] and generally if the number of persons dying each year be multiplied by $£ 100$ and the product is divided by the number living at the beginning of that year the quotient will be the premium for insuring $£ 100$ for one year on a life of that age as is done in the following table:

## [p.10, ms]

| [One Year Term Insurance Premiums] |  |  |
| :--- | :--- | :--- |
| Age | the premium | [in decimals] |
| 40 | $3.3 .2 ~ 1 / 2$ | $[3.160]$ |
| 41 | 3.5 .3 | $[3.263]$ |
| 42 | $3.7 .51 / 2$ | $[3.373]$ |
| 43 | 3.9 .10 | $[3.490]$ |
| 44 | $3.13 .01 / 2$ | $[3.603]$ |
| 45 | 3.14 .9 | $[3.738]$ |
| 46 | 3.17 .8 | $[3.883]$ |
| 47 | $4.0 .93 / 4$ | $[4.040]$ |
| 48 | $4.4 .21 / 2$ | $[4.210]$ |
| 49 | 4.7 .11 | $[4.394]$ |

Now although it appeared before that the premium for insuring during any part of this period was $£ 3.24 .6$ [3.725] yet it is now plain that there ought to be a great distinction between the premiums payable in the youngest and oldest extream thereof the first being but $£ 3.3 .22$ [3.160] and the latter 4.7.11 [4.396]. By
this method of calculating the premiums on insurance of lives of any age for a single year above explained no greater sum will be taken than is barely adequate to the Haz[ard] and consequence if the receipt of the premiums and the payment of the claims occassioned by death were to be made at the same time it might be reasonably expected that the one would nearly ballance the other and the corperation would always remain without being masters of any considerable property but by the provision made in the proposal for the receipt of the premiums at the beginning of the year and delaying the payment of the claims occassioned by deaths till the expiration thereof twelve months interest on most of sums received for premiums may be made which if the corperation has good business will soon amount to a considerable sum.

## [p. Il, ms.]

Besides which as the Bills of Mortality contain the deaths of all kinds of people healthy and unhealthy and as care will be taken not to insure those lives which are likely to be soon extinct therefore in all probability fewer of the persons insured will die in proportion to their number than of those not insured which will also contribute to the gain of the corperation since the premiums are proportioned by the Bill

## [2. SAMPLE PREMIUM CALCULATION ]

Before I proceed to show the methods of calculating the proper annual premium for insuring a life for 3,5 , or 7 years certain or during its whole continuance it will be necessary to explain further the manner of applying calculation to questions relating to chance.

1. If a piece of money be thrown up between two players, one of whom will upon the fall if it shown one of its faces which shall be particularly by the assigned be entitled to receive $£ 2$ when if before the event he should choose to sell his chance to another it is evident that if there is nothing in the form of the money that will incline it to show one face instead of the other, he ought to receive one pound for his chance that is one half of the stakes.
2. If a common dye be thrown and the player will if he throws an Ace or any other face assigned be entitled to six pounds then because there are 6 faces one of which only is favorable to him he ought to receive one point that is one-sixth of the stake if he parts with his chance to another.
3. If he were entitled to receive the $£ 6$ if he threw either the Ace or Duce then because there are two faces out of the 6 that are favorable to him he ought to receive $£ 2$ that is $2 / 6$ of the stake if he parts with his chance to another.

## [p.12, ms.]

The above fractions $1 / 2,1 / 6,2 / 6$ are called by calculaters, the probabilities of the happening of the several events and if we assume unity to denote certainty that $1 / 2 \& 1 / 2,1 / 5,2 / 6,4 / 6$ being severally added together produce unity for in case of the dice it is certain that the ace will or will not be thrown and therefore the probabilities of those events happening or failing will be unity.

Now to apply this to lives by the table above given there appear to be 3190 persons alive at the age of 62 of which 164 die in the year and 3026 remain alive at the age of 63 whence the probability of any particular person dying in that year is $164 / 3190$ and that of his living is $3026 / 3190$ the sum of which two fractions will be $3190 / 3190$ or unity.

If therefore $£ 100$ would become due if a person age 62 should die within the year and that debt were be sold at the beginning of the year then $164 / 3190$ parts of $£ 100$ would be the value thereof if the interest of money be not considered and the numerical process would be just the same as that before directed to find the premium of insurance for a single year. And if $£ 5$ would become due if a person of that age should not die within the year then the value of that debt estimated at the beginning of the year would be $3026 / 3190$ or $£ 5$.

Suppose now that a person aged 62 would purchase an annuity of one pound for his life and let the number alive at the several ages $62,63,64,65,66$, and to the extremity of old age be as in the table 3190,3025 , $2862,2698,2535$ and then by reasoning as before , 1 due at the end of one year if a life of 62 continue in being so long will be worth $340267 / 3190$

## [p.13, ms.]

parts of $£ 1$ at the end of 2 years 2862 / 3190 of $£ 1$ due at the end of 3 years 2698/3190 at the end of 4 years $2535 / 3190$ and the sum of those fractions would be the true value of the annuity for the whole life if the purchaser were not to be allowed interest for his money out on account of that the 3026/3190 must be discounted for 1 year the 2862/3190 must be discounted for 2 years; the 2698 for 3 years and the present worth of those several payments will be the worth of the annuity. According to this, the finding the value of an annuity for a single life especially if it be a young one, is a very tedious process but as the present intention is only to show the principles upon which such calculations are founded, it will be sufficient to inform the reader that the algebraic art will furnish the means of abbreviating it greatly the manner of doing which cannot be easily explained.

Contenting ourselves, therefore, with the method above mentioned it appears that the calculating the value of the whole life and that of a few years certain if the life would continue so long depends upon the same principle that the latter is attended with the least trouble.

Suppose then it were demanded to know what will be the value of two annual payments of $£ 1$ each which will become due at expirations of one and two years if a person aged 62 should live so long. I answer, it has already been that the first of them will on account of the chance of death be reduced to $3026 / 3190$ or $£ 0.18 .111 / 2$ [.948] and the second to $2862 / 3190$ or $£ 0.17 .111 / 4$ [.897] which sums discounted $4 \%$ the first for one and the second for two years will be further reduced to $£ 0.18 .3$ [0.913] and $£ 0.16 .7$ [0.829].

Therefore three annual premiums of insurance upon a life of 62 of $£ 1$ each where if the first is paid down will at the time of insurance be worth as follows:

## [p.14, ms.]

The first payment 1.0.0 [1.000] The second 0.18 .3 [ 0.913 ] The third 0.16 .7 [ 0.829 ] Total 2.14.10 [2.742] And let the annual premium be what it will, the above total multiplied by such premium will be the value of three such thereof.

From what has been said before it is evident that the premiums payable for insuring the same life for a single year increase yearly: whence it follows if a person could insure a life for any number of years and would pay annually the same premium that he ought to pay a greater sum for the first years insurance that he would if he had insured for that single year, and consequently, if he lives to pay the premium for that last year such premium would be less than he would have paid if he had insured for that year only.

The annual premium for insuring $£ 100$ one life of 62 years for 3 years certain, may be computed in the following manner.

The premium for insuring this life for a single year will by what has been before advanced be $164 / 3191$ of $£$ 100 or 5.2.10 [5.141] which sum is to be received at the time of insuring if the first year were past. That is if the insured were 63 years of age the premium for insuring his life next year would be 164/3026 or 5.8.5 [5.420] but this sum will not be payable to the insurer unless the insured survives the first year. Now the probability of this is $3026 / 3190$ and therefore the expectations of receiving another premium at the end of the first year will be worth 3126/3190 of

## [p.15, ms.]

£5.8.5 [5.420] or 5.2.10 [5.141].
If the second year were also past then the premium for insuring the third year would be 164/2862 of ,100 or 5.14 .7 [5.730] but this will not become payable unless the insured survive both the first and second years. The probability of which is $2862 / 3190$ of $5.14 .71 / 2$ [5.730] that is also 5.2.10 [5.141]. Therefore if this interest of money be not considered the whole exception of the insurer will be to receive in hand at the end of one year and at the end of two years which two last sums ought to be discounted for the times at which they are payable in order to find their present value.

Now if interest be allowed at 4 per cent 4.18.10 $1 / 2$ [4.944] will appear to be the present value of 5.2.10 [5.141] due at the end of one year and 4.15.0 $1 / 2$ [4.754] that of the like sum due at the end of two years and consequently the present worth of the three premiums which the insurer may receive if the life be insured only from year to year will be as follows:

| For the first year | $£ 5.2 .10$ | $[5.141]$ |
| :--- | :--- | :--- |
| Second | $4.18 .101 / 2$ | $[4.944]$ |
| Third | $4.15 .01 / 2$ | $[4.754]$ |
| Total | 4.16 .9 | $[14.839]$ |

Now if instead of insuring from year to year the life is to be insured by paying a constant annual payment then the above mentioned sum of [14.839] ought to be equal to the present worth of those three annual payments in order to render the two different methods of paying the premiums equally advantageous to the insurer and insured.

## [p.16, ms.]

But it was found before that three such annual premiums of $£ 1$ each will at the time of insuring be worth $£ 2.14 .10$ [2.742] as often therefore as $£ 2.14 .10$ [2.742] is contained in $£ 14.16 .9$ [14.839] so many pounds should be paid as an actual premium which will therefore be $£ 5.10 .3$ [5.412]. It is very easy to conceive from what has been said now the life annual premiums may be found either for the whole continuance of life or any number of years certain and therefore I hope that the methods of calculating one of the principal questions that will arise in the course of the corperation business is set in a light sufficiently clear.

From what has been said there also clearly follows the method of finding that sum which a person ought to pay in the hands to be insured for any number of years certain or even for life. Thus the above $£ 14.16 .9$ [14.839] should be paid down in order for a life aged 62 to be insured for three years from the time of payment and this may be continued to any number of years by finding the present worth of $£ 5.2 .10$ [5.141] due at the end of $3,4,5$ years and adding their result.

## [3A. FUND - 10 year renewable term, with greatest deaths for 10 years ]

In order to determine what fund it may be necessary to raise in order fully to secure to those persons who shall choose to insure without running any risk of loss, the immediate payment of their claims without a call on the society it will be necessary to promise the following things.

First, since a medium of 23 years was taken as the basis of these calculation [see page 1], that is reasonable to suppose that alto' the corperation should have a run upon them for some of the first years, yet that

## [p.17, ms.]

afterwards before the expiration of 23 , the number dying that whole time of those whose lives are insured with, will be proportional to the mean numbers.

Hence out of any number of persons insured, if we suppose that there die yearly for 10 years successively as many persons as would be purported enable to the greatest number of deaths which has been found to happen in any one year, it will necessarily follow that for the remaining 13 years the number dying will be to these living in a proportion less than those of the mean numbers and consequently if such a loss be provided for the Premiums received cannot afterwards as proved insufficient to answer the claims.

Second, it has appeared that between $40 \& 50$ the greatest number of deaths exceed the mean number more than in any other period in which it is proposed to insure therefore if a calculation of the loss which the corperation will sustain by such a ruse for those 10 years be made and provided for that provisions will be more than sufficient to and intended purpose when all the other periods are included.

Third, let us therefore suppose that all the 8165 persons alive at 40 are insured for $\$ 100$ each at the mean rate during that period which for them that are liable to bear a proportion of the loss has been found to be $£ 3.14 .6$ [3.725] and this being increased by its $1 / 4$ part 18/7 1/2 [0.931] will become $£ 4.13 .11 / 2$ [4.65625] then out of 8165 , if the greatest number of deaths in the whole period 3731 be taken the remainder $\mathbf{4 4 3 4}$ will be the number of persons alive at the beginning of the succeeding period who are than to pay a greater premium and consequently as the proportion all number

## [p.18, ms.]

of deaths will afterwards be less than are provided for in the succeeding periods the corperation will than begin to recover their loss.

Fourth, let us suppose that of the 3731 persons dying in the ten years $40 \& 50374$ die in the first year and that in the remaining 9 years there die yearly 373 then the number of the living and will for that period be as follows.

| Age | Living | of which die | age | Living | of which die |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 40 | 8165 | 374 | 45 | 6299 | 373 |
| 41 | 7791 | 373 | 46 | 5926 | 373 |
| 42 | 7418 | 373 | 47 | 5553 | 373 |
| 43 | 7045 | 373 | 48 | 5180 | 373 |
| 44 | 6672 | 373 | 49 | 4807 | 373 |
|  |  |  | 50 | 4434 | 373 |

Fifth, let us suppose that the numbers dying in the next period of life viz from 50 to 60 will be such that at the age of 60 the mean number of the living 3518 above found may be then alive and that for the remaining three years the numbers proceed as in the table. This is indeed reducing the numbers to the mean in twenty instead of 23 years but the supposition of there being a constant. Mortality for 10 successive years equal to the greatest that happened in any of the 23 years from whence the mean numbers were derived will greatly over ballance it. Now by the last process we suppose 4434 persons to be alive at the age of 50 and above, that there will be 3518 alive at the age of 60 therefore only 916 persons will die in the 2 nd ten years which may be disposed yearly as follows:

| Age | Living | Of which die | Age | Living | Of which die |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 50 | 4434 | 92 | 55 | 3914 | 92 |
| 51 | 4342 | 92 | 56 | 3882 | 91 |
| 52 | 4250 | 92 | 57 | 3791 | 91 |
| 53 | 4158 | 92 | 58 | 3700 | 91 |
| 54 | 4066 | 92 | 59 | 3609 | 91 |
|  |  |  | 60 | 3518 |  |

[The total deaths in the age groups 40-59 are 3731 plus 916]

## [p. 19, ms.]

Now the mean premium payable during this period [ages 50-59] by those who are liable to bear their proportion on the loss was found to be $£ 4.11 .0$ [4.550] to which adding its $1 / 4$ part viz $£ 1.2 .9$ [1.1375] the sum ,5.13.9 [5.6875] will be the premium payable by these who are not liable to any such loss. Let us now state the account of the corperation profit and loss for those 20 years.

