NATIONAL ENGINEERING HANDBOOK

SECTION 4

HYDROLOGY

CHAPTER 14. STAGE-DISCHARGE RELATIONSHIPS

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NATIONAL ENGINEERING HANDBOOK

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HYDROLOGY

CHAPTER 14. STAGE DISCHARGE RELATIONS

Introduction

In planning and evaluating the structural measures of watershed protection, it is necessary for SCS engineers and hydrologists to develop stage discharge curves at selected locations on natural streams.

Many hydraulics textbooks and handbooks, as well as NEH-5, contain methods for developing stage discharge curves assuming non-uniform steady flow. Some of these methods are elaborate and time consuming. The type of available field data and the use to be made of these stage discharge curves should dictate the method used in developing the curve.

This chapter presents alternate methods of developing these curves at selected points on a natural stream.

Manning's formula has been used to develop stage discharge curves for natural streams assuming the water surface to be parallel to the slope of the channel bottom. This can lead to large errors, since this condition can only exist in long reaches having the same bed slope without a change in cross section shape or retardance.

This condition does not exist in natural streams.

The rate of change of discharge for a given portion of the stage discharge curves differs between the rising and falling sides of a hydrograph. Some streams occupy relatively small channels during low flows, but overflow onto wide flood plains during high discharges. On the rising stage the flow away from the stream causes a steeper slope than that for a constant discharge and produces a highly variable discharge with distance along the channel. After passage of the flood crest, the water re-enters the stream and again causes an unsteady flow, together with a stream slope less than that for a constant discharge. The effect on the stage-discharge relation is to produce what is called a loop rating for each flood.[1/] Generally in the work performed by the SCS the maximum stage the water reached is of primary interest. Therefore, the stage discharge curve used for routing purposes is a plot for the maximum elevation obtained during the passage of flood hydrographs of varying magnitudes. This results in the plot being a single line.

1/ Handbook of Applied Hydrology, Ven Te Chow, page 15-37.

Development of Stage Discharge Curves

Direct Measurement

The most direct method of developing stage discharge curves for natural streams is to obtain velocities at selected points through a cross section. The most popular method is to use a current meter though other methods include the use of the dynamometer, the float, the Pitot tube and chemical and electrical methods. From these velocities and associated cross sectional areas, the discharge is computed for various stages on the rising and falling side of a flood flow and a stage discharge curve developed.

The current meter method is described in detail in USGS Water Supply Paper 888, "Stream Gaging Procedure", and in "Handbook of Hydraulics," by King and Brater, McGraw-Hill, 1963, Fifth edition (generally referred to as King's Handbook).

The velocity head rod (Figure 14-1) may be used to measure flows in small streams or baseflow in larger streams. In making a measurement with a velocity head rod, a tape is stretched across the flowing stream, and both depth and velocity head readings are taken at selected points that represent the cross section of the channel. Table 14-1 is an example of a discharge determined by the velocity head rod. The data is tabulated as shown in columns 1, 2 and 3 of the table and the computation made as shown.

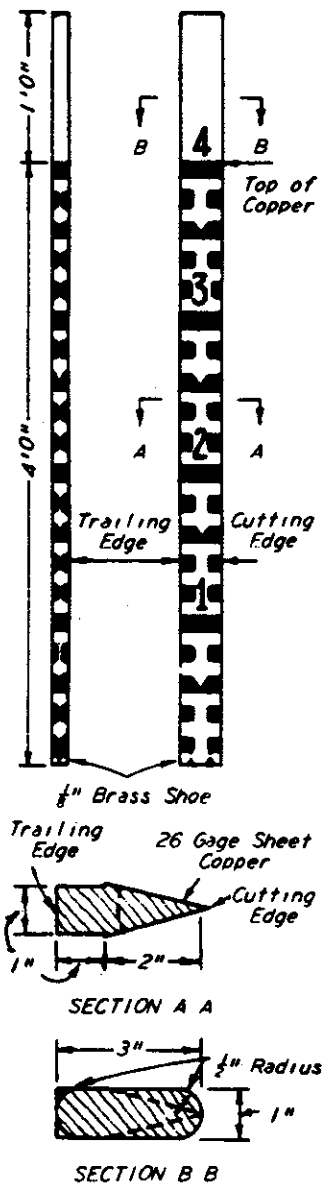
The total area of flow in the section is shown in column 9 and the total discharge in column 10. The average velocity is 45.19/15.00 or 3.01 ft/sec.

Indirect Measurements

Indirectly, discharge is measured by methods such as slope-area, contracted-opening, flow over dam, flow through culvert, and critical depth. These methods, which are described in "Techniques of Water Resources Investigations of the United States Geological Survey," Book 3, Chaps. 3-7, utilize information on the water-surface profile for a specific flood peak and the hydraulic characteristics of the channel to determine the peak discharge.

It should be remembered that no indirect method of discharge determination can be of an accuracy equal to a meter measurement.

Fairly accurate discharges may be computed from measurements made of flows over different types of weirs by using the appropriate formula and coefficients selected from King's "Handbook of Hydraulics," Sections 4 and 5. Overfall dams or broad-crested weirs provide an excellent location to determine discharges. Details on procedures for broad-crested weirs may be found in King's Handbook or USGS Water Supply Paper No. 200, entitled "Weir Experiments, Coefficients, and Formulas" by R. E. Horton.



VELOCITY HEAD ROD
Developed at San Dimas Experimental Forest

The rod is first placed in the water with its foot on the bottom and the sharp edge facing directly upstream. The stream depth at this point is indicated by the water elevation at the sharp edge, neglecting the slight ripple or bow wave. If the rod is now revolved 180 degrees, so that the flat edge is turned upstream a hydraulic jump will be formed by the obstruction to the flow of the stream. After the depth or first reading has been subtracted from the second reading, the net height of the jump equals the actual velocity head at that point. Velocity can then be computed by the standard formula,

$$v = \sqrt{2gh} = 8.02\sqrt{h}$$

in which v = Velocity in ft. per sec.
g = Acceleration of gravity (32.16 ft. per sec. per sec.)
h = Velocity head, in ft.

The average discharge for the stream is obtained by taking a number of measurements of depth and velocity throughout its cross section. Q = AV. in which Q = discharge cfs; A = cross sectional area, sq. ft. V = velocity, ft. per sec.

VELOCITY FOR DIFFERENT VALUES OF "h"

$$v = 8.02\sqrt{h}$$

h, Velocity Head in Ft.	Velocity, Ft. per sec.
.05	1.8
.10	2.5
.15	3.1
.20	3.6
.25	4.0
.30	4.4
.35	4.8
.40	5.1
.45	5.4
.50	5.7

Figure 14-1. Velocity head rod for measuring stream flow.

Table 14-1. Computation of discharge using Velocity Head Rod (VHR) measurements.

	Depths of flow using VHR			Velocity					
Distance along Section (ft.)	Cut-ting edge (ft.)	Flat edge (ft.)	Δh Col 3 - Col 2	At point[1] (fps)	Average for section (fps)	Mean depth (from Col 2) (ft.)	Width (from Col 1) (ft.)	Area (Col 7 x Col 8) ($ft.^2$)	Discharge (Col 9 x Col 6) (cfs)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
3.5	0	0		0					
					0.9	0.68	1.00	0.68	0.61
4.5	1.35	1.40	.05	1.8					
					2.15	2.05	0.40	0.82	1.76
4.9	2.75	2.85	.10	2.5					
					3.35	2.90	1.00	2.90	9.72
5.9	3.05	3.32	.27	4.2					
					4.05	3.03	1.50	4.54	18.39
7.4	3.01	3.25	.24	3.9					
					3.40	2.60	.50	1.30	4.42
7.9	2.18	2.31	.13	2.9					
					2.35	1.48	2.80	4.14	9.75
10.7	.78	.83	.05	1.8					
					.90	.39	1.60	.62	.56
12.3	0	0		0					
							Totals	15.00	45.19

[1] Column 5 is read from Figure 14-1 using the Δh in column 4.

Slope-Area Estimates

Field measurements taken after a flood are used to determine one or more points on the stage-discharge curve at a selected location. The peak discharge of the flood is estimated using high water marks to determine the slope.

Three or four cross sections are usually surveyed so that two or more independent estimates of discharge, based on pairs of cross sections, can be made and averaged. Additional field work required for slope-area estimates consists of selecting the stream reach, estimating "n" values and surveying the channel profile and high water profile at selected cross sections. The work is guided by the following:

1. The selected reach is as uniform in channel alignment, slope, size and shape of cross section, and factors affecting the roughness coefficient "n" as is practicable to obtain. The selected reach should not contain sudden breaks in channel bottom grade, such as shallow drops or rock ledges.

2. Elevations of selected high water marks are determined on both ends of each cross section.

3. The three or more cross sections are located to represent as closely as possible the hydraulic characteristics of the reach. Distances between sections must be long enough to keep small the errors in estimating stage or elevation.

The flow in a channel reach is computed by one of the open-channel formulas. The most commonly used formula in the slope area method is the Manning equation

$$Q = \frac{1.49}{n} AR^{2/3}S^{1/2} \qquad \text{(Eq. 14-1)}$$

Where Q is the discharge, n is the coefficient of roughness, A is the cross sectional area, R is the hydraulic radius, and S is the slope of the energy gradient. Rearranging Eq. 14-1 gives

$$\frac{Q}{S^{1/2}} = \frac{1.49}{n} AR^{2/3} \qquad \text{(Eq. 14-2)}$$

The right side of Eq. 14-2 contains only the physical characteristics of the cross section and is referred to as the conveyance factor Kd. The slope is determined from the elevations of the highwater mark and the distances between the high water marks along the direction of flow.

Modified Slope Area Method

The following equations based on Bernoulli's theorem are discussed fully in NEH-5, Supplement A.

$$\frac{q^2}{2g} = \frac{E_1 - E_2}{U_2^{+} - U_1^{-}} \qquad \text{(Eq. 14-3)}$$

where

q = discharge, in cfs
E_1 = elevation of the water surface at the upstream section
E_2 = elevation of the water surface at the downstream section
U_2^+ = and U_1^- = symbols used by Doubt for certain computed values; (See NEH-5, page A.14)

The working equation is derived from equation 14-1,

$$q = \left(\frac{2g(E_1 - E_2)}{U_2^+ - U_1^-}\right)^{1/2} \qquad \text{(Eq. 14-4)}$$

Also from NEH-5 -

$$U_2^+ = \frac{1}{a_2^2} + \frac{\ell g\ s_o}{q_n^2, d_2} \qquad \text{(Eq. 14-5)}$$

and:

$$U_1^- = \frac{1}{a_1^2} - \frac{\ell g\ s_o}{q_n^2, d_1} \qquad \text{(Eq. 14-6)}$$

where ℓ is the length of the reach between sections 1 and 2, and the other symbols are as defined in NEH-5. The nomographs shown in NEH-5, Supplement A as standard drawings ES-75, 76, and 77 are expedient working tools used to solve Equations 14-4, 14-5 and 14-6.

The following example illustrates the modified slope area method and the use of Eq. 14-2. The example is based on data taken from USGS Water Supply Paper 816 (Major Texas Floods of 1936).

Example 14-1 - Using data for the Concho River near San Angelo, Texas, for the September 17, 1936, flood compute the peak discharge that occurred. Figure 14-2 shows Section A and B with the high water mark profile along the stream reach between the two sections.

1. Draw a water surface through the average of the high water mark. From Figure 14-2 the elevation of the water surface at the lower cross section B is 55.98 designated in the example as E_2. The elevation of the water surface at cross section A is 56.50 designated as E_1.

2. Compute the length of reach between the two sections. From Figure 14-2 the length of reach is 680 feet.

3. Divide each cross section into segments as needed due to different "n" values as shown in Figure 14-2.

   In computing the hydraulic parameters of a cross section on a natural stream when flood plain flow exists, it is desirable to divide the cross section into segments. The number of segments will depend on the irregularity of the cross section and

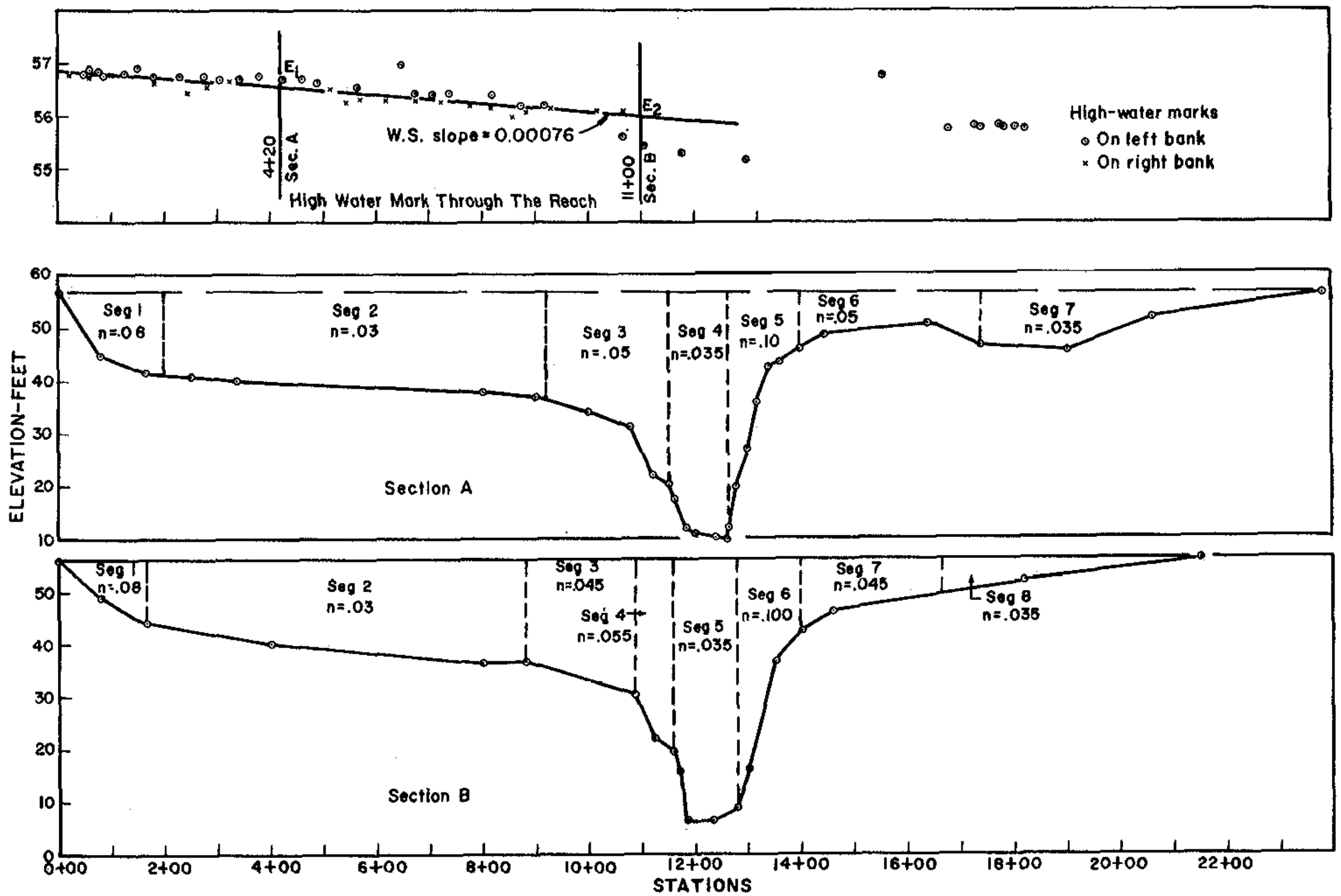


Figure 14-2. High water mark profile and cross sections, Concho River near San Angelo, Texas. Example 14-1.

the variation in "n" values assigned to the different portions. NEH-5, supplement B, gives a method of determining "n" values for use in computing stage discharge curves.

4. Compute the cross sectional area and wetted perimeter for each segment of each cross section. Tabulate in columns 2 and 3 of Table 14-2(a) for cross section A and Table 14-2(b) for cross section B.

5. Compute $F = 1.486\ AR^{2/3}$ for each segment. Using standard drawing ES-76 (NEH-5), compute F and tabulate in column 4, Table 14-2(a) and 14-2(b).

6. Compute $Q/S^{1/2} = 1.486\ AR^{2/3}$. Tabulate the "n" value assigned to each segment in column 5 of Table 14-2(a) and 14-2(b). Column 6 is $A/S^{1/2}$ and is computed by dividing column 4 by column 5 or by using ES-77 (NEH-5). This is commonly called the flow factor of conveyance and is generally designated as Kd.

7. Compute the total area and the total Kd. Sum columns 2 and 6 of Table 14-2(a) and 14-2(b).

8. Compute U^-. Using Eq. 14-6 or ES-77 compute U^- for the downstream cross section A using data from Table 14-2(a).

$$\text{From Eq. 14-6: } U^- = \frac{1}{a_1^2} - \frac{1gs}{q_1^2}$$

$$\frac{1}{a_1^2} = \frac{1}{(34729)^2} = 8.29 \times 10^{-10}$$

$$\frac{s}{q_1^2} = \frac{1}{(91.88 \times 10^5)^2} = 1.18 \times 10^{-14}$$

$$1g\left(\frac{s}{q_1^2}\right) = (680)\ (32.2)\ (1.18 \times 10^{-14}) = 2.58 \times 10^{-10}$$

$$U^- = (8.29 \times 10^{-10}) - (2.58 \times 10^{-10}) = 5.71 \times 10^{-10}$$

9. Compute U^+ Using Eq. 14-5 or ES-77 compute U^+ for upstream cross section B using data in Table 14-2(b).

$$\frac{1}{a_2^2} = \frac{1}{(32771)^2} = 9.31 \times 10^{-10}$$

$$\frac{s}{q_2^2} = \frac{1}{(87.11 \times 10^5)^2} = 1.32 \times 10^{-14}$$

Table 14-2(a) Data for computing discharge from modified slope-area measurements; Cross Section A at Station 4+20. Example 14-1

Segment	Area	Wetted Perimeter	F	n	$\frac{q}{s_o^{1/2}}$
(1)	(2)	(3)	(4)	(5)	(6)
1	2354	252	1.55×10^4	0.080	1.94×10^5
2	12691	735	12.60×10^4	.030	42.00×10^5
3	5862	231	7.50×10^4	.050	15.00×10^5
4	5385	167	8.1×10^4	.035	23.14×10^5
5	2523	135	2.64×10^4	.100	2.64×10^5
6	2498	350	1.38×10^4	.050	2.76×10^5
7	3416	645	1.54×10^4	.035	4.40×10^5
	34729				91.88×10^5

Table 14-2(b) Data for computing discharge from modified slope-area measurements; Cross Section B at Station 11+100. Example 14-1

Segment	Area	Wetted Perimeter	F	n	$\frac{q}{s_o^{1/2}}$
(1)	(2)	(3)	(4)	(5)	(6)
1	1598	236	0.85×10^4	0.080	1.06×10^5
2	11750	725	11.18×10^4	.030	37.27×10^5
3	4750	227	5.37×10^4	.045	11.93×10^5
4	2486	78	3.71×10^4	.055	6.75×10^5
5	4944	153	7.43×10^4	.035	21.23×10^5
6	3455	134	4.47×10^4	.100	4.47×10^5
7	2270	273	1.38×10^4	.045	3.07×10^5
8	1518	513	0.465×10^4	.035	1.33×10^5
	32771				87.11×10^5

$$1g\left(\frac{s}{q_2^2}\right) = (680)\ (32.2)\ (1.32 \times 10^{-14}) = 2.89 \times 10^{-10}$$

$$U^{+} = (9.31 \times 10^{-10} + 2.89 \times 10^{-10}) = 12.20 \times 10^{-10}$$

10. Compute q. Using Eq. 14-4. $q = \left(\frac{2g\ (E_1 - E_2)}{U_2^{+} - U_1^{-}}\right)^{1/2}$

$$q = \sqrt{\frac{(2)\ (32.2)\ (56.50 - 55.98)}{(12.20 - 5.71) \times 10^{-10}}} = 10^5 \times \sqrt{\frac{33.3}{6.49}}$$

$q = 2.265 \times 10^5$ or $q = 226{,}500$. This compares with the discharge of 230,000 cfs computed by USGS in Water Supply Paper 816.

Synthetic methods

There are various methods which depend entirely on data which may be gathered at any time. These methods establish a water surface slope based entirely on the physical elements present such as channel size and shape, flood plain size and shape and the roughness coefficient. The method generally used by the SCS is the modified step method.

This method bases the rate of friction loss in the reach on the elements of the upstream cross section. Manning's equation is applied to these elements and the difference in elevation of the water surface plus the difference in velocity head between the two cross sections is assumed to be equal to the total energy loss in the reach. This method, ignoring the changes in velocity head, is illustrated in Example 14-6.

Selecting Reach Lengths

The flow distance between one section and the next has an important bearing on the friction losses between sections. For flows which are entirely within the channel the channel distance should be used. On a meandering stream the overbank portion of the flow may have a flow distance less than the channel distance. This distance approaches but does not equal the floodplain distance due to the effect of the channel on the flow.

From a practical standpoint the water surface is considered level across a cross section. Thus the elevation difference between two cross sections is considered equal for both the channel flow portion and the overbank portion.

It has been common practice to compute the conveyance for the total section then compute the discharge by using a given slope with this conveyance, where the slope used is an average slope between the slope of the channel portion and the overbank portion. The average slope is computed by the formula:

$$S_a = \frac{H}{L_a} \qquad \text{(Eq. 14-7)}$$

where: S_a = average slope of energy gradient in reach
H = elevation difference of the energy level between sections
L_a = average reach length

The reach length L_a can be computed as follows:

$$q_c = Kd_c \times S_c^{1/2} \quad \text{(Eq. 14-8)}$$

$$q_f = Kd_f \times S_f^{1/2} \quad \text{(Eq. 14-9)}$$

$$q_t = Kd_t \times S_a^{1/2} \quad \text{(Eq. 14-10)}$$

where q_c = discharge in channel portion
Kd_c = conveyance in channel portion
S_c = energy gradient in channel portion
q_f = discharge in floodplain portion
Kd_f = conveyance in floodplain portion
S_f = energy gradient in floodplain portion
q_t = total discharge
Kd_t = total conveyance
S_a = average slope of energy gradient

The total discharge in a reach is equal to the flow in channel plus the flow in the overbank.

Then $q_t = q_c + q_f$ (Eq. 14-11)

Substituting from Equations 14-8, 14-9 and 14-10

$$Kd_t \times S_a^{1/2} = Kd_c \times S_c^{1/2} + Kd_f \times S_f^{1/2} \quad \text{(Eq. 14-12)}$$

Let $S = \frac{H}{L}$

where H = elev. of reach head - elev. of reach foot
L = length of reach

Then substituting into Eq. 14-12 using the proper subscripts

$$Kd_t \times \left(\frac{H}{L_a}\right)^{1/2} = Kd_c \times \left(\frac{H}{L_c}\right)^{1/2} + Kd_f \times \left(\frac{H}{L_f}\right)^{1/2}$$

Divide both sides by $H^{1/2}$

$$\frac{Kd_t}{L_a^{1/2}} = \frac{Kd_c}{L_c^{1/2}} + \frac{Kd_f}{L_f^{1/2}}$$

$$L_a = \left(\frac{Kd_t}{Kd_c/L_c^{1/2} + Kd_f/L_f^{1/2}}\right)^2 \quad \text{(Eq. 14-13)}$$

If the average reach length is plotted vs. elevation for a section then it is possible to read the reach length directly to use with the Kd for any desired elevation. The data will plot in a form as shown in Figure 14-3.

This procedure is somewhat difficult to use as each time a new elevation is selected for use a new reach length must also be used.

The procedure can be modified slightly and a constant reach length used in all computations.

