Basics of the Dipole Antenna

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Contents

1 Outline of Presentation

This lecture note reviews the mathematical basis for linear antennas, closely following the presentation in Chapter 10 of *Elements of Engineering Electromagnetics* by N.N. Rao [1]. The behavior of a linear antenna begins with an analysis of an "incremental" antenna, i.e., an antenna of incremental length (substantially less than the wavelength of the E&M signal being generated. Given the electromagnetic fields of such an incremental antenna (called a *Hertzian dipole*) at a distance from the antenna substantially larger than the wavelength of the signal (i.e., the so-called "far-field" pattern, the electromagnetic field produced by a linear antenna of larger length (e.g., comparable to or substantially greater than the wavelength) can be obtained by suitable integration over that antenna. The discussion is restricted to linear antennas (i.e., an antenna corresponding to a straight wire driven by an electrical current).

In the presentation, it is necessary to draw upon background in E&M and to develop the analysis using spherical (corresponding to the electromagnetic field caused by a point source) rather than rectangular (x, y, and z) coordinates. Transformations to convert from rectangular to spherical coordinates are provided.

2 The Incremental Antenna: Hertzian Dipole

2.1 The Basic Hertzian Dipole Antenna

The Hertzian dipole is shown in Figure 1 and corresponds to a short length of straight wire (of length $dl \ll \lambda$ where λ is the wavelength of the sinusoidal signal) driven by an AC current of frequency ω. The wire is shown as open-ended (i.e., without termination resistances or the usual return path), with the current leading to charge increasing and decreasing at the two ends of the wire. The wire is taken as along the z-axis in a rectangular coordinate system.

The current $I(t)$ is taken as a single frequency sine wave, i.e., $I(t) = I_0 \cos(\omega t)$. Signals corresponding to multifrequency signals (e.g., a passband signal as might appear in a true mobile radio system) can be considered to be composed of several sinusoids through use of the Fourier transform. However, we shall see that the radiation pattern of a linear antenna is not uniformly omnidirectional, with the directionality depending on the wavelength. You may want to give this some thought when the pattern for a given frequency is given later.

2.2 Electromagnetic Field from a Hertzian Dipole Antenna

To determine the electromagnetic field of the Hertzian antenna, some digression into the general topic of E&M is necessary. If your mind is hazy regarding the basics of E&M and Maxwell's equations, a quick review of your course book would be helpful.

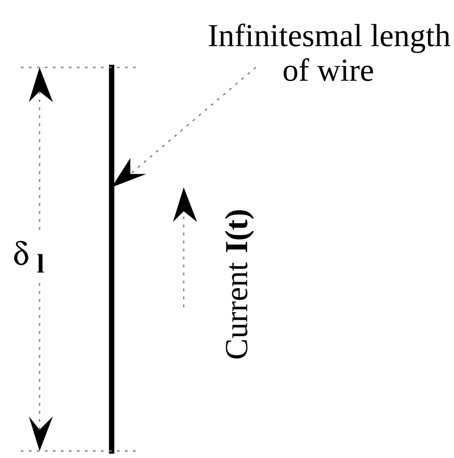

Figure 1: Hertzian antenna

2.3 Quick Review of Fields and Maxwell's Equations

The electric field $\vec{E}$ (volts/m) and magnetic field or flux density $\vec{B}$ (Wb/m^2) are vectors and were the main topics of your E&M course. When you considered media other than free space, it was necessary to take into account electric and magnetic dipoles, leading to the vectors $\vec{D}$ and $\vec{H}$, respectively. The *displacement flux density* $\vec{D}$ was related to the dielectric constant ϵ_r, the permittivity of free space ϵ_0 and the electric polarization $\vec{P}$ of the dielectric material by

$$\begin{aligned} \vec{D} &= \epsilon_r \epsilon_0 \vec{E} + \vec{P} \\ \vec{D}(\text{free space}) &= \epsilon_0 \vec{E} \end{aligned} \tag{1}$$

where $\vec{D}$ in free space results from $\epsilon_r \equiv 1$ in free space. The *magnetic field intensity* $\vec{H}$ is related to the relative permeability μ_0 and the magnetic polarization $\vec{M}$ by

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}. \tag{2}$$

The electric and magnetic fields are not independent, making it more convenient to work with a variable that represents both (namely, the vector field $\vec{A}$). Recall that Maxwell's equations relate the electric and magnetic fields as follows.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\begin{aligned} \vec{\nabla} \times \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\ \vec{\nabla} \cdot \vec{D} &= \rho \\ \vec{\nabla} \cdot \vec{B} &= 0 \end{aligned} \tag{4}$$