|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 1st year | 374 claims | 37400 | By 8165 Prem. at 4.65625 | 38018.5 .7 |
|  |  |  | at $[\mathbf{4 . 6 5 6 2 5 ]} 4.13 .11 / 2$ |  |
|  | Ballance of Profit | 1758.5 .7 | 1st Year Interest on 38018 at 3\% | 1140 |
|  |  | 39158.5 .7 |  | 39158.5 .7 |


|  | Dr Profit \& Loss | P.S.D. | Contra Cr | P.S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 2nd year | 373 claims | 37300 | Ball Brot. down | 1758.5 .7 |
|  | Ballance of Profit | 1875.17 .5 | Interest on do 3\% | 52.10 |
|  |  |  | by 7791 Premium | 36276.16 .10 |
|  |  |  | Interest on do 3\% | 1088.5 |
|  |  | 39175.17 .5 |  | 39175.17 .5 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |


| 3rd year | 373 claims | 37300 | Ball | 1875.17 .5 |
| :--- | :--- | :--- | :--- | :--- |
|  | Ballance of Profit | 208.7 .8 | Interest on do | 56.5 |
|  |  |  | by 7418 Premium | 34520.4 .3 |
|  |  |  | Interest on do | 1036.4 |
|  |  | 37508.7 .8 |  | 37508.7 .8 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 4th year | 373 claims | 37300 | Ballance | 208.7 .8 |
|  |  |  | Interest on do | 6 |
|  |  |  | By 7045 premiums | 32803.5 .7 |
|  |  |  | Interest on do | 984 |
|  |  |  | Ballance | 3298.6 .9. |
|  |  | 37300 |  | 37300 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 5th year | 373 claims | 37300 | Ballance by 6672 Premiums | 31066.10 |
|  | Ballance of Profit | 3298.6 .9 | Int. on do | 931.166 |
|  | Int on $3300 @ 3 \%$ | 99 | Ballance | 8699.0 .9 |
|  |  | 40697.6 .9 |  | 40697.6 .9 |

## [p. 20, ms.]

|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 6th year | 373 claims | 37300 | Ballance by 6299 <br> Premiums | 39329.14 .4 |
|  | Ballance of Profit | 8699.9 | Interest on 29300 | 879.18 |
|  | Int on 8700 at 3\% | 261 | Ballance | 16050.8 .5 |
|  |  | 46260.9 |  | 46260.9 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 7th year | 373 claims | 37300 | By 5926 Premiums | 27592.18 .9 |
|  | Ballance of Profit | 16050.8 .5 | Int | 820 |
|  | Int | 481.10 | Ballance | 16050.8 .5 |
|  |  | 53831.18 .5 |  | 53831.18 .5 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 8th year | 373 claims | 37300 | By 5555 Premiums | 25856.0 .1 |


|  | Ballance of Profit | 25410.19 .8 | Interest | 775.10 |
| :--- | :--- | :--- | :--- | :--- |
|  | Interest | 762 | Ballance | 36841.9 .7 |
|  |  | 63472.19 .8 |  | 63472.19 .8 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 9th year | 373 claims | 37300 | By 5180 Premiums | 24119.7 .6 |
|  | Ballance of Profit | 36841 | Ballance | 50658.7 .1 |
|  | Int on ,12725 | 636.5 |  |  |
|  |  | 74777.14 .7 |  | 74777.14 .7 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 10th year | 373 claims | 37300 | By 4807 Premiums | 22383.11 .10 |
|  | Ballance of Profit | 50658.7 .1 | Ballance | 66989.10 .3 |
|  | Int on ,28275 | 1413.15 |  |  |
|  |  | 89372.2 .1 |  | 89372.2 .1 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 11th year | 92 claims | 9200 | By 4434 Premiums | 25218.7 .6 |
|  | Ballance of Profit | 66989.10 .3 | at 5.13.9 $[\mathbf{5 . 6 8 7 5}]$ |  |
|  | Interest on ,41775 | 2088.15 | Ballance | 53059.17 .9 |
|  |  | 78278.2 .1 |  | 78278.15 .3 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 12th year | 92 claims | 9200 | By 4342 Premiums | 24695.2 .6 |
|  | Ballance of Profit | 53039.17 .9 | Ballance | 38983.0 .3 |
|  | Interest on ,28365 | 1418.5 |  |  |
|  |  | 63678.2 .9 |  | 63678.2 .9 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 13th year | 92 claims | 9200 | By 4250 Premiums | 24171.17 .6 |
|  | Ballance of Profit | 38983.0 .3 | Ballance | 24757.12 .9 |
|  | Interest on ,14810 | 740.10 |  |  |
|  |  | 48923.10 .3 |  | 48923.10 .3 |

## [p. 21, ms.]

|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |


| 14th year | 92 claims | 9200 | By 4158 Premiums | 23648.12 .6 |
| :--- | :--- | :--- | :--- | :--- |
|  | Ballance of Profit | 24757.19 | Ballance | 10358.0 .3 |
|  | Interest on, 4100 | 55 |  |  |
|  |  | 34006.12 .9 |  | 34006.12 .9 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 15th year | 92 claims | 9200 | By 4066 Premiums | 23125.7 .6 |
|  | Ballance of Profit | 10358.0 .3 | Interest on $£ 12700$ | 381 |
|  | Ballance | 3948.7 .3 |  |  |
|  |  | 23506.7 .6 |  | 23506.7 .6 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 16th year | 92 claims | 9200 | By Ballance | 3948.7 .3 |
|  | Ballance of Profit | 18146.19 .9 | 3974 Premiums | 22602.2 .6 |
|  |  |  | Interest on $£ 26550$ | 796.10 |
|  |  | 27346.19 .9 |  | 27346.19 .9 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 17th year | 91 claims | 9100 | By Ballance | 18146.19 .9 |
|  | Ballance of Profit | 32332.12 .3 | 3082 Premiums | 22078.17 .6 |
|  |  |  | Interest on 40225 | 1206.15 |
|  |  | 41432.12 .3 |  | 41432.12 .3 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 18th year | 91 claims | 9100 | By Ballance | 32332.12 .3 |
|  | Ballance of Profit | 46140.12 .6 | 3791 Premiums | 21516.16 .3 |
|  |  |  | Interest on 53890 | 1616.14 |
|  |  | 55510.12 .6 |  | 55510.12 .6 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 19th year | 91 claims | 9100 | By Ballance | 60377.17 .6 |
|  | Ballance of Profit | 60377.17 .6 | 3700 Premiums | 21043.15 |
|  |  |  | Interest on 80900 | 2427 |
|  |  | 69477.17 .6 |  | 69477.17 .6 |


|  | Dr Profit \& Loss | P S.D. | Contra Cr | P S.D. |
| :--- | :--- | :--- | :--- | :--- |
| 20th year | 91 claims | 9100 | By Ballance | 60377.17 .6 |
|  | Ballance of Profit | 74331.1 .3 | 3609 Premiums | 20526.3 .9 |


|  |  |  | Interest on 80900 | 2427 |
| :--- | :--- | :--- | :--- | :--- |
|  |  | 83331.1 .3 |  | 83331.1 .3 |

## [p.22, ms.]

Hence it appears that at the beginning of the 11th year, the corperation may be in debt near $£ 67,000$ not with transacting the payment of which and its interest at $5 \%$ there will be at the end of the 20th years gain of more than $£ 74,000$. But since the Corperation may be under a necessity to borrow money, it will be more easily obtained when their credit is good than when such a run has happened as has been above supposed for in such a case, most people will be afraid of venturing their money for fear the corperation should not be able to extricate itself.

Besides since there may be such a call for money to pay claims the persons who come to insure without being liable to the risk of loss have a right to demand the establishment of such a fund as may render the payment of the claims which may arise upon their insurance both certain and prompt and this I think may be said to be done if for every 8165 persons so insured $£ 67,000$ was deposited in the Public Funds for that purpose which is something less than $£ 8.5$ [8.250] on every ,100 insured but to put the security past all contradiction. Let us suppose that double the sums of $£ 67000$ viz, was raised and deposited in the Public Funds for the purpose above recited. Then the next thing to be considered will be to explain the manner in which the persons who advance the sum are to be recompensed for the hazard they run that the half or greater part thereof shall be applied to pay the claims of the insured.

## [3B FUNDS with Initial Public Funds of $£ 134,000]$

Let us suppose that the Public Funds bear no more than 3 percent interest and that the corperation pay 5 percent the 2 percent will be annually allowed to them. in consideration of that hazard and let us neither suppose that the corperation pay this for 20 years then each person who advances $£ 100$

## [p.23, ms.]

will have an annuity of $£ 2$ for 20 years more than he can make in the Public Funds, the present worth of which at $3 \%$ upwards of $£ 29.15$ and if it can be proved by a second computation that the corperation will offer paying the interest be in cash to pay off the whole debt in 20 years this must be sufficient consideration.

Then the yearly state of the Society's Funds \& Profit \& Loss so far as relates to the 8165 persons insured at the age of 40 will stand as follows:

| Dr Profit \& Loss | P.S.D | Contra Cr | P.S.D. |
| :--- | :--- | :--- | :--- |
| to Int. of $134,000 @ 5 \%$ | 6700 | By 8165 Premiums | 38018.5 .7 |
|  |  | At 4.13.1 $1 / 24.65625]$ |  |
| to 374 Claims @ ,100 <br> each | 37400 | Int on 173,000 at $3 \%$ | 5160 |
|  |  | Ballances [Asset - Public funds] | 921.14 .5 |
|  | 44100 |  | 44100 |

[In the chart below what Dodson defined as "Ballances Lost" is "Assets less Public Funds." Dodson uses
different rounding than used in the last three columns.]

| Year | Ballances Lost | [Assets - Public] | [Assets] | [Public Funds] |
| :--- | ---: | ---: | ---: | ---: |
| [First] | 921.14 .5 | $(921.719)$ | $133,078.281$ | 134,000 |
| Second | 2742.13 .2 | $(3,663.656)$ | $130,336.344$ | 134,000 |
| Third | 4613.19 .9 | $(8,277.302)$ | $125,722.698$ | 134,000 |
| Fourth | 6540.19 .6 | $(14,818.241)$ | $119,181.759$ | 134,000 |
| Fifth | 8526.0 .0 | $(23,244.293)$ | $110,755.707$ | 134,000 |
| Sixth | 10570.17 .0 | $(33,712.012)$ | $100,287.988$ | 134,000 |
|  |  |  |  |  |

Before we proceed let us see if the surviving persons insured have sufficient security left now the survivors are 5926 in number. And as 8165:134000::5926:97,300 whence their security is better by upwards of , 3000 whence the corperation may pay off the same which will reduce the principal to , 131000 the interest of which at $5 \%$ is 6550 .

| Dr Profit \& Loss | P.S.D. | Contra Cr | P.S.D. |
| :--- | :--- | :--- | :--- |
| 7 th year Claims | 37,400 | By 5926 Premiums | 27952.10 .9 |
| Int at $5 \%$ of $134,000=6700]$ | 6,700 | $27953+100,084=128,037$ |  |
| $[37,400$ in claims $]$ |  | $128,037 @ 3 \%$ | 3840.10 |
|  |  | Ballance | 12306.3 .3 |
|  | 44100 |  | 44100 |


| Dr Profit \& Loss |  | Contra Cr |  |
| :--- | :--- | :--- | :--- |
| 8th year to Int. \& Claims | 44100 | By 5553 Premiums | 25856.3 .1 |
|  |  | $87778+25856=113634$ |  |
|  |  | $113634 @ 3 \%$ | 3840.18 |
|  |  | Ballance | 14834.10 .11 |
|  | 44100 |  | 44100 |

## (p.24, ms.)

9th year the ballance of loss is 17060.16 .6
10th 19369.18.2

Hence it appears that the $£ 134000$ advanced may be reduced to $£ 36500$ but even then there will be left a sufficient security for the payment of the claims of the survivors. For it was before proved that $£ 67000$ was a sufficient deposit for 8165 insured of whom only 4344 are now supposed to survive. Then as 8165:67000::4344:36385.

Again if it be considered that the persons who advanced the $£ 134,000$ have received 2 percent more than they could have made in the funds amounting to $£ 2680$ annually for 10 years with annuity at 3 percent annually amounts to $£ 30,700$ and upwards will not in this worst state of the corperation affairs be loosers
more than $1 / 2$ the sum advanced. [ $2 \%$ of 134,000 equals 2680 and an annuity immediate of 2680 for 10 years is approximately 30,700 ]

| For to the sum remaining in the Funds of | 36,500 |
| :--- | :--- |
| Add the value of the said Annuity | 30,700 |
|  | 6,720 |

But we proceed to show the repayment in the following 10 years.

| $11^{\text {th }}$ Year | Dr |  | Cr |
| :--- | :--- | :--- | :--- |
| To Int [at 5\%] on 134000 | 6700 | By 4434 Premiums |  |
| 92 Claims | 9200 | At 5.13.9 [5.6785] | 25218.7 .6 |
| Ballance | 11169.19 .6 | $36504+25,210=61722$ |  |
|  |  | 61722 at 3 pct. | 1851.12 .0 |
|  | 27069.19 .6 |  | 27069.19 .6 |

## (p.25, ms.)

| 12th year Profit | 10965.18 .6 |  |  |
| :--- | :--- | :--- | :--- |
| $13^{\text {th }}$ | 10756.3 .6 |  |  |
| $14^{\text {th }}$ | 10539.16 .6 |  |  |


| $15^{\text {th }}$ year Dr | P.S.D. | Cr | P.S.D. |
| :--- | :--- | :--- | :--- |
| to Int. on 121000 at $5 \%$ | 6050 | By 4066 Premiums [5.6875] | 23125.7 .6 |
|  |  |  |  |
| 92 claims | 9200 | $66936+23125=90061$ |  |
| Ballance | 10577.3 .6 | $90061 @ 3$ pct. | 2701.16 |
|  | 25827.3 .6 |  | 25827.3 .6 |

The survivors being reduced to 3974,65513 will be sufficient security for them for as 8165:134000::3974:65219.

And therefore $£ 12,000$ more may be paid off and the debt will be thereby reduced to $£ 109,000$.
It will now appear upon inquiry that the, 79936 in the Public Funds is more than a sufficient security to the 4066 survivors by $£ 13208$.... for as $8165: 13400:: 4066: 66728$.

Therefore $£ 13,000$ may be paid off the $£ 134,000$ and then interest will be payable only for $£ 121,000$ and no more than 66936 will remain in the public funds.

| $16^{\mathrm{th} \text { Dr }}$ | P.S.D | Cr | P.S.D. |
| :--- | :--- | :--- | :--- |
| Int on 134,000 at $5 \%$ | 5450 | By 3974 Premiums | 22602.2 .6 |


|  |  | $[5.6785]$ |  |
| :--- | :--- | :--- | :--- |
| 91 claims | 9100 |  |  |
| Ballance | 10695.11 .6 | $65513+22602=88,115$ |  |
|  |  | $88115 @ 3 \%$ | 2643.9 |
|  | 25245.11 .6 |  | 25245.11 .6 |

The number of survivors being now 3882,63778 will be a sufficient security for them and ,12500 may be again paid off which will reduce the debt to $, 96,500$. The 17 years profit will be ,10727.8.6

## \{p.26, ms.\}

The number of survivors being now but $3791, £ 62326$ will be a sufficient security for them and, 12200 more may be paid off which will reduce the debt to $£ 84300$. The 18th year the profit will be ,10793.19.

