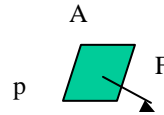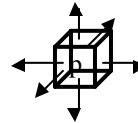# Pressure

- Pressure is force per unit area:

$$p = \frac{F}{A}$$
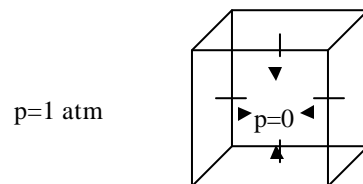
A

p F

- units: Newton/(meter)^2=Pascal (Pa)
  - pressure exerted by the weight of a dollar bill is about 1 Pa
  - 1 atmosphere is about 10^5 Pa

- direction: always perpendicular to surface
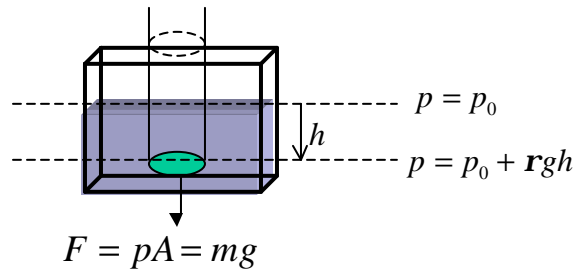
1

Pressure

- **Example:** You want to build a vacuum vessel that is a cube 20 cm on a side. How much force does each side have to withstand?

p=1 atm

p=0

2

Pressure and Depth

- Fluid pressure corresponds to the weight of the column of fluid above the point of interest:

$$p = p_0$$

$$h$$

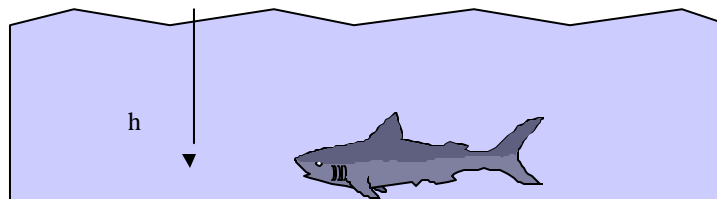$$p = p_0 + rgh$$

$$F = pA = mg$$

r is the density of the fluid (kg/m^3)

- pressure only depends on the depth h, not on the shape of the vessel
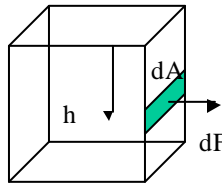
3

Pressure and depth

- Example: At about what depth in the sea does the pressure reach 2 atmospheres?
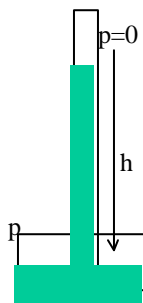
h

4

Pressure, depth and force

- **Example:** A 20 cm cube is filled with mercury (density 13.6×10^3 kg/m^3).
  - What is the force that the base of the cube must withstand?
  - What is the force that a side of the cube must withstand?

Pressure and Depth

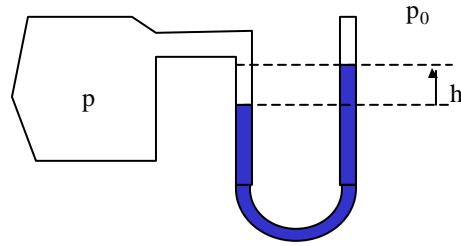- Barometers measure absolute pressure at the exposed surface:

- Example: If you wanted to make a barometer with water, how tall would it have to be?

Pressure

- Open-tube manometers measure "gauge pressure" (difference from ambient pressure).
- Absolute pressure must always be positive or zero ("nothing sucks")
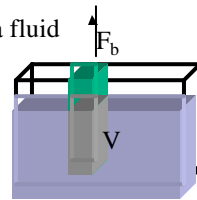- Gauge pressure can be negative

$$p_g = p - p_0 = \rho g h$$

Buoyancy

- Supporting force from (partial or total) immersion in a fluid
- Direction: up (opposite gravity)
- Magnitude:

$$F_b = \rho_f g V$$

Buoyant force =
(density of fluid)(accel. of gravity)(volume of fluid displaced)

- Note:
  - the relevant density is that of the fluid, not of the object itself
  - the relevant volume is that of the fluid which the object has displaced (equal to the volume of the object for total immersion)
  - the buoyant for is not the total force; you need to add (at least) the weight of the object iself

- FL**1.** Imagine holding two bricks under water. Brick *A* is just beneath the surface of the water, while brick *B* is at a greater depth. The force needed to hold brick *B* in place is
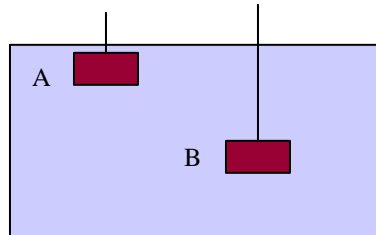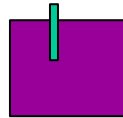

  - 1. larger
  - 2. the same as
  - 3. smaller

than the force required to hold brick *A* in place.