Multiply both sides of Equation 14-9 by $\left(\frac{S_c}{S_c}\right)^{1/2}$

This gives:

$$q_f \left(\frac{S_c}{S_c}\right)^{1/2} = (Kd_f)(S_c{}^{1/2}) \left(\frac{S_f}{S_c}\right)^{1/2} \qquad \text{(Eq. 14-14)}$$

The $\left(\frac{S_c}{S_c}\right)^{1/2}$ on the left hand side drops out with a value of 1 giving

$$q_f = (Kd_f)(S_c)^{1/2} \left(\frac{S_f}{S_c}\right)^{1/2} \qquad \text{(Eq. 14-15)}$$

S_f and S_c can be represented as follows

$$S_f = \frac{H}{L_f} \quad \text{or} \quad (S_f)^{1/2} = \left(\frac{H}{L_f}\right)^{1/2} \qquad \text{(Eq. 14-16)}$$

$$S_c = \frac{H}{L_c} \quad \text{or} \quad (S_c)^{1/2} = \left(\frac{H}{L_f}\right)^{1/2} \qquad \text{(Eq. 14-17)}$$

Divide Equation 14-16 by Equation 14-17

$$\left(\frac{S_f}{S_c}\right)^{1/2} = \frac{\left(\frac{H}{L_f}\right)^{1/2}}{\left(\frac{H}{L_c}\right)^{1/2}} = \left(\frac{L_c}{L_f}\right)^{1/2} \qquad \text{(Eq. 14-18)}$$

Equation 14-15 becomes by substitution:

$$q_f = (Kd_f)(S_c)^{1/2} \left(\frac{L_c}{L_f}\right)^{1/2} \qquad \text{(Eq. 14-19)}$$

The term $\left(\frac{L_c}{L_f}\right)$ is commonly referred to as the meander factor.

Then substituting Equation 14-19 and 14-8 into Equation 14-11 we get

$$q_t = (Kd_c)(S_c)^{1/2} + (Kd_f)(S_c)^{1/2} \left(\frac{L_c}{L_f}\right)^{1/2}$$

Rearranging we get

$$q_t = \left(Kd_c + (Kd_f)\left(\frac{L_c}{L_f}\right)^{1/2}\right)(S_c)^{1/2} \qquad \text{(Eq. 14-20)}$$

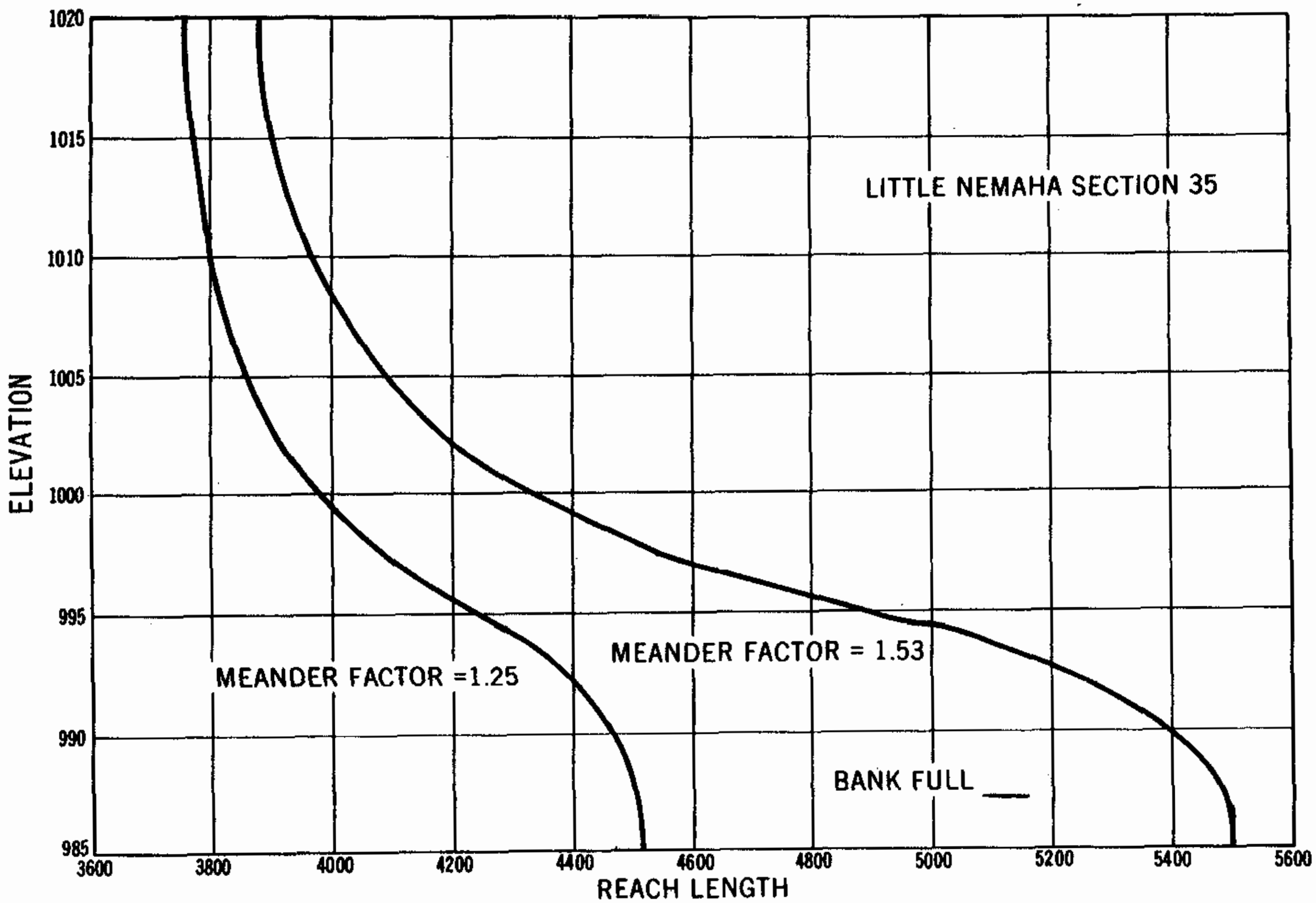


Figure 14-3. Reach length vs. elevation, Little Nemaha Section 35.

Equation 14-20 can be used to compute the total stage discharge at a section by using the channel reach length rather than a variable reach length. Example 14-5 illustrates the use of modifying the flood plain conveyance by the square root of the meander factor in developing a stage discharge curve.

Discharge vs. Drainage Area

It is desirable for the water surface profile to represent a flow which has the same occurrence interval throughout the watershed. The CSM (cubic feet per second per square mile) values for most floods vary within a channel system having a smaller value for larger drainage areas. Thus when running a profile the 50 CSM of the outlet, the actual CSM rate will increase as the profile progresses up the watershed.

The rate of discharge at any point in the watershed is based on the formula[1/]

$$Q = 46C\,A^{\left(\frac{.894}{A^{.048}} - 1\right)} \qquad \text{(Eq. 14-21)}$$

where Q is discharge in CSM
A is the drainage area
and C is a coefficient depending on the characteristics of the watershed

Assuming that C remains constant for any point in the watershed, then the discharge at any point in the watershed may be related to the discharge of any other point in the watershed by the formula

$$\frac{Q_1}{Q_2} = K = \frac{A_1^{\left(\frac{.894}{A_1^{.048}} - 1\right)}}{A_2^{\left(\frac{.894}{A_2^{.048}} - 1\right)}} \qquad \text{(Eq. 14-22)}$$

where Q_1 and A_1 represent the discharge rate in CSM and drainage area of one point in the watershed and Q_2 and A_2 represent the CSM and drainage area at another.

In practice Q_2 and A_2 usually represent the outlet of the watershed and remain constant and A_1 is varied to obtain Q_1 at other points of interest.

Equation 14-22 is plotted in Exhibit 14-1 for the case where A_2 is 400 square miles. This curve may be used directly to obtain the CSM

1/ Engineering For Dams, Vol. 1 page 125, Creager, Justin & Hines.

discharge of the outlet if the outlet is at 400 square miles as shown in Example 14-2. Example 14-3 shows how to use Exhibit 14-1 if the drainage area at the outlet is not 400 square miles.

Example 14-2

Find the CSM value to be used for a reach with a drainage area of 50 square miles when the CSM at the outlet is 80 CSM. The drainage area at the outlet is 400 square miles.

1. Determine K for a drainage area of 50 square miles. From Exhibit 14-1 with a drainage area of 50 square miles read K = 2.61.

2. Determine CSM rate for 50 square miles. Multiply CSM at the outlet by K computed in step 1.

   (80) (2.61) = 209 CSM @ 50 square miles.

Example 14-3

Find the CSM rate to be used at a reach with a drainage area of 20 square miles if the drainage area at the outlet is 50 square miles. The CSM rate at the outlet is 60 CSM.

1. Determine K for a drainage area of 20 square miles. From Exhibit 14-1 for a drainage area of 20 square miles read K = 3.66.

2. Determine K for a drainage area of 50 square miles. From Exhibit 14-1 for a drainage area of 50 square miles read K = 2.61.

3. Compute a new K value for a drainage area of 20 square miles. Divide step 1 by step 2.

   $$\frac{3.66}{2.61} = 1.40$$

4. Determine CSM rate for the 20 square mile drainage area. Multiply K obtained in step 3 by the CSM at the outlet.

   (1.40) (60) = 84 CSM

Computing Profiles

When using water surface profiles to develop stage discharge curves for flows at more than critical depth, it is necessary to have a stage discharge curve for a starting point at the lower end of a reach. This starting point may be a stage discharge curve developed by current meter measurements or one computed from a control section where the flow passes through critical discharge; or it may be one computed from the elements of the cross section and an estimate of the slope. The latter case is the most commonly used by SCS since the more accurate stage discharge curves are not generally available on small watersheds. In most cases it is advisable to locate three or four cross sections close together in order to eliminate part of the error in estimating the slope used in developing the stage discharge curve at the lower or first cross section on a watershed.

Example 14-4

Develop the starting stage discharge curve for cross section M-1 (Figure 14-4) shown as the first cross section at the outlet end of the watershed, assuming an energy gradient of .001 ft/ft.

1. Plot the surveyed cross section. From field survey notes, plot the cross section, Figure 14-5(a) noting the points where there is an apparent change in the "n" value.

2. Divide the cross section into segments. An abrupt change in shape or a change in "n" is the main factor to be considered in determining extent and number of segments required for a particular cross section. Compute the "n" value for each segment using NEH-5, Supplement B, or the "n" may be based on other data or publications.

3. Plot the channel segment on an enlarged scale. Figure 14-5(b), for use in computing the area and measuring the wetted perimeter at selected elevations in the channel. The length of the segment at selected elevations is used as the wetted perimeter for the flood plain segments. The division line between each segment is not considered as wetted perimeter.

4. Tabulate elevations to be used in making computations. Starting at an elevation equal to or above any flood of record, tabulate in column 1 of Table 14-3 the elevations that will be required to define the hydraulic elements of each segment.

5. Compute the wetted perimeter at each elevation listed in step 4. Using an engineer's scale and starting at the lowest elevation in column 1, measure the wetted perimeter of each segment at each elevation and tabulate in columns 3, 7, 11, and 15 of Table 14-3. Note that the maximum wetted perimeter for the channel segment is 62 at elevation 94.

6. Compute the cross sectional area for each elevation listed in step 4. Starting at the lowest elevation, compute the accumulated cross sectional area for each segment at each elevation in column 1 and tabulate in columns 2, 6, 10, and 14 of Table 14-3.

7. Compute F factor. $F = 1.486AR^{2/3}$ for each elevation. Using standard drawing ES-76, compute the F factor for each segment

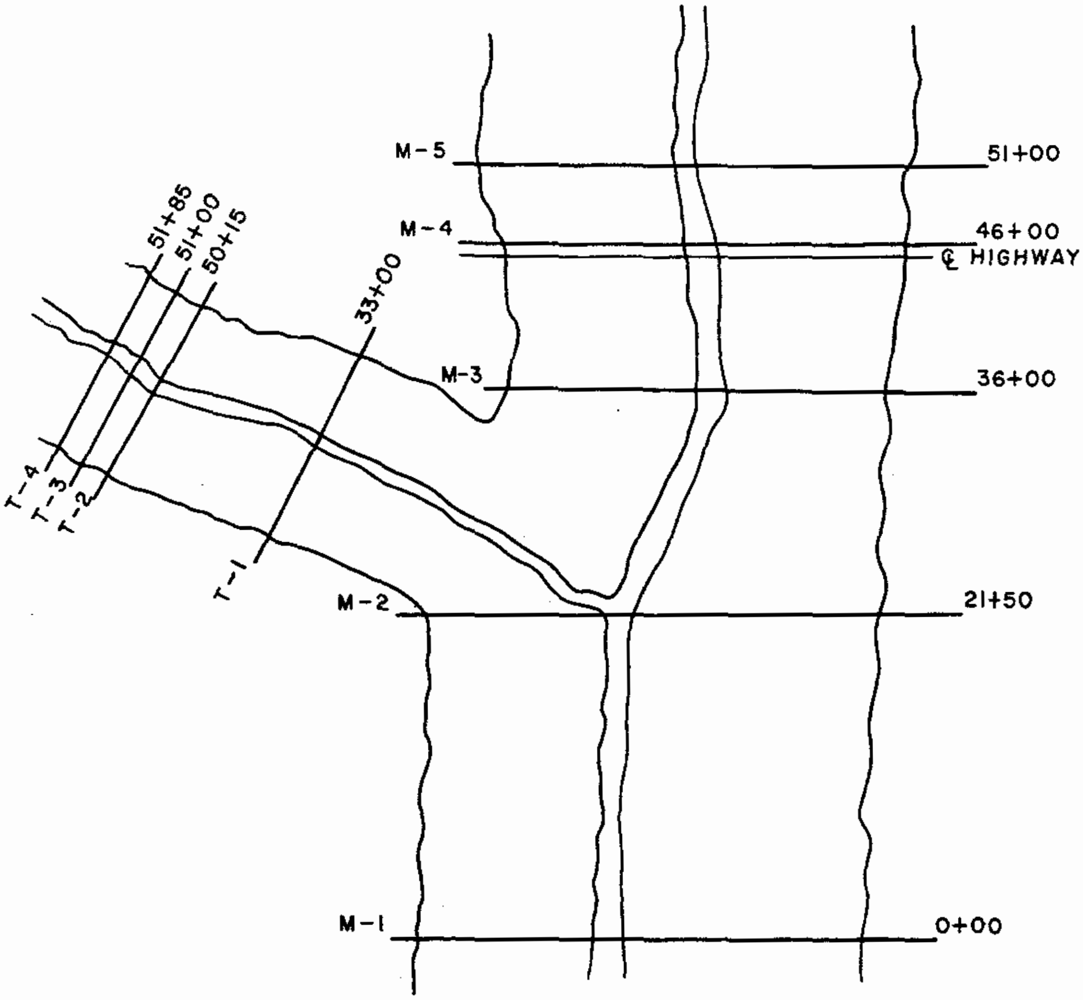


Figure 14-4. Schematic of Watershed for Examples 14-4, 14-5, and 14-6.

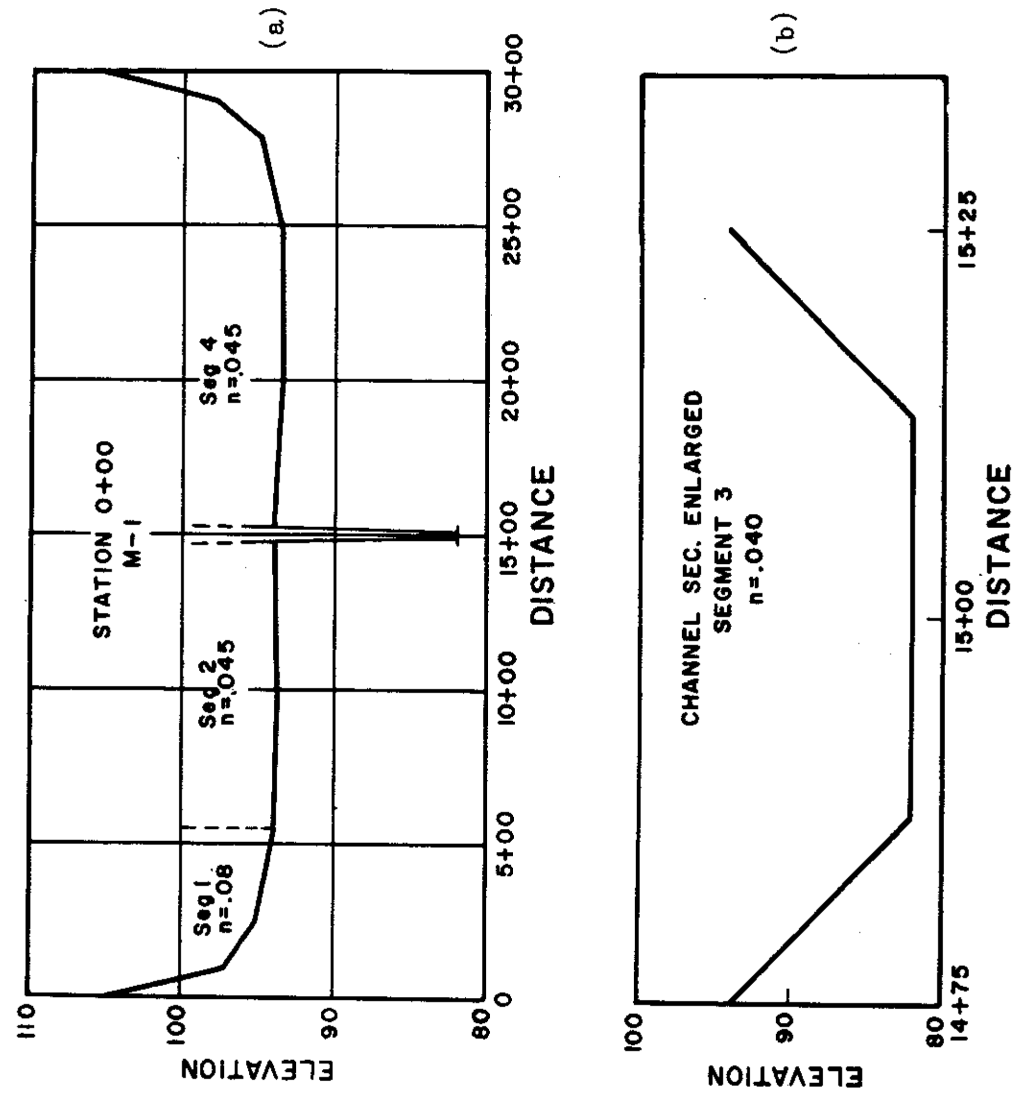


Figure 14-5. Cross section M-1, Examples 14-4 and 14-5.

Table 14-3. Hydraulic parameters for starting cross section M-1, Example 14-4.

	n = .08 Segment 1				n = .045 Segment 2				n = .040 Segment 3				n = .045 Segment 4					
Elev	Area	WP	F	$q_{nd}/S_o^{1/2}$	A	WP	F	$q_{nd}/S_o^{1/2}$	A	WP	F	$q_{nd}/S_o^{1/2}$	A	WP	F	$q_{nd}/S_o^{1/2}$	$\Sigma\ q_{nd}/S_o^{1/2}$	Σ Area
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
105	4862	550	30600	3.80×10^5	10268	925	75500	1.67×10^6	1006	62	9560 1/	2.38×10^5	15555	1460	112000 2/	2.49×10^6	4.78×10^6	31691
102	3272	510	17000	2.12×10^5	7493	925	44700	9.83×10^5	856	62	7300	1.82×10^5	11175	1440	65000	1.44×10^6	2.81×10^6	22796
100	2272	490	9350	1.17×10^5	5643	925	27900	6.20×10^5	756	62	5940	1.48×10^5	8325	1400	40800	9.06×10^5	1.79×10^6	16996
98	1322	460	3970	4.96×10^4	3793	925	14400	3.20×10^5	656	62	4700	1.17×10^5	5523	1380	20800	4.60×10^5	9.47×10^5	11294
96	487	375	860	1.07×10^4	1943	925	4740	1.05×10^5	556	62	3560	8.9×10^4	2833	1300	7040	1.56×10^5	3.61×10^5	5819
95	150	300	140	1.75×10^3	1018	925	1615	3.59×10^4	506	62	3040	7.6×10^4	1543	1275	2600	5.78×10^4	1.72×10^5	3217
94	0	0	0	0	93	925	30	6.67×10^2	456	62	2560	6.4×10^4	378	1050	284	6.32×10^3	7.10×10^4	927
93					0	0	0	0	407	58	2250	5.61×10^4	0	0	0	0	5.61×10^4	407
91									315	52	1560	3.92×10^4					3.92×10^4	315
89									231	46	1080	2.51×10^4					2.51×10^4	231
87									155	41	560	1.40×10^4					1.40×10^4	155
85									87	35	236	5.90×10^3					5.90×10^3	87
82									0	26	0	0						0

1/ To solve this on ES-77 divide F by 2, then double results read from Sheet 3, ES-77.
2/ In order to solve this on ES-76 it is necessary to divide both area and WP by 2 and then double the F factor read from Sheet 3, ES-76.

NOTE: $q_{nd}/S_o^{1/2}$ is the same as Kd or commonly referred to as the conveyance factor.

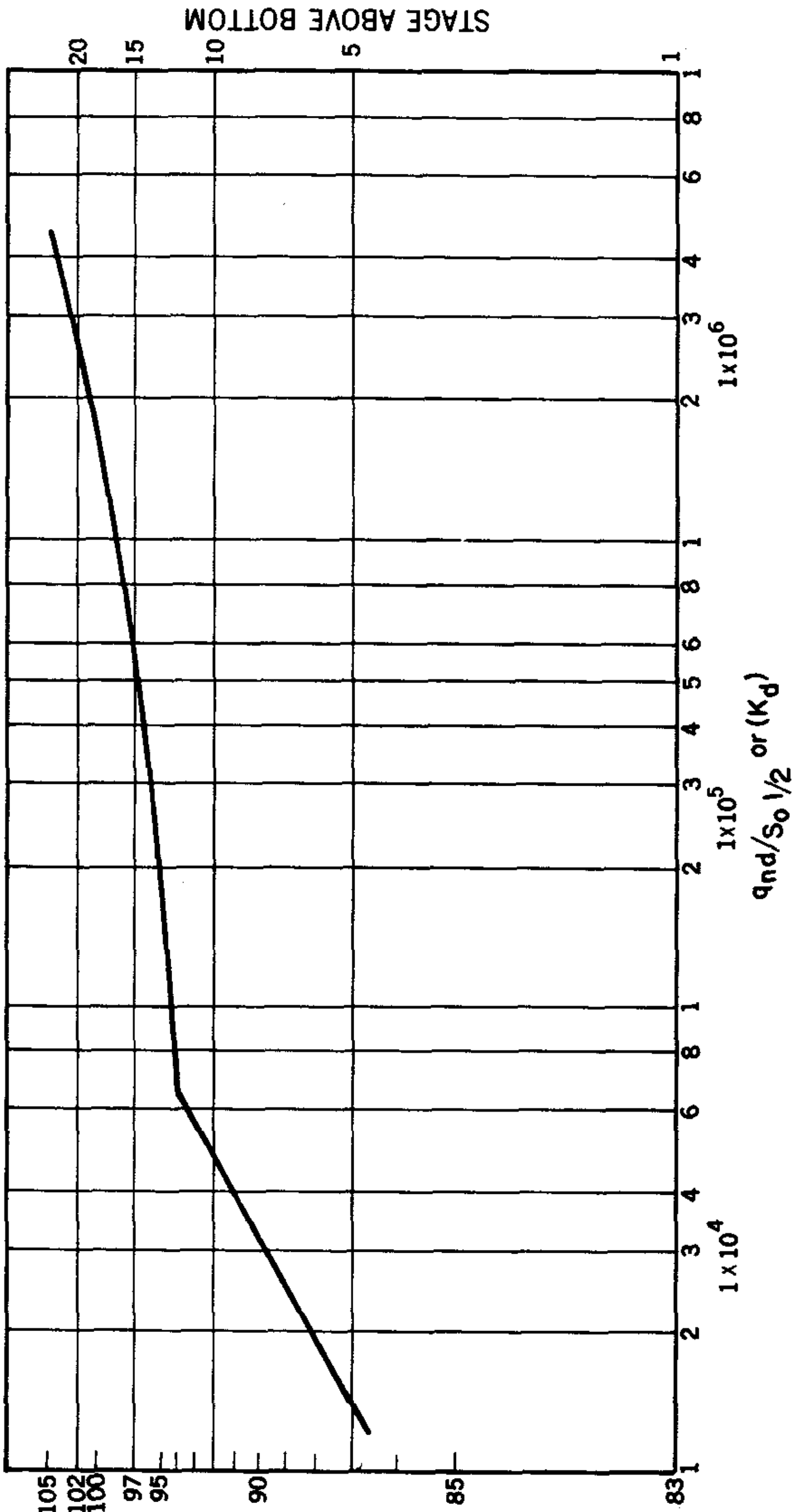


Figure 14-6. Conveyance values section M-1, Example 14-4.