The gradient of the curl of a vector necessarily vanishes,i.e., $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) \equiv 0$ for any vector $\vec{A}$. From (4), we can therefore express the magnetic field in terms of a vector as

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{5}$$

Using (5) in (3), we see that we can express $\vec{E}$ in terms of this vector $\vec{A}$ as

$$\vec{E} = -\vec{\nabla}\Phi - \frac{\partial \vec{A}}{\partial t} \tag{6}$$

where the *electric scalar potential* Φ is introduced since the curl of the divergence of any scalar vanishes. This vector $\vec{A}$ is called the magnetic vector potential. Equations 5 and 6 show that, if we use the magnetic vector potential $\vec{A}$, we can readily obtain the electric and magnetic fields directly while automatically satisfying Maxwell's equations.

2.4 Magnetic Vector Potential of Hertzian Antenna

We now return to the Hertzian dipole antenna. The current in the wire leads to a magnetic field encircling the wire. We will start using here spherical coordinates (r, θ, and *phi*) since we will be interested in electromagnetic waves propagating along the line from the transmitting antenna to a receiving antenna (i.e., radially away from the transmitting antenna). In general, for a length of line $\vec{\delta_l}$ carrying a current I, the magnetic field is given by

$$\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \left(\frac{\vec{\delta_l} \times \vec{i_r}}{r^2} \right) \tag{7}$$

where $\vec{i_r}$ is the unit vector along the line joining the current element and the point in space of interest and r is the distance from the current element to the point of interest. Note that the magnetic field is perpendicular to both the direction of the line and the direction from the current element to the point of interest, as you would expect from the "right hand rule" with three fingers.

Equation 7 can be simplified somewhat by noting that

$$\vec{i}_r \equiv -\vec{\nabla}\frac{1}{r}, \tag{8}$$

a result that you should check back in one of your E&M reference or vector algebra books. Using this equivalence, (7) becomes

$$\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \vec{\delta}_l \times \left(-\vec{\nabla}\frac{1}{r}\right) \tag{9}$$

Continuing the vector calculus razz-ma-taz, the following vector identity always holds.

$$\vec{V} \times \vec{\nabla} S = S\vec{\nabla} \times \vec{V} - \vec{\nabla} \times S\vec{V} \tag{10}$$

allowing (9) to be rewritten as

$$\vec{B} = \frac{\mu_0 I}{4\pi r}\vec{\nabla} \times \vec{\delta}_l + \vec{\nabla} \times \frac{\mu_0 I \vec{\delta}_l}{4\pi r} \tag{11}$$

which, since $\vec{\delta}_l$ is constant, becomes

$$\vec{B} = \vec{\nabla} \times \frac{\mu_0 I \vec{\delta}_l}{4\pi r} \tag{12}$$

Comparing (12) with (5), the magnetic vector potential due to the current element is

$$\begin{aligned} \vec{A} &= \frac{\mu_0 I \vec{\delta}_l}{4\pi r} \\ &= \left(\frac{\mu_0 I \delta_l}{4\pi r}\right)\vec{i}_z. \end{aligned} \tag{13}$$

where $\vec{i}_z$ is the unit vector in the z-direction (i.e., along the wire).

2.5 Magnetic Vector Potential for Sinusoidal Current

The current I in (13) was assumed at the start to be a time-varying current given by $I(t, r = 0) = I_0 \cos(\omega t)$ at the dipole. At distances away from the dipole, there is a retardation effect due to the time required for the electromagnetic field to propagate to the point $r \neq 0$, i.e., the time varying part becomes $\cos(\omega\{t - r/c)\} = \cos(\omega t - \beta r)$ where c is the propagation velocity (speed of light assuming free space propagation) of the electromagnetic signal and the *phase constant* $\beta = \omega/c$. Therefore,

$$\vec{A} = \left(\frac{\mu_0 I_0 \delta_l}{4\pi r}\right)\cos(\omega t - \beta r)\vec{i}_z. \tag{14}$$

Next, some more mathematical manipulation regarding use of spherical coordinates is necessary. The relationship between the unit vector $\vec{i}_z$ in a rectangular coordinate system and the unit vectors ($\vec{i}_r$, $\vec{i}_\theta$, and $\vec{i}_\phi$) in a spherical coordinate system is:

$$\vec{i}_z = \cos(\theta) \cdot \vec{i}_r - \sin(\theta) \cdot \vec{i}_\theta \tag{15}$$

