Shaping Demand to Match Anticipated Supply

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1. Introduction

With the expanding reach and connectivity of information systems and increasing visibility of transactions across the supply chain, manufacturing firms are focusing on how to exploit the available information to streamline their operations and maximize profits. Many recent papers (e.g., Gavirneni et al. 1999, Lee et al. 2000) have addressed the benefits of information sharing by downstream supply chain partners (e.g., conveying retail demand information to suppliers). This paper addresses an opportunity for firms to exploit information from upstream suppliers to better match their demand with anticipated supply. Companies such as Dell have access to information from suppliers and shippers on the status of pending orders for raw material and components, and can therefore accurately predict when they will receive each order. Using this information, they can maximize profits by shaping demand through dynamic pricing. For instance, if a computer manufacturer anticipates shortage of a particular disk drive or video card, it can adjust the relative prices of its computer models to "steer" consumers to alternate computer configurations that do not require the scarce parts. Using a stylized economic model that combines concepts from revenue management and market segmentation, we address these benefits of upstream information sharing in the context of a firm selling vertically differentiated products. The revenue management literature (e.g., Bitran and Caldentey 2003, Talluri and Van Ryzin 2004) studies dynamic pricing, assuming that a fixed capacity is available at the beginning of the planning horizon (whereas we permit periodic replenishments). Marketing researchers (e.g., Moorthy and Png 1992, Desai 2001) address product line design issues for vertically differentiated products. We focus on a multi-period pricing problem, with deterministic demand and known replenishment quantities in each period; the pricing decisions must account for the tradeoff between exhausting the inventory in each period versus carrying forward inventory for future sale, and also the impact of each product's price on other products' demand. Our analysis reveals some interesting properties of optimal dynamic pricing strategies when component availability is limited; we also develop an efficient solution algorithm based on these properties. This work serves as the foundation for more advanced models incorporating features such as component commonality and time varying customer arrival rates.

2. Demand Shaping Model

Consider a monopolist manufacturer who produces and sells two vertically differentiated products, a high quality product h and a low quality product l. The manufacturer relies on suppliers to provide various materials and components needed for the products. Let C_h and C_l denote the sets of components needed for the high and low quality products, respectively. Without loss of generality, we assume that each unit of a product requires one unit of each component in its component set. For this paper, we will assume that the two products do not share any common components. For each component $j \in C_h \cup C_l$, the manufacturer knows, through the upstream information sharing mechanisms, exactly how many units of the component will arrive in each of the following m periods. Let A_{jt} denote the number of units of component j that the manufacturer will receive at the start of period t, for $t = 1, 2, \ldots, m$; without loss of generality, we can assume that the initial inventory of each component is zero (if not, we can add the actual initial inventory to the quantity received in the first period). Since each unit of finished product requires one unit of each component, we can determine the number of "kits" (or full sets of components to produce finished products) that become available in each period. Specifically, for all $t = 1, 2, \ldots, m$, and for product $k = h$ or l, let $B_{kt} = \min_{j \in C_k} \left\{ \sum_{t'=1}^{t} A_{jt'} \right\}$ denote the *cumulative availability* of kits for product k until time t. Then, $R_{kt} = B_{kt} - B_{k,t-1}$ is the additional number of component kits that become available for product k in period t; we will refer to this value as product k's *kit replenishment* in period t.

To model demand, we assume that λ potential customers arrive in each period. All customers value the high quality product more than the low quality product; however, they differ in the incremental price they are willing to pay for this quality differential. Let q_h and q_l, with $q_h > q_l > 0$, denote the *quality levels* of the high and low quality products, respectively. Each customer has an associated nonnegative *quality sensitivity* parameter θ that determines her valuation of quality. In particular, if p denotes the price of a product with quality level q, then the customer's utility or surplus from purchasing the product is $\theta q - p$. The customers arriving in each period differ in their quality sensitivity θ. Suppose θ follows a Uniform distribution from 0 to 1, i.e., in any period, the proportion of arriving customers with quality sensitivity greater than or equal to θ is $(1-\theta)$. Observe that customers with lower quality sensitivity assign a lower value to the quality differential $(q_h - q_l)$ between the two products compared to those with higher quality sensitivity. So, the price differential and the customer's quality sensitivity together determine whether she will purchase product l or h (or no product at all).