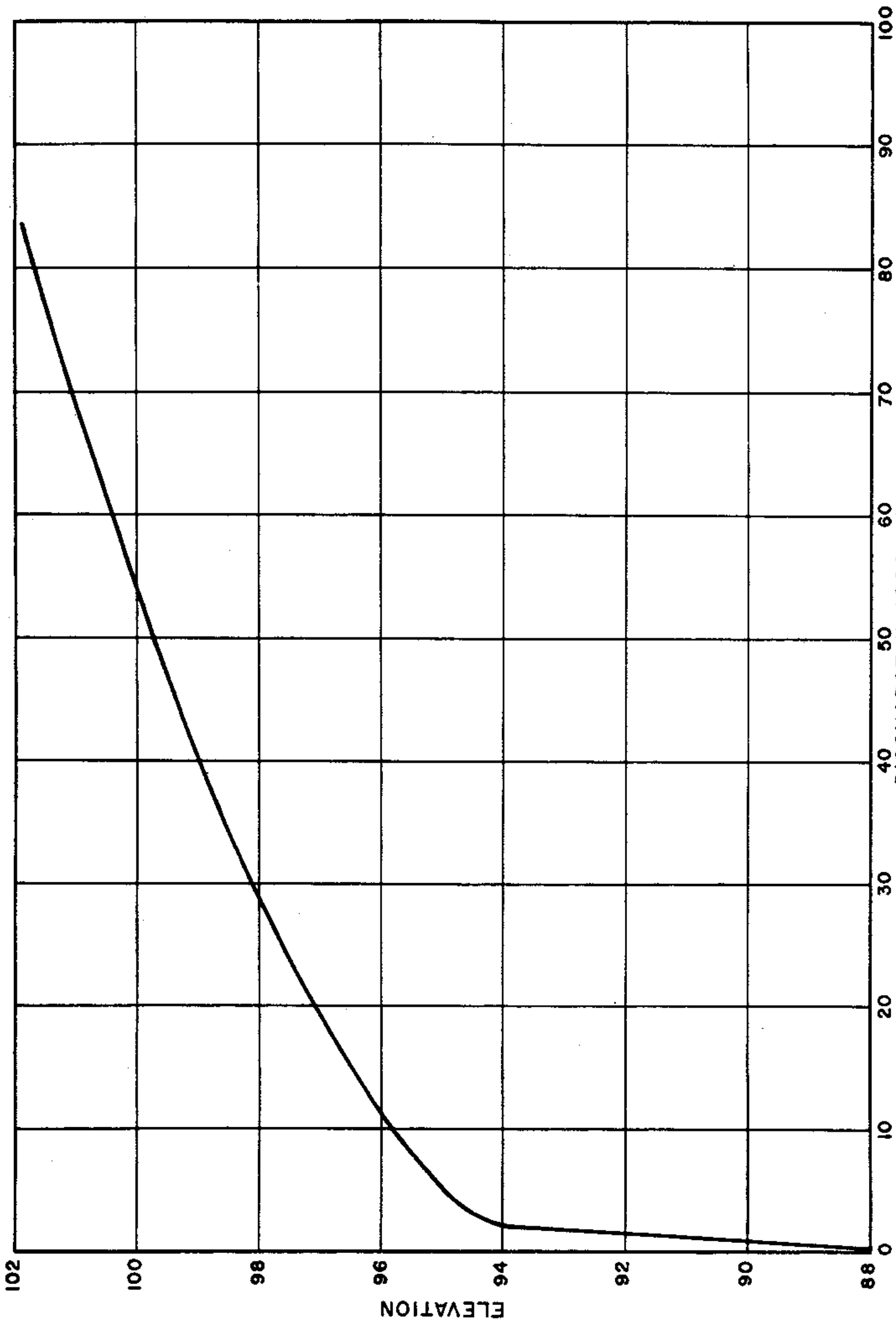


Figure 14-7. Stage discharge section M-1, Example 14-4.

at each elevation in column 1 and tabulate in columns 4, 8, 12, and 16, Table 14-3.

8. Compute the conveyance factor $q_{nd}/S_o^{1/2}$ for each elevation.

   Using standard drawing ES-77 and the assigned "n" value for each segment compute $q_{nd}/S_o^{1/2}$ for each segment at each elevation in column 1 and tabulate in columns 5, 9, 13, and 17 of Table 14-3. This can also be done by dividing F by n using a slide rule or desk calculator.

9. Sum columns 5, 9, 13, and 17 and tabulate in column 18. A plot of column 18 on log-log paper is shown on Figure 14-6. The elevation scale is selected based on feet above the channel bottom.

10. Compute the discharge for each elevation. Using the average slope at cross section M-1, S = .001, develop stage discharge for cross section M-1, $q = S^{1/2} \times q_{nd}/S_o^{1/2}$, or $q = S^{1/2} \times Kd$. The stage discharge curve for cross section M-1 is shown on Figure 14-7.

The next example shows the effect of a meandering channel in a floodplain on the elevation discharge relationship. Equation 14-20 will be used to determine the discharge.

Example 14-5

Develop the stage discharge curve for cross section M-1 (Figure 14-4) if M-1 represents a reach having a channel length of 2700 feet and a floodplain length of 2000 feet. The energy gradient of the channel portion is 0.001 ft./ft.

1. Compute the total floodplain conveyance Kd_f.

   Figure 14-5 shows segments 1, 2 and 4 of section M-1 are floodplain segments. Table 14-3 of Example 14-4 was used to develop the hydraulic parameters for section M-1 for each segment. From Table 14-3 add the $Qnd/So^{1/2}$ values for each elevation from columns 5, 9, and 17 and tabulate as Kd_f in column 2 of Table 14-4.

2. Determine the meander factor L_c/L_f. For the channel length of 2700 feet and the floodplain length of 2000 feet the meander factor is:

$$\frac{2700}{2000} = 1.35$$

3. Determine $L_c/L_f^{1/2}$.

$$(1.35)^{1/2} = 1.16$$

Table 14-4. Stage discharge for Section M-1 with meander correction, Example 14-5

Elevation	Floodplain Kd_f	$Kd_f \left(\frac{I_c}{L_f}\right)^{1/2}$	Channel Kd_c	Col. 3 + Col. 4	Discharge Q_t
(1)	(2)	(3)	(4)	(5)	(6)
105	4.54×10^6	5.27×10^6	2.38×10^5	5.51×10^6	174000
102	2.64×10^6	3.06×10^6	1.82×10^5	3.24×10^6	102000
100	1.64×10^6	1.90×10^6	1.48×10^5	2.05×10^6	64800
98	8.30×10^5	9.63×10^5	1.17×10^5	1.08×10^6	34100
96	2.72×10^5	3.16×10^5	8.9×10^4	4.05×10^5	12800
95	9.55×10^4	1.11×10^5	7.6×10^4	1.87×10^5	5910
94	6.99×10^3	8.11×10^3	6.4×10^4	7.21×10^4	2280
93	0.	0.	5.61×10^4	5.61×10^4	1770
91	0.	0.	3.92×10^4	3.92×10^4	1240

4. Compute $(Kd_f)\ (L_c/L_f)^{1/2}$. For each elevation in column 1 of Table 14-4 multiply column 2 by $(L_c/L_f)^{1/2}$ and tabulate in column 3.

$$(4.54 \times 10^6)\ (1.16) = 5.27 \times 10^6$$

5. Compute the channel conveyance Kd. From Figure 14-4 the channel is segment 3 and the conveyance has been calculated in column 13 of Table 14-3. Tabulate Kd_c in column 4 of Table 14-4.

6. Compute $Kd_c + (Kd_f)\ (L_c/L_f)^{1/2}$. From Table 14-4 add columns 3 and 4 and tabulate in column 5.

7. Compute the discharge for each elevation. Use $S_c = .001$ and Equation 14-20. Multiply columns by $S_c^{1/2}$ and tabulate in column 6.

$$Q_t = (Kd_c + (Kd_f)\ (L_c/L_f)^{1/2})\ (S_c)^{1/2}$$

$$Q_t = (5.51 \times 10^6)\ (3.16 \times 10^{-2}) = 1.74 \times 10^5 = 174{,}000 \text{ cfs.}$$

The next example will show the use of the modified step method in computing water surface profiles. It is a trial and error procedure based on estimating the elevation at the upstream section, determining the conveyance, Kd, for the estimated elevation and computing $S^{1/2}$ by using Mannings equation in the form $S^{1/2} = \frac{Q}{Kd}$ where $Kd = \frac{1.486}{n} AR^{2/3}$. S is the head loss per foot (neglecting velocity head) from the downstream to the upstream section. This head loss added to the downstream water surface elevation should equal the estimated upstream elevation.

Example 14-6

Using the rating curve developed in Example 14-4 for cross section M-1 and parameters plotted on Figures 14-8 and 14-10 for cross sections M-2 and T-1, compute the water surface profiles required to develop stage discharge curves for cross sections M-2 and T-1. The changes in velocity head will be ignored for these computations. The drainage area at section M-1 is 400 sq. mi., at M-2 is 398 sq. mi. and at T-1 is 48 sq. mi. The reach length between M-1 and M-2 is 2150 feet and between M-2 and T-1 is 1150 feet. Assume the meander factor for this example is 1.0.

1. Determine the range of csm needed to define the stage discharge curve. One or more of the csm's selected should be contained within the channel. Tabulate in column 1, Table 14-5(a).

2. Compute the discharge in cfs for each csm at the two cross sections M-1 and M-2. At section M-1 the drainage area is 400 sq. mi. Using Exhibit 14-1 the K factor is 1.0 and the cfs for 2 csm is 2 x 400 x 1.0 = 800 cfs. At section M-2 the drainage

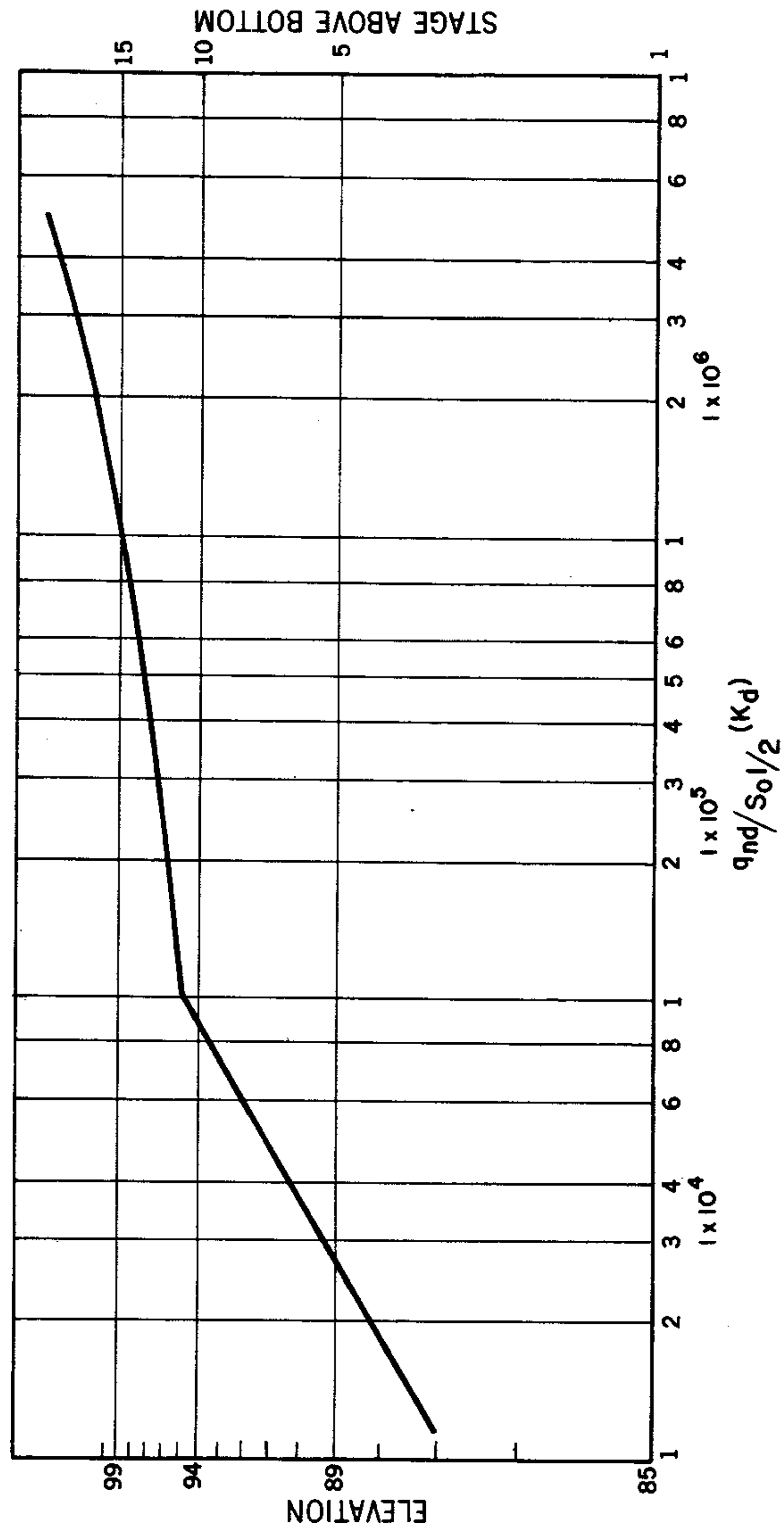


Figure 14-8. Conveyance values section M-2, Example 14-6.

Table 14-5(a). Water Surface profiles from cross section M-1 to M-2, Example 14-6.

CSM	Discharge in cfs M-1	M-2	ℓ_C	Elev. @ M-1	Assumed elev. @ M-2	Kd_{M-2}	$\left(\frac{q_{M-2}}{Kd_{M-2}}\right)^2 = S_f$	$S_f \times \ell$	Col 5 + Col 9 estimate elev @ M-2	Computed elev. @ M-2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2	2 x 400 x 1.000 = 800									
	2 x 398 x 1.002 1/ =	798	2150 2/	89.22	90.0	3.70×10^4	.00046	.99	90.21^+	
					90.2	3.89×10^4	.00044	.95	90.17^-	90.2
					90.1	3.75×10^4	.00045	.97	90.19^+	
10	10 x 400 x 1.000 = 4000									
	10 x 398 x 1.002 =	3990	2150	94.66	95.2	1.20×10^5	.00107	2.30	96.96^+	
					95.6	1.60×10^5	.00062	1.34	96.00^+	95.7
					95.8	1.90×10^5	.00044	.95	95.61^-	
20	20 x 400 x 1.000 = 8000									
	20 x 398 x 1.002 =	7980	2150	95.52	96.7	3.50×10^5	.00052	1.12	96.62^-	96.7
					96.6	3.30×10^5	.00059	1.26	96.78^+	
50	50 x 400 x 1.000 = 20000									
	50 x 398 x 1.002 =	19950	2150	97.12	98.3	7.80×10^5	.00065	1.40	98.52^+	
					98.4	8.00×10^5	.00062	1.34	98.46^+	98.4
					98.5	8.20×10^5	.00059	1.27	98.39^-	
100	100 x 400 x 1.000 = 40000									
	100 x 398 x 1.002 =	39900	2150	98.96	100.3	1.60×10^6	.00062	1.33	100.29^-	100.3
200	200 x 400 x 1.000 = 80000									
	200 x 398 x 1.002 =	79800	2150	101.68	103.2	3.20×10^6	.00062	1.33	103.01^-	103.1
					103.1	3.00×10^6	.00071	1.52	103.20^+	

1/ Computed from equation shown on Exhibit 14-1.
2/ Where the channel length is different from the flood plain length, Kd values for flood plain portion of section are modified so channel length may be used in all calculations.

area is 398 sq. mi. and from Exhibit 14-1 the K factor is 1.002. For 2 csm the discharge at M-2 is 2 x 398 x 1.002 = 798 cfs. Tabulate the discharges at M-1 and M-2 on Table 14-5(a), columns 2 and 3 of Table 14-5(a).

3. Tabulate the reach length between the two cross sections in column 4. The reach length between section M-1 and M-2 is 2150 feet.

4. Determine the water surface elevation at M-1. For the discharge listed in column 2 read the elevation from Figure 14-7 and tabulate in column 5 of Table 14-5(a).

5. Assume a water elevation at section M-2. For the smallest discharge of 798 cfs assume an elevation of 90.0 at M-2 and tabulate in column 6 of Table 14-5(a).

6. Determine Kd for assumed elevation. Read $Qnd/So^{1/2}$ or Kd_{M-2} of 3.70×10^4 at elevation 90.0 from Figure 14-8 and tabulate in column 7 of Table 14-5(a).

7. Determine S_f. $S_f = \frac{(Q_{M-2})^2}{(Kd_{M-2})}$. Divide column 3 by column 7 and square the results $(798/37000)^2 = .00046$ and tabulate in column 8 of Table 14-5(a).

8. Determine $S_f \times \ell$. Multiply column 8 by column 4, .00046 x 2150 = .99, and tabulate in column 9 of Table 14-5(a).

9. Compute elevation at M-2. Add column 9 (S_f) to column 5 (elevation at M-1) and tabulate in column 10 of Table 14-5(a).

10. Compare computed elevation with assumed elevation. Compare column 10 with column 6 and adjust column 6 up if column 10 is greater and down if it is less. For 2 csm discharge the computed elevation is 90.12 and the estimated elevation is 90.0. Since column 10 is greater a revision in the estimated elevation at M-2 in column 6 must be made.

    Repeat steps 5 through 10 until a reasonable balance between column 10 and 6 is obtained. A tolerance of 0.1 foot was used in this example.

11. Repeat steps 5 through 10 for each csm value selected.

12. Plot stage discharge curve, columns 3 and 11 as shown on Figure 14-9.

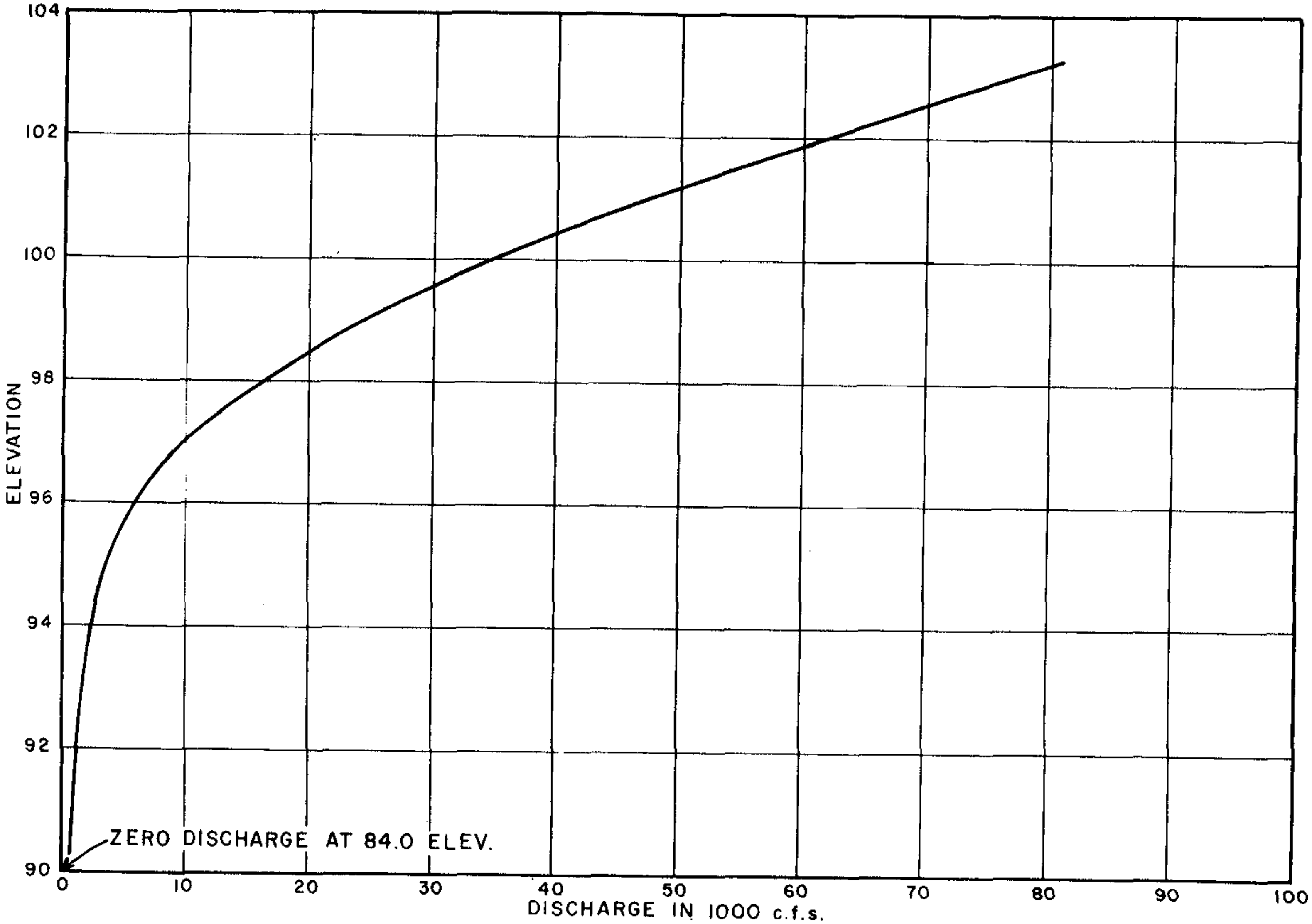


Figure 14-9. Stage discharge section M-2, Example 14-6.

Table 14.5(b). Water surface profiles from cross section M-2 to T-1. Example 14-6.

CSM	Discharge in cfs	M-2	T-1	ℓ_C	Elev. @ M-2	Assumed elev. @ T-1	Kd_{T-1}	$\left(\frac{q_{T-1}}{Kd_{T-1}}\right)^2 = S_f$	$S_f \times \ell$	Col 5 + Col 9 estimate elev @ T-1	Computed elev. @ T-1
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
2	2 x 398 x 1.002 =	798									
	2 x 48 x 2.68[1] =		260	1150	90.15	93.0	1.7×10^3	.0234	26.9	116.9+	
						94.0	3.4×10^3	.0058	6.70	96.8+	
						94.5	4.4×10^3	.0035	4.02	94.2−	94.4
						94.4	4.2×10^3	.0038	4.42	94.6+	
10	10 x 398 x 1.002 =	3990									
	10 x 48 x 2.68 =		1290	1150	95.70	97.0	1.65×10^4	.0062	7.05	102.75+	
						98.0	5.7×10^4	.00051	.59	96.29−	97.5
						97.5	3.2×10^4	.00163	1.87	97.57+	
20	20 x 398 x 1.002 =	7980									
	20 x 48 x 2.68 =		2580	1150	96.65	98.0	5.7×10^4	.00205	2.36	99.01+	
						98.5	9.0×10^4	.00082	.94	97.59−	98.2
						98.2	7.0×10^4	.00135	1.56	98.21+	
50	50 x 398 x 1.002 =	19950									
	50 x 48 x 2.68 =		6450	1150	98.45	100.0	2.32×10^5	.00077	.89	99.34−	
						99.5	1.85×10^5	.00122	1.40	99.85+	99.7
						99.65	2.00×10^5	.00104	1.20	99.65−	
100	100 x 398 x 1.002 =	39900									
	100 x 48 x 2.68 =		12900	1150	100.3	101.0	3.6×10^5	.00128	1.47	101.77+	
						101.2	4.0×10^5	.00104	1.2	101.5+	101.3
						101.3	4.4×10^5	.00086	.99	101.29−	
200	200 x 398 x 1.002 =	79800									
	200 x 48 x 2.68 =		25800	1150	103.15	104.0	9.2×10^5	.00078	.90	104.05	104.0

[1] Taken from Exhibit 14-1.