Using (15) in (14) gives

$$\vec{A} = \left(\frac{\mu_0 I_0 \delta_l}{4\pi r}\right)\cos(\omega t - \beta r)\left(\cos(\theta) \cdot \vec{i}_r - \sin(\theta) \cdot \vec{i}_\theta\right) \tag{16}$$

$$= A_r \cdot \vec{i}_r + A_\theta \cdot \vec{i}_\theta \tag{17}$$

where the two components of $\vec{A}$ are given by (compare equations 16 and 17)

$$A_r = \left(\frac{\mu_0 I_0 \delta_l}{4\pi r}\right)\cos(\omega t - \beta r)\cos(\theta) \tag{18}$$

$$A_\theta = -\left(\frac{\mu_0 I_0 \delta_l}{4\pi r}\right)\cos(\omega t - \beta r)\sin(\theta) \tag{19}$$

2.6 E and H fields for Hertzian Dipole

We are now at the point where we can determine $\vec{B}$ and $\vec{E}$. However, we need to take the curl of $\vec{A}$ in spherical coordinates. Again, some spherical razz-ma-taz is used. Referring to your book on E&M or a vector calculus book, the curl in spherical coordinates is given by

$$\begin{aligned}\vec{\nabla}\times\vec{A} &= \frac{1}{r\sin(\theta)}\left[\frac{\partial}{\partial\theta}\left(A_\phi\sin(\theta)\right)-\frac{\partial A_\theta}{\partial\phi}\right]\vec{i}_r \\ &+\frac{1}{r}\left[\frac{1}{\sin(\theta)}\frac{\partial A_r}{\partial\phi}-\frac{\partial}{\partial r}(rA_\phi)\right]\vec{i}_\theta \\ &+\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial A_r}{\partial\theta}\right]\vec{i}_\phi\end{aligned} \tag{20}$$

Using (20), $\vec{H}$ and $\vec{B}$ are given by

$$\begin{aligned}\vec{H} &= \frac{\vec{B}}{\mu_0}=\frac{1}{\mu_0}\vec{\nabla}\times\vec{A} \\ &= \frac{1}{\mu_0 r}\left[\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial A_r}{\partial\theta}\right]\cdot\vec{i}_\phi \\ &= \frac{I_0\delta_l\sin(\theta)}{4\pi}\left[\frac{cos(\omega t-\beta r)}{r^2}-\frac{\beta\sin(\omega t-\beta r)}{r}\right]\vec{i}_\phi \\ &\text{or} \\ H_\phi &= \frac{1}{\mu_0 r}\left[\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial A_r}{\partial\theta}\right] \\ &= \frac{I_0\delta_l\sin(\theta)}{4\pi}\left[\frac{cos(\omega t-\beta r)}{r^2}-\frac{\beta\sin(\omega t-\beta r)}{r}\right]\end{aligned} \tag{21}$$

Given $\vec{H}$, we can use Maxwell's equations, given earlier, to obtain the electric field (with $\vec{J}=0$ at the point away from the Hertzian dipole where we are observing the fields). Therefore

$$\begin{aligned}\frac{\partial\vec{E}}{\partial t} &= \frac{1}{\epsilon_0}\vec{\nabla}\times\vec{H} \\ &= \frac{1}{\epsilon_0 r^2\sin(\theta)}\frac{\partial}{\partial\theta}\left(r\sin(\theta)H_\phi\right)\vec{i}_r-\frac{1}{\epsilon_0 r\sin(\theta)}\frac{\partial}{\partial r}\left(r\sin(\theta)H_\phi\right)\vec{i}_\theta\end{aligned}$$

Taking the derivatives of H_ϕ and integrating over time yields

$$\begin{aligned}\vec{E} &= \frac{2I_0\delta_l\cos(\theta)}{4\pi\epsilon_0\omega}\left[\frac{\sin(\omega t-\beta r)}{r^3}+\frac{\beta\cos(\omega t-\beta r)}{r^2}\right]\vec{i}_r \\ &+\frac{I_0\delta_l\sin(\theta)}{4\pi\epsilon_0\omega}\left[\frac{\sin(\omega t-\beta r)}{r^3}+\frac{\beta\cos(\omega t-\beta r)}{r^2}-\frac{\beta^2\sin(\omega t-\beta r)}{r}\right]\vec{i}_\theta\end{aligned} \tag{22}$$