The survivors are now 3700 in number and $£ 60796.12 .6$ will be a sufficient security for them, therefore, , 12200 may be again paid off by which means the debt will be reduced to $£ 72100$. The 19th year the profit will be $£ 10793.19$

There are now but 3609 persons surviving for whom $£ 59290$ will be sufficient security for them, therefore, $£ 12300$ may again be paid off by which means the debt will be reduced to $£ 59800$. The 20th year the profit will be $£ 10830.15 .9$.

| Dr Public | Funds Contra Cr |  |
| :--- | :--- | :--- |
| 1st year to the principal | By Loss Paid | 921.14 .5 |
|  | Ballance | 133070.5 .7 |
| 134,000 |  | 134000 |


| Year | Payable on the <br> Ballance | The Ballance will be |
| :--- | :--- | :--- |
| $[1$ st year $]$ |  | 133070.5 .7 |
| 2nd year | 0 | 130335.12 .5 |
| 3rd | 0 | 125721.18 .8 |
| 4th | 0 | 119180.19 .3 |
| 5th | 0 | 110650.19 .3 |
| 6th | 0 | 100114.1 .7 |
| $7^{\text {th }}$ | 0 | 87777.10 .4 |
| 8th | 0 | 72942.19 .5 |
| $9^{\text {th }}$ | 0 | 55874.2 .11 |
| $10^{\text {th }}$ | 0 | 36504.4 .9 |
| $11^{\text {th }}$ | 0 | 11169.19 .6 |
| $12^{\text {th }}$ | 0 | 58640.2 .9 |
| $13^{\text {th }}$ | 0 | 69396.6 .3 |


| $14^{\text {th }}$ | 0 | 79936.2 .9 |
| :--- | :--- | :--- |
| $15^{\text {th }}$ | 13000 | 77513.6 .3 |
| $16^{\text {th }}$ | 12000 | 76208.17 .9 |
| $17^{\text {th }}$ | 12500 | 74436.6 .3 |
| $18^{\text {th }}$ | 12200 | 72996.12 .6 |
| $19^{\text {th }}$ | 12200 | 71590.11 .6 |
| $20^{\text {th }}$ | 12300 | 70121.7 .3 |
| Paid off | 74200 |  |

## \{p.27, ms.)

It appears therefore that the end of the 20th year there will be in the public funds upwards of ,70121 and that the debt of $£ 134000$ will be reduced to $£ 59800$ whence the corperation will be able to pay the same and will have upwards of $£ 10121$ in hand.

So that the corperation will upon the above supposition gain about 10,000 by insuring the 8155 persons in 20 years which is about 1.5 .1 [1.254] by each a sum so small that it cannot be expected to be lessened. From what has been said therefore, it may be concluded that a fund of $£ 134000$ upon every 8163 persons insured for $£ 100$ each or if 1641 upon every 100 insurers of that sum will be more than sufficient to answer all the demands that will happen in consequence of them and therefore, about , 1650 should be borrowed in the above manner for every $£ 100$ such insurances.

Again it is also evident that the persons who lend the said sum may be very easy with regard to their principal since it appears by the account above stated that even the worst circumstances that can be supposed the corperation will be to repay their principal with all their interest and be considerable gainers themselves within the space of 20 years.

In the last calculations made on purpose to see the worst situations of the corperation affairs we made the extravagant supposition that for 10 successive years after the establishment of the corperation there will annually die of the persons insured therein, a number equal to the greatest that has appeared by the Bills of Mortality to have died in anyone years since the great plague in 1665 but this is by no means to be admitted as a true state of their affairs.

## [3C ASSETS with Public Funds and 1731-1750 Deaths]

I shall therefore, now proceed to consider what that could have been if the

## \{p.28, ms.)

corperation had been established at the beginning of the year 1730, esteeming in such a case as this experience to be the best guide. In order to this we will again suppose that all the 8165 persons alive of the age of 40 became insured as before and that the several persons dying in each year may be found from the Bills of Mortality for the several years from 1730 to 1750 in the following manner.

| $[\underline{\text { Year }]}$ | $\underline{\text { Death between 40 and 50] }}$ | $\underline{\text { Deaths }]}$ | $\underline{\text { Year of their }}$ |
| :--- | :--- | :--- | :--- |


| 1731 | $2161<1>$ | $227<3>$ | 41 |
| :--- | :--- | :--- | :--- |
| 1732 | 2121 | 213 | 42 |
| 1733 | 2685 | 269 | 43 |
| 1734 | 2154 | 216 | 44 |
| 1735 | 2138 | 214 | 45 |
| 1736 | 2357 | 236 | 46 |
| 1737 | 2578 | 258 | 47 |
| 1738 | 2363 | 238 | 48 |
| 1739 | 2378 | 237 | 49 |
| 1740 | 2866 | 287 | 50 |


| $[$ Year $]$ | $[$ Deaths between 50 and 60] | [Deaths] | [Year of their Age] |
| :--- | :--- | :--- | :--- |
| 1741 | $2851<2>$ | 286 | 51 |
| 1742 | 2333 | 234 | 52 |
| 1743 | 2004 | 201 | 53 |
| 1744 | 1637 | 164 | 54 |
| 1745 | 1741 | 175 | 55 |
| 1746 | 2245 | 225 | 56 |
| 1747 | 2079 | 208 | 57 |
| 1748 | 2160 | 216 | 58 |
| 1749 | 2113 | 212 | 59 |
| 1750 | 2107 | 211 | 60 |

<1> Between the age of 40 and 50 there dyed 2261 in the year of 1731 . <2> Between the age of 50 and 60 there dyed [2852] in the year of [1741<3> In 1731 died 2261 persons of whom let us suppose that there dyed 227 in their 41st year of age.

## \{p.29, ms.\}

Then the state of the persons living and dying during the said term will be as follows:

| $\underline{\text { Age }}$ | $\underline{\text { Living }}$ | $\underline{\text { Out of whom die }}$ |
| :--- | :--- | :--- |
| 40 | 8165 | 227 |
| 41 | 7938 | 213 |
| 42 | 7725 | 269 |
| 43 | 7456 | 216 |
| 44 | 7240 | 214 |
| 45 | 7026 | 236 |
| 46 | 6790 | 258 |
| 47 | 6532 | 237 |


| 48 | 6295 | 237 |
| :--- | :--- | :--- |
| 49 | 6057 | 287 |
| 50 | 5770 | 286 |
| 51 | 5484 | 234 |
| 52 | 5250 | 201 |
| 53 | 5049 | 164 |
| 54 | 4895 | 175 |
| 55 | 4710 | 225 |
| 56 | 4485 | 208 |
| 57 | 4277 | 216 |
| 58 | 4061 | 212 |
| 59 | 3849 | 211 |
| 60 | 3638 | -- |

## \{p.30,ms.\}

But because 3638 , the number of the living at the age of 60 according to calculations, exceeds 3518 by 120, let us therefore suppose that for the first 12 years there die annually 10 more than in the last supposition then the numbers will stand thus.

| Age | Living | Out of whom die |
| :--- | :--- | :--- |
| 40 | 8165 | 237 |
| 41 | 7928 | 223 |
| 42 | 7705 | 279 |
| 43 | 7426 | 226 |
| 44 | 7200 | 224 |
| 45 | 6976 | 246 |
| 46 | 6730 | 268 |
| 47 | 6462 | 247 |
| 48 | 6215 | 248 |
| 49 | 5967 | 297 |
| 50 | 5670 | 296 |
| 51 | 5374 | 244 |
| 52 | 5130 | 201 |
| 53 | 4929 | 164 |
| 54 | 4765 | 175 |
| 55 | 4950 | 225 |
| 56 | 4365 | 208 |
| 57 | 4157 | 216 |
| 58 | 3941 | 212 |


| 59 | 3729 | 211 |
| :--- | :--- | :--- |
| 60 | $\underline{3518}$ | --- |
|  | 120943 | 4647 |

## \{p.31, ms.\}

Upon these suppositions the yearly state of the Society's Fund and Profit Loss will be as follows:

| DR Profit \& Loss | P.S.D. | Contra Cr | P.S.D. |
| :--- | :--- | :--- | :--- |
| Int on $134000 @ 5 \%$ | 6700 | By 8165 Premiums |  |
| 237 claims | 23700 | $@ 4.13 .11 / 2[4.65625]$ | 38018.5 .7 |
| Ballance | 12778.0 .7 | $134000+38010=172010$ |  |
|  |  | $172010 @ 3 \%$ | 5160 |
|  | 43178.5 .7 |  | 43278.5 .7 |


| Dr Public Funds | P.S.D. | Contra Cr | P.S.D. |
| :--- | :--- | :--- | :--- |
| To Principal | 134,000 | Ballance | 146778.5 .7 |
| Add | $12,778.5 .7$ |  |  |
|  | $146,778.5 .7$ |  | 146778.5 .7 |


| 2nd year To Int | 6700 | By 7928 Premiums | 36914.15 |
| :--- | :--- | :--- | :--- |
| 223 Claims | 22,300 | $146778+36950=183693$ |  |
|  |  |  |  |
| Ballance | $13,425.9$ | $183693 @ 3$ | 5510.14 |
|  | 42425.9 |  | 42425.9 |


| Dr Public Funds |  | Contra Cr |  |
| :--- | :--- | :--- | :--- |
| Principal | 146778.5 .7 | Ballance | 160203.14 .7 |
| Add | 13425.9 |  |  |
| 3rd year gain | 7150.4 .3 | Ballance | 167362.7 .10 |
| 4th year gain | 11335.4 .3 | Ballance | 178697.11 .10 |
| 5th year gain | 10791.12 .0 | Ballance | 189489.3 .10 |
|  |  |  |  |

The sum now vested in the funds is now by much to great a security for the 6976 persons remaining alive for 8165:134000::6976:114487 but because there shall be no difficulty upon that account let us suppose that the original $£ 134000$ shall remain entirely in the funds and that once in 5 years the surplus, if any, shall be applied in the discharge of the Society's debts.

## \{p.32, ms.\}

Then from 189000 take $£ 134000$ and the remainder $£ 55000$ may now be paid off which will reduce the corperation's debt to ,79000, the interest of which is at $5 \%--3950$.

| Dr Profits \& Loss |  | Contra Cr |  |
| :--- | :--- | :--- | :--- |
|  |  | 6th year |  |
| To Int on 79000@5\% | 3950 | By 6976 Premiums | 32482 |
| 246 Claims | 24600 | Int at 3\% |  |
|  |  | on Prin. \& Prem. | 4994.8 |
| Ballance | 8926.8 |  |  |
|  | 37476.8 |  | 37476.8 |


|  |  | of Prin. paid off | 55000.0 .0 |
| :--- | :--- | :--- | :--- |
| To $6^{\text {th }}$ yrs gain | 8926.8 | Ballance | 143415.11 .10 |
| 7 th | 5829 | Ballance | 149244.13 .1 |
| $8^{\text {th }}$ | 6818.11 .9 | Ballance | 156063.4 .10 |
| $9^{\text {th }}$ | 5738.11 .10 | Ballance | 161801.16 .8 |


|  |  | $10^{\text {th }}$ year |  |
| :--- | :--- | :--- | :--- |
| To Interest | 3950 | by 5967 Prem[4.65625] | 27783.16 .10 |
| 297 claims | 29700 | 189585 at $3 \%$ | 5687.8 .10 |
|  |  | Ballance | 178.15 .2 |
|  | 33650.0 |  | 33650.0 |


| Dr Public Funds |  | $10^{\text {th }} \mathrm{Yr}$ Contra Cr |  |
| :--- | :--- | :--- | :--- |
| to rest 9th year | 161801.16 .8 | By Loss | 178.15 .2 |
|  |  | Ballance 161623 | 161623.1 .6 |
|  | 161801.16 .8 |  | 161801.16 .8 |
|  |  |  |  |


| The second period of 5 years being now expired |  |
| :--- | :--- |
| From the Ballance of Public Funds | 161623.1 .6 |
| Take the Security | 134000.0. |
| And there remains | 27623.16 |

And therefore $\$ 27000$ may again be paid off the Society's remaining debt of $\$ 79000$ which will reduce it to
$\$ 52000$, the interest of which at $5 \%$ is $£ 2600$.

## \{p.33, ms.\}

| Dr Profit \& Loss | P.S.D. | Contra Cr | P.S.D. |
| :--- | :--- | :--- | :--- |
| To Interest | 2600.0 | by 5670 Premiums | 32248.2 .6 |
|  |  | at 5.13.9. [5.6875] |  |
| 296 claims | 29600 | 13400 |  |
| Ballance | 5035.12 .6 | 32248 |  |
|  |  | 166248 at 3\% | 4907.10 |
|  | 37235.12 .6 |  | 37235.12 .6 |


| Dr Public Funds | P.S.D. | Contra Cr | P.S.D. |
| :--- | :--- | :--- | :--- |
| To Ballance of the 10 <br> yrs | 161623.1 .6 | By Principal | 27000 |
| Add | 5035.12 .6 | Ballance | 139658.14 |
|  | 166658.14 |  | 166658.14 |


| Year | Profit | Ballance |
| :--- | :--- | :--- |
| 12 | 8671.4 .6 Ballance | 148329.10 .6 |
| 13 | 11801.17 .6 | 160131.16 |
| 14 | 14678.9 .9 | 174110.5 .9 |
| 15 | 13858.4 .9 | 187868.10 .6 |
|  |  |  |
|  | Now if from the Sum resting | 187868.10 .6 |
|  | There be taken the Security | 134000 |
|  |  | 53868.10 .6 |

Which will more than pay off the remaining part of the debt viz ,52000. Let us then suppose that the same is now discharged and there will then remain ,135868.10.6 in the Funds which is all clear profit and from this time it will be proper to keep an account of the Profit and Loss only which will be found to be as follows:

| 16 years profit | 14435.5 .0 |
| :--- | :--- |
| 17 Do | 153433.13 .9 |
| 18 Do | 160788.27 .6 |
| 19 Do | 167499.6 .3 |
| 20 Do | $\mathbf{1 7 3 , 2 6 9}$ |

Hence it appears that if the Corperation had been established at the beginning of the years 1730 and

## [p.34, ms.]

that 8165 persons had then insured with them of the age of 40 upon terms above specified for 20 years, that the gain of the Corperation would not withstanding the having paid their debt of ,134,000 and its interest at $5 \%$ be possessed of $£ \mathbf{1 7 3 , 2 6 9}$ and since upon the above calculation, it is supposed that 3518 persons out of the 8165 remain alive at the end of 20 years, if the half of them were to die in the next year viz 1759 persons, the corperation would be able to pay the claims arising.