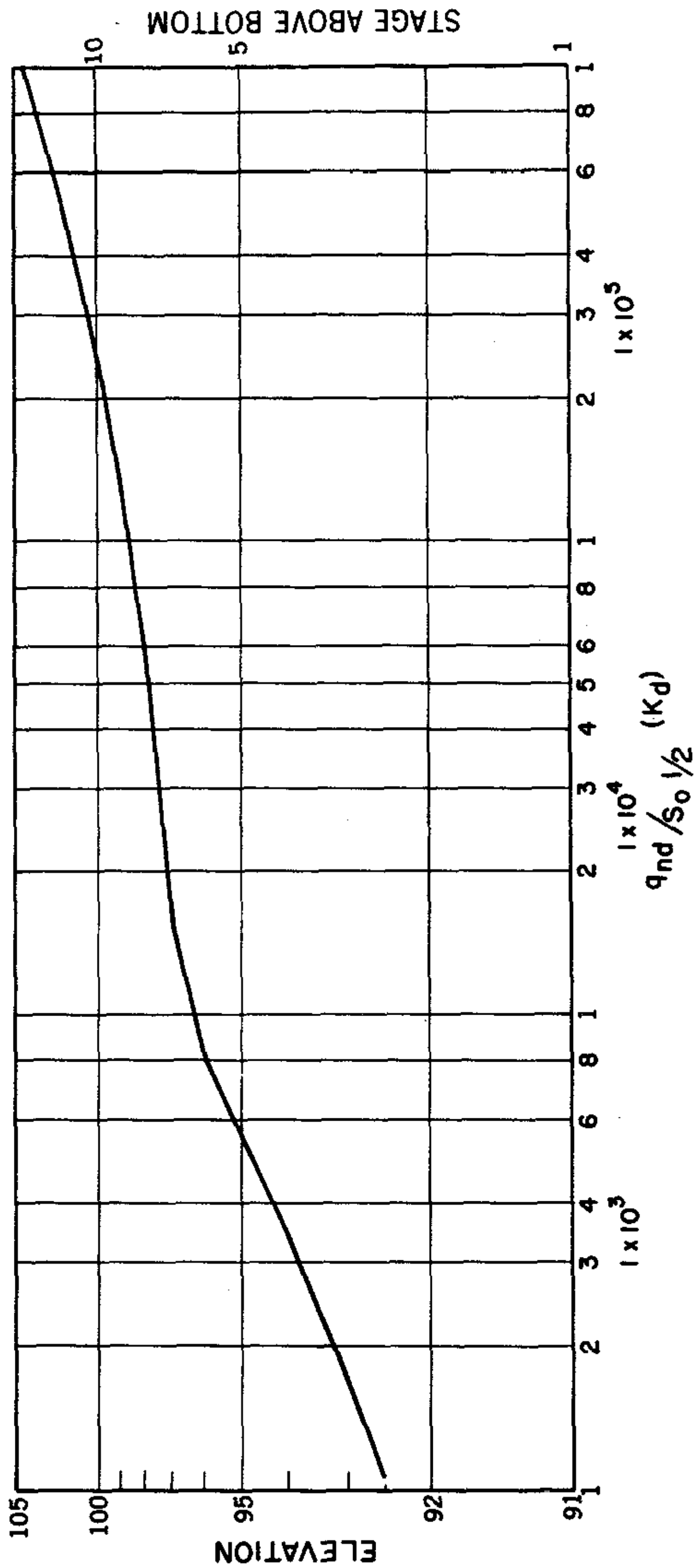


Figure 14-10. Conveyance values section T-1, Example 14-6.

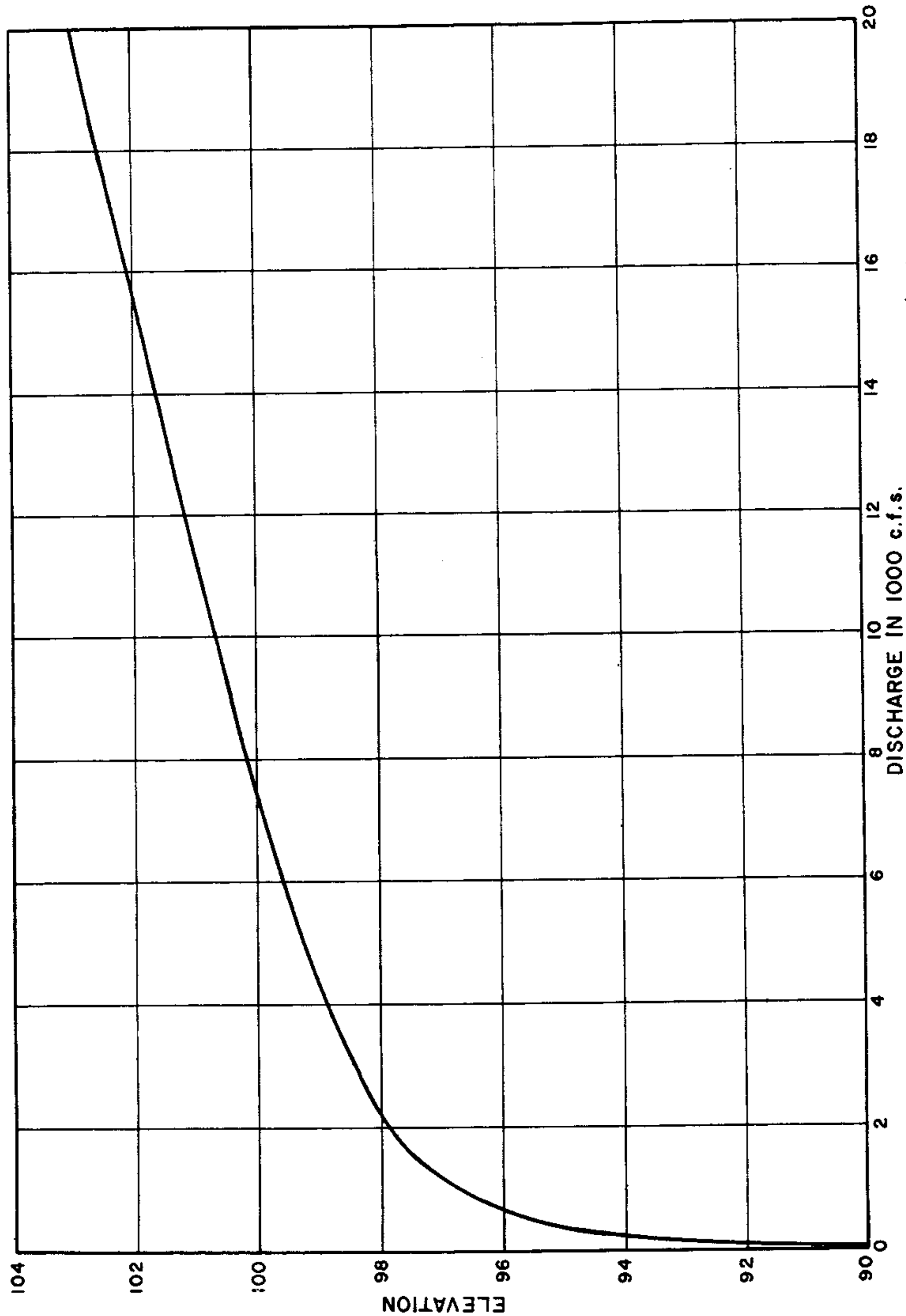


Figure 14-11. Stage discharge section T-1, Example 14-6.

Table 14-5(b) shows computations similar to step 1 through step 11 computing water surface profiles between cross section M-2 on the main stem and T-1 the first cross section on a tributary. Kd values are shown on Figure 14-10. Figure 14-11 was plotted from Table 14-5(b).

Road Crossings

Bridges

In developing the hydraulics of natural streams, bridges of all types and sizes are encountered. These bridges may or may not have a significant effect on the stage discharge relationship in the reach above the bridge. Many of the older bridges were designed without regard to their effect on flooding in the reach upstream from the road crossing.

The Bureau of Public Roads (BPR) in cooperation with Colorado State University initiated a research project with Colorado State University in 1954 which culminated in the investigation of several features of the bridge problem. Included in these investigations was a study of bridge backwater. The laboratory studies, in which hydraulic models served as the principal research tool, have been completed and since then considerable progress has been made in the collection of field data by the U.S. Geological Survey to substantiate the model results and extend the range of application. The procedure developed is explained in the publication "Hydraulics of Bridge Waterways," U. S. Department of Transportation, Federal Highway Administration, Bureau of Public Roads, 1970. This is one method which is recommended by the Soil Conservation Service for use in computing effects of bridges in natural channels and floodplains.

The FHWA document may be obtained from the Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. and it should be included in the working files of any engineer concerned with the effect of bridges on stream hydraulics.

The Bureau of Public Roads (BPR) Method has been formulated by applying the principle of conservation of energy between the point of maximum backwater upstream from the bridge and a point downstream from the bridge at which normal stage has been re-established. The general expression for the computation of backwater upstream from a bridge constricting the flow is:

$$h_1^* = K^* \frac{\alpha_2 V_{n2}^2}{2g} + \left(\frac{\alpha_4 V_4^2}{2g} - \frac{\alpha_1 V_1^2}{2g} \right) \qquad \text{(Eq. 14-23)}$$

where h_1^* = total backwater, in feet

K^* = total backwater coefficient

$\alpha_1, \alpha_2, \alpha_4$ = velocity head energy coefficients at the upstream, constriction, and downstream section.

V_{n2} = average velocity in constriction or $\frac{Q}{A}$ in feet per second.

V_4 = average velocity at section 4 downstream in feet per second.

V_1 = average velocity at section 1 upstream in feet per second.

(For a more detailed explanation of each term and the development of the equation refer to "Hydraulics of Bridge Waterways.")

Equation 14-23 is reasonably valid if the channel in the vicinity of the bridge is essentially straight, the cross sectional area of the stream is fairly uniform, the gradient of the bottom is approximately constant between sections 1 and 4, the flow is free to expand and contract, there is no appreciable scour of the bed in the constriction and the flow is in the subcritical range.

This procedure relates the total backwater effect to the velocity head caused by the constriction times the total backwater coefficient. The total backwater coefficient is comprised of the effect of constriction as measured by the bridge opening coefficient, M, type of bridge abutments, size, shape and orientation of piers, and eccentricity and skew of bridge.

For a detailed discussion of the backwater coefficient and the effect of constriction, abutments, piers, eccentricity and skew of bridges refer to "Hydraulics of Bridge Waterways."

A preliminary analysis may be made to determine the maximum backwater effect of a bridge. If the analysis shows a significant bridge effect then a more detailed procedure should be used. If the analysis shows only a minor effect then the bridge may be eliminated from the backwater computation.

The examples shown in this chapter are based on the approximate equation to compute bridge head losses taken from the BPR report:

$$h^* = K^* \frac{V^2}{2g} \qquad \text{(Eq. 14-24)}$$

where: h^* = total backwater, in feet

K^* = total backwater coefficient

V = average velocity in constriction $\frac{Q}{A}$

A = gross water area in constriction measured below normal stage.

The following data are the minimum needed for estimating the maximum backwater effect of a bridge using Equation 14-24.

1. Total area of bridge opening.

2. Length of bridge opening.

3. Cross section upstream from the bridge a distance approximately equal to the length of the bridge opening.

4. Area of approach section at elevation of the bottom of bridge stringers or at the low point in the road embankment.

5. Width of flood plain in approach section.

6. Estimate of the velocity of unrestricted flow at the elevation of the bottom of the bridge stringers or at the low point in the road embankment.

A preliminary analysis to determine an estimate of the maximum backwater effect of a bridge is shown in Example 14-7. Exhibits 14-2 and 14-3 were developed only for use in making preliminary estimates and should not be used in a more detailed analysis.

Example 14-7.
Estimate the backwater effect of a bridge with 45° wingwalls given the following data: area of bridge = 4100 sq. ft., length of bridge = 400 ft., area of approach = 11850 sq. ft., width of flood plain = 2650 ft., estimated velocity in the natural stream = 2.5 ft./sec.

1. Compute the ratio of the area of the bridge to the area of approach section. From the given data: 4100/11850 = .346

2. Compute the ratio of length of bridge to the width of the flood plain. From the given data: 400/2650 = .151

3. Determine the change in velocity head. Using the results of step 1 (.346) and the estimated velocity in the natural stream (2.5 ft/sec), read the velocity head, h, from Exhibit 14-2. This is the velocity head, $\frac{V^2}{2g}$ in Equation 14-24 and (from Exhibit 14-2) is 0.8 ft.

4. Estimate the constriction ratio, M. Using the results from step 1 (.346) and step 2 (.151) read M = .67 from Exhibit 14-3.

5. Estimate the total backwater coefficient. Using M = .67 from step 4 read from Exhibit 14-4 curve 1, K_b = .6. K_b is the BPR base curve backwater coefficient and for estimating purposes is considered to be the total backwater coefficient, K*, in Eq. 14-24.

6. Compute the estimated total change in water surface, h*. From Equation 14-24 the total change in water surface is $h^* = K^* \frac{V^2}{2g} = (.6)(.8) = .48$ ft.

If the estimate shows a change in water surface that would have an appreciable effect on the evaluation or level of protection of a plan or the design and construction of proposed structural measures, a more detailed survey and calculation should be made for the bridge and flood in question.

Example 14-8 shows a more detailed solution to the backwater loss using Equation 14-24. In order to use the BPR method it is necessary to develop stage discharge curves for an exit and an approach section assuming no constriction between the two cross sections.

The exit section should be located downstream from the bridge a distance approximately twice the length of the bridge. The approach section should be located upstream from the upper edge of the bridge a distance approximately equal to the length of the bridge.

If the elevation difference between the water surface at the exit section and the approach section prior to computing head loss is relatively small the bridge tailwater may be taken as the elevation of the exit section and the bridge head loss simply added to the water elevation of the approach section. However, if this difference is not small the bridge tailwater should be computed by interpolation of the water elevation at the approach section and exit section and the friction loss from the bridge to the approach section recomputed after the bridge headwater is obtained.

In Example 14-8 it is assumed that all preliminary calculations have been made. The profiles are shown on Figure 14-12a and the stage discharge curve for cross section M-5 is shown on Figure 14-13, Natural Condition.

<u>Example 14-8</u>

Develop stage discharge curves for each of four bridges located at cross section M-4 (Figure 14-4), 300, 400, 500, and 700 feet long (Figure 12c) with 45° wingwalls. The elevation of the bottom of the bridge stringer is 10^3 for each trial bridge length. The main span is 100 feet with the remaining portion of the bridge supported by 24" H-columns on 25 foot centers. Assume the fill is sufficiently high to prevent over topping for the maximum discharge (70000 cfs) studied. It is assumed that water surface profiles have been run for present conditions through section M-5 and that this information is available for use in analyzing the effects of bridge losses.

1. <u>Select a range of discharges that will define the rating curve</u>. For this problem select a range of discharges from 5000 to 70000 cfs for each bridge length and tabulate in column 1 of Table 14-6.

2. <u>Determine present condition elevation for each discharge at the bridge section M-4</u>. For this example water surface profiles have been computed from section M-3 to M-5 without the bridge in place. The results are plotted in Figure 14-12a. From Figure 14-12a read the normal elevation for each discharge at cross section M-4 and tabulate in column 2 of Table 14-6.

3. <u>Compute the elevation vs. gross bridge opening area</u>. The gross area of the bridge is the total area of the bridge opening at a given elevation without regard to the area of

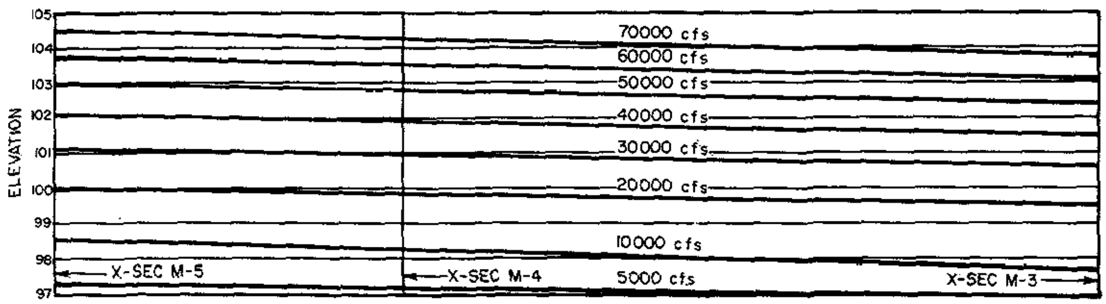


Figure 14-12a. Water surface profile without constriction. Example 14-8.

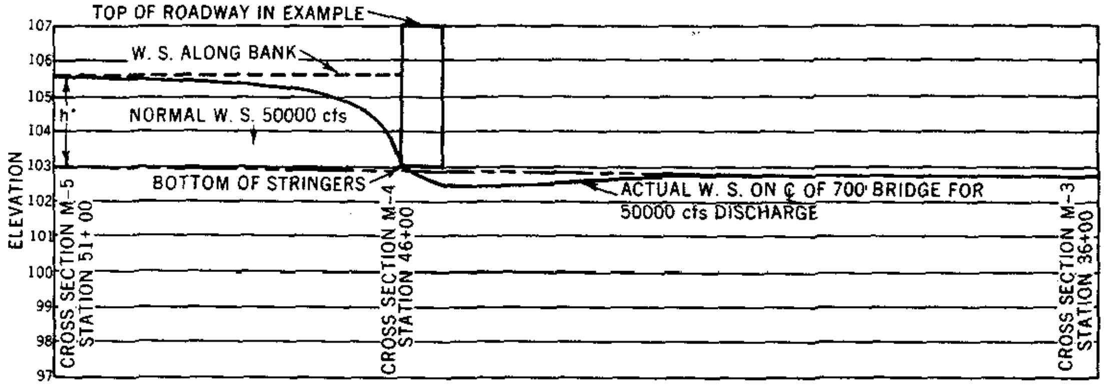


Figure 14-12b. Water surface profile with constriction. Example 14-8.

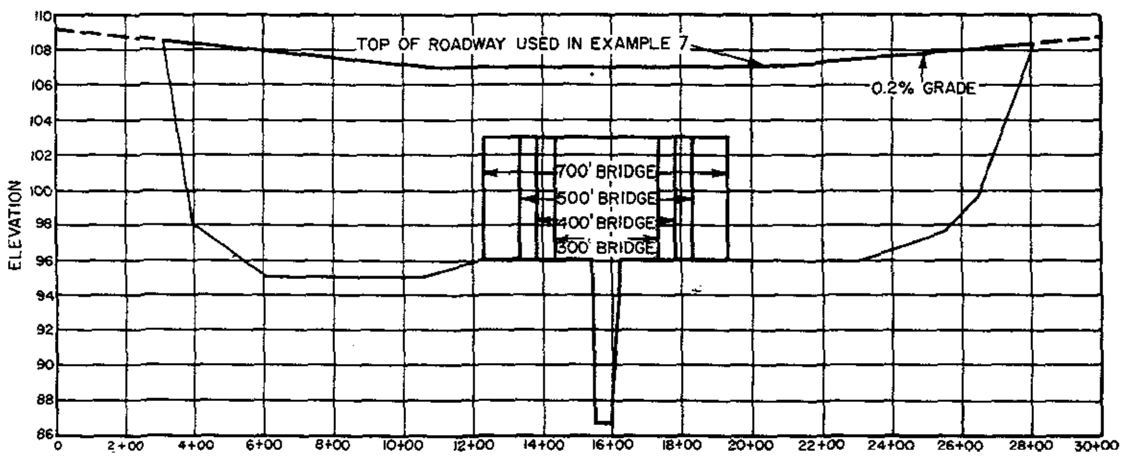


Figure 14-12c. Cross section of road at section M-4, Example 14-8.

Table 14-6. Backwater computations through bridges, Example 14-8.

	Discharge in 1000 cfs	Normal el. @ x-sec M-4	Restricted area A_{n2}	Velocity through bridge openings	Normal el. @ x-sec M-5	M 1/	K_b 1/	J 1/	ΔK_p 1/	K* 1/	$\frac{V_{n2}^2}{2g}$ 1/	h*	Elev. with 24" H. col. 25' on cen.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
300' Bridge 45° Wingwalls	5	97.20	885	5.65	97.35	.470	1.24	.020	.06	1.30	.495	.64	97.99
	10	98.25	1280	7.81	98.55	.350	1.74	.028	.08	1.82	.947	1.72	100.27
	20	99.82	1720	11.63	100.00	.276	2.08	.035	.08	2.16	2.10	4.53	104.53
	30	100.95	2060	14.56	101.15	.243	2.24	.038	.08	2.32	3.29	7.63	108.78
	40	101.90	2340	17.10	102.10	.222	2.34	.040	.08	2.42	4.54	10.99	113.09
	50	102.80	2600	19.23	103.00	.208	2.41	.041	.08	2.49	5.74	14.29	117.29
	60	103.55	2660	22.56	103.75	.183	2.54	.042	.08	2.62	7.90	20.70	124.45
	70	104.25	2660	26.32	104.50	.160	2.66	.042	.07	2.73	10.76	29.37	133.87
400' Bridge 45° Wingwalls	5	97.20	1030	4.85	97.35	.510	1.09	.027	.10	1.19	.365	.43	97.78
	10	98.25	1470	6.80	98.55	.385	1.59	.036	.12	1.71	.718	1.23	99.78
	20	99.82	2070	9.66	100.00	.315	1.90	.043	.12	2.02	1.45	2.93	102.93
	30	100.95	2540	11.81	101.15	.282	2.05	.046	.12	2.17	2.17	4.71	105.86
	40	101.90	2950	13.56	102.10	.265	2.13	.048	.12	2.25	2.86	6.44	108.54
	50	102.80	3300	15.15	103.00	.250	2.21	.049	.12	2.33	3.56	8.29	111.29
	60	103.55	3380	17.72	103.75	.220	2.35	.049	.11	2.46	4.89	12.03	115.78
	70	104.25	3380	20.71	104.50	.192	2.49	.049	.10	2.59	6.66	17.25	121.75
500' Bridge 45° Wingwalls	5	97.20	1160	4.31	97.35	.525	1.03	.032	.13	1.16	.288	.33	97.88
	10	98.25	1670	5.99	98.55	.420	1.44	.042	.15	1.59	.557	.89	99.44
	20	99.82	2550	7.84	100.00	.350	1.74	.049	.16	1.90	.955	1.81	101.81
	30	100.95	3050	9.84	101.15	.325	1.85	.052	.16	2.01	1.50	3.02	104.17
	40	101.90	3520	11.36	102.10	.310	1.92	.054	.16	2.08	2.00	4.16	106.26
	50	102.80	3950	12.66	103.00	.298	1.98	.055	.16	2.14	2.49	5.33	108.33
	60	103.55	4050	14.81	103.75	.262	2.15	.055	.14	2.29	3.41	7.81	111.56
	70	104.25	4050	17.28	104.50	.230	2.30	.055	.13	2.43	4.64	11.28	115.78
700' Bridge 45° Wingwalls	5	97.20	1420	3.52	97.35	.580	0.84	.040	.19	1.03	.192	.20	97.55
	10	98.25	2170	4.61	98.55	.480	1.20	.050	.21	1.41	.330	.47	99.02
	20	99.82	3300	6.06	100.00	.415	1.46	.056	.21	1.67	.570	.95	100.95
	30	100.95	4080	7.35	101.15	.394	1.55	.058	.21	1.76	.839	1.48	102.63
	40	101.90	4750	8.42	102.10	.377	1.62	.059	.21	1.83	1.10	2.01	104.11
	50	102.80	5380	9.29	103.00	.367	1.67	.060	.20	1.87	1.34	2.51	105.47
	60	103.55	5520	10.87	103.75	.325	1.85	.061	.19	2.04	1.83	3.73	107.48
	70	104.25	5520	12.68	104.50	.285	2.04	.061	.17	2.21	2.50	5.53	110.03

1/ These letters and symbols are the same as used in Hydraulics of Bridge Waterways, U. S. Dept. of Transportation, Bureau of Public Roads, 1970. This publication is for sale by Superintendent of Documents.