Equations 21 and 22 look rather horrendous but they include terms close to the antenna (i.e., the so-called "near-field terms"). As we move away from the dipole antenna, the terms involving r^{-2} and r^{-3} become negligible relative to the terms involving r^{-1} and can be ignored (in the "far-field region"). In fact, the distance r at which these "lower order terms" can be neglected is defined by $\beta/r \gg 1/r^2$ or $r \gg 1/\beta$. We can relate all this to the wavelength λ. Recall that β was defined as $\beta=\omega/c$. The wavelength, on the other hand, is defined as $\lambda=c/f$. Since $\omega=2\pi f$, $\beta/(2\pi)=f/c$ or $2\pi/\lambda=\beta$. Therefore, the far field region corresponds to $r\gg\lambda/2\pi$, where the fields are given by

$$\vec{H}(r\gg\lambda/2\pi) = -\left[\frac{I_0\delta_l\beta}{4\pi}\right]\left[\frac{\sin(\theta)}{r}\right]\sin(\omega t-\beta r)\vec{i}_\phi \tag{23}$$

and

$$\vec{E}(r \gg \lambda/2\pi) = -\left[\frac{I_0 \delta_l \beta^2}{4\pi\epsilon_0\omega}\right]\left[\frac{\sin(\theta)}{r}\right]\sin(\omega t - \beta r)\vec{i}_\theta$$
$$= -\left[\frac{\eta I_0 \delta_l \beta}{4\pi}\right]\left[\frac{\sin(\theta)}{r}\right]\sin(\omega t - \beta r)\vec{i}_\theta \quad (24)$$

where

$$\eta \equiv \beta/(\epsilon_0\omega). \quad (25)$$

Note that in the far field region, both $\vec{H}$ and $\vec{E}$ are in the plane normal to the direction $\vec{i}_r$, i.e., the direction along the line from the dipole antenna to the point of observation.

2.7 Radiated Power and Radiation Resistance for Hertzian Dipole

We are now in a position to calculate the radiation power at some point in the far field region. The power is given by the *Poynting vector* ($\vec{P}$), i.e.,

$$\vec{P} = \vec{E} \times \vec{H}$$
$$= E_\theta \vec{i}_\theta \times H_\phi \vec{i}_\phi$$
$$= \left[\frac{\eta\beta^2 I_0^2(\delta_l)^2\sin^2(\theta)}{16\pi^2 r^2}\right]\sin^2(\omega t - \beta r)\vec{i}_r \quad (26)$$

The total radiated power P_{rad} is obtained by integrating $\vec{P}$ over the surface of a sphere with radius r, i.e.,

$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\left[(\vec{P}\cdot\vec{i}_r\right] r^2\sin(\theta)\, d\theta d\phi$$
$$= \frac{\eta\beta^2 I_0^2(\delta_l)^2}{6\pi}\sin^2(\omega t - \beta r) \quad (27)$$
$$= \frac{2\pi\eta I_0^2}{3}\left(\frac{\delta_l}{\lambda}\right)^2\sin^2(\omega t - \beta r) \quad (28)$$

where we have introduced the wavelength λ into the equation. Next, we can do a time average of the radiated power to obtain the time-averaged power. Since the average value of $\sin^2(\omega t - \beta r) = 1/2$, the time averaged, radiated power is

$$< P_{rad} > = \frac{\pi\eta I_0^2}{3}\left(\frac{\delta_l}{\lambda}\right)^2 < \sin^2(\omega t - \beta r) >$$
$$= \frac{I_0^2}{2}\left[\frac{2\pi\eta}{3}\left(\frac{\delta_l}{\lambda}\right)^2\right] \quad (29)$$

The bracketed term in (29) defines the *radiation resistance* (R_{rad}), defined as the value of a resistor driven by the current in the dipole antenna which would lead to the same average power dissipation as the average radiated power. Therefore,

$$R_{rad} = \frac{2\pi\eta}{3}\left(\frac{\delta_l}{\lambda}\right)^2 \quad (30)$$

The term η above is called the *impedance of free space* and has a standard value
Impedance of free space $= \eta = 120\pi$ ohms.