Given the anticipated kit replenishments and the characteristics of demand, we seek the dynamic pricing strategy that maximizes the manufacturer's total profits during the m-period planning horizon. For

$t = 1, 2, \ldots, m$, let p_{ht} and p_{lt} denote, respectively, the prices of the high and low quality products. Then, for any period t, a consumer with quality sensitivity θ maximizes her utility by adopting the following purchasing strategy:

Buy the high quality product	if $\theta q_h - p_{ht} > \theta q_l - p_{lt}$ and $\theta q_h - p_{ht} > 0$,
Buy the low quality product	if $\theta q_l - p_{lt} > \theta q_h - p_{ht}$ and $\theta q_l - p_{lt} > 0$, and
Do not buy either product	otherwise.

Since $q_h > q_l$, customers who purchase product h must all have higher quality sensitivity than those who purchase product l; in turn, these latter customers have higher quality sensitivity than those who do not purchase the product. Let $\theta_{ht} = (p_{ht} - p_{lt})/(q_h - q_l)$ denote the threshold value of quality sensitivity at which a customer is indifferent between purchasing the high or low quality product; similarly, let $\theta_{lt} = p_{lt}/q_l$ be the threshold value at which a customer is indifferent between purchasing the low quality product and not purchasing any product. (We can show that the manufacturer will price product h high enough relative to product l to satisfy $p_{ht}/p_{lt} \geq q_h/q_l$, and so $0 \leq \theta_{lt} \leq \theta_{ht} \leq 1$.) Then, all customers with $\theta \in [\theta_{ht}, 1]$ purchase the high quality product, those with $\theta \in [\theta_{lt}, \theta_{ht})$ purchase the low quality product, while the rest do not purchase any product. Therefore, in period t, the demand for high and low quality products is $(1-\theta_{ht})\lambda$ and $(\theta_{ht} - \theta_{lt})\lambda$, respectively. So, if the manufacturer has only limited high quality kits available in inventory or wishes to preserve these kits for later sale, it can regulate the demand for high quality product by raising the price p_{ht}, thereby increasing θ_{ht}.

To formulate the multi-period demand shaping (pricing) problem as an optimization model, we define the following inventory and sales variables. For $k = h$ or l, and $t = 1, 2, \ldots, m$, let S_{kt} be the quantity of product k sold in period t, and let I_{kt} be the number of kits for product k carried forward at the end of period t. Using these variables, together with variables for the prices and quality sensitivity thresholds, the demand shaping problem has the following nonlinear programming formulation [DSP]:

[DSP] $$\text{Maximize} \sum_{t=1}^{m} \left(p_{ht} S_{ht} + p_{lt} S_{lt} \right) \tag{1}$$

subject to:

Inventory balance:	$S_{ht} = I_{h,t-1} + R_{ht} - I_{ht}$	for $t = 1, \ldots, m$,	(2)
	$S_{lt} = I_{l,t-1} + R_{lt} - I_{lt}$	for $t = 1, \ldots, m$,	(3)
Incentive compatibility:	$\theta_{ht} q_h - p_{ht} \geq \theta_{ht} q_l - p_{lt}$	for $t = 1, \ldots, m$,	(4)
	$\theta_{lt} q_l - p_{lt} \geq \theta_{lt} q_h - p_{ht}$	for $t = 1, \ldots, m$,	(5)
Individual rationality:	$\theta_{ht} q_h - p_{ht} \geq 0$	for $t = 1, \ldots, m$,	(6)
	$\theta_{lt} q_l - p_{lt} \geq 0$	for $t = 1, \ldots, m$,	(7)
Sales:	$S_{ht} = \lambda(1 - \theta_{ht})$	for $t = 1, \ldots, m$,	(8)