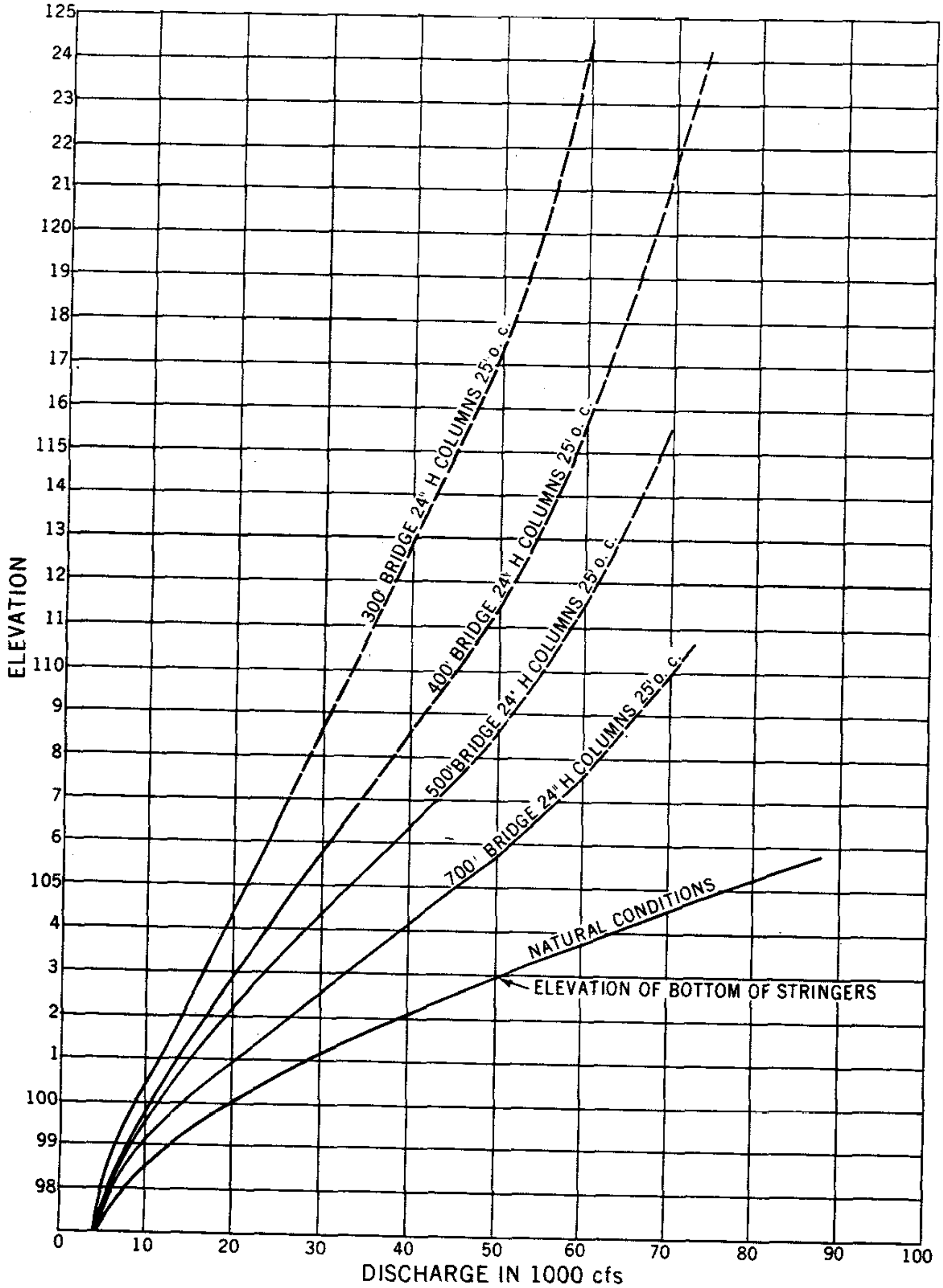


Figure 14-13. Stage discharge without embankment overflow. Section M-5, Example 14-8.

piers. The channel area is 600 ft.^{2} and for the 300 ft. long bridge the gross bridge area is:

Elevation	Bridge Area
96	600
97	900
99	1500
103	2700

Plot the elevation vs. gross bridge opening area as shown in Figure 14-14.

4. Determine the gross area of the bridge opening at each water surface elevation. Using Figure 14-14 read the gross area at each elevation tabulated in column 2 and tabulate in column 3 of Table 14-6.

5. Compute the average velocity through the bridge opening. Divide column 1 by column 3 and tabulate in column 4 of Table 14-5. For the 300 ft. long bridge:

$$V = \frac{Q}{A} = \frac{5000}{885} = 5.65 \text{ ft./sec.}$$

6. Compute the velocity head $(V^2)/2g$. Using the velocities from column 4 compute the velocity head for each discharge and tabulate in column 11 of Table 14-6. For a discharge of 5000 cfs and a bridge length of 300 feet the velocity head is $\frac{(5.65)^2}{(2)(32.2)} =$ .495

7. Determine the elevation for each discharge at section M-5 under natural conditions. Using Figure 14-12a or Figure 14-13 (natural condition curve) read the elevation for each discharge at cross section M-5 and tabulate in column 5 of Table 14-6.

8. Compute M vs. elevation for each bridge size. M is computed as outlined in "Hydraulics of Bridge Waterways." It is computed as the ratio of that portion of the discharge at the upstream section computed for a width equal to the length of the bridge to the total discharge of the channel system. If Q_b is the discharge at the upstream section computed for a flood plain or channel width equal to the length of the bridge and Q_a and Q_c is the remaining discharge on either side of Q_b then $M = \frac{Q_b}{Q_a+Q_b+Q_c} = \frac{Q_b}{Q}$.

   The bridge opening ratio, M, is most easily explained in terms of discharges, but it is usually determined from conveyance relations. Since conveyance (Kd) is proportional to discharge, assuming all subsections to have the same slope, M can be expressed also as:

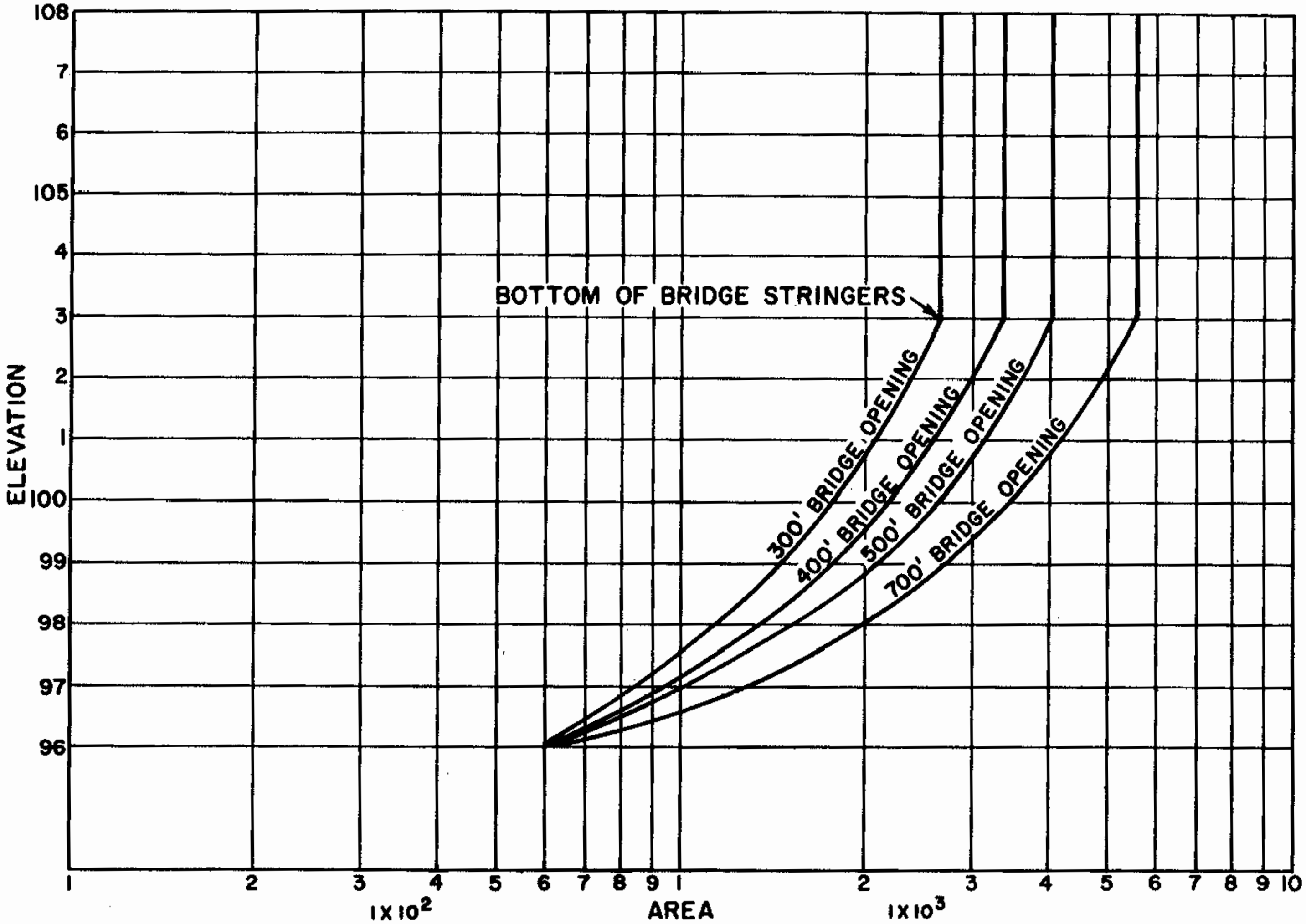


Figure 14-14. Bridge opening areas, Example 14-8.

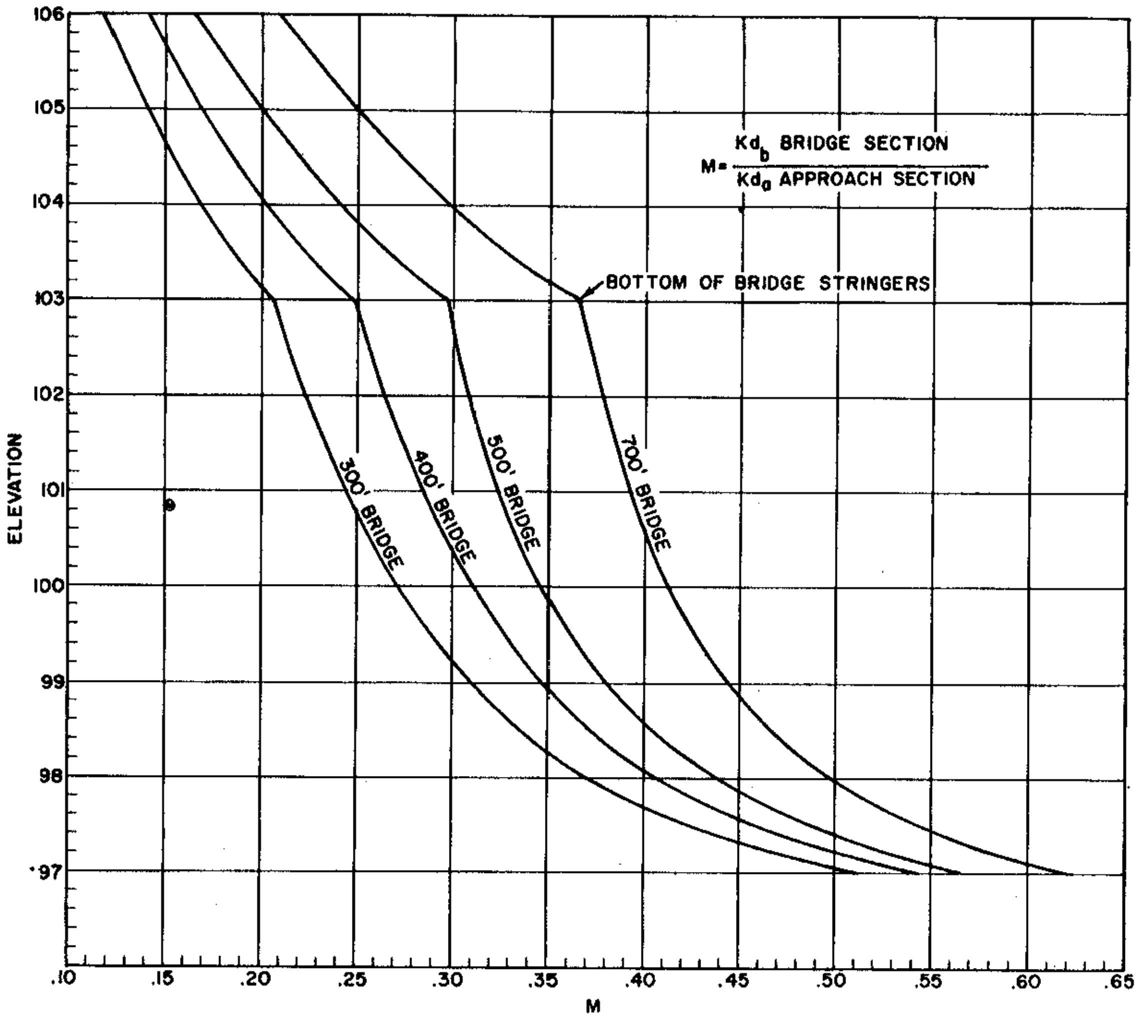


Figure 14-15. M values for bridge, Example 14-8.

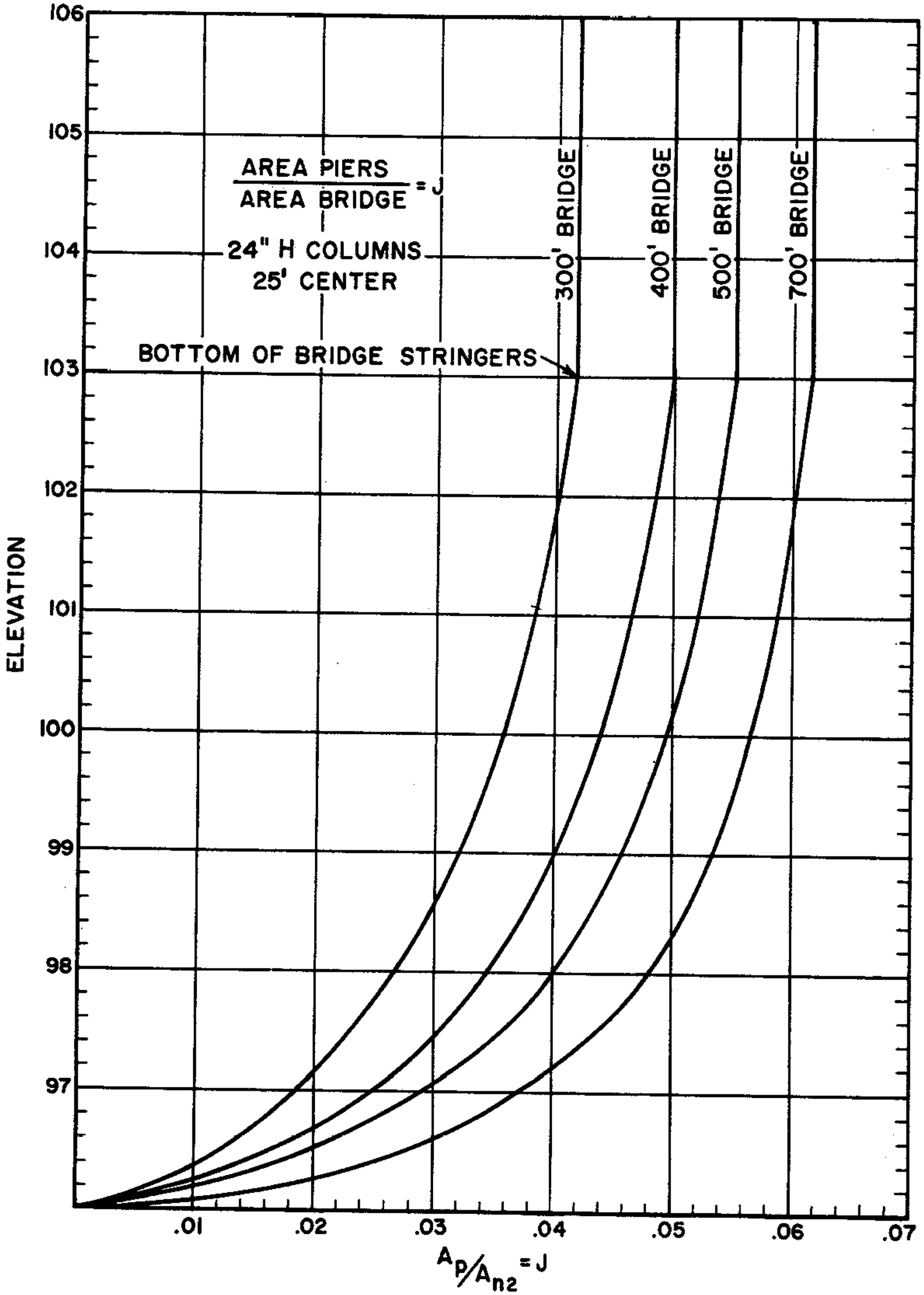


Figure 14-16. J values for bridge, Example 14-8.

$$M = \frac{Kd_b}{Kd_a+Kd_b+Kd_c} = \frac{Kd_b}{Kd}$$

The approach section information is not shown for this example.

Plot M vs. elevation for each bridge size as shown in Figure 14-15.

9. Read M for each elevation. Using Figure 14-15 prepared in step 8 read M for each elevation in column 2 and tabulate in column 6 of Table 14-6.

10. Determine the base backwater coefficient K_b. Using M from step 9, read K_b Exhibit 14-4 for bridges having 45° wingwalls and tabulate in column 7 of Table 14-6.

11. Compute the area of pier/area of bridge vs. elevation.

$$\frac{\text{area of piers}}{\text{area of bridge}} = \frac{Ap}{A_{n2}} = J$$

For the 300' bridge the piers are located in an area 200' wide. (300' - 100' clear span = 200'). The piers are on 25 foot centers and are 2 feet wide. Within the 200 foot width the piers will occupy $\left(\frac{200}{25}\right)$ (2) = 16 feet.

At an elevation of 103 the piers will occupy an area 25 feet wide by 7 feet deep (103-96 = 7 feet). From Figure 14-14 the gross area of the bridge opening is 2700 feet.

Then: $$\frac{Ap}{A_{n2}} = \frac{(16)(7)}{2700} = .41$$

Compute and plot Ap/A_{n2} vs. elevation for each bridge length as shown in Figure 14-16.

12. Determine J for each elevation. Read J from Figure 16-16 for each elevation in column 2 and tabulate in column 8 of Table 14-6.

13. Determine the incremental backwater coefficient ΔK_p.

Using J from step 12 read ΔK from the appropriate curve (for this example curve 1) from Exhibit 14-5a. Using M from step 9 read σ from the appropriate curve (curve 1) from Exhibit 14-5b. Multiply ΔK by σ and tabulate as ΔK_p in column 9 of Table 14-6.

for 5000 cfs and a 300' bridge:

$$\Delta K = .105 \qquad \sigma = .59$$

$$\Delta K_p = \Delta K \sigma = (.105)\ (.59) = .06$$

14. Determine the total backwater coefficient K*. Add columns 7 and 9 and tabulate as K* in column 10. This is the total backwater coefficient for the bridge that will be considered for this example. If there are other losses that appear to be significant, the user should follow the procedure shown in the BPR report for computing their effects.

15. Determine the total change in water surface h*. Multiply column 10 by column 11 and tabulate in column 12. From Eq. 14-24:

$$h^* = K^* \frac{V^2}{2g}$$

for 5000 cfs and a 300 foot bridge with piers:

$$h^* = (1.30)\ (.495) = .64 \text{ feet}$$

If the example did not include piers or if the effect of eliminating the piers are desired the h* could be determined by multiplying column 7 by column 11.

for 5000 cfs and a 300 foot bridge without piers:

$$h^* = (1.24)\ (.495) = .61 \text{ feet}$$

16. Determine the elevation with bridge losses. Add column 5 and column 12 and tabulate in column 13. Column 13 is plotted on Figure 14-13 which shows the stage discharge curve for cross section M-5, assuming the fill to be high enough to force all of the 70,000 cfs discharge through the bridge opening.

* *

Full bridge flow

The analysis of flood flows past existing bridges involves flows which submerge all or a part of the bridge girders. When this condition occurs the computation of the head loss through the bridge must allow for the losses imposed by the girders. This may be accomplished in several ways.

One method is to continue using the BPR method but hold the bridge flow area and Kd constant for all elevations above the bridge girder. Example 14-8 uses this procedure. (See Figure 14-14).

Another approach commonly taken is to compute the flow through the bridge opening by the orifice flow equation.

$$q = CA\sqrt{2g\Delta h} \qquad \text{(Eq. 14-25)}$$

where q = discharge, in cfs
Δh = the difference in water surface elevation between headwater and tailwater, in feet
A = flow area of bridge opening, in square feet
g = acceleration of gravity
C = coefficient of discharge

In estimating C, if conditions are such that flow approaches the bridge opening with relatively low turbulence, the appropriate value of C is about 0.90. In the majority of cases C probably is in the 0.70 to 0.90 range. For very poor conditions (much turbulence), it may be as low as 0.40 to 0.50. In judging a given case, consider the following.

(1) Whether the abutments are square-cornered or shaped so as to reduce turbulence
(2) the number and shape of piers
(3) the degree of skew
(4) the number and spacing of pile bents since closely-spaced bents increase turbulence
(5) the existence of trees, drift, or other types of obstruction at the bridge or in the approach reach.

Using a C value of 0.8 has given approximately the same results as the BPR method for Example 14-7. However, the corresponding C value varied with discharge.
* *

Overtopping of bridge embankment

When the fill of a bridge is overtopped the total discharge at the bridge section is equal to the discharge through the bridge opening plus the discharge over the embankment. A reliable estimate of the effect of the bridge constriction on stages upstream under these conditions is difficult to obtain.

A generally accepted procedure to use in analyzing flows over embankments is to consider the embankment as acting as a broad crested weir. The broad crested weir equation is:

$$Q = CLH_e^{3/2} \qquad \text{(Eq. 14-26)}$$

where L = length of weir, in feet
H_e = energy head which is comprised of the velocity head at the upstream section plus the depth of flow over the weir, in feet
C = a coefficient

The following approximate ranges of C values for flows over embankments are recommended for use in Eq. 14-26. For road and highway fills, C = 2.5 to 2.8; for single-track railroad fills, C = 2.2 to 2.5; for double-track railroad fills, C = 1.9 to 2.2.

Equation 14-26 was developed for use in rectangular weir sections. Since road profiles encountered in the field seldom represent rectangular sections it becomes difficult to determine the weir length to use. Many approaches have been formulated to approximate this length. One approach suggests measuring the top width at the maximum depth of flow over the road and computing $H_e = d_c + \frac{A}{2T}$ for each depth.

Another method suggests measuring the weir length from the cross section at an elevation equal to 5/6 of h above the low point on the embankment.

A method suggested for use in this chapter substitutes the flow area A for the weir length and flow depth over the weir in Eq. 14-26.

Then: $$Q = C'Ah^{1/2} \qquad \text{(Eq. 14-27)}$$

where:
- A = flow area over the embankment at a given depth, h, in square feet
- h = flow depth measured from the low point on the embankment, in feet
- C' = coefficient which accounts for the velocity of approach.