To each of the above mentioned 3518 persons each aged 60 which are supposed to remain alive, the sum of $£ 100$ is to be paid at their decease. Now let us suppose that the corperation will contract with some other body, take the insuring of these off their hands by a payment of a sum in hand, being allowed discounts for the money so advanced them by computing the sum for which , 100 may be insured for a life of 60 on the manner already planned, it will amount to about $£ 75.11 .9[75,588]$. Therefore, the sum for which the whole 3518 persons may be so insured will be $£ 265916.76 .6$ and this may be considered at the present value of all the sums which the Corperation has to pay.

But they are also to receive the Annual Premium due from the insured survivors which because they are now entered into the Period of Life between $60 \& 70$ will be advanced to 6.5 .0 [ 6.250 ] upon those who are members of the corpertaion, which increased by its $1 / 4$ part $£ 1.11 .3$ [1.563 will amount to $£ 7.16 .3$ [7.8125]. Those whose payments are now computing and 3518 annuities of

## [p. 35, ms.]

$£ 7.16 .3$ each during the Lives of like number of persons each age 60 years will be 138 years, purchase which on a mean of the London Bills of Mortality is the value of such a Life amounts to $£ 223667.15 .0$ [223667 = (7.8125)(8.138)(3518) and the state of the Account will be as follows:

| Dr Profit \& Loss |  | Contra Cr |  |
| :--- | :--- | :--- | :--- |
| To 3418 Pymts due | 25616.16 .6 | By ballance of 20 yrs | 173269 |
| As before mentioned |  | Account |  |
| Profit on the Whole | 131019.18 .6 | 3518 annuities | 223667.15 |
|  | 395936.15 |  | 3969636.15 |

Whence upon the insuring 8165 persons of 40 years each for their whole lives the Corpertaion would if it had been established in the year 1730 have gained $£ 131019.18 .6$ which is one with another about $£ 17.6$ [17.300] each.

## [3D. FUNDS with MEAN MORTALITY]

Having above made the calculations relating to the insurance of such persons, as shall not chance [sp?] to become members of the Corperation upon a supposition of its establishment in the years 1730, it seems necessary to consider the case of those who shall become members upon the same supposition.

Here there will be no occasion to borrow, since if necessary, every person is liable to pay his share of the loss and the Premiums of Insurance will be but $4 / 5$ of those in the preceeding case.

| Dr Profit \& Loss |  | Contra Cr |  |
| :--- | :--- | :--- | :--- |
| Year 1 |  |  |  |
| 237 Claims | 23700 | By 8165 Premiums | 30414.12 .6 |
| Ballance | 7614.12 .1 | at 3.14.6 [3.725] |  |
|  |  | Int. on $\$ 30,000$ at $3 \%$ | 900 |
|  | 31314.12 .6 |  | 31314.12 .6 |

## [p.36, ms.]

|  | Ballance after <br> Claims |
| :--- | :--- |
| $[1]$ | $[7,614.12 .1]$ |
| 2 | $15,960.12 .6$ |
| 3 | $10,101.11$ |
| 4 | $24,536.4$ |
| 5 | $30,496.14$ |
| 6 | $33,576.14$ |
| 7 | $33,605.6$ |
| 8 | $34,706.10$ |
| 9 | $34,792.17 .6$ |
| 10 | $29,030.5$ |
| 11 | $26,873.10$ |
| 12 | $28,464.19$ |
| 13 | $33,260.9$ |
| 14 | $40,957.19$ |
| 15 | $47,017.15$ |
| 16 | $47,439.5$ |
| 17 | 48,519 |
| 18 | $47,856.5$ |
| 19 | $46,561.4$ |
| 20 | $44,334.3$ |

It remains to compute the expectation of the corperation has of farther receipts from the 3518 persons left alive at the age of 60 and for distinctions sake, we shall divide the remainders of life into two periods viz 60 \& 70 from 70 upwards it was before found that the yearly premium for insuring one life with anthers between the ages of $60 \& 70$ was about ,6.5. And because we would compute it at the most reasonable value let us suppose the premium for insuring a life of $70 \&$ upwards to be one with another, $14 \&$ lastly because we would suppose the worst let only half of the 3518 persons alive at the agree of 60 be presumed to be alive at the beginning of the 70 years.

Hence the expectation that the corperations will have on the 3518 persons alive at 60 will be 3518 annuities of $, 14 \&, 6.5,7.5[14,6.250,7.750]$ each for the lives of as many persons aged 70 . The first age being worth about 8.138
[p.37, ms.]
years per above (see page 35 ) \& the latter 4.977 years purchase if computed at $3 \%$ compound interest and the Acct will stand thus:

| Dr Profit \& Loss |  | Contra Cr |  |
| :--- | :--- | :--- | :--- |
| The deaths of as many |  | By Ballance at the end of 20 <br> years | $44,333.1 .0$ |
| Persons [3158] each aged <br> 60 at 75.11.9 each Claims | $265,916.16 .6$ | 3518 annuities of $£ 6.5$ for <br> lives at 60 | $178,934.4 .0$ |
| Profit | $25,198.10 .6$ | 1759 Annuities of $£ 7.15$ for <br> the lives of as man persons of <br> age 70 | $67,048.2 .0$ |
|  | $291,115.7$ |  | $291,115.7$ |

Whence it appears that if the corperations had been established in 1730 that the 8165 persons each aged 40 had become members thereof then the gains of the corperation might reasonably be estimated on that account ,25198.10.6 which would be upwards of , 3 on each person. But it maybe objected to the above account that many of not most of the members of the corperation will insure for their whole life at a constant annual premium.

## [4A. WHOLE LIFE INSURANCE PROJECTION]

I shall therefore proceed to show the state of the corperations affairs upon that supposition retaining the same ages and numbers except in the premium for I have found that the constant annual premium for a life of 40 should be $£ 4.12 .6$ [4.625]

| Dr Profit \& Los |  | Contra Cr |  |
| :--- | :--- | :--- | :--- |
| Year 1 |  |  |  |
| To 237 Claims @ 100 | 23700 |  |  |
| Ballance | 15194.2 .6 | By 8165 Premium <br> at 4.12.6 | 37763.2 .6 |
|  |  | Int 37700 @3\% | 1131 |
|  | 38894.2 .6 |  | 38894.2 .6 |

## [p. 38, ms.]

Second year the Ballance will be after the claims are paid as above and the premium received on 7928 persons left alive and interest made on the cash [sp?] then in hand 31115.2.6. [We have added columns for premium, interest, claims, cash flow and assets. The assets do not equal the "Ballance" amounts because of
rounding.]

| Year | $[$ Ballance $]$ | $[$ Premium $]$ | [Interest] | [Claims] | [Cash Flow] | [Assets] |
| :--- | :--- | ---: | ---: | :--- | ---: | ---: |
| $[1]$ | $15,194.2 .6$ | $37,763.13$ | $1,132.89$ | $(23,700)$ | $15,196.02$ | $15,196.02$ |
| 2 | 31115.2 .6 | $36,667.00$ | $1,555.89$ | $(22,300)$ | $15,922.89$ | $31,118.91$ |
| 3 | 40851.15 | $35,635.63$ | $2,002.64$ | $(27,900)$ | $9,738.26$ | $40,857.17$ |
| 4 | 54850 | $34,345.25$ | $2,256.07$ | $(22,600)$ | $14,001.32$ | $54,858.49$ |
| 5 | 68393 | $33,300.00$ | $2,644.75$ | $(22,400)$ | $13,544.75$ | $68,403.25$ |
| 6 | 79075 | $32,264.00$ | $3,020.02$ | $(24,600)$ | $10,684.02$ | $79,087.27$ |
| 7 | 86707.5 | $31,126.25$ | $3,306.41$ | $(26,800)$ | $7,632.66$ | $86,719.92$ |
| 8 | 95391 | $29,886.75$ | $3,498.20$ | $(24,700)$ | $8,684.95$ | $95,404.87$ |
| 9 | 103058.7 .6 | $28,744.38$ | $3,724.48$ | $(24,800)$ | $7,668.85$ | $103,073.72$ |
| 10 | 104873.15 | $27,597.38$ | $3,920.13$ | $(29,700)$ | $1,817.51$ | $104,891.23$ |
| 11 | 105427.10 | $26,223.75$ | $3,933.45$ | $(29,600)$ | 557.20 | $105,448.43$ |
| 12 | 109788.5 | $24,854.75$ | $3,909.10$ | $(24,400)$ | $4,363.85$ | $109,812.28$ |
| 13 | 117522.10 | $23,726.25$ | $4,006.16$ | $(20,100)$ | $7,632.41$ | $117,444.68$ |
| 14 | 128128.2 .6 | $22,796.63$ | $4,207.24$ | $(16,400)$ | $10,603.86$ | $128,048.55$ |
| 15 | 137169.5 .0 | $22,038.13$ | $4,502.60$ | $(17,500)$ | $9,040.73$ | $137,089.27$ |
| 16 | 140647 | $22,893.75$ | $4,799.49$ | $(22,500)$ | $5,193.24$ | $142,282.51$ |
| 17 | 144859.2 .6 | $20,188.13$ | $4,874.12$ | $(20,800)$ | $4,262.24$ | $146,544.76$ |
| 18 | 147405.5 | $19,226.13$ | $4,973.13$ | $(21,600)$ | $2,599.25$ | $149,144.01$ |
| 19 | 149400.7 .6 | $18,227.13$ | $5,021.13$ | $(21,200)$ | $2,048.26$ | $151,192.27$ |
| 20 | 150545 | $17,246.63$ | $5,053.17$ | $(21,100)$ | $1,199.79$ | $152,392.06$ |

## [4B. Accumulation of individual Premiums]

The credit side of the account of a member of the intended corperation insuring for the remainder of a life of the age of 40 [is shown below].

| 1 year premium [4.625] | $£ 4.12 .6$ |  |  |
| :--- | :--- | :--- | :--- |
| 2nd yr int on 4.12.6@3\% |  | $0.2 .9^{1 / 4}$ | $4.15 .3 .1 / 4$ |
| Premium |  | $4.12 .6^{1 / 4}$ | $9.7 .9^{1 / 4}$ |
|  |  |  |  |
| 3rd yr int on 4.15.3 $1 / 4$ |  | $0.5 .7 .1 / 2$ |  |
| Premium |  | 4.12 .6 | $4.18 .1^{1 / 2} 2$ |
| [Total] |  |  | 14.5 .10 |

## [p.39, ms.]

[In the chart below we add columns for premium, interest, and ending asset in $1 / 1000$ of $£$. The "accumulation" in $£$.S.P differs from decimal numbers because of rounding.]

| Year | [Accumulation] | [Premium | [Interest] | [Accumulation] |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4.15.3 1/4 | 4.625 | 0.139 | 4.764 |
| 2 | 9.7.9 1/4 | 4.625 | 0.282 | 9.670 |
| 3 | 14.5.10 3/4 | 4.625 | 0.429 | 14.724 |
| 4 | 19.6.11 3/4 | 4.625 | 0.580 | 19.930 |
| 5 | 24.11.1 1/4 | 4.625 | 0.737 | 25.291 |
| 6 | 29.18.4 | 4.625 | 0.897 | 30.814 |
| 7 | 35.8.9 1/4 | 4.625 | 1.063 | 36.502 |
| 8 | 41.2.6 1/2 | 4.625 | 1.234 | 42.361 |
| 9 | 46.19 .8 3/4 | 4.625 | 1.410 | 48.395 |
| 10 | 53.0.5 | 4.625 | 1.591 | 54.611 |
| 11 | 59.4.8 3/4 | 4.625 | 1.777 | 61.013 |
| 12 | $65.12 .91 / 4$ | 4.625 | 1.969 | 67.607 |
| 13 | 72.4.7 3/4 | 4.625 | 2.167 | 74.399 |
| 14 | 79.0.5 3/4 | 4.625 | 2.371 | 81.395 |
| 15 | 86.0.4 3/4 | 4.625 | 2.581 | 88.601 |
| 16 | 93.4.6 | 4.625 | 2.797 | 96.022 |
| 17 | 100.12.11 1/4 | 4.625 | 3.019 | 103.667 |
| 18 | 108.5.10 | 4.625 | 3.249 | 111.541 |
| 19 | 116.3.3 3/4 | 4.625 | 3.485 | 119.650 |
| 20 | 124.5.6 1/4 | 4.625 | 3.728 | 128.004 |
| 21 | 132.12.7 | 4.625 | 3.979 | 136.608 |
| 22 | 141.4 .8 | 4.625 | 4.237 | 145.470 |
| 23 | 150.1.11 | 4.625 | 4.503 | 154.597 |
| 24 | 159.4.5 1/2 | 4.625 | 4.777 | 163.999 |
| 25 | 168.12 .6 | 4.625 | 5.059 | 173.683 |
| 26 | 178.6.2 | 4.625 | 5.349 | 183.657 |
| 27 | 188.5.7 1/4 | 4.625 | 5.648 | 193.931 |
| 28 | 198.11.1 3/4 | 4.625 | 5.957 | 204.512 |
| 29 | 209.2.8 1/4 | 4.625 | 6.274 | 215.411 |
| 30 | 220.0.8 1/4 | 4.625 | 6.601 | 226.637 |
| 31 | 231.5.3 | 4.625 | 6.938 | 238.200 |
| 32 | 242.16 .6 | 4.625 | 7.285 | 250.110 |
| 33 | 254.14.8 1/4 | 4.625 | 7.642 | 262.377 |
| 34 | 267.0.0 1/2 | 4.625 | 8.010 | 275.012 |
| 35 | 279.12 .9 | 4.625 | 8.389 | 288.026 |
| 36 | 292.13.0 1/4 | 4.625 | 8.780 | 301.431 |
| 37 | 306.1.1 1/2 | 4.625 | 9.182 | 315.237 |


| 38 | 319.7 .3 | 4.625 | 9.596 | 329.458 |
| :--- | :--- | ---: | ---: | ---: |
| 39 | 334.1 .8 | 4.625 | 10.022 | 344.106 |
| 40 | 348.14 | 4.625 | 10.462 | 359.193 |
| 41 | $363.16 .41 / 4$ | 4.625 | 10.915 | 374.732 |
| 42 | $379.7 .13 / 4$ | 4.625 | 11.381 | 390.738 |
| 43 | 395.7 .3 | 4.625 | 11.861 | 407.224 |
| 44 | $411.16 .11 / 2$ | 4.625 | 12.355 | 424.204 |
| 45 | $428.16 .61 / 4$ | 4.625 | 12.865 | 441.694 |
| 46 | $446.6 .41 / 2$ | 4.625 | 13.390 | 459.709 |
| 47 | 464.6 .8 | 4.625 | 13.930 | 478.264 |

It appears from the above account that a member of the corperation who insures at the age of 40 will if compound interest be allowed him at $3 \%$ have paid the , 100 which is insured to him at the time of his paying the 17 th premium and about $£ 0.13 .0[0.650]$ more and consequently the corperation will gain by all those who live to pay 17 or more premiums. It appears also that a member who lives to pay the 47 premium at which time he will be aged 86 will have overpaid the $£ 100$ insured by upwards of $£ 364$.