Buoyancy

- **Example 1:** A special float of mass 20 g and volume 30 cm^3 is used to measure the density of wine to judge when to stop the fermentation. What fraction of the float is above the surface when the density is 0.960 g/cm^3?


- **Example 2:** What is the tension in the rope holding a styrofoam float (volume 0.0050 m^3, mass 1.0 kg) under water?
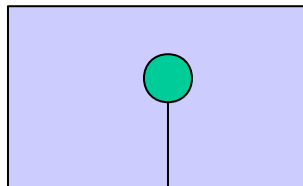
- FL2: A 200-ton ship enters the lock of a canal. The fit between the sides of the lock and the ship is tight so that the weight of the water left in the lock after it closes is much less than 200 tons. Can the ship still float if the quantity of water left in the lock is much less than the ship's weight?
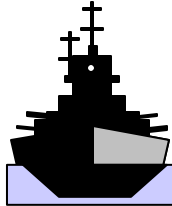
  - 1. Yes, as long as the water gets up to the ship's waterline.
  - 2. No, the ship touches bottom because it weighs more than the water in the lock.

- **21.** Which weighs more?
  - a bathtub filled to the brim with water
  - the same bathtub brim full with water with a battleship floating in it
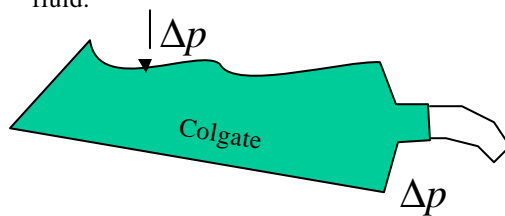 - both weigh the same

- **10.** A boat carrying a large boulder is floating on a lake. The boulder is thrown overboard and sinks. The water level in the lake (with respect to the shore)

    – rises.

    – drops.

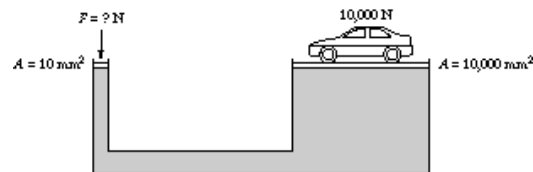   – remains the same.

13

Pascal's principle

- Any increase in pressure is transmitted undiminished to all parts of the fluid.

Δp

Colgate

Δp

    – Toothpaste tubes
    – hydrolic levers

14

7

- **3.** A container is filled with oil and fitted on both ends with pistons. The area of the left piston is 10 mm^2; that of the right piston 10,000 mm^2. What force must be exerted on the left piston to keep the 10,000-N car on the right at the same height?
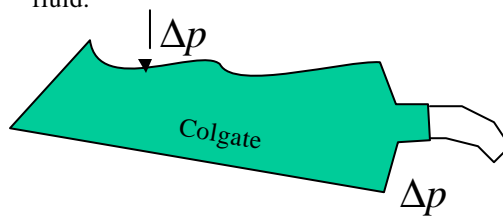

- – 1. 10 N
- – 2. 100 N
- – 3. 10,000 N
- – 4. 10^6 N
- – 5. 10^8 N

Pascal's principle

- Any increase in pressure is transmitted undiminished to all parts of the fluid.
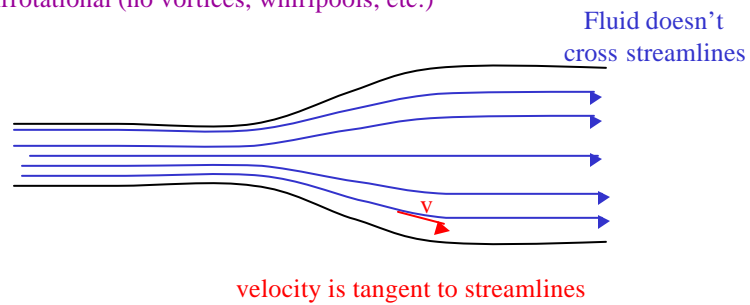

- – Toothpaste tubes
- – hydrolic levers

Fluid Flow

- Simplifying assumptions for flow of an **ideal fluid**:
  - steady (not turbulent)
  - incompressible (density does not change appreciably with pressure)
  - non-viscous
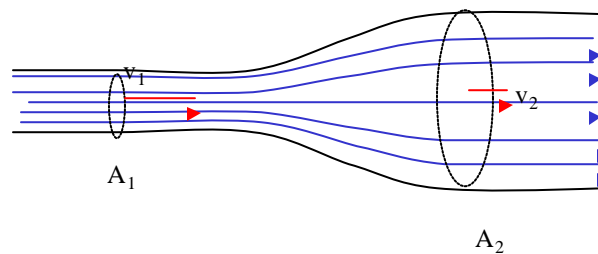 - irrotational (no vortices, whirlpools, etc.)