The coefficient C' can be computed by equating Equations 14-26 and 14-27 and solving for C'.

$$C' = C\left(\frac{1}{\frac{\text{depth}}{\text{depth+velocity head}}}\right)^{3/2} \qquad \text{(Eq. 14-28)}$$

In Eq. 14-28 the depth is measured from the low point on the embankment of the bridge section and the velocity head is computed at the upstream section for the same elevation water is flowing over the embankment. The approach velocity may be approximated by V = Q/A where Q is the total discharge and A is the total flow area at the upstream section for the given elevation. In cases where the approach velocity is sufficiently small C' will equal C and no correction for velocity head will be needed to use Equation 14-27.

The free discharge over the road computed using Eq. 14-27 must be modified when the tailwater elevation downstream is great enough to submerge the embankment of the bridge section. The modification to the free discharge, Q_f, is made by computing a submergence ratio, H_2/H_1, where H_2 and H_1 are the depths of water downstream and upstream, respectively, above the low point on the embankment. A submergence factor, R, is read from Figure 3-4, NEH-11, Drop Spillways, and the submerged discharge is computed as $Q_s = RQ_f$. Then the total discharge at the bridge section is equal to the discharge through the bridge opening plus the submerged discharge over the embankment.

Example 14-9 shows the use of Eq. 14-27 and Eq. 14-28 in computing flows over embankments using a trial and error procedure to determine C'.

Example 14-9.
Develop a stage discharge curve for the overflow section of the highway analyzed in Example 14-8 (see Figure 14-12c) for the bridge opening of 300 feet. The top of embankment is at elevation 107. Assume a C value of 2.7.

1. Select a range of elevations that will define the rating curve over the road. Tabulate in column 1 of Table 14-7. The low point on the road is at elevation 107.

2. Compute the depth of flow, h, over the road. For each elevation listed in column 1 compute h and list in column 2 of Table 14-7.

3. Compute $h^{1/2}$. Tabulate in column 3 of Table 14-7.

4. Compute the flow area, A, over the road. For each elevation listed in column 1 compute the area over the road and tabulate in column 4 of Table 14-7.

Steps 5 through 11 are used to calculate the modified coefficient, C' to account for the approach velocity head. If it is determined that no modification to the coefficient C is required these steps may be omitted.

5. Compute the flow area at the upstream section. For each elevation listed in column 1 compute the total area at the upstream section and tabulate in column 5 of Table 14-7. The flow area can be obtained from the Kd computations at the upstream section or computed directly from the surveyed cross section.

6. Determine the discharge through the bridge. For the elevation in column 1 read the discharge through the bridge opening previously computed using bridge loss equations and tabulate in column 6 of Table 14-7.

7. Estimate the discharge over the road. Tabulate in column 7 of Table 14-7.

8. List the total estimated discharge going past the bridge section. Sum columns 6 and 7 and tabulate in column 8 of Table 14-7.

9. Compute the average velocity at the upstream section. The velocity can be estimated by using the total upstream area from column 5 and the estimated discharge from column 8 for the elevations listed in column 1 in the equation V = Q/A. For example for elevation 107.5:

$$V = \frac{28250 \text{ ft}^3/\text{sec}}{26700 \text{ ft}^2} = 1.06 \text{ ft/sec.}$$

Tabulate the velocity in column 9 of Table 14-7.

Table 14-7. Stage discharge over roadway at cross section M-4 without submergence. Example 14-9.

Elevation	h ft	$h^{1/2}$	A over road ft^2	A up- stream ft^2	Q through bridge cfs	Q est.over road cfs	Q est. total cfs	V ft/sec	$\frac{V^2}{2g}$	C'	Q over road cfs	Q total cfs
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
107.0	0.	0.	0.	25500.	26000.	0	26000	1.02	.016	0	0	26000
107.5	.5	.707	625	26700.	27000.	1300	28300	1.06	.0175	2.85	1300	28300
108.0	1.0	1.000	1500	28000.	28000.	4300.	32300	1.15	.021	2.79	4200	
						4200.	32200	1.15	.021	2.79	4200	32200
108.5	1.5	1.225	2525.	29200.	29300.	8500.	37800	1.30	.027	2.76	8500	37900.
109.0	2.0	1.414	4000.	30400.	30300.	12000	42000	1.38	.030	2.76	15600	
						15600	45900	1.51	.035	2.76	15600	45900.
110.0	3.0	1.730	7500.	32800.	32800.	35000	67800	2.06	.066	2.79	36200	
						36200	69000	2.10	.068	2.79	36200	69000

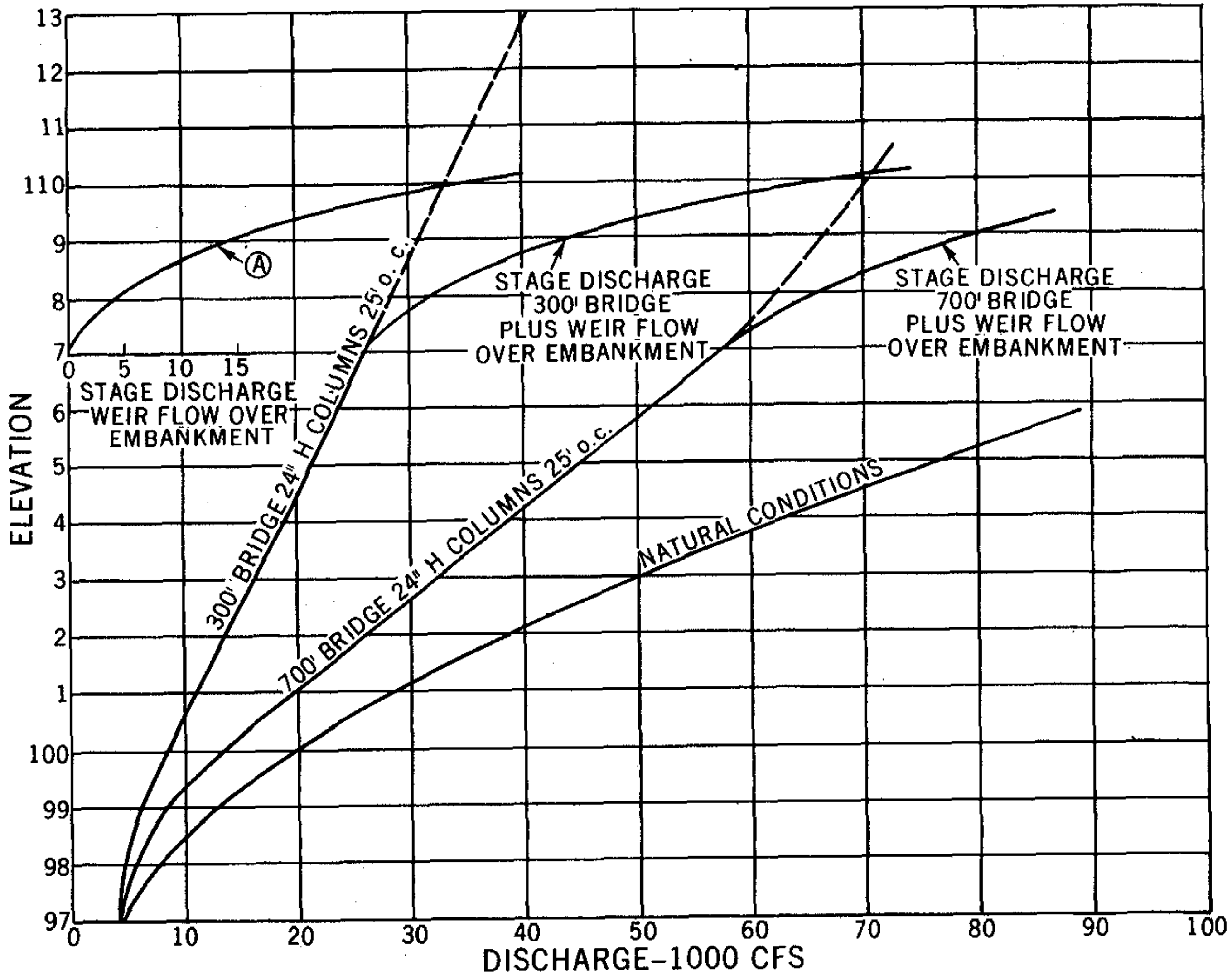


Figure 14-17. Stage discharge with embankment overflow, section M-5, Example 14-9.

10. Compute the velocity head. Using the velocity from column 9 compute $V^2/2g$ and tabulate in column 10 of Table 14-7.

11. Compute C'. Using equation 14-28 and data from Table 14-7 compute C'. For example at elevation 107.5:

$$C' = 2.7 \frac{1}{\left(\frac{.5}{.5+.0175}\right)^{3/2}} = \frac{2.7}{(.966)^{3/2}} = 2.85$$

List C' in column 11 in Table 14-7.

12. Compute discharge over the road. Using equation 14-27 and data from Table 14-7 compute the discharge over the road. For example at elevation 107.5:

$$Q = C'Ah^{1/2} = 2.85(625)(.707) = 1260 \text{ cfs}$$

Round to 1300 cfs and list in column 12. Compare this discharge value to the estimated discharge listed in column 7. If the computed discharge is less than or greater than the estimated discharge modify the estimated discharge in column 7 and recompute C' following steps 8 through 12.

13. List the total discharge going past the bridge section. Sum columns 6 and 12 and Tabulate in column 13 of Table 14-7.

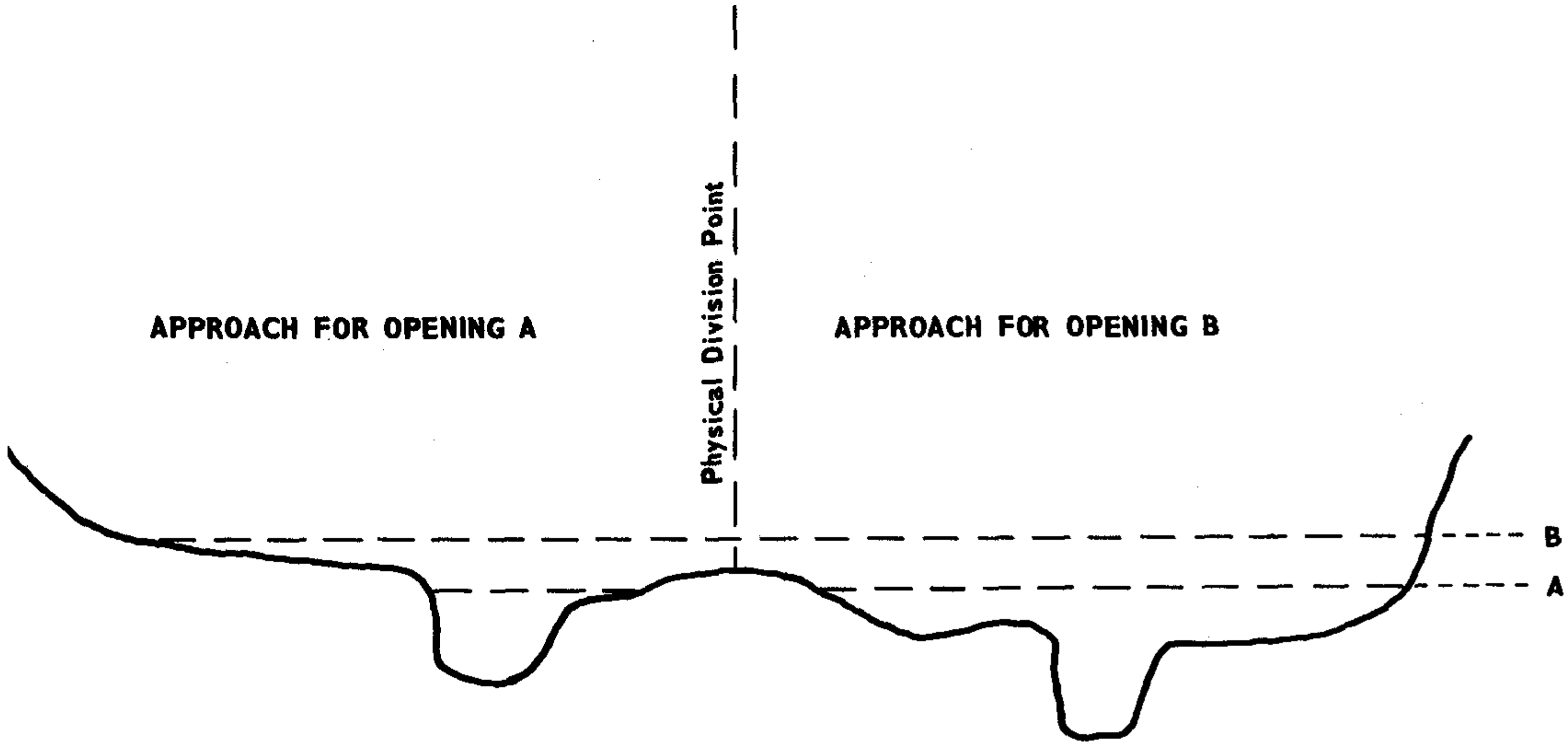
14. Plot the stage discharge curve. Using the computations shown in columns 1 and 13 of Table 14-7 plot the elevation versus discharge. The portion of the discharge flowing over the road (column 12) and the total discharge curve is shown in Figure 14-17 for the 300 foot bridge. This is the total stage discharge curve for the approach section (M-5).

Multiple bridge openings

Multiple openings in roads occur quite often and must be considered differently from single openings. The M ratio in the BPR procedure is defined as:

$$\frac{\text{Kd Bridge}}{\text{Kd Approach}}$$

When multiple openings are present the proper ratio must be assigned to each opening and then the capacity computed accordingly. If the flow is divided on the approach, the porblem is then one of divided flow with single openings in each channel. In many cases the flow is not divided



When water elevation is at A approaches act as directed by the physical division point.

When water elevation is at B approaches act according to the ratio of KD's of openings.

Figure 14-18. Approach section for a bridge opening.

for overbank flows. In these cases the headwater elevation must be considered to be the same elevation for each opening and the solution becomes trial and error until the head losses are equal for each opening and the sum of the flows equals the desired total.

The approaches are divided as shown in Figure 14-18. When the headwater is below the physical dividing point as illustrated by Level A then the M ratio is computed as in a single opening.

When the headwater is above the physical dividing point cross flow can occur. When this occurs the approach used to compute the M ratio and J is as follows:

1. Compute the Kd value for each bridge opening.

2. Compute the Kd value for the total approach section.

3. Proportion the approach Kd value for each opening by the relationship:

$$\text{Kd appr}_x = \frac{\text{Kd}_{\text{bridge}_x}}{\text{Kd}_{\text{bridge}_1} \ldots \text{Kd}_{\text{bridge}_2} \ldots + \text{Kd}_{\text{bridge}_n}} \text{ x total approach Kd.}$$

4. Compute M as before using the Kd value computed in step 3 for the approach.

5. Compute the approach area contributing to this opening by the relationship:

$$\text{Area appr}_x = \frac{\text{Kd}_{\text{bridge}_x}}{\text{Kd}_{\text{bridge}_1} \ldots + \text{Kd}_{\text{bridge}_2} \ldots \text{Kd}_{\text{bridge}_n}} \text{ x total approach area}$$

6. Compute J as before using the area computed in step 5 for the approach area.

Culverts

Culverts of all types and sizes are encountered when computing stage discharge curves in natural streams. These culverts may or may not have a significant effect on the development of a watershed work plan. However, in many cases they present a problem in evaluating a plan and must be analyzed to determine if an acceptable plan can be installed without enlarging or replacing the existing culvert.

The Bureau of Public Roads has developed procedures based on research data for use in designing culverts. This document, Hydraulic Charts for the Selection of Highway Culverts, Hydraulic Engineering Circular No. 5, December 1965, is available from the Superintendent of Documents, Washington, D. C.

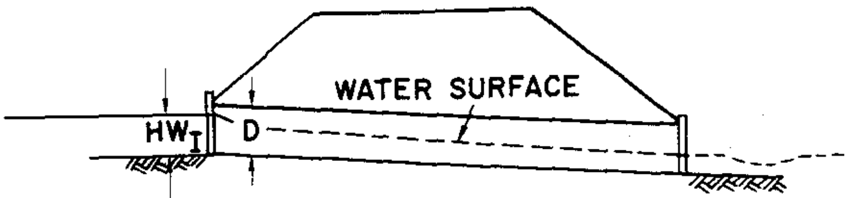


Figure 14-19a. Unsubmerged inlet

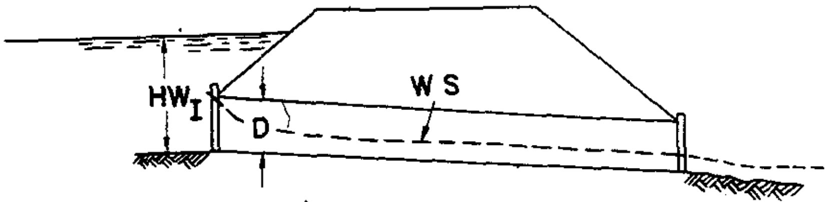


Figure 14-19b. Submerged inlet

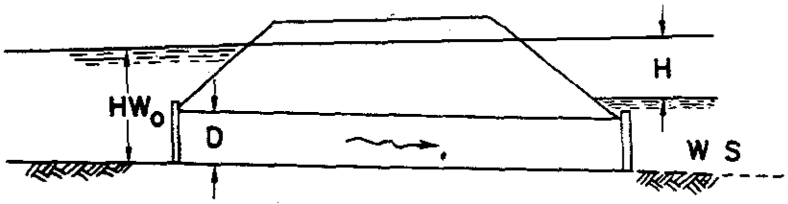


Figure 14-19c. Submerged outlet

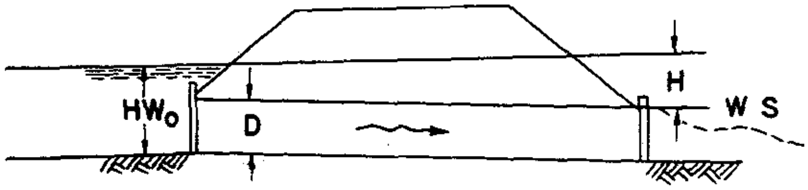


Figure 14-19d. Outlet flowing full

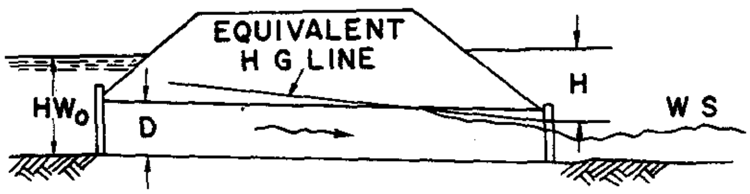


Figure 14-19e. Pipe full part way

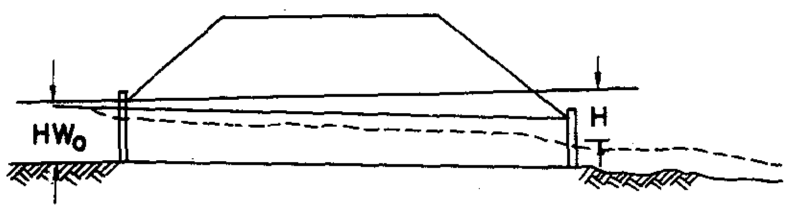


Figure 14-19f. Open flow through pipe

Culverts of various types, installed under different conditions, were studied in order to develop procedures to determine the backwater effect for the two flow conditions: 1) culverts flowing with inlet control; 2) culverts flowing with outlet control.

Inlet Control
Inlet control means that the capacity of the culvert is controlled at the culvert entrance by the depth of headwater (HW_I) and the entrance geometry of the culvert including the barrel shape and cross sectional area and the type of inlet edge, shape of headwall, and other losses. With inlet control the entrance acts as an orifice and the barrel of the culvert is not subjected to pressure flow. Figure 14.19a and 14.19b show sketches of two types of inlet controlled flow.

The nomographs shown on Exhibits 14-6 through 14-10 were developed from research data by the Division of Hydraulic Research, Bureau of Public Roads research data. They have been checked against actual measurements made by USGS with favorable results.

Types of Inlets. - The following descriptions are taken from "Electronic Computer Program for Hydraulic Analysis of Circular Culverts" Bureau of Public Roads, February 1969. Some of the types of inlets are illustrated in Figure 14-20.

a. Tapered - This inlet is a type of improved entrance with can be made of concrete or metal. Shapes are shown in Figure 14-20a.

b. Bevel A and Bevel B - These bevels, a type of improved entrance, can be formed of concrete or metal.

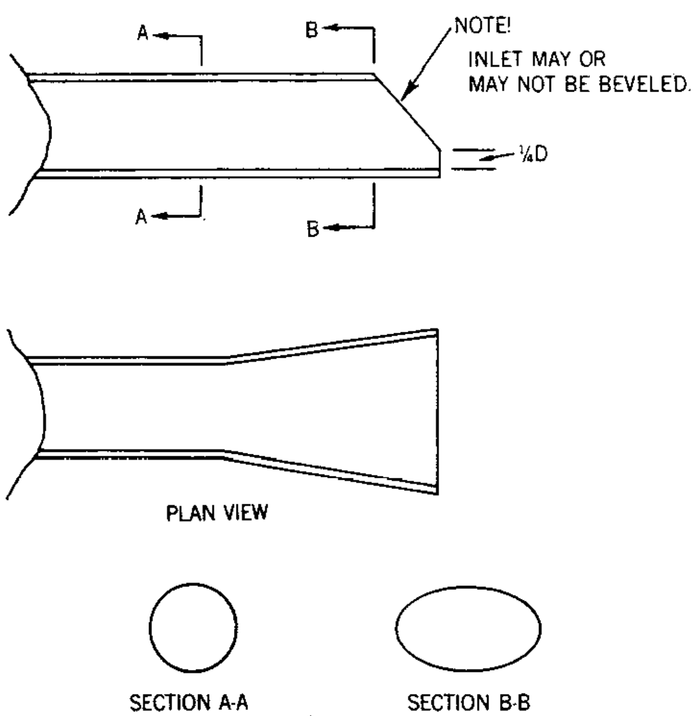
c. Angled wingwall - Similar to headwall but at an angle with culvert.

d. Projecting - The culvert barrel extends from the embankment. The transverse section at the inlet is perpendicular to the longitudinal axis of the culvert.

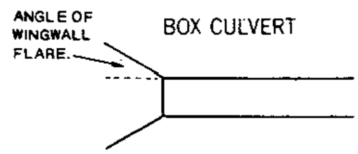
e. Headwall - A headwall is a concrete or metal structure placed around the entrance of the culvert. Headwalls considered are those giving a flush or square edge with the outside edge of the culvert barrel. No distinction is made for wingwalls with skewed alignment.

f. Mitered - The end of the culvert barrel is on a miter or slope to conform with the fill slope. All degrees of miter are treated alike since research data on this type of inlet are limited. Headwater is measured from the culvert invert midway into the mitered section.

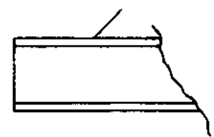
g. End section - This section is the common prefabricated end made of either concrete or metal and placed on the inlet or outlet ends of a culvert. The closed portion of the section, if present, is not tapered. (Not illustrated)



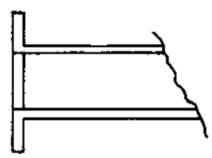
Tapered inlet



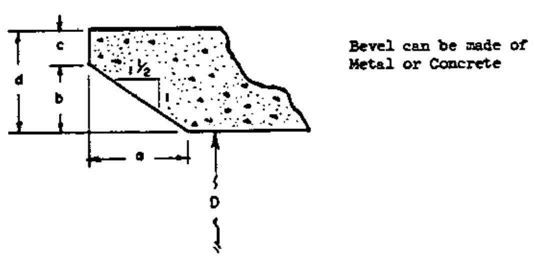
Angled wing wall



Projecting

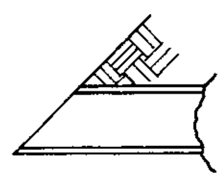


Headwall



BEVEL	$\frac{b}{D}$	$\frac{a}{D}$	$\frac{c}{D}$	$\frac{d}{D}$
A	0.042	0.063	0.042	0.083
B	0.083	0.125	0.042	0.125

Bevel A and Bevel B



Mitered

Figure 14-20. Types of culvert inlets.

h. Grooved edge - The bell or socket end of a standard concrete pipe is an example of this entrance. (Not illustrated)

Outlet Control

Culverts flowing with outlet control can flow with the culvert barrel full or part full for part of the barrel length or for all of it. Figures 14-19c, 14-19d, 14-19e, and 14-19f show the various types of outlet control flow. The equation and graphs for solving the equation give accurate results for the first three conditions. For the fourth condition shown in Figure 14-19f, the accuracy decreases as the head decreases. The head H, Figure 14-19c and 14-19d, or the energy required to pass a given discharge through the culvert flowing in outlet control with the barrel flowing full throughout its length consists of three major parts: 1) velocity head H_v, 2) entrance loss H_e, and 3) friction loss H_f, all expressed in feet. From Figure 14-21a:

$$H = H_v + H_e + H_f \qquad \text{(Eq. 14-29)}$$

$H_v = \frac{V^2}{2g}$ when V is the average velocity in the culvert barrel.