2.8 Directionality and Directivity of Hertzian Dipole Antenna

Equation 26 shows that the radiated power has a maximum at $\theta = \pi/2$ (i.e., along a direction normal to the wire) and has a minimum (equal to zero) at $\theta = 0, \pi$ (i.e., along a line extending from the wire and parallel to the wire). A plot of the radiated power in fact looks like a pair of eggs, extending from the right and left of the center of the wire in a direction normal to the wire. The maximum radiated power density $[P_r]_{max}$ is, from (26) with $\theta = \pi/2$,

$$[P_r]_{max} = \left[\frac{\eta\beta^2 I_0^2(\delta_l)^2}{16\pi^2 r^2}\right] \sin^2(\omega t - \beta r) \tag{31}$$

The average power density, $[P_r]_{av}$, is obtained by dividing the radiated power (equation 27) by the surface area $4\pi r^2$ of the sphere at distance r, i.e.,

$$[P_r]_{av} = \left[\frac{\eta\beta^2 I_0^2(\delta_l)^2}{24\pi^2 r^2}\right] \sin^2(\omega t - \beta r) \tag{32}$$

The *directivity* (D) of the antenna is defined as the ratio of the maximum power density to the average power density which, using (31) and (32), is given by

$$D = \frac{[P_r]_{max}}{[P_r]_{av}} = 3/2 = 1.5 \tag{33}$$

3 Half-Wavelength Dipole Antenna

With the background above, it is relatively easy to determine the radiation characteristics of a half-wave dipole antenna. The basic structure of a dipole antenna (see Figure 2a) is two wires, one extending in the $+z$ direction and the other extending in the -z direction, with the sum of the lengths of these two sections being $L = \lambda/2$. Current $I(t, z = 0) = I_0 \cos(\omega t)$ is fed into the center (i.e., at the two endpoints at $z = 0$). The current $I(t, z)$ vanishes at each wire's end (i.e., at $z = \pm L$)[1]. The current at any point on the antenna is given by

$$I(t, z) = I_0 \cos(\pi z/L) \cos(\omega t) \tag{34}$$

where $\cos \pi z/L$ represents the peak current at any point z along the wire.

To determine the fields well away from the antenna, each point on the wire can be viewed as a point source (Hertzian dipole antenna) and the fields for these Hertzian dipole at some distance from the antenna, given above. can be integrated over the finite length dipole antenna. In those equations above, we replace the current term I_0 with the current $I_0 \cos(\pi z/L)$, giving the various fields a z-dependence. The fields far away from the antenna are then obtained by integrating the expressions for the Hertzian dipole antenna (with the substitution for I_0 above) from $z = -L/2$ to $z = +L/2$. Some approximations come into play during the derivation, which are not presented here. However, the final results for the fields are summarized as follows (for frequency ω and phase constant β matched to the antenna length $L \equiv \lambda/2$).

$$\begin{aligned}
E_\theta &= -\left[\frac{\eta I_0}{2\pi r}\right]\left[\frac{\cos[(\pi/2)\cos(\theta)]}{\sin(\theta)}\right]\sin(\omega t - \beta r) \\
H_\phi &= -\left[\frac{I_0}{2\pi r}\right]\left[\frac{\cos[(\pi/2)\cos(\theta)]}{\sin(\theta)}\right]\sin(\omega t - \beta r) \\
\vec{P} &= \vec{E} \times \vec{H} \\
&= \left[\frac{\eta I_0^2}{4\pi^2 r^2}\right]\left[\frac{\cos^2[(\pi/2)\cos(\theta)]}{\sin^2(\theta)}\right]\sin^2(\omega t - \beta r) \\
P_{rad} &= \left[\frac{0.609\eta I_0^2}{\pi}\right]\sin^2(\omega t - \beta r) \\
< P_{rad} > &= \frac{1}{2} \cdot I_0^2 \cdot \left[\frac{0.609\eta}{\pi}\right] \\
R_{rad} &= \frac{0.609\eta}{\pi} \quad \text{ohms}
\end{aligned}$$

[1]This is not intuitive but involves a special "folded" model of the line, not discussed here.