It seems, therefore, unreasonable to divide any part of the profit among those who have not paid as much as they receive because they have evidently a profit by being paid their claims which profit is very great in first

## [ $\mathrm{p} .40, \mathrm{~ms}$ ]

years especially as those who live at the extremity of old age will be such losers at seems reasonable therefore to divide the profit amongst the persons to have paid more their claims and to do this in proportion to the sums so overpaid. In order to do this we should consider that the number of persons dying each year is different therefore in order to find the whole sum which has been advanced by each particular person must for each year be multiplied by the number so dying in that year as in the following table.

The corperation will have gained by each of the persons who have been supposed to die in that year in which they had paid the 17 th premium the sum of $£ \mathbf{0 . 1 2 . 1 1} \mathbf{1} / \mathbf{4}$ therefore the whole gain by the 208 persons dying will be $£ 134.12 .11$ 1/4.

| Persons dying | Year | Will gain by each | [Total gain] |
| :--- | :--- | :--- | :--- |
| 208 | 17 | $0.12 .11^{1 / 2}$ | $\mathbf{1 3 4 . 1 2 . 4 ~ 1 / 4}$ |
| 216 | 18 | 8.5 .10 | $1791.0 .53 / 4$ |
| 212 | 19 | $16.3 .53 / 4$ | $3427.1 .101 / 2$ |
| 211 | 20 | $24.5 .61 / 4$ | 5122.2 .8 |
| 176 | 21 | 32.12 .7 | $5742.13 .31 / 4$ |
| 176 | 22 | 41.4 .8 | $7256.18 .103 / 4$ |
| 176 | 23 | 50.1 .11 | $8816.13 .01 / 4$ |
| 176 | 24 | 59.4 .5 | 10423.3 .0 |
| 176 | 25 | 68.12 .6 | 12077.16 .10 |


| 176 | 26 | 78.6 .2 | 13782.3 .6 |
| :--- | :--- | :--- | :--- |
| 176 | 27 | $88.5 .7^{1 / 4}$ | 15537.12 .9 |
| 176 | 28 | $98.11 .1^{1 / 4}$ | 17345.15 .3 |
| 176 | 29 | $109.2 .8^{1 / 4}$ | $19098.19 .101 / 4$ |
| 175 | 30 | $120.0 .8^{1 / 4}$ | $21006.9 .81 / 2$ |
| 157 | 31 | 131.5 .3 | $19820.12 .01 / 4$ |
| 157 | 32 | 142.16 .6 | 21566.11 .7 |
| 151 | 33 | $154.14 .8^{1 / 2}$ | 23364.19 .2 |
| 151 | 34 | $167.0 .01 / 3$ | $25217.5 .61 / 2$ |
| 151 | 35 | 179.12 .9 | $27125.3 .31 / 2$ |
| 151 | 36 | $192.13 .01 / 4$ | $29090.5 .91 / 4$ |
| 150 | 37 | $206.1 .1^{1 / 2}$ | $30908.6 .21 / 2$ |
| 150 | 38 | 219.17 .3 | $32979.6 .11 / 2$ |
| 126 | 39 | $234.1 .8^{1}$ | 29443.8 .10 |
| 101 | 40 | $248.14 .7^{1 / 4}$ | 25121.15 .3 |
| 100 | 41 | $263.16 .4^{1 / 4}$ | $26381.14 .43 / 4$ |
| 75 | 42 | $279.7 .1^{1 / 2}$ | $20951.15 .01 / 4$ |
| 50 | 43 | $295.7 .3^{2}$ | $14768.2 .41 / 4$ |
| 26 | 44 | $311.16 .11^{1 / 2}$ | $8108.1 .01 / 4$ |
| 26 | 45 | $328.16 .6^{1 / 4}$ | $8549.10 .101 / 2$ |
| 25 | 46 | $346.6 .4^{1 / 2}$ | $8657.19 .21 / 4$ |
| 25 | 47 | $368.6 .8^{2}$ | 9108.6 .6 |
| $[$ Total $]$ |  |  | 502.777 |
|  |  |  |  |

## [p.41, ms.]

Whence it appears that the whole sum advanced more than have been paid in claims is about $£ \mathbf{5 0 2 , 7 7 7}$ in proportion to which sum the net gain viz $\mathbf{7 1 , 7 1 6}$ is to be divided this may be done by the method commonly taught in schools called fellowship by saying as the whole sum advanced by each particular set of persons to their share in the said sum to be divided which share must again be divided by the number of persons in each set in order to find the share of each particular person the results of these operations are inserted in the following table.

The proportional share of real gain viz $£ 71716.7 .6$ which ought to be equally divided among the persons who have been supposed to die in that year in which they have paid the premium will by calculation appear to be $£ 19.4 .1$ [19.205] whence each individual of the said particular set of persons ought to receive $£ 0.1 .10$ 1/4 [0.093].

| Persons dying | year | Total gain | to each |
| :--- | :--- | :--- | :--- |
| 208 | 17 | 19.4 .1 | $0.12 .111 / 2$ |
| 216 | 18 | $255.9 .53 / 4$ | 1.3 .8 |
| 212 | 19 | $488.16 .101 / 4$ | $2.6 .11 / 2$ |


| 211 | 20 | 730.12.5 1/2 | 3.9.3 |
| :---: | :---: | :---: | :---: |
| 176 | 21 | 810.2.8 1/2 | 4.13 .1 |
| 176 | 22 | 1035.2.8 | 5.17.7 1/2 |
| 176 | 23 | 1257.12.2 1/4 | 7.2.11 |
| 176 | 24 | 1486.15.3 1/4 | 8.8.11 1/4 |
| 176 | 25 | 1722.15.9 1/2 | 9.10.9 1/4 |
| 176 | 26 | 1965.17.11 | 11.3.4 1/4 |
| 176 | 27 | 2216.5.10 1/4 | 12.11.10 1/4 |
| 176 | 28 | 2474.4.2 | 14.1.2 |
| 175 | 29 | 2724.5.9 1/4 | 15.11.4 1/4 |
| 175 | 30 | 2996.7.0 3/4 | 17.2.5 1/4 |
| 151 | 31 | 2827.4.4 1/2 | 18.14.5 1/2 |
| 151 | 32 | 3076.5.4 | 20.7.5 1/2 |
| 151 | 33 | 3332.15.8 1/2 | 22.1.5 $1 / 4$ |
| 151 | 34 | 3597.0.0 3/4 | 23.16 .5 |
| 151 | 35 | 3869.2.11 | 25.12.5 1/2 |
| 151 | 36 | 4149.9.0 1/2 | 27.9.7 1/2 to each |
| 150 | 37 | 4408.15.6 3/4 | 29.7.10 |
| 150 | 38 | 4704.3.8 1/2 | 31.7.2 3/4 |
| \{42 m |  |  |  |
| 126 | 39 | 4204.3.0 1/2 | 33.7.9 1/2 |
| 101 | 40 | 3583.7.6 3/4 | 35.9.7 |
| 100 | 41 | 3763.2.0 1/4 | 37.12.7 1/2 |
| 75 | 42 | 2988.11.4 1/4 | 39.16.11 1/4 |
| 50 | 43 | 2106.10.7 1/2 | 42.2.6 1/4 |
| 26 | 44 | 1156.10 .9 | 44.9.7 3/4 |
| 26 | 45 | 1219.10.2 3/4 | 46.18.1 |
| 25 | 46 | 1234.19.6 1/4 | 49.7.11 3/4 |
| 25 | 47 | 1299.4.3 1/4 | 51.19.4 1/2 |
| Tot | --- | 71716.7.5 ${ }^{\text {/4 }}$ |  |

It appears from the foregoing calculations that the mean profit bears but small proportion of the sum advanced more than has been paid in claims the former being less than $1 / 7$ of the latter, how inequitable then will it be to take away any part thereof in order to give the same to those who have received more than they have advanced in all.

## [4B AMICABLE SOCIETY]

It plainly appears also from the calculations that the Amicable Society takes a premium too great by much of
most of the persons whose lives they insure for it is clear that less than $£ 5$ is a sufficient premium for a life of 40 yet their practice is to take $£ 5$ even for those who are not 20 if any such insure with them and refuse to insure any lives older than 45 so that it should seem that every person who is not near that period has disadvantage in such insurance.

Again those calculations will serve to show in a very strong light the absurdity of that custom which they have established of dividing among the persons who die any particular year all the premiums that they received in that year.

For first by that means if a person dies in the first year and that

## [p.43, ms.]

should happen to be one of those in which but few die his nominee might be entitled to receive $£ 180$ or upwards at the expense of a single $£ 5$ and that the person who have paid for many years it may be double or treble the $£ 100$ yet if he die in a year in which many others die he shall receive no more that a single 4100 which is as I think not only absurd but unjust. Secondly if they were not to have a sufficient supply of young persons insure with them nothing but their boosted $£ 29000$ in stocks could enable them towards the close of life to divide even the $£ 100$ this will appear very evident by comparing their numbers with those above used as well as by what was formerly their practice. By their own accounts the whole number of persons insured with them amount of about 800 which being nearly the tenth part of 8165 the number of persons whose insurance was before considered average will the better to enable us to compute/suppose 816 to be the tenth part of consequently reckoning them to be one with another at the age of 40 we may for the number dying each year safely take $1 / 10$ part of those above computed by the manner of calculating according to their custom will be different to the preceeding on two accounts for:

First when but a small number die in anyone year they pay the full premiums received and...
Secondly as they receive their premiums quarterly then cannot make up same interest of them as we shall.
Suppose for instance , 400 were to be received of 800 persons insured then

## [P.44, ms.]

$£ 1000$ only could be put to interest the first quarter-- $£ 2000$ the 2 nd quarter -- $£ 3000$ the 3 rd as computed on the other side.

| Interest for $£ 1000$ for $1 / 4$ year at $3 \%$ | 7.10 .0 | 7.5 |
| :--- | :--- | :--- |
| 2000 Do | 15 | 15 |
| 3000 Do | 22.10 .0 | 22.5 |
| 4000 Do | 30 | 30 |
|  | 75 | 75 |

Which upon the whole $£ 4000$ is but $£ 1.17 .6$ [1.875] percent per annum. Let us then rate their case at the age of 70 at which age we before supposed that out of our 8165 there remains 1760 alive whence we may suppose they have 176 left of their 800 and since 151 our of our 1760 die that year let 15 of their 176 do so and their account will stand thus the 31 year.

| 15 Claims @ $£ 100$ each --1500-- | by 176 Premiums at $£ 5$ | 880 |
| :--- | :--- | :--- |


|  | Int @ $1.875 \%$ | 16.500 |
| :--- | :--- | :--- |
|  | Ballance | 603.500 |
| 1500 |  | 1500 |

Now since the loss cannot be provided for as with us by the savings out of those years when the savings overpaid the claims, it must necessarily fall upon their capital of the interest they each make upon their capital will barely defray the expense of house rent $\&$ servents as will appear by seeing the stake of their accounts when the shore 816 persons were supposed alive.

| To 24 claims at $£ 170$ each | 4080 | By 816 Premiums at $£ 5$ | 4080 |
| :--- | :--- | :--- | :--- |
|  |  | Int @ $1.875 \%$ | 76.10 |
| House Rent | 300 |  |  |
|  |  | Ballance | 223.10 |
|  | 4380 |  | 4380 |

## [p.45, ms.]

It is quite happy for them therefore that they have this fund of ,29000 to support them for without it their management would soon ruin them. But a question very naturally arises viz how did they get this capital. It is difficult to be very certain in the answer but this much is known.

First that in the beginning there was a stipulation in the charter that the nominees of them that died in some number of years limited should not receive the full $£ 100$.

Secondly they took $£ 6.4 .0$ [6.200] annual premiums for a good many years after their first establishment and if I understand right divided only $£ 5$ therefore in the manner above described.

Thirdly if as it should seem they did they took the $£ 7.0 .0$ [7.000] as a deposit from their first members which those who insure with them now are abridged to pay for their blank or vacant numbers or even ,6 thereof than we may presume that they were possessed of 800 times $£ 6$ or 4800 about 40 years of agoe or sooner the growing interest of which they have all along enjoyed and this at $4 \%$ and that they may be supposed to have made of it one time will amount to $£ 23000$ of the money calculations for members supposed to insure in the year 1730 .