	$S_{lt} = \lambda(\theta_{ht} - \theta_{lt})$	for $t = 1,\ldots, m$,	(9)
Nonnegativity:	$\theta_{lt}, \theta_{ht} \geq 0$	for $t = 1,\ldots, m$, and	(10)
	$I_{l0} = I_{h0} = 0$, $I_{lt}, I_{ht}, S_{lt}, S_{ht} \geq 0$	for $t = 1,\ldots, m$.	(11)

The objective function (1) maximizes the total profits for high and low quality products over all periods. Constraints (2) and (3) are the inventory balance identities. The incentive compatibility constraints (4) and (5) ensure that consumers purchasing the high quality product are not better off by switching to the low quality product, and vice versa. The individual rationality constraints (6) and (7) specify that all consumers who purchase either product get nonnegative utility. Constraints (8) and (9) define the sales variables, and constraints (10) and (11) impose nonnegativity. Note that constraints (8) and (9), together with nonnegativity of sales, imply that $\theta_{lt} \leq \theta_{ht} \leq 1$, while nonnegativity of the θ variables ensure that the total sales in each period never exceed the number of arriving customers λ.

3. DSP Properties and Solution Method

We now identify some properties of optimal DSP solutions, and later describe a polynomial solution procedure based on these properties. We omit the formal proofs for the results discussed in this section.

First, formulation [DSP] has an optimal solution in which constraints (4) and (7) are binding, i.e.,

$$p_{lt} = \theta_{lt} q_l \text{ and } p_{ht} = p_{lt} + \theta_{ht}(q_h - q_l) \text{ for all } t = 1, 2,\ldots, m. \tag{12}$$

Otherwise, we can raise the product prices in period t without decreasing total profits. Furthermore, these prices automatically satisfy constraints (5) and (6). Therefore, we can replace constraints (4) to (7) with equalities (12), converting all constraints (except the nonnegativity requirements) of the DSP model to equality constraints (henceforth, we will assume this equality form for formulation [DSP]). So, we can express the model in terms of any one set of variables—the θ variables, inventory variables, sales variables, or price variables—by substituting for the other variables in the objective function and nonnegativity constraints.

We can also show that an optimal DSP solution sets $\theta_{lt} \geq \frac{1}{2}$ for all periods t. So, the total sales (and demand) for both the high and low quality products in any period never exceeds $\lambda/2$. Therefore, if we define the cumulative future availability of kits for product $k = h$ or l as $W_{kt} = \sum_{t'=t}^{m} R_{kt'}$, we can assume without loss of generality that $W_{ht} + W_{lt} \leq \lambda(m - t + 1)/2$ for all $t = 1, 2, \ldots, m$.

Let us now consider a "relaxation" of the problem in which all the replenishments occur in the first period, i.e., the manufacturer receives all W_{h1} kits for the high quality product and W_{l1} kits for the low quality product at the start of the first period. Then, the manufacturer maximizes profits by maintaining a constant price for each product throughout the planning horizon so as to sell high and low quality products at constant rates of W_{h1}/m and W_{l1}/m units per period, respectively. The total profit for this strategy is an upper bound on the optimal value of model [DSP]. The manufacturer must deviate from this constant pricing strategy if the actual replenishments are not adequate to sustain the desired constant sales rates; in this case, profit maximization requires deviating as little as possible from the target rates. These observations suggest a close connection between pricing decisions in any period and whether or not the manufacturer runs out of inventory during that period.