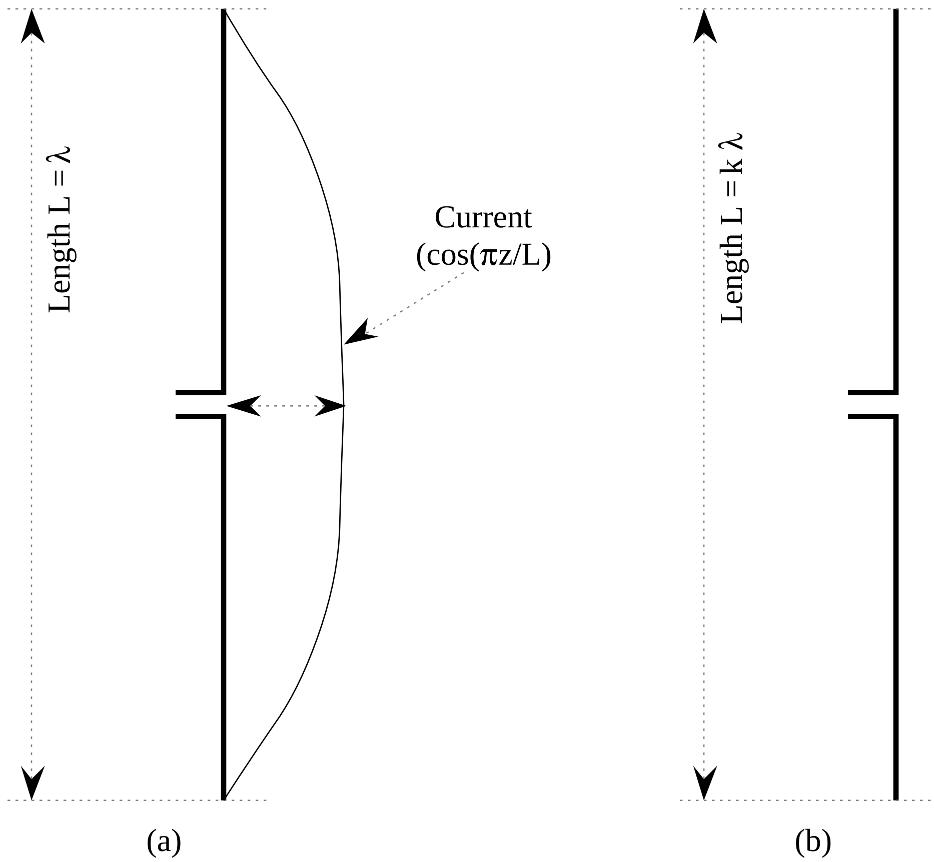

Figure 2: Dipole antenna. (a) Half wavelength antenna ($L = \lambda$). (b) Arbitrary length (L) antenna.

$$D = 1.642$$

4 Dipole Antenna with Longer Length

Next, we consider a dipole antenna as in the previous section but with an arbitrary length L and the length of each of the two sections equal to $L/2$. Again, we don't worry about the detailed derivations here, and give only the main results.

$$\begin{aligned}
E_\theta &= -\left[\frac{\eta I_0}{2\pi r}\right] f(\theta)\sin(\omega t - \beta r) \\
H_\phi &= -\left[\frac{I_0}{2\pi r}\right] f(\theta)\sin(\omega t - \beta r) \\
R_{rad} &= \frac{\eta}{\pi}\int_{\theta=0}^{\pi/2} f^2(\theta)\sin(\theta)d\theta \\
D &= \frac{\left[f^2(\theta)\right]_{max}}{\int_{\theta=0}^{\pi/2} f^2(\theta)\sin(\theta)d\theta}
\end{aligned}$$

where the *radiation pattern* $f(\theta)$ is given by

$$\begin{aligned} f(\theta) &= \frac{\cos\left(\frac{\beta L}{2} cos(\theta)\right) - \cos(\beta L/2)}{\sin(\theta)} \\ &= \frac{\cos\left(\frac{\pi L}{\lambda} cos(\theta)\right) - \cos(\pi L/\lambda)}{\sin(\theta)}. \end{aligned} \quad (35)$$

For the case when the length is an exact integer multiple of the wavelength (i.e., $L \equiv k\lambda$, k an integer), the expression for $f(\theta)$ above becomes

$$f(\theta) = \frac{\cos\left(k\pi cos(\theta)\right) - \cos(k\pi)}{\sin(\theta)}. \quad (36)$$

5 Homework

1. Calculate and plot the radiation pattern $f(\theta)$ given by equation 35 for values of $\alpha = L/\lambda$ equal to 1, 1.5, 2, 3, and 5. You can use MatLab or any other mathematics program you wish.

2. Assuming that the antenna has a length $L = \lambda$ for a frequency $f = 500$ MHz, calculate the radiation pattern given by equation 35 for frequencies $f = 300$, 400, 500, 600 and 700 MHz. If a wideband signal (band extending from 300 to 700 MHz) is used to carry channels, does your result suggest that the different frequencies will have different reception effects for an antenna designed for the center frequency of 500 MHz?

References

[1] N.N. Rao, *Elements of Engineering Electromagnetics*, 3rd Edition, Prentice Hall: Englewood Cliffs, NJ, 1991.