H_e = entrance loss which depends on the geometry of the inlet. The loss is expressed as a coefficient K_e (Exhibit 14-21) times the barrel velocity head.

$$H_e = K_e \frac{V^2}{2g} \qquad \text{(Eq. 14-30)}$$

H_f = friction loss in barrel

$$H_f = \frac{29n^2 L}{R^{1.33}} \times \frac{V^2}{2g} \qquad \text{(Eq. 14-31)}$$

n = Mannings friction factor
L = length of culvert barrel (ft)
V = velocity in culvert barrel (ft/sec)
g = acceleration of gravity (ft/sec^2)
R = hydraulic radius (ft)

Substituting in Equation 14-23:

$$H = \left(1 + K_e + \frac{29n^2 L}{R^{1.33}}\right) \frac{V^2}{2g} \qquad \text{(Eq. 14-32)}$$

Figure 14-21a shows the terms of Eq. 14-29, the hydraulic gradeline, the energy gradeline, and the headwater depth HW_o.

The expression for H is derived by equating the total energy upstream from the culvert to the energy just inside the culvert outlet.

$$H = d_1 + \frac{V_1{}^2}{2g} + LS_o - d_2 = H_v + H_e + H_f \qquad \text{(Eq. 14-33)}$$

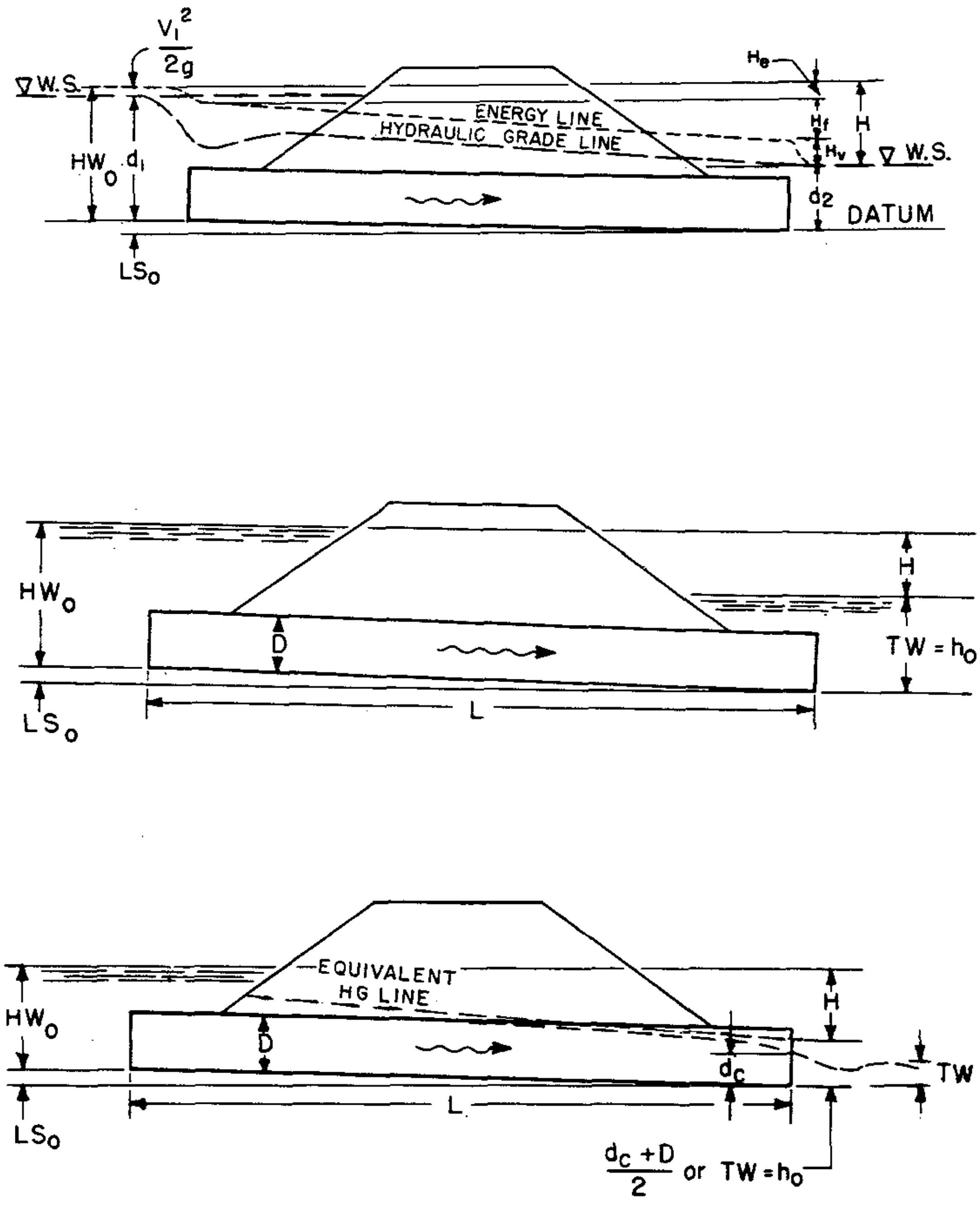


Figure 14-21. Elements of culvert flow.

From Figure 14-21a:

$$HW_o = H + d_2 - LS_o \qquad \text{(Eq. 14-34)}$$

If the velocity head in the approach section $(\frac{V_1^2}{2g})$ is low it can be ignored and HW_o is considered to be the difference between the water surface and the invert of the culvert inlet.

The depth, d_2, for culverts flowing full is equal to the culvert height Figure 14-19d, or the tailwater depth (TW) whichever is greater, Figure 14-21b.

The hydraulic gradeline for culverts flowing with the barrel part full for part of the barrel length passes through a point where the water breaks with the top of the culvert and if extended as a straight line will pass through the plane of the outlet end of the culvert at a point above the critical depth. This point is approximately halfway between d_c and the crown of the culvert, or equal to $\frac{d_c + D}{2}$. The depth d_2 or h_o (see Figure 14-21c) for this type of flow is equal to $\frac{d_c + D}{2}$ or TW whichever is greater.

With the above definition of d_2 which will be designated as h_o, an equation common to all outlet control conditions can be written:

$$HW_o = H + h_o - LS_o \qquad \text{(Eq. 14-35)}$$

This equation was used to develop the nomographs shown on Exhibits 14-11 through 14-15 which can be used to develop stage discharge curves for the approach section to culverts flowing with outlet control.

Exhibit 14-16 shows d_c for discharge per foot of width for rectangular sections. Exhibits 14-17 to 14-20 show d_c for discharges for various non-rectangular culvert sections.

Example 14-10

Develop a stage discharge curve for cross section T-4 (Figure 14-4) showing the backwater effect of eight 16' x 8' concrete box culverts for each of three conditions: 1) inlet control, 2) outlet control, present channel, and 3) outlet control, improved channel. Figure 14-22a shows a cross section along the centerline of the roadway at cross section T-3. Figure 14-22b shows a section through the roadway with water surface profiles prior to and after the construction of the culverts and roadway embankment.

The culvert headwalls are parallel to the embankment with no wingwalls, and the entrance is square on three edges.

The following are given in this example: a stage discharge curve for cross section T-2, present condition and with proposed channel improvement

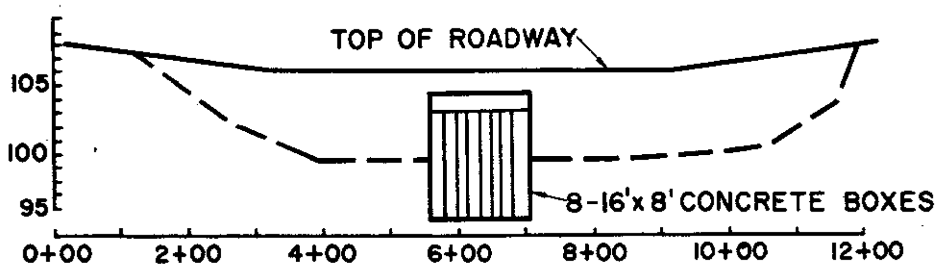


Figure 14-22a. Cross section T-3.

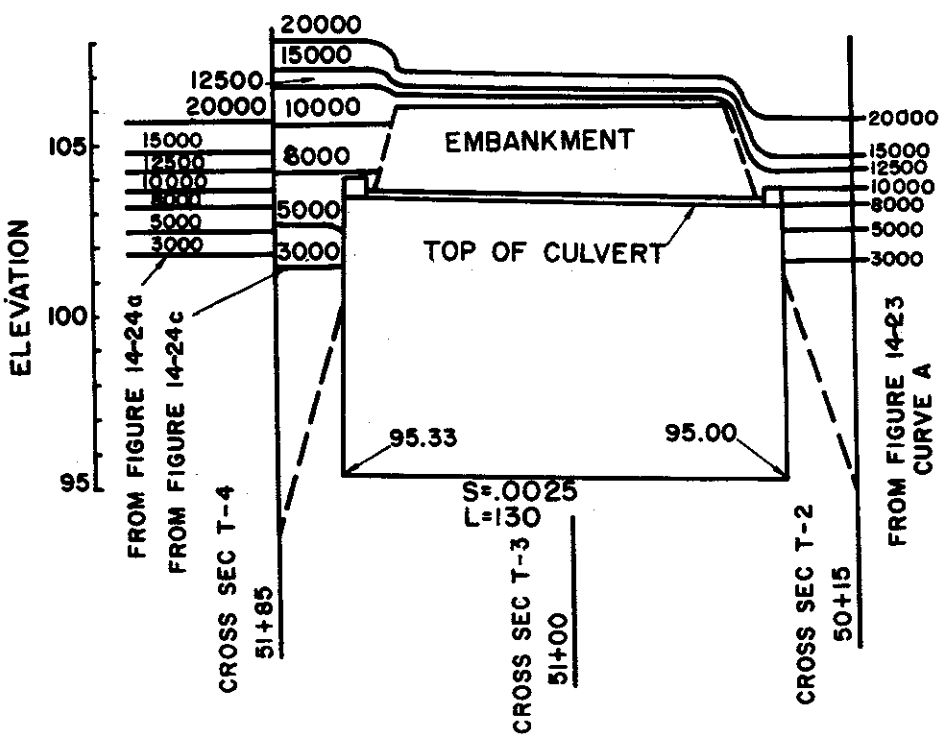


Figure 14-22b. Profile through culvert, Example 14-10.

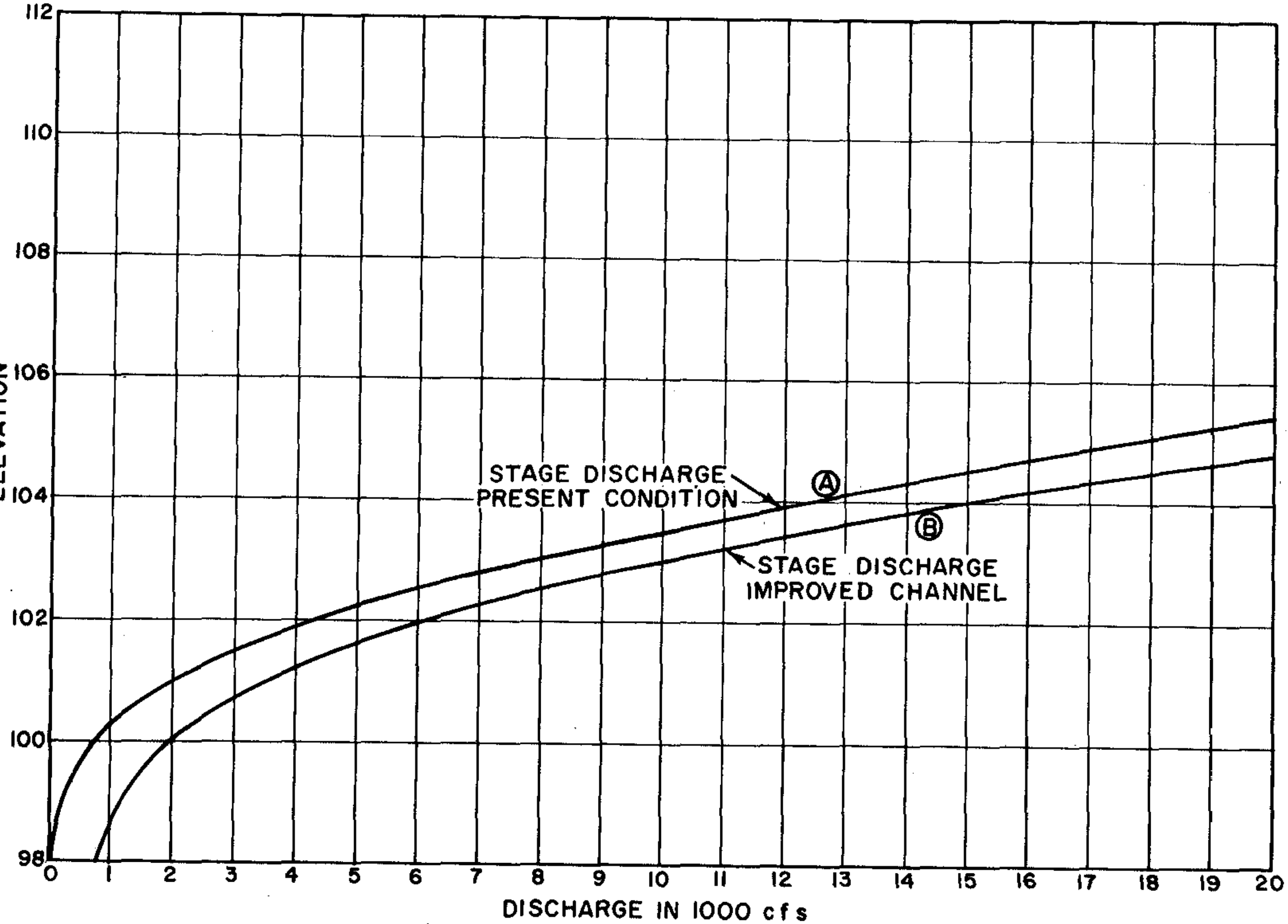


Figure 14-23. Stage discharge exit section T-2, Example 14-10.

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(Figure 14-23, curves A and B). Also given is a stage discharge curve for cross section T-4 disregarding the effect of the culverts and roadway fill (Figure 14-24a).

Condition 1--Inlet Control

1. Select a range of discharges sufficient to define the new stage discharge curve. Tabulate in column 1 of Table 14-8.

2. Determine the discharge for each culvert. Divide the discharges in column 1 by the number of culverts (8) and tabulate in column 2 of Table 14-8.

3. Determine the discharge per foot of width. Divide the discharges in column 2 by the width of each culvert (16 feet) and tabulate in column 3 of Table 14-8.

4. Compute $\frac{HW}{D}$. Using the nomograph, Exhibit 14-6, read HW/D for each discharge per foot of width in column 3 and tabulate in column 4 of Table 14-8. Referring to Exhibit 14-6 project a line from the depth of culvert (8 feet) through the discharge per foot of width (line q/B) to the first HW/D line, then horizontal to line (3), which is the HW/D for the type of culvert in this example.

5. Compute HW. Multiply column 4 by the depth of the culvert (8 feet) and tabulate in column 5 of Table 14-8.

6. Add the invert elevation at the entrance to the culvert (elev. 95.33) to column 5. Tabulate in column 6 of Table 14-8.

7. Plot the stage discharge curve assuming inlet control. Plot column 1 and column 6 of Table 14-8 as the stage discharge curves for cross section T-4 (see Figure 14-24b curve A). This assumes inlet control with the road sufficiently high to prevent over topping.

Condition 2--Outlet Control, Present Channel

1. Compute the entrance loss coefficient, K_e. Read $K_e = 0.5$ from Exhibit 14-21 for the type of headwall and entrance to box culvert and tabulate in column 7 of Table 14-8.

2. Compute the head loss, H, for the concrete box culvert flowing full. Using the nomograph on Exhibit 14-11, draw a line from L = 130 feet on the $K_e = 0.5$ scale to the cross sectional area scale, 16' x 8' = 128 square feet, and establish a point on the turning line. Draw a line from the discharge (q) line for each of the discharges shown in column 2, through the turning point to the head (H) line. Tabulate H in column 8 of Table 14-8.

Table 14-8. Headwater computations for eight 16' x 8' concrete box culverts, headwalls parallel to embankment (no wingwalls), square edged on three sides, Example 14-10.

Total Discharge q	Discharge for Each Culv.	Discharge per foot of Width	Inlet Control			Outlet Control, Present Channel								Outlet Control, Improved Channel	
			$\frac{HW}{D}$	HW	HW_I 1/	K_e	H	d_c	$\frac{d_c + D}{2}$	h_o 2/ Elev.	TW Elev.	LS_o	HW_o 3/ Elev.	TW Elev.	HW_o Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
3000	375	23.5	0.55	4.40	* 4/	0.5	0.22	2.6	5.30	100.30	101.4	0.33	* 4/	100.7	* 4/
5000	625	39.1	0.77	6.15	* 4/	0.5	0.60 5/	3.6	5.80	100.80	102.3	0.33	102.57	101.6	101.87
8000	1000	62.5	1.08	8.65	103.98 6/	0.5	1.55	4.9	6.45	101.45	103.0	0.33	104.22 6/	102.6	103.82 6/
10000	1250	78.0	1.31	10.46	105.79	0.5	2.50	5.7	6.85	101.85	103.5	0.33	105.67	103.0	105.17
12500	1565	98.0	1.61	12.88	108.21	0.5	3.90	6.7	7.35	102.35	104.0	0.33	107.57	103.6	107.17
15000	1875	117.0	2.01 7/	16.08	111.41	0.5	5.60	7.5	7.75	102.75	104.5	0.33	109.77	104.0	109.27
20000	2500	156.3	--	--	--	0.5	10.00	9.0 8/	8.00 9/	103.00	105.5	0.33	115.77	104.8	114.47

1/ $HW_I = HW + 95.33$ (invert elevation at entrance end of culvert = 95.33).

2/ $h_o = \frac{d_c + D}{2} + 95.00$ (invert elevation at outlet end of culvert = 95.00).

3/ $HW_o = H + TW - LS_o$ or $H + h_o - LS_o$, whichever is greater.

4/ Tailwater elevation is higher than the computed elevation and open channel flow exists.

5/ See example on Exhibit 14-11.

6/ Note: with channel improvement the control switches from outlet to inlet between 5000 and 8000 cfs.

7/ See example on Exhibit 14-6.

8/ If $d_c \geq D$, the outlet always controls.

9/ $\frac{d_c + D}{2}$ cannot exceed D.

3. Compute the critical depth, d_c, for each discharge per foot of width. Using Exhibit 14-16, read d_c for each discharge per foot of width shown in column 3 and tabulate in column 9 of Table 14-8.

4. Compute $\frac{d_c + D}{2}$. Tabulate in column 10 of Table 14-8

   Note: $\frac{d_c + D}{2}$ cannot exceed D.

5. Compute h_o. Add the invert elevation of the outlet end of the culvert (elev. 95.00) to $\frac{d_c + D}{2}$ and tabulate as h_o in column 11 of Table 14-8.

6. Compute the TW elevation for each discharge in column 1. Using Figure 14-23, curve A, read the elevation for each discharge in column 1 and tabulate as TW elevation in column 12 of Table 14-8.

7. Compute the difference in elevation of the inlet and outlet inverts of the culverts. Multiply $L \times S_o = 130 \times .0025 = 0.33$ and tabulate in column 13 of Table 14-8.

8. Compute the water surface elevation, HW_o, assuming outlet control. Add values in column 8 to the larger of column 11 or column 12 minus column 13 and tabulate as HW_o in column 14 of Table 14-8.

9. Plot the stage discharge curve assuming outlet control. Plot column 1 and column 14 on Figure 14-24c curve A assuming outlet control with the roadway sufficiently high to prevent over topping.

Condition 3-- Outlet Control, Improved Channel.

1. Compute the tailwater elevation at the culvert for the improved channel condition.
   Using Figure 14-23, curve B, read the elevation for each discharge in column 1 and tabulate as TW elevation in column 15 of Table 14-8.

2. Compute the elevation assuming outlet control, improved channel. Add column 8 plus the layer of column 15 or column 15 minus column 13 and tabulate in column 16 of Table 14-8.

3. Plot the stage discharge curve assuming outlet control with improved channel. Plot column 1 and column 16 on Figure 14-24d, curve A, as the stage discharge curve for cross section T-4 assuming outlet control with improved channel and the roadway sufficiently high to prevent over topping.

Condition for flow over roadway.
Assume the approach velocity head for this example is negligable and the coefficient C will equal C^1 used in Eq. 14-26. If the velocity head is significant and a correction to the coefficient C is desired by using Eq. 14-27 follow steps 5 through 9 of Example 14-9.

1. Select a range of elevations that will define the rating curve over the road. Tabulate in column 1 of Table 14-9. The low point on the road is at elevation 106.

2. Compute the depth of flow, H, over the road. For each elevation in column 1 compute H and list in column 2 of Table 14-9.

3. Compute $H^{1/2}$. Tabulate in column 3 of Table 14-9.

4. Compute the flow area, A, over the road. For each elevation listed in column 1 compute the area over the road and tabulate in column 4 of Table 14-9.

5. Determine coefficient, C. Assume C = 2.7 for this example and assume $C = C^1$. Tabulate C^1 in column 5 of Table 14-9.

6. Compute the discharge over the roadway using Eq. 14-26.

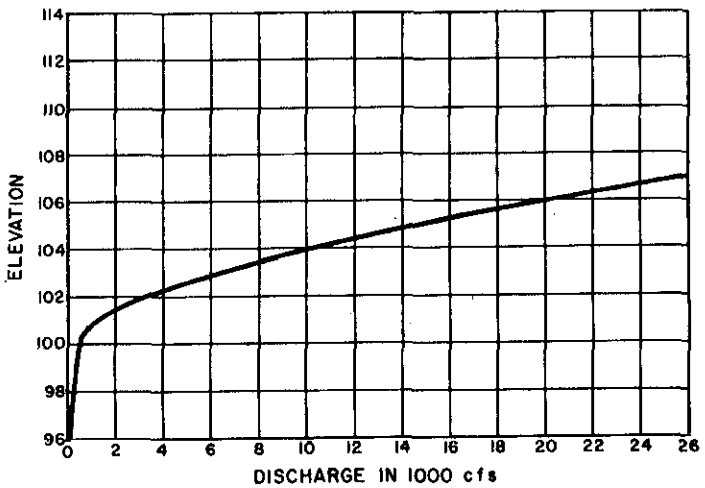
7. Plot the stage discharge curve. Using the computations shown in Table 14-9 plot column 1 and column 6 shown on Figure 14-24b, c, and d as curve B.

8. Graphically combine curves A and B on Figures 14-24b, c and d to form the stage discharge curve for the culverts and weir flow over the roadway.