Fluid doesn't cross streamlines

v

velocity is tangent to streamlines

17

Fluid Flow

- Conservation of mass, conservation of volume

$$R = v_1 A_1 = v_2 A_2$$

Units of R = (m^3/s)

Volume flow rate = constant
=(fluid velocity)(cross-sectional area)

v_1

v_2

A_1

A_2

strictly true only for incompressible fluids;
constant mass flow rate is more true in general:

$$r_1 v_1 A_1 = r_2 v_2 A_2$$

18

9

Fluid Flow

- **Example:** Water is flowing at 0.60 liters/min through a pipe that starts with a diameter of 3/4 inch (1.91 cm), and then narrows to 1/2 inch (1.27 cm)
  - What is the velocity of the water in the wide section?
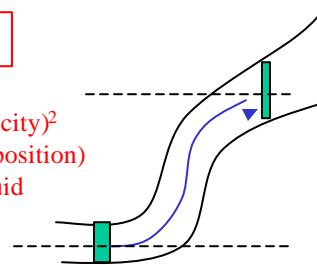 - What is the velocity of the narrow section?
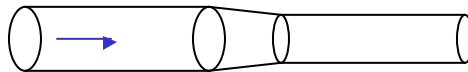
Bernoulli's Equation

- Ideal fluid flow:

$$p + \tfrac{1}{2}\,\boldsymbol{r}v^2 + \boldsymbol{r}gy = \text{constant}$$

Pressure + (1/2)(density)(fluid velocity)^2
+(density)(accel. of gravity)(vertical position)
is constant in all points of the fluid

- Really just a statement of conservation of energy:

$$\Delta p + \tfrac{1}{2}\,\boldsymbol{r}\Delta v^2 + \boldsymbol{r}g\Delta y = 0$$

Change in gravitational potential energy per unit volume

Work per unit volume

Change in kinetic energy per unit volume

Bernoulli's Equation

- Special case #1, v=0

$$p + \rho g y = \text{constant}$$ $\Leftrightarrow$ $$p = p_0 + \rho g h$$

- Special case #2, y=constant (horizontal flow)

$$p_1 + \tfrac{1}{2}\rho v_1^{\,2} = p_2 + \tfrac{1}{2}\rho v_2^{\,2}$$

"If the speed of a fluid particle increases as it travels
along a horizontal streamline, then the pressure of the fluid
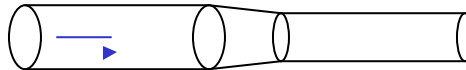must decrease, and conversely"

Reading quiz, question 2: lower speed $\Leftrightarrow$ higher pressure

21

Bernoulli's Equation

- **Example:** Water is flowing at 0.60 liters/min through a horizontal pipe
  that starts with a diameter of 3/4 inch (1.91 cm), and then narrows to
  1/2 inch (1.27 cm). The gauge pressure in the wide section is 80 kPa.

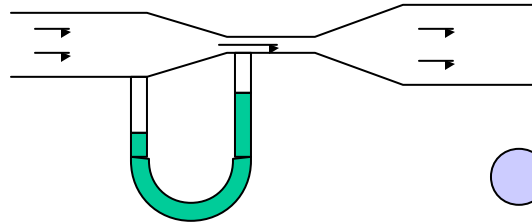 – What is the pressure in the narrow section?

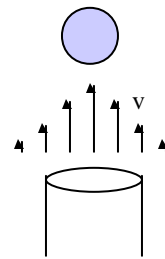- (Recall that earlier we found that v_{wide}= 3.5 m/s and v_{narrow}=7.9 m/s)

22

Bernoulli's Equation

- Note that the pressure is **lower** where the fluid is moving faster.
- Examples of interesting consequences:
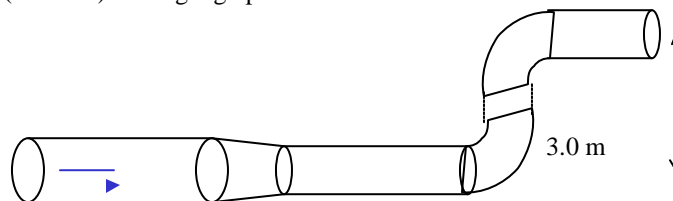 - Venturi effect: pressure is lower in the neck of the tube:

  - ping pong ball in air stream:

23

Bernoulli's Equation

- **Example:** Water is flowing at 0.60 liters/min through a horizontal pipe that starts with a diameter of 3/4 inch (1.91 cm), and then narrows to 1/2 inch (1.27 cm). The gauge pressure in the wide section is 80 kPa.

3.0 m

  - What is the water pressure on the second floor, 3.0 meter above?
  - (Recall that earlier we found that v_{wide}= 3.84 m/s and v_{narrow}=7.90 m/s)

24