## [4C ADEQUATE PREMIUM WITH CALLS VS. AN INITIAL FUND]

| Dr Profit \& Loss |  | Contra Cr |  |
| :--- | :--- | :--- | :--- |
| 1 year |  |  |  |
| To 237 claims £ 100 each | 23700 | By 8165 persons at | 37763.2 .6 |
| Ballance | 15194.2 .6 | \#4.12.6 [4.625]7 |  |
|  |  | Int on \#37770 @3 | 1131 |


|  | 38894.2 .6. |  | 38894.2 .6 |
| :--- | :--- | :--- | :--- |
| \{page 46, ms $\}$ |  |  |  |
| 2 year |  |  |  |
| To 233 Claims | 22300 | By Ballance | 15194.2 .6 |
| Ballance | 31115.2 .6. | 7928 Premiums | 36667 |
|  | 53415.2 .6 | Int on 51800 | 53415.2 .6 |


| Years | Claims | Premiums | [Amt. Premium $]$ | [Interest] | Ballance |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $[1]$ | 237 | 8165 | 37763.2 .6 | 1131 | 38894.2 .6 |
| $[2]$ | 223 | 7928 | 15194.2 .6 | 1554 | 53415.2 .6 |
| 3 | 279 | 7705 |  |  | 40851.15 |
| 4 | 226 | 7426 |  |  | 54850 |
| 5 | 224 | 7200 |  |  | 68393 |
| 6 | 246 | 6976 |  |  | 79075 |
| 7 | 268 | 6730 |  | 86707.5 |  |
| 8 | 247 | 6462 |  |  | 95391 |
| 9 | 248 | 6215 |  |  | 103058.7 .6 |
| 10 | 297 | 5967 |  |  | 104873.15 |
| 11 | 296 | 5670 |  |  | 105247.10 |
| 12 | 244 | 5374 |  |  | 109788.5 |
| 13 | 201 | 5130 |  |  | 117419.10 |
| 14 | 164 | 4929 |  |  | 137022.2 .6. |
| 15 | 175 | 4765 |  |  | 140535 |
| 16 | 225 | 4590 |  |  | 144744.2 .6. |
| 17 | 208 | 4365 |  |  | 147287.5 |
| 18 | 216 | 4157 |  |  | 149297.7 .6 |
| 19 | 212 | 3941 |  |  | 150241 |
| 20 | 211 | 3729 |  |  |  |

At the end of the 20 years from 1730 to 1750 , the corperations affairs with regard to the insurance of the following 8165 members will stand assumed.

| To 3518 claims of ,100 <br> each |  | By ballance on hand | 150421 |
| :--- | :--- | :--- | :--- |
| Due on the death of |  | 3518 annuities of |  |
| As many persons each | 265916.16 | 4.12 .6 for the lives |  |
| Aged 60 at 75.11 .9 |  | of as many persons |  |
| 75.588 |  | each aged 60. | 134411.7 .3 |


| Ballance | 16915.10 .9 |  | 282832.7 .3 |
| :--- | :--- | :--- | :--- |
|  | 282832.7 .3 |  |  |

## [p.47, ms.]

Calculations for members on the mortality 1741 continued to co-successive years.

| Dr Stocks |  | Contra Cr |  |
| :--- | :--- | :--- | :--- |
| 1 year to 373 claims | 37300 | by 8165 Prem 4.12.6 | 37763.2 .6 |
| Ballance | 1594.2 .6 | Int on 37700 @ 3\% | 1131 |
|  | 38894.2 .6. |  | 38894.2 .6 |


| 2 yrs to 373 claims | 37300 | By Ballance | 1594.2 .6 |
| :--- | :--- | :--- | :--- |
| Ballance | 1460.2 .6 | 7792 premiums | 36038 |
|  |  | Int on 37632 | 1128 |
|  | 38760.2 .6 |  | 38760.2 .6 |


| 3 yrs to 373 claims | 37300 | By Ballance | 1460.2 .6 |
| :--- | :--- | :--- | :--- |
| Ballance | 100.8 .6 | 7419 premiums | 34312.17 .6 |
|  |  | Int on 35772 | 1071 |
|  |  | a call of 1.6 on every, 100 | 556.8 .86 |
|  | 37400.8 .6 |  | 37400.8 .6 |


| 4 yrs to 373 claims | 37300 | by ballance | 100.8 .6 |
| :--- | :--- | :--- | :--- |
| Ballance | 254.6 .6 | 7046 premiums | 32587.15 |
|  |  | int on 32660 | 978 |
|  |  | a call of 0.10 .6 | 3888.8 |
|  | 37554.6 .6 |  | 37554.6 .6 |


| 5 yrs to 373 claims | 37300 | by ballance | 100.8 .6 |
| :--- | :--- | :--- | :--- |
| Ballance | 79.7 | 6673 | 30862.12 .6 |
|  |  | int. on 30800 | 978 |
|  |  | a call of 0.16 .0 | 5338.8 |
|  | 37379.7 |  | 37379.7 |


| 6 yrs to 373 claims | 37300 | by ballance | 79.7 |
| :--- | :--- | :--- | :--- |
| Ballance | 34.17 | 6300 premiums | 29137 |


|  |  | int. on 29100 | 873 |
| :--- | :--- | :--- | :--- |
|  |  | a call of 1.3.0 | 7245.10 |
|  | 37334.17 |  | 37334.17 |
|  |  |  |  |

## \{48, ms.\}

| 7 yrs to 373 claims | 37300 | by ballance | 34.17 |
| :--- | :--- | :--- | :--- |
| Ballance | 466.1 .6 | 5927 premiums | 27421.7 .6 |
|  |  | Int on 27400 | 832 |
|  |  | A call of 1.11.0 | 9186.17 |
|  | 37466.1 .6 |  | 37466.1 .6 |


| 8 yrs to 373 claims | 37300 | by ballance | 166.1 .6 |
| :--- | :--- | :--- | :--- |
| Ballance | 157.12 .6 | 5554 premiums | 25687.5 |
|  |  | Int on 25800 | 774 |
|  |  | A call of 1.19 .0 | 10830.6 |
|  | 37457.12 .6 |  | 3757.12 .6 |


| 9 yrs to 373 claims | 37300 | By ballance | 157.12 .6 |
| :--- | :--- | :--- | :--- |
| ballance | 106.9 .6 | 5181 premiums | 23962.2 .6 |
|  |  | Int on 24100 | 723 |
|  |  | A call of 2.8.6 | 12563.14 .6 |
|  | 37406.9 .6 |  | 37406.5 .6 |


| 10 yrs to 373 claims | 37300 | by ballance | 106.9 .6 |
| :--- | :--- | :--- | :--- |
| ballance | 16.5 .6 | 4808 premiums | 22237 |
|  |  | int on 22300 | 669 |
|  | 37316.5 .6 |  | 37316.5 .6 |

It has been supposed that James Dodsons Scheme for Establishing a Corperation for Insuring Lives is defective because no fund is supposed to be provided to answer claims more than what arises from the premiums of insurance to be received but the fallacy of this kind of reasoning may be evident two ways.

First let the consequence of raising fund be considered even in the most favorable light. Suppose the friends of the design would advance a sufficient sum upon a moderate interest for instance 4 pet.

## [P.49, ms.]

which is about $1 / 2$ pct. more than the government now give that would occasion the premiums of insurance to rise near 1 pct. for it will readily be allowed to conceive that a fund of life than, 20 on every $£ 100$ insured would hardly be sufficient to answer those purposes which are proposed by it. To illustrate this suppose at the first beginning of the corperation 100 persons were each to insure , 100 then the whole sum insured would be , 10000 and the fund , 2000 at $4 \%$ being , 80 together with the charges of bonds other deeds and additional servants would be born by the 100 persons and consequently would not amount to much life than , 1 each.

But the design of this undertaking is to render the expense of insuring as small as possible which design would be hereby frustrated perhaps it may be said that there is another method of raising a fund viz by taking a deposit of the members at the time of their insuring which may be kept during the continuance of their policies to answer such demands as they would be, liable to as members and to be returned with interest if not so applied when the policy determines. But this is also making the insurance expensive at first which ought to be considered as much as possible as it may prevent many from becoming members who otherwise would. This the very case we condemn in the Amicable Society for such a deposit would act in the same manner against us as the payment of ,7.0.0 upon every ,100 to purchase one of their blank or vacant numbered accounts against them.

For arguments sake suppose that a deposit of, 5 only on every , 100 would prevent a call during the whole time that a person continued a member

## [P.50, ms.]

of the corperation yet that payment at first would be more grievous than occasional calls of 10/at or 20/at a time tho' repeated often enough to make the whole sum greater than the said $£ 5$.

Again let this fund be advanced how or by whom it will its support can be nothing else but the credit of the corperation or the particular members thereof and consequently is in fact no more security than the persons insured will have without for instance admit that by the death of 20 persons out of the 100 the fund of ,2000 would be near exhausted must not the consequence be that either the corperation must borrow more money to reestablish the Fund and thereby increase the premiums or else they must make a call upon their members for that purpose, the former will perhaps be difficult after such ill success and the latter is what is proposed in the present scheme. But there will on consideration appear a wide difference between a call to reestablish a fund already exhausted and a call to pay claims as they become due. The former will show the credit of the corperation is lost and the other readily paid would establish as much credit as the corporation can want besides it will fall doubly severe on the members after having paid an advance of $1 \%$ on their premiums for some time to be abridged to pay the same call as would have secured them from that advance and yet not get rid of it.

Again for whose security is it that this fund is necessary to be raised is it for the members of the corperation surely not, they are engaged in partnership and ought to be satisfied with their own mutual security, no director can be chosen and consequently no business transacted but by their participation

## [p.51, ms.]

unless they chose to be idle spectators instead of useful members and they are the farther security of an application to the visitors of any such measures are taken as may render their properly precarious of, therefore, the fund is necessary at all it must be for the security of the persons who may chose to be insured and not become members be it so then, the only will consequence at times, the want of it will be that fewer of that sort will be concerned, that is, those that are not satisfied with the security of the corperation and that if its particular members will insure elsewhere, let them do so. For the sake of becoming more generally useful,
the original scheme was extended to those who would not become members, but then it is by not means reasonable that the members should put to so much hazard and expense for their convenience who are but consideration nor need it be apprehended that many insurances will cost for want of the fund, for the corperation will in most cases insure on cheaper terms than is done elsewhere and the security of private underwriters or even of the other corperations is not enterely free from objections.

Me thinks I hear the counsel on the other side begin to open up on this. Then there's the case you are going under the presence of insuring cheaper than others to draw in the unwary, and such a measure is chimerical and must fail, many persons will become sufferers and this leads me to prove secondly that the premiums now proposed to be taken for insurance are sufficient security for the payment of the claims and here the experience past times is the best guide to form conclusions for the future.

In doing this the Bills of Mortality in London have been carefully consulted

## [p.52, ms.]

and from them it will appear that upon a mean of 23 years, beginning with 1728 and ending with 1750 there have died annually \{page 4\}

| Under 2 Years of Age | 9513 persons |
| :--- | :--- |
| Between 2 \& 5 | 2232 |
| 5 \& 10 | 926 |
| $10 \& 20$ | 813 |
| $20 \& 30$ | 2058 |
| $30 \& 40$ | 2500 |
| $40 \& 50$ | 2574 |
| $50 \& 60$ | 2074 |
| $60 \& 70$ | 1635 |
| $70 \& 80$ | 1174 |
| $80 \& 90$ | 598 |
| Upwards of 90 | 110 |

Here it evidently appears that a greater number of persons die in the period between 40 \& 60 than in any such period in which persons may be insured whence the foregoing example of the success attending that period may be satisfactory as to the rest.

From the number of persons actually dying during the different periods it is easy to compute how many of them must be actually alive at the beginning of any one period for instance if there die 110 persons yearly aged 90 and upwards there must be 110 persons of the age or 90 always alive between $80 \& 90$ there die annually 598 there must be 598 and 110 persons, that is 700 always alive at the age of 80 and so on, whence it will appear from one of the foregoing tables that at the age of 40 there are 8165 persons alive.

I have supposed therefore, that all the 8165 persons alive at of 40 insured each , 100 for life and became members of the corperations in the
[p.53, ms.]
year 1730 and that there dies each year between that and 1750, a number of them proportionate to the numbers which died within each particular year in the respective periods between those ages according to the yearly bills of those work times so that only 3518 remained alive at the end of the year 1750 and these numbers are the ground of the foregoing calculations.

It is proposed that the premiums of insurance shall be paid at the beginning and that the claims shall not become due till the end of the year, so interest is computed at $3 \%$ for all the premiums in hand yearly and the annual premiums for insuring , 100 at that age is by my tables , 4.12.6 [4.625].

The calculation at length is in the foregoing sheets, I shall here only mention the results.
At the end of the 20 years, the Corperation would have had upwards of $£ 150,000$ in hand and upon supposition that anybody of means have then undertaken to insure the 3518 persons then alive on similar terms they would have gained more that ,16000.

If it be said that the whole number of persons of the age of 40 viz 8165 would not have become members and therefore the computation should be made for a less number let it be so but then it must not be supposed that a greater number of them would have died in the time than would be proportionate to the numbers above for in the above we have taken nearly all the inhabitants in London of that age, sickly as will as healthy whereas it should be presumed that the corperations take a good deal of care

## [p.54, ms.]

to insure only those that are healthy. Suppose then that only 100 persons of that age had been so insured then the number alive would be only 43 in the sum in hand ,1800 and the gain about ,200.

Now if we resume, the sum above fixed as proper to be established as a fund confining it however to the 43 remaining survivers it will appear that 20 times 43 or , 860 would be the proper proportions for them and consequently that the corperation would in the 20 years have accumulated a fund greater than the proposed one by near, 7000 .

But there are some persons who having determined to oppose this scheme may not admit that we ought not to calculate upon the principle of proportioning the deaths in a small number according to that of the whole they will tell us that more may die in proportions out of our 100 then died out of the 8165 or that though they may come pretty near that proportion in a length of 20 years, yet if some of the first years there should die a greater number than usual we may be oblidged to make such calls upon our members as they will be unable not to say unwilling to pay will not withstanding the improbability of this, we will admit that it may be so and endeavour to vestigate the consequence. It appears from the above quoted Bills of Mortality that a greater number of persons died in the 1741 than in any other year since 1665, the year of the great plague and that in 1741--13731 persons died between the ages of $40 \& 50$.

Let us therefore suppose for sure they will not desire to state mortality at a higher rate than ever experience trade made it that out of the

## [p.55, ms.]

above 8165 persons of the age of 40 insured there should die yearly for the first ten years 373 making in the whole 3730 which is much greater than can every reasonably be supposed to happen and let us also suppose the Corperation should make a call upon their members every time that the income of the current year will not
pay its expense then it will appear by the foregoing calculations that a call will be necessary at the end of:

| The Third Year | 0.1 .6 on every ' 100 insured | $[0.075]$ |
| :--- | :--- | :--- |
| 4 | 0.10 .6 | $[0.525]$ |
| 5 | 0.16 .0 | $[0.800]$ |
| 6 | 1.3 .0 | $[1.150]$ |
| 7 | 1.11 .0 | $[1.550]$ |
| 8 | 1.19 .0 | $[1.950]$ |
| 9 | 2.8 .6 | $[2.425]$ |
| 10 | 2.19 .6 | $[2.975]$ |
| Total | 11.9 .0 | $[11.450]$ |

Which upon an average is $1.3 \%$ annum not much more than must have been advanced in the Annual premium if a fund had been established and the corperation had been by that means 8165 times ,20, $£ 16300$ in debt during the whole 10 years and remained so at the end of that time. Thus it appears upon taking the most mortal period upon allowing the greatest degree of mortality for 10 years together that has ever been experienced in any one year and without supposing any new insurance to be granted that year the premiums of which might assist in defraying the expences that the calls will not greatly exceed the interest of a sufficient fund and consequently that the premiums of insurance strengthened by the power of making calls is security enough for the payment of the claims.