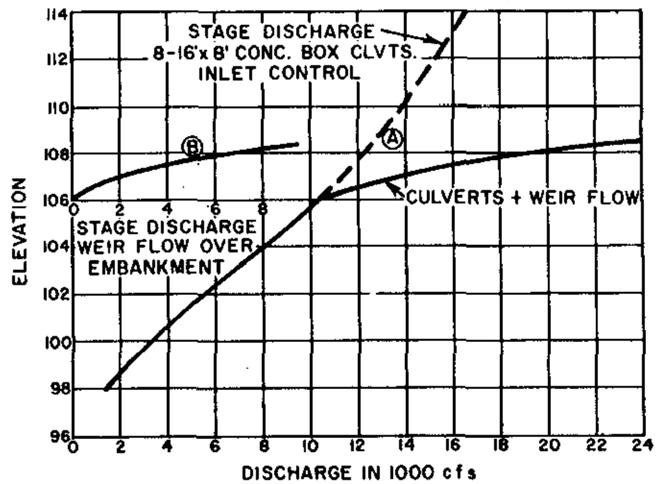
Table 14-9. Stage discharge over roadway at cross section T-3, Figure 14-4. Example 14-10.

Elevation (1)	H (2)	$H^{1/2}$ (3)	A (4)	C^1 (5)	q (6)
106.	0.	0.	0	2.7	0
106.5	.5	.707	340	2.7	650
107.	1.0	1.	750	2.7	2020
107.5	1.5	1.225	1230	2.7	4070
108	2.0	1.414	1790	2.7	6830

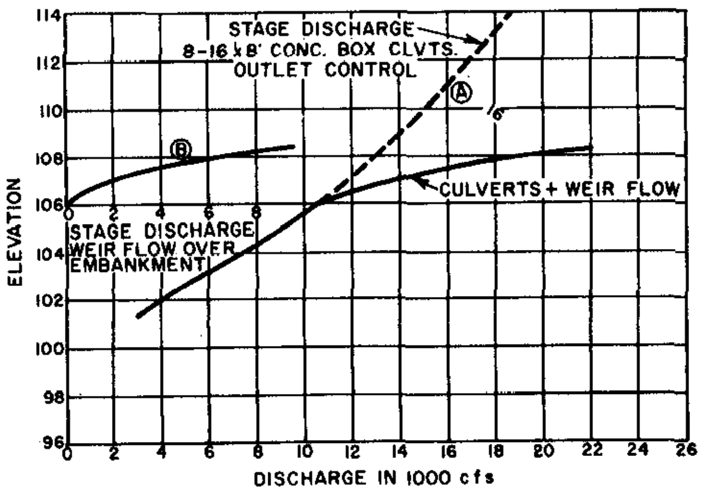
Each of the 3 flow conditions were computed independent of each other. The flow condition that actually controls is that which requires the greater upstream elevation for the discharge being considered. By comparing elevations for the same discharge for the 3 conditions tabulated on Table 14-8 and plotted on Figure 14-24 b, c and d the type of control at any given discharge can be determined. It may be advantageous to plot all the curves on one graph to better define points of intersection.



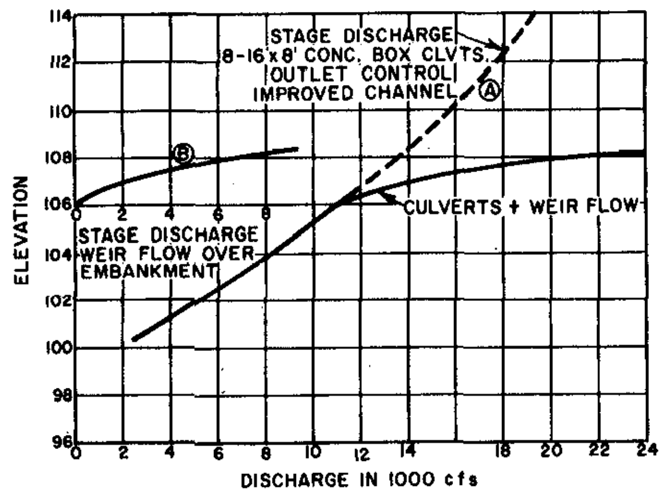
(a) assuming no roadway in place.



(b) inlet control



(c) outlet control.



(d) improved channel-outlet control

Figure 14-24. Rating curves cross section T-4, Example 14-10.

Under the old channel conditions it can be determined that open channel flow conditions exist for discharges less than about 4000 cfs, outlet control governs between about 4000 and 9000 cfs and inlet control governs for discharges greater than 9000 cfs.

Under new channel conditions open channel flow exists for discharges less than 3800 cfs, outlet control governs for discharges between 3800 and 7300 cfs and inlet control governs for discharges greater than 7300 cfs. Also, in both cases, discharges greater than 10,200 cfs flow will occur over the road embankment.

If the actual profile for discharges occurring under open channel flow conditions is desired water surface profiles should be run through the culverts.

It can also be seen from Figure 14-24a and 14-24b that by constructing the highway with 8 - 16' x 8' concrete box culverts elevations upstream will increase over present conditions for discharges greater than 5000 cfs. for improved outlet conditions upstream elevations will not be increased above present conditions until a discharge of 7200 cfs occurs.

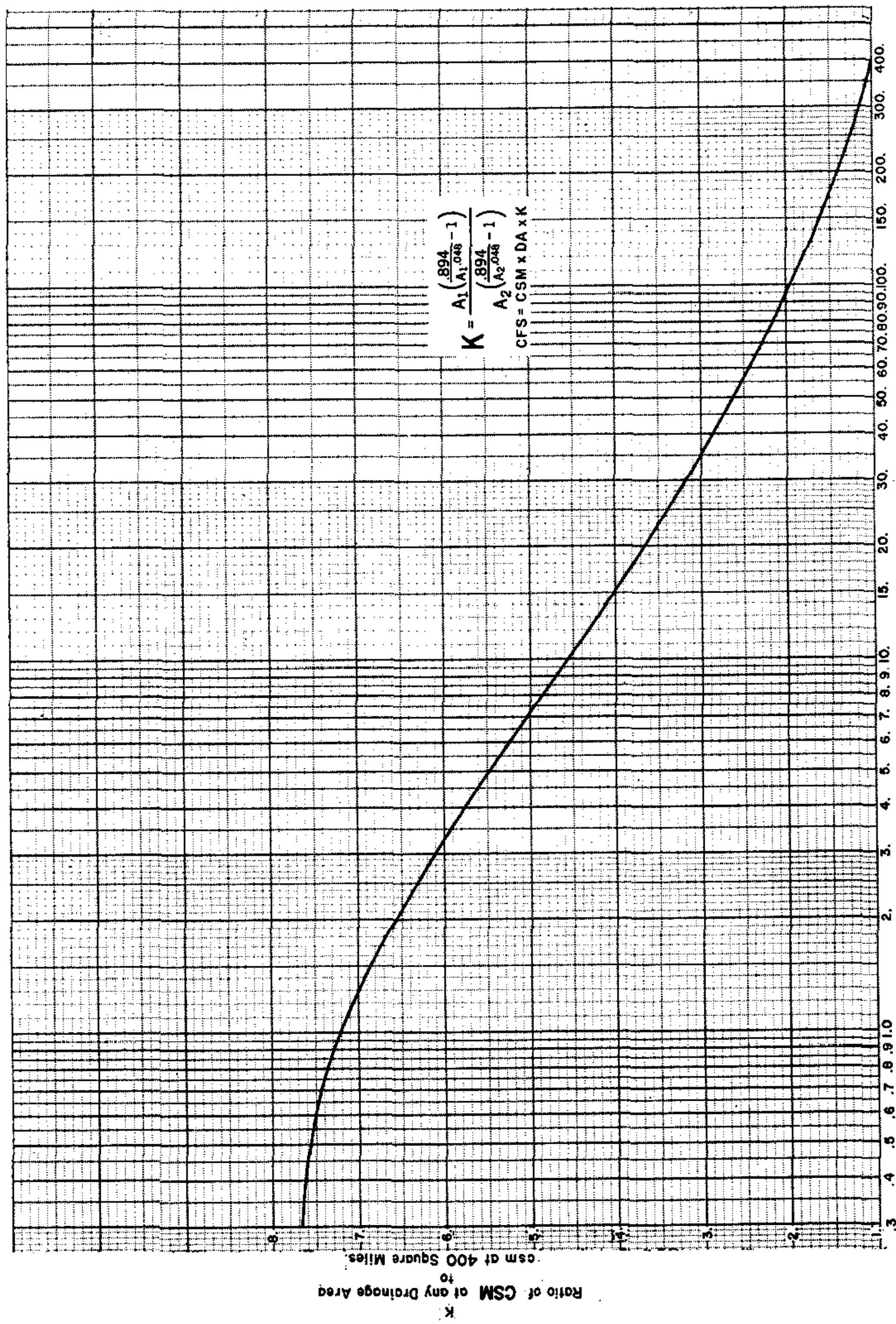


Exhibit 14-1. K values for converting CSM to CFS.

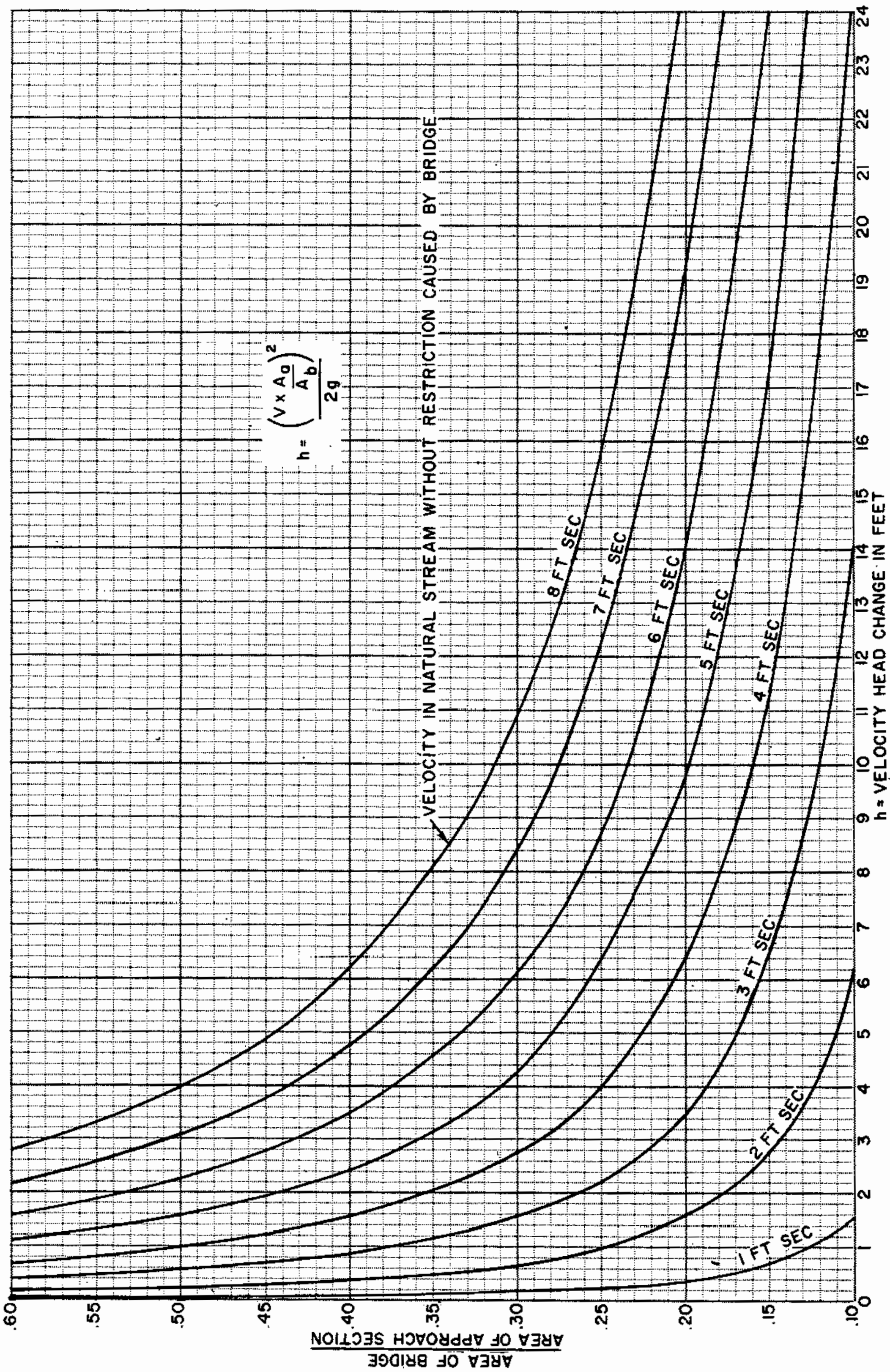


Exhibit 14-2. Estimate of head loss in bridges.

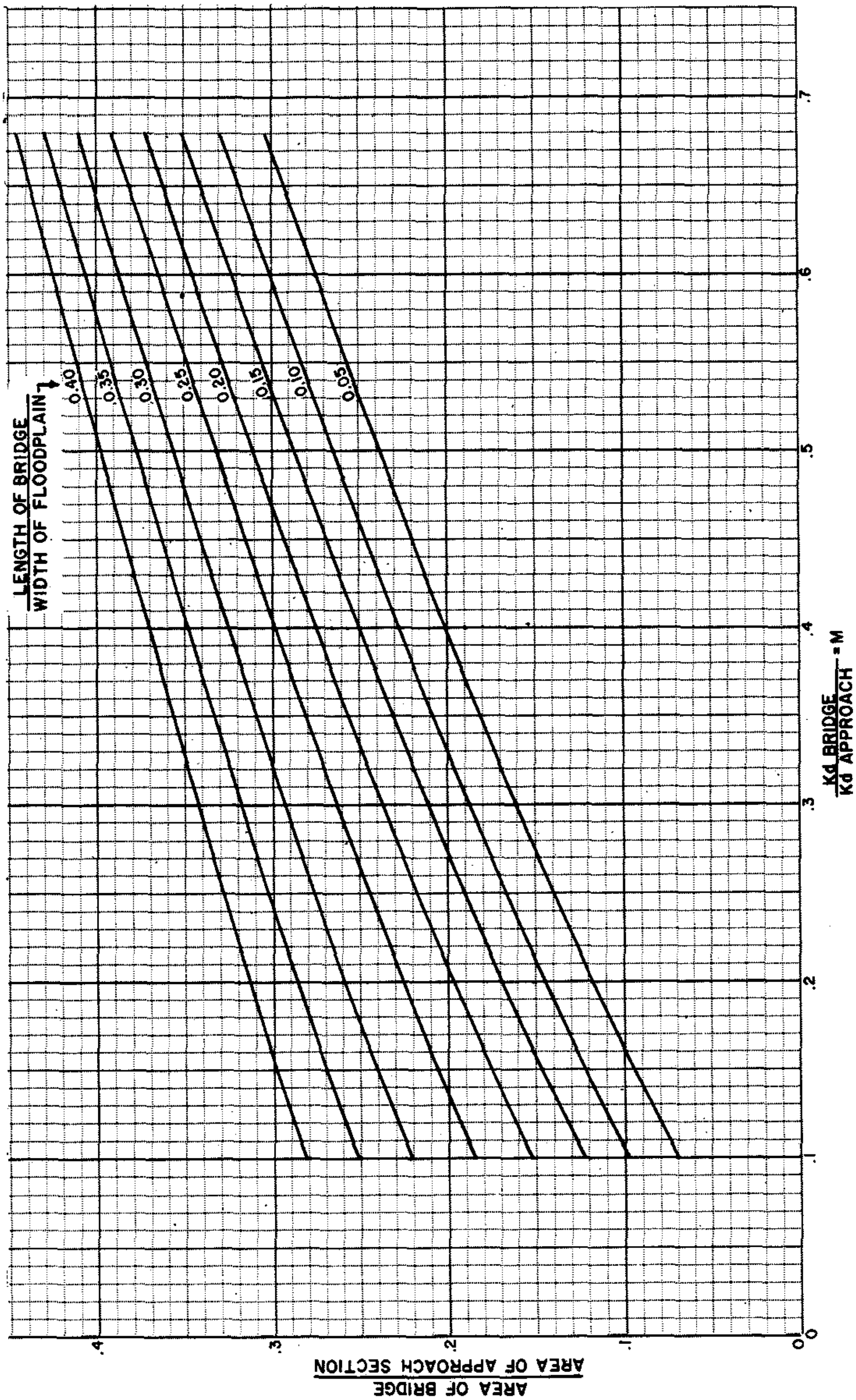


Exhibit 14-3. Estimate of M for use in BPR equation.

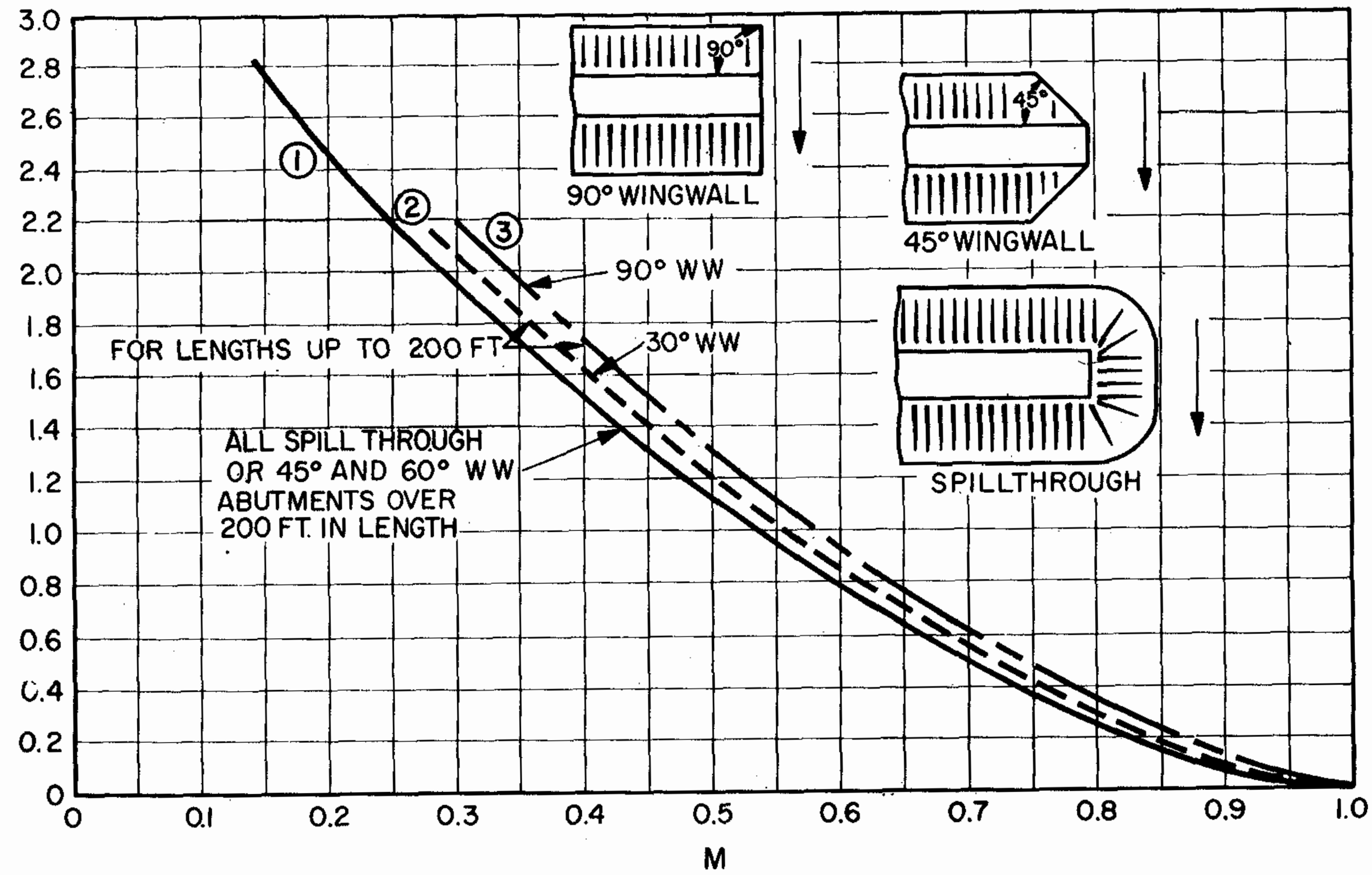


Exhibit 14-4. BPR base curve for bridges (K_b).

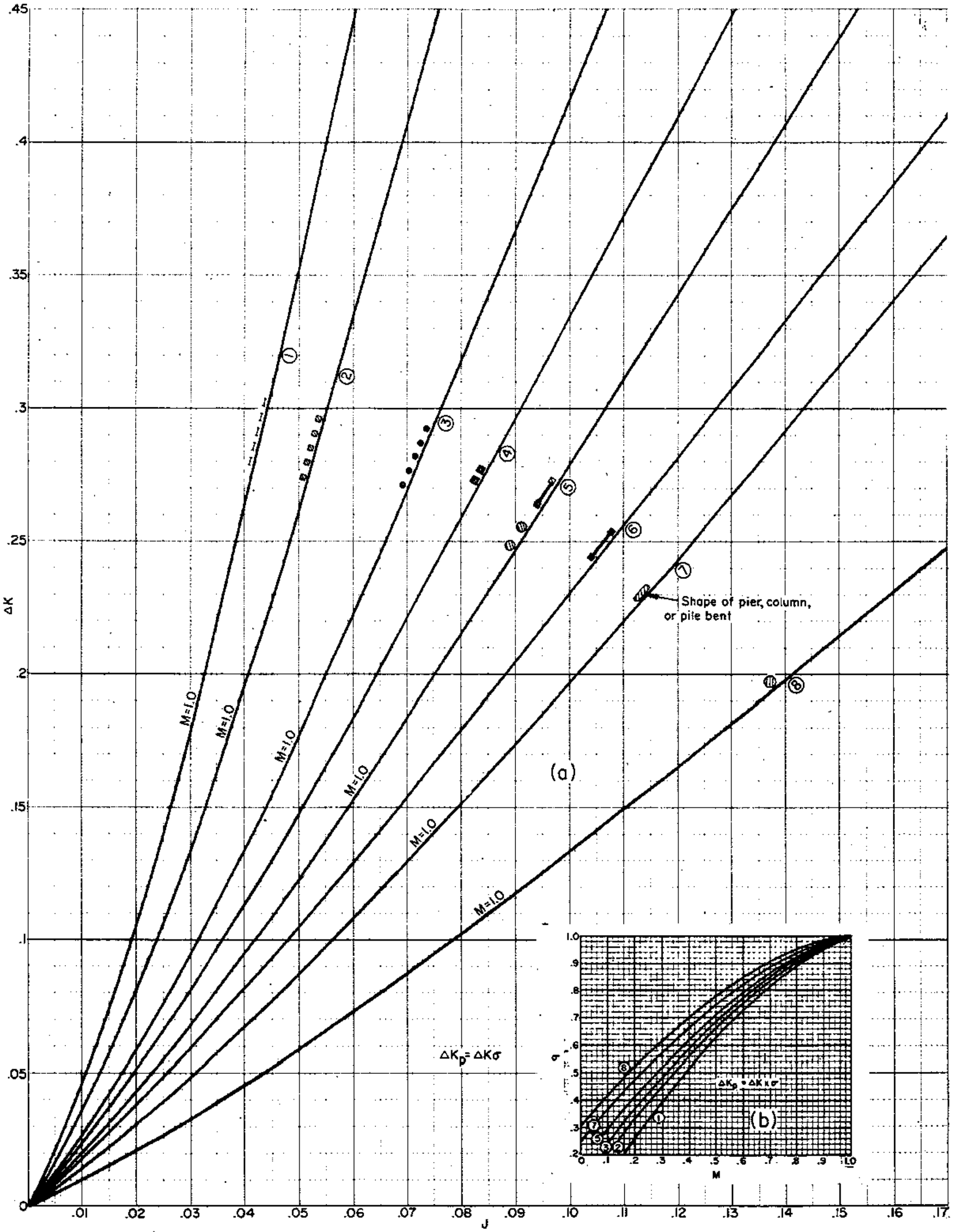


Exhibit 14-5. Incremental backwater coefficient for the more common types of columns, piers and pile bents.