Given any feasible solution to model [DSP], we say that a period t is an *exhaust* period for the high (low) quality product if the solution does not transfer or carry forward any inventory of the high (low) quality product to the next period, i.e., if $I_{ht} = 0$ ($I_{lt} = 0$). We refer to periods that are not exhaust periods as *transfer* periods. The following result characterizes the optimal solution in terms of exhaust periods, and identifies some interesting and useful properties of the optimal pricing strategy.

Theorem 1 (optimality conditions): Any feasible DSP solution is optimal if and only if:

(a) the prices for both the high and low quality products are non-increasing with time; and,

(b) between consecutive exhaust periods of the high (low) quality product, the price for the product is constant.

Now, suppose we know the optimal sets of *transfer* periods $T_h^* \subseteq \{1,2,...,m\}$ and $T_l^* \subseteq \{1,2,...,m\}$ for the high and low quality products. We next discuss how to determine the optimal solution to [DSP].

Theorem 2 (**simultaneous equations**): Given the set of optimal transfer periods T_h^* and T_l^*, we can find the optimal solution (i.e., inventory values I_{kt}) to [DSP] by solving the following simultaneous equations:

$$2I_{ht} - I_{h,t-1} - I_{h,t+1} + q_l/q_h(2I_{lt} - I_{l,t-1} - I_{l,t+1}) = R_{ht} - R_{h,t+1} + q_l/q_h(R_{lt} - R_{l,t+1}) \quad \text{for all } t \in T_h^*\text{, and} \tag{13}$$

$$2I_{ht} - I_{h,t-1} - I_{h,t+1} + (2I_{lt} - I_{l,t-1} - I_{l,t+1}) = R_{ht} - R_{h,t+1} + (R_{lt} - R_{l,t+1}) \quad \text{for all } t \in T_l^*\text{, and} \tag{14}$$

$$I_{ht} = 0 \quad \text{for } t \notin T_h^*\text{, and } I_{lt} = 0 \text{ for } t \notin T_l^*. \tag{15}$$

Equations (13) and (14) are equivalent to setting $p_{ht} = p_{h,t+1}$ for all $t \in T_h^*$, and $p_{lt} = p_{l,t+1}$ for all $t \in T_l^*$, corresponding to the optimality requirement that price must be constant between consecutive exhaust periods. Equation (15) ensures that inventory transfer is zero during the specified exhaust periods. Given the inventory transfer solution to this system of equations, we can compute the desired sales in each period (from equations (2) and (3)), the corresponding θ values (using equations (8) and (9)), and the optimal prices (using equation (12)).

These properties of optimal DSP solutions motivate our efficient solution algorithm. Starting with an empty set of transfer periods, the method iteratively identifies additional periods that must be transfer periods in the optimal solution by solving the simultaneous equations (13) to (15) for the current set of transfer periods. A formal statement of the algorithm follows:

Demand shaping algorithm

Step 1: Initialize $T_h = \varnothing$ and $T_l = \varnothing$

Step 2: Solve equations (13) to (15) with respect to the current sets of transfer periods T_h and T_l, and use the inventory solution values I_{kt} to compute the corresponding prices p_{kt} (using equations (2), (3), (8), (9), and (10)) for $k = h$ and l, and $t = 1, 2, \ldots, m$.

Step 3: If the prices for both products are non-increasing with t, Stop. The current set of transfer periods is optimal. Otherwise, suppose $p_{kt} < p_{k,t+1}$ for $k = h$ or l and some $t = 1, 2, \ldots, m-1$. Update $T_k \leftarrow T_k \cup \{t\}$, and repeat Step 2.

The procedure requires $O(m)$ iterations, with the profit increasing at each iteration. We can develop additional properties to efficiently identify periods that must necessarily be exhaust periods.

In summary, this paper has addressed opportunities to maximize profits by shaping demand through price adjustments when the manufacturer has advance information about the deliveries of components. The principles and results we have discussed in this paper also extend readily to situations where the customer arrival rate varies by time period, and the two products use certain common components whose availability is also limited.

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