## (p.56, ms.)

## [5. TABLE OF DECREMENTS ]

A table of decrements wherein the hazard of life is esteemed to be as great as any author has conceived it to be or as can be deduced from any bills of mortality hitherto made public.

| Ages | Persons <br> Living | D | Ages | Persons <br> Living | D | Ages | Persons <br> Living | D | Ages | Persons <br> Living | D |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| born | 1400 | 448 | 22 | 493 | 8 | 44 | 284 | 11 | 66 | 98 | 7 |
| 1 | 952 | 186 | 23 | 485 | 8 | 45 | 273 | 11 | 67 | 91 | 7 |
| 2 | 766 | 71 | 24 | 477 | 8 | 46 | 262 | 11 | 68 | 84 | 7 |
| 3 | 695 | 38 | 25 | 469 | 8 | 47 | 252 | 10 | 69 | 77 | 7 |
| 4 | 657 | 22 | 26 | 460 | 9 | 48 | 242 | 10 | 70 | 70 | 7 |
| 5 | 635 | 17 | 27 | 451 | 9 | 49 | 232 | 10 | 71 | 64 | 6 |
| 6 | 618 | 14 | 28 | 442 | 9 | 50 | 222 | 10 | 72 | 58 | 6 |
| 7 | 604 | 11 | 29 | 433 | 9 | 51 | 213 | 9 | 73 | 52 | 6 |
| 8 | 593 | 10 | 30 | 424 | 9 | 52 | 204 | 9 | 74 | 46 | 6 |
| 9 | 583 | 8 | 31 | 414 | 10 | 53 | 195 | 9 | 75 | 40 | 6 |
| 10 | 575 | 7 | 32 | 404 | 10 | 54 | 186 | 8 | 76 | 34 | 6 |
| 11 | 568 | 7 | 33 | 394 | 10 | 55 | 178 | 8 | 77 | 28 | 6 |
| 12 | 561 | 7 | 34 | 384 | 10 | 56 | 170 | 8 | 78 | 22 | 5 |
| 13 | 554 | 6 | 35 | 374 | 10 | 57 | 162 | 8 | 79 | 17 | 5 |


| 14 | 548 | 6 | 36 | 364 | 10 | 58 | 154 | 8 | 80 | 13 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 15 | 542 | 6 | 37 | 354 | 10 | 59 | 147 | 7 | 81 | 9 | 4 |
| 16 | 536 | 6 | 38 | 344 | 10 | 60 | 140 | 7 | 82 | 6 | 3 |
| 17 | 530 | 7 | 39 | 334 | 10 | 61 | 133 | 7 | 83 | 4 | 2 |
| 18 | 523 | 7 | 40 | 324 | 10 | 62 | 126 | 7 | 84 | 3 | 1 |
| 19 | 516 | 7 | 41 | 314 | 10 | 63 | 119 | 7 | 85 | 2 | 1 |
| 20 | 509 | 8 | 42 | 304 | 10 | 64 | 112 | 7 | 86 | 1 | 1 |
| 21 | 501 | 8 | 43 | 294 | 10 | 65 | 105 | 7 | 87 | 0 |  |

A short account of the tables on which the computation of the premium which ought to be paid for assurance for lives at different ages is founded to which is subjoined the terms upon which the gentlemen petioning for a charter to enable themselves to make such assurances are willing to be abridged to assure healthy lives per Cauncellor De Whall. The Tables of the Decrements

## [P.57, ms.]

of Life on the other side was composed by Mr. Dodson from his observations on the bills of Mortality for 23 years from the year 1728 to 1750 both inclusive in which periods is concluded the year 1741 in which was that of the greatest mortality since 1665 the year of the great plague. It contains the numbers out of 1400 persons supposed to be born from year to year till the age of 87 when they are all supposed to be extinct. That in the forming this Table the Hazard of Life is estimated to be as great as can reasonably be conceived to be appears by comparing it with the Table exhibited by Mr. Hodgson in his Valuation of Annuities on Lives which is founded on the London bills of Mortality for the ten first years of the above mentioned period. For according to the last mentioned author out of 1000 persons born, there are 501 that survive the age of 8 years whereas according to Mr. Dodson out of 1400 persons born there are only 593 living at the age of 8 years but according to the proportion of 501 in 1000 there ought to be at least 701 living out of 1400 .

The difference is still more considerable in the subsequent years viz according to Mr. Dodson, out of 593 living at the age of 8 years there will be only 501 living at the age of $20 ; 92$ persons having died in the interim, but according to Hodgson out of 501 living at 8 , there will be 459 living at 20 and only 42 will have died in the interval.

And the same will be true in all ages tho' the difference will not be quite so great.
And upon the bare inspection of the two Tables it appears that Dodson supposes the Mortality to be greater than the other Author

## [p.58, ms.]

for he makes the whole number of 1400 to be extinct at 87 whereas the other does not suppose a smaller number of 1000 to be all extinct at 95 .

The Breslaw Tables on which Dr. Halley and Mr. DeMoivre have founded their calculation make the probabilities of living still greater for according to those tables, out of 1000 persons born there will be 680 living at 8 years old whereas according Dodson; out of 1400 there will be 593 lives at the same age of 8 .

This being sufficiently established it is very easy to compute what ought to be the premium for assuring a life of any age for a single year, it being evident that the premium ought to bear the same proportion to the sum assured that the number of persons dying in the year does to the whole number of persons living at the age of

9 years, there die 8 within the year and the remaining 575 survive the premium for assuring $£ 100$ ought to bear the same proportions to the sum as 8 does to 583 this will be found to be less than $£ 1.7 .6$ [1.375], the premium will amount to, 801.12 .6 [ 801.625 ] whereas only $£ 800$ is to be paid on the claims of the 8 persons dying.

The computation of the premium to be paid for the assurance of a life for any number of years or during the whole continuances of life tho' depending upon the same principals is of higher dequesition and would require too much time to explain, it will therefore be sufficient if it appears that the premium herein after proposed for making assurances in these cases are very moderate and reasonable.

## [P.59, ms.]

If the Corperation petitioned for propose out of the premiums to raise a fund for payment of all future claims it is but reasonable they should take a larger premium than according to the strict calculation which will establish the security of the Society on an undoubted foundation and even in this case the premium will appear to be very small, especially during the younger part of life when compared with those now taken by any Corperation or any private assurers.

The petitioners therefore, humbly propose that the future Corperation shall be oblidged to make assurances with all such persons as shall become members upon the following premiums to be paid to them. 8 to 14 -for assurance of any healthy boy whose age shall not be less than 8 years and not exceed 14 years for a single year, the premium not to exceed ,1.15 [1.750] for every sum of 100 so insured.

For the assurance of , 100 on such a life for any term of years not exceeding 5 years the annual premium not exceed $£ 2.0 .0$ [2.000] to be paid at the beginning of each year or the totall premium for such assurance for 5 years not to exceed $£ 9.5 .0$ [9.250] to be paid at the time of making such assurance.

For the life assurance for any term not exceeding 10 years, the annual sum not to exceed $£ 2.10$ [2.500] or the total premium for assuring $£ 100$ for 10 years not to exceed, 21 to be paid as aforesaid. For assurance of such a life during the whole continuance thereof the annual premium of ,2.15.0 [2.750].

| Ages | One years <br> payment - 5 <br> years | Annual <br> Payment | Single <br> Payment- <br> 10 years | Annual <br> Payment | Single <br> Payment | Life <br> Prem |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $14 \& 30$ | 2.0 .0 | 2.5 .0 | 10.10 .0 | 2.15 .0 | 35.0 .0 | 3.3 |
| $20 \& 30$ | 2.10 .0 | 3.0 .0 | 13.10 .0 | 3.10 .0 | 30.0 .0 | 4.0 |
| $30 \& 40$ | 3.10 .0 | 4.0 .0 | 18.10 .0 | 4.10 .0 | 40.0 .0 | 5.0 |
| $40 \& 50$ | 4.10 .0 | 5.0 .0 | 23.0 .0 | 5.10 .0 | 50.0 .0 | 6.6 |
| $50 \& 55$ | 5.10 .0 | 6.6 .0 | 30.0 .0 | 7.0 .0 | 63.0 .0 | 7.10 |
| $55 \& 60$ | 6.6 | 7 |  | 8 |  | 8.10 |

## [p.60, ms.]

As the terms of the last assurance a little exceed these expressed for a life not exceeding 60 contained in the head of the charter laid before the attorney and solicitor general, the petitioners humbly beg leave to inform
them that upon a review of Mr. Dodsons calculations they find that assurances cannot, with safety, be made on such elderly lives under the terms here proposed and as they have fixed such moderate teens for the assurance of younger lives, they hope they shall be indulged in making this small advance on the older lives especially as they are not inhibited from insuring lives at a lower rate, but only not to take a higher than is exhibited. They have fixed no times for the totall premiums to be had at once for the assurance of a [life] during its whole continuance nor for assuring for a term of years where the age exceeds 55, the reason of which is that they apprehend that such assurances will sooner or never happen to be made as the premium will too great a proportion to the sum assured that it will scarce be worth anyone's while to make such assurance and therefore in a case that is so unlikely to happen they think they may be left to settle the terms with the assured in such manner as they shall agree.

The terms above proposed are confined to such persons as shall become members of the corperation who are to sign an obligation to become liable to pay all claims that shall be made on them as the first intention of the gentlemen who proposed this plan was to assure no other persons but as some persons may think it advantageous to be assured by the corperation without being members thereof it is but reasonable they should

## [P.61, ms.]

pay a larger premium than those who are bound to make good the claims, however, that this increase of premium may not be left to the arbitration of the corperation, they are willing to be obliged to assure all such persons applying to them upon payment of an increased premium not exceeding $5 \%$ upon the premium of a single year under 40 years of age and $10 \%$ above that age as the annual premium above specified for assurance for a term of years.

They are willing and desirous to assure the lives of all persons within the limits as also of women on such equitable terms as shall be agreed on and also to submit to all such necessary and proper regulations as shall most tend to the safety of all persons assuring with them. Finis. When neither the occupation of the person nor the circumstances of the life whereon the assurance is made shall be attended with any peculiar hazard, there shall be demanded and taken for the assurance of the son of ONE HUNDRED POUNDS for single year upon the life of any healthy boy or man of the ages is on the other side

```
[p.62, ms.]
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TABLE FOR ANNUAL PREMIUMS FOR A SINGLE YEAR FROM 8 YEARS TO 40 \& 50 TO 67 YEAR

| Age | Premium | Age | Premium | Age | Premium | Age | Premium |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 1.10 .6 | 23 | 1.17 .9 | 38 | 2.17 .7 | 52 | 4.15 .1 |
| 9 | 1.10 .8 | 24 | 1.18 .6 | 39 | 2.19 .9 | 53 |  |
| 10 | 1.10 .10 | 25 | 1.19 .4 | 40 | 3.2 .0 | 54 |  |
| 11 | 1.11 .0 | 26 | 2.0 .3 | 41 | 3.4 .4 | 55 |  |
| 12 | 1.11 .3 | 27 | 2.1 .3 | 42 | 3.6 .9 | 56 |  |
| 13 | 1.11 .6 | 28 | 2.2 .3 | 43 | 3.9 .3 | 57 |  |
| 14 | 1.11 .9 | 29 | 2.3 .3 | 44 | 3.11 .10 | 58 |  |
| 15 | 1.12 .6 | 30 | 2.4 .6 | 45 | 3.14 .5 | 59 |  |


| 16 | 1.12 .9 | 31 | 2.5 .9 | 46 | 3.17 .1 | 60 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 17 | 1.13 .3 | 32 | 2.7 .0 | 47 | 3.19 .10 | 61 |  |
| 18 | 1.14 .0 | 33 | 2.8 .6 | 48 | 4.2 .8 | 62 |  |
| 19 | 1.14 .9 | 34 | 2.10 .1 | 49 | 4.5 .7 | 63 |  |
| 20 | 1.1 .56 | 35 | 2.10 .10 |  |  | 64 |  |
| 21 | 1.16 .3 | 36 | 2.13 .8 | 50 | 4.8 .8 | 65 |  |
| 22 | 1.17 .0 | 37 | 2.15 .7 | 51 | 4.11 .10 | 66 |  |
|  |  |  |  |  |  | 67 | 7.18 .1 |

NB without the addition of 10/part of the charter fund to be paid by the Society.

## [p.63, ms.]

TABLE FOR ANNUAL PREMIUMS FOR THE WHOLE CONTINUANCE OF LIFE FROM THE AGE OF 8 TO 49 \& 50 TO 67 YEARS

| Age | $\underline{\text { Premium }}$ | Age | Premium | Age | $\underline{\text { Premium }}$ | $\underline{\text { Age }}$ | $\underline{\text { Premium }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 | 2.4 .10 | 23 | 2.19 .1 | 38 | 4.7 .9 | 53 | 6.7 .8 |
| 9 | 2.5 .0 | 24 | 3.1 .9 | 39 | 4.9 .11 | 54 | 6.11 .3 |
| 10 | 2.5 .3 | 25 | 3.3 .5 | 40 | 4.12 .2 | 55 | 6.15 .6 |
| 11 | 2.5 .7 | 26 | 3.5 .2 | 41 | 4.14 .6 | 56 | 6.19 .3 |
| 12 | 2.6 .0 | 27 | 3.6 .11 | 42 | 4.17 .0 | 57 | 7.5 .7. |
| 13 | 2.6 .8 | 28 | 3.8 .8 | 43 | 4.19 .6 | 58 | 7.11 .6 |
| 14 | 2.7 .7 | 29 | 3.10 .5 | 44 | 5.2 .1 | 59 | 7.18 .0 |
| 15 | 2.8 .9 | 30 | 3.12 .3 | 45 | 5.4 .8 | 60 | 8.5 .2 |
| 16 | 2.10 .0 | 31 | 3.14 .1 | 46 | 5.7 .3 | 61 | 8.13 .2 |
| 17 | 2.11 .3 | 32 | 3.15 .11 | 47 | 5.9 .10 | 62 | 9.2 .0 |
| 18 | 2.12 .7 | 33 | 3.17 .9 | 48 | 5.12 .7 | 63 | 9.11 .8 |
| 19 | 2.13 .11 | 34 | 3.19 .7 | $49 *$ | 5.15 .5 | 64 | 10.2 .2 |
| 20 | 2.15 .4 | 35 | 4.1 .6 | 50 | 5.18 .4 | 65 | 10.13 .6 |
| 21 | 2.16 .10 | 36 | 4.3 .6 | 51 | 6.1 .4 | 66 | 11.5 .8 |
| 22 | 2.18 .5 | 37 | 4.5 .7 | 52 | 6.4 .5 | 67 | 11.18 .8 |

* Man or Woman of age


## [6 Contingent Insurance]

To find the probability that an older life shall survive the younger and to find the present worth of a sum depending upon such survivorships let the complement of the older life be $n$ and that of the Younger $m$ then

| $(2 \mathrm{n}-1) / 2 \mathrm{n}$ | $(2 \mathrm{n}-3) / 2 \mathrm{n}$ | $(2 \mathrm{n}-5) / 2 \mathrm{n}$ | $(2 \mathrm{n}-7) / 2 \mathrm{n}$ etc. to $1 / \mathrm{n}$ | Etc to $1 / \mathrm{n}$ |
| :--- | :--- | :--- | :--- | :--- |

will be the expectations of the life of the older or the probabilities of its continuance and $1 / \mathrm{m}$ will be the probability of the younger life failing in any particular year during the continuance or the life of older which two events being independent on each other. [The sum]

$$
\begin{array}{c|c|c|c|l}
\hline(2 n-1 / 2 n)(1 / m)+ & (2 n-3 / 2 n)(1 / m)+ & (2 n-5 / 2 n)(1 / m)+ & (2 n-7) / 2 n(1 / m)+ & \text { Etc to }(1 / n)(1 / m) \\
\hline
\end{array}
$$

will be the probability of the older surviving the younger.

## \{p.64, ms.\}

Then for the value of a sum depending upon that survivorship it has been proved that the probability thereof is consequently the present worth of a sum P payable upon the contingency will be

$$
\begin{array}{|l|l|l|l|l|}
\hline(2 \mathrm{n}-1) / 2 \mathrm{n}(\mathrm{P} / \mathrm{m})+ & (2 \mathrm{n}-3) / 2 \mathrm{n}(\mathrm{P} / \mathrm{m})+ & (2 \mathrm{n}-5) / 2 \mathrm{n}(\mathrm{P} / \mathrm{m})+ & (2 \mathrm{n}-7) / 2 \mathrm{n}(\mathrm{P} / \mathrm{m})+ & \text { Etc to }(1 / \mathrm{n})(\mathrm{P} / \mathrm{m}) \\
\hline
\end{array}
$$

and in each of which terms $\mathrm{P} / \mathrm{m}$ is a constant factor and therefore
$(\mathrm{P} / \mathrm{m})\left\{\mathrm{V}(2 \mathrm{n}-1) / 2 \mathrm{n}+\mathrm{V}^{2}(2 \mathrm{n}-3) / 2 \mathrm{n}+\mathrm{V}^{3}(2 \mathrm{n}-5) / 2 \mathrm{n}+\ldots\right\}$
will be the present value required in which expression P denotes the value of the older life.
NB. The complement of the older life signified by n is the chance of the number of years he is supposed to live, $m$ the same of the younger grounded on the Decrements of Life.

## POSTSRCIPT by Thomas Kabele

On page 64 Dodson is apparently using DeMoivre's approximation to compute probabilities of death. Let " $x$ " be the older life and " $y$ " be the younger life. Let $S(x)$ be the survivors to age $x$. Then by DeMoivre's formula:

$$
\begin{aligned}
& S(x+t)=S(x)(1-t / n) \text { for } 0<t<n \\
& S(y+t)=S(y)(1-=t / m) \text { for } 0<t<m .
\end{aligned}
$$

Let ${ }_{t} p_{x}=S(x+t) / S(x)$ be the probability of a life aged $x$ surviving $t$ years, and let $p_{x}={ }_{1} p_{x}$ and $q_{x}=1-$ $p_{x}$. Let ${ }_{t| |} \mathrm{q}_{\mathrm{y}}=\mathrm{S}(\mathrm{x}+\mathrm{t})-\mathrm{S}(\mathrm{x}+\mathrm{t}+1) / \mathrm{S}(\mathrm{x})=$ probability of living t years and then dying in the next year. Let "I" be the discount rate, and $\mathrm{V}=1 /(1+\mathrm{I})$ and assume the benefit of " P " is paid at the end of the year. Under the DeMoivre assumption ${ }_{t \mid} \mathrm{q}_{\mathrm{y}}=1 / \mathrm{m}$.

Then the present value is:

$$
\text { Value }=P\left\{\begin{array}{llll}
V_{0| |} q_{y} & 0.5 \\
p_{x}
\end{array}+V^{2}{ }_{1| |} \mathbf{q}_{y} \quad{ }_{1.5} p_{x}+V^{3}{ }_{2| |} \mathbf{q}_{y} \quad{ }_{2.5} p_{x}+\ldots\right\}
$$

The following are some values computed from the Dodson paper.

| Dodson London Table P. 56 |  |  |  | Dodson's Mean Deaths P.30 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| Age | Living | Death | Rate | Living | Death | Rate |
| 40 | 8165 | 252 | 0.03086 | 8165 | 237 | 0.02903 |
| 41 | 7913 | 252 | 0.03185 | 7928 | 223 | 0.02813 |
| 42 | 7661 | 252 | 0.03289 | 7705 | 279 | 0.03621 |
| 43 | 7409 | 252 | 0.03401 | 7426 | 226 | 0.03043 |
| 44 | 7157 | 277 | 0.03873 | 7200 | 224 | 0.03111 |
| 45 | 6880 | 277 | 0.04029 | 6976 | 246 | 0.03526 |
| 46 | 6603 | 252 | 0.03817 | 6730 | 268 | 0.03982 |
| 47 | 6351 | 252 | 0.03968 | 6462 | 247 | 0.03822 |
| 48 | 6099 | 252 | 0.04132 | 6215 | 248 | 0.03990 |
| 49 | 5847 | 252 | 0.04310 | 5967 | 297 | 0.04977 |
| 50 | 5595 | 227 | 0.04054 | 5670 | 296 | 0.05220 |
| 51 | 5368 | 227 | 0.04225 | 5374 | 244 | 0.04540 |
| 52 | 5141 | 227 | 0.04412 | 5130 | 201 | 0.03918 |
| 53 | 4914 | 227 | 0.4615 | 4929 | 164 | 0.03327 |

Function Dodson
Premium And Annuity Factors

| LIVING AND DEATHS FROM DODSON'S LONDON TABLE PAGE 56 ASSUME PREMIUM PAID AT BEGINNING OF YEAR AND DEATHS PAID AT END OF YEAR |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age | Living | Deaths | Mortality Rate | Net Annual Premium | Net Single Premium | Annuity Due |
| 0 | 1400 | 448 | 32.00000 | 6.21176 | 68.07870 | 10.95965 |
| 1 | 952 | 186 | 19.53782 | 3.71607 | 56.06038 | 15.08594 |
| 2 | 766 | 71 | 9.26893 | 2.63324 | 47.48116 | 18.03147 |
| 3 | 695 | 38 | 5.46763 | 2.25948 | 43.68588 | 19.33451 |
| 4 | 657 | 22 | 3.34855 | 2.09318 | 41.81512 | 19.97681 |
| 5 | 635 | 17 | 2.67717 | 2.03217 | 41.09718 | 20.22330 |
| 6 | 618 | 14 | 2.26537 | 2.00267 | 40.74371 | 20.34466 |
| 7 | 604 | 11 | 1.82119 | 1.99250 | 40.62086 | 20.38684 |
| 8 | 593 | 10 | 1.68634 | 2.00408 | 40.76063 | 20.33885 |
| 9 | 583 | 8 | 1.37221 | 2.02305 | 40.98831 | 20.26068 |
| 10 | 575 | 7 | 1.21739 | 2.05891 | 41.41403 | 20.11452 |
| 11 | 568 | 7 | 1.23239 | 2.10479 | 41.94976 | 19.93058 |
| 12 | 561 | 7 | 1.24777 | 2.15277 | 42.49962 | 19.74180 |
| 13 | 554 | 6 | 1.08303 | 2.20300 | 43.06418 | 19.54797 |
| 14 | 548 | 6 | 1.09489 | 2.26508 | 43.74686 | 19.31358 |
| 15 | 542 | 6 | 1.10701 | 2.33072 | 44.45107 | 19.07180 |

## LIVING AND DEATHS FROM DODSON'S LONDON TABLE PAGE 56 ASSUME PREMIUM PAID AT BEGINNING OF YEAR AND DEATHS PAID AT END OF YEAR

| Age | Living | Deaths | Mortality <br> Rate | Net Annual <br> Premium | Net Single <br> Premium | Annuity <br> Due |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | 536 | 6 | 1.11940 | 2.40022 | 45.17771 | 18.82232 |
| 17 | 530 | 7 | 1.32075 | 2.47392 | 45.92776 | 18.56480 |
| 18 | 523 | 7 | 1.33843 | 2.54176 | 46.60031 | 18.33389 |
| 19 | 516 | 7 | 1.35659 | 2.61343 | 47.29287 | 18.09611 |
| 20 | 509 | 8 | 1.57171 | 2.68925 | 48.00632 | 17.85116 |
| 21 | 501 | 8 | 1.59681 | 2.75829 | 48.63926 | 17.63385 |
| 22 | 493 | 8 | 1.62272 | 2.83091 | 49.28868 | 17.41089 |
| 23 | 485 | 8 | 1.64948 | 2.90741 | 49.95526 | 17.18203 |
| 24 | 477 | 8 | 1.67715 | 2.98812 | 50.63972 | 16.94703 |
| 25 | 469 | 9 | 1.91898 | 3.07339 | 51.34286 | 16.70562 |
| 26 | 460 | 9 | 1.95652 | 3.15045 | 51.96130 | 16.49329 |
| 27 | 451 | 9 | 1.99557 | 3.23119 | 52.59260 | 16.27654 |
| 28 | 442 | 9 | 2.03620 | 3.31588 | 53.23719 | 16.05523 |
| 29 | 433 | 9 | 2.07852 | 3.40482 | 53.89553 | 15.82920 |
| 30 | 424 | 10 | 2.35849 | 3.49834 | 54.56809 | 15.59829 |
| 31 | 414 | 10 | 2.41546 | 3.58112 | 55.14728 | 15.39943 |
| 32 | 404 | 10 | 2.47525 | 3.66696 | 55.73243 | 15.19853 |
| 33 | 394 | 10 | 2.53807 | 3.75597 | 56.32330 | 14.99567 |
| 34 | 384 | 10 | 2.60417 | 3.84827 | 56.91959 | 14.79094 |
| 35 | 374 | 10 | 2.67380 | 3.94399 | 57.52095 | 14.58447 |
| 36 | 364 | 10 | 2.74725 | 4.04322 | 58.12698 | 14.37640 |
| 37 | 354 | 10 | 2.82486 | 4.14609 | 58.73720 | 14.16690 |
| 38 | 344 | 10 | 2.90698 | 4.25268 | 59.35104 | 13.95614 |
| 39 | 334 | 10 | 2.99401 | 4.36308 | 59.96784 | 13.74437 |
| 40 | 324 | 10 | 3.08642 | 4.47735 | 60.58684 | 13.53185 |
| 41 | 314 | 10 | 3.18471 | 4.59552 | 61.20714 | 13.31888 |
| 42 | 304 | 10 | 3.28947 | 4.71758 | 61.82767 | 13.10583 |
| 43 | 294 | 10 | 3.40136 | 4.84345 | 62.44721 | 12.89312 |
| 44 | 284 | 11 | 3.87324 | 4.97304 | 63.06431 | 12.68125 |
| 45 | 273 | 11 | 4.02930 | 5.07684 | 63.54422 | 12.51648 |
| 46 | 262 | 10 | 3.81679 | 5.17799 | 64.00000 | 12.36000 |
| 47 | 252 | 10 | 3.96825 | 5.30760 | 64.56762 | 12.16512 |
| 48 | 242 | 10 | 4.13223 | 5.43791 | 65.12054 | 11.97528 |
| 49 | 232 | 10 | 4.31034 | 5.56785 | 65.65494 | 11.79180 |
| 50 | 222 | 9 | 4.05405 | 5.69600 | 66.16624 | 11.61626 |
| 51 | 213 | 9 | 4.22535 | 5.86179 | 66.80550 | 11.39678 |
| 52 | 204 | 9 | 4.41176 | 6.03102 | 67.43362 | 11.18112 |
| 53 | 195 | 9 | 4.61538 | 6.20269 | 68.04694 | 10.97055 |
| 54 | 186 | 8 | 4.30108 | 6.37537 | 68.64101 | 10.76659 |
| 55 | 178 | 8 | 4.49438 | 6.60059 | 69.38340 | 10.51170 |
| 56 | 170 | 8 | 4.70588 | 6.83578 | 70.12207 | 10.25809 |
|  |  |  |  |  |  |  |

(Table continued)

| LIVING AND DEATHS FROM DODSON'S LONDON TABLE PAGE 56 ASSUME PREMIUM PAID AT BEGINNING OF YEAR AND DEATHS PAID AT END OF YEAR |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Age | Living | Deaths | Mortality Rate | Net Annual Premium | Net Single Premium | Annuity Due |
| 57 | 162 | 8 | 4.93827 | 7.08065 | 70.85416 | 10.00674 |
| 58 | 154 | 7 | 4.54545 | 7.33448 | 71.57614 | 9.75886 |
| 59 | 147 | 7 | 4.76190 | 7.66802 | 72.47216 | 9.45123 |
| 60 | 140 | 7 | 5.00000 | 8.02830 | 73.37864 | 9.14000 |
| 61 | 133 | 7 | 5.26316 | 8.41821 | 74.29474 | 8.82547 |
| 62 | 126 | 7 | 5.55556 | 8.84098 | 75.21933 | 8.50803 |
| 63 | 119 | 7 | 5.88235 | 9.30012 | 76.15097 | 8.18817 |
| 64 | 112 | 7 | 6.25000 | 9.79943 | 77.08771 | 7.86655 |
| 65 | 105 | 7 | 6.66667 | 10.34286 | 78.02703 | 7.54405 |
| 66 | 98 | 7 | 7.14286 | 10.93429 | 78.96555 | 7.22183 |
| 67 | 91 | 7 | 7.69231 | 11.57710 | 79.89871 | 6.90144 |
| 68 | 84 | 7 | 8.33333 | 12.27334 | 80.82031 | 6.58503 |
| 69 | 77 | 7 | 9.09091 | 13.02226 | 81.72173 | 6.27554 |
| 70 | 70 | 6 | 8.57143 | 13.81766 | 82.59072 | 5.97719 |
| 71 | 64 | 6 | 9.37500 | 14.92187 | 83.66861 | 5.60711 |
| 72 | 58 | 6 | 10.34483 | 16.18512 | 84.74887 | 5.23622 |
| 73 | 52 | 6 | 11.53846 | 17.63490 | 85.82495 | 4.86677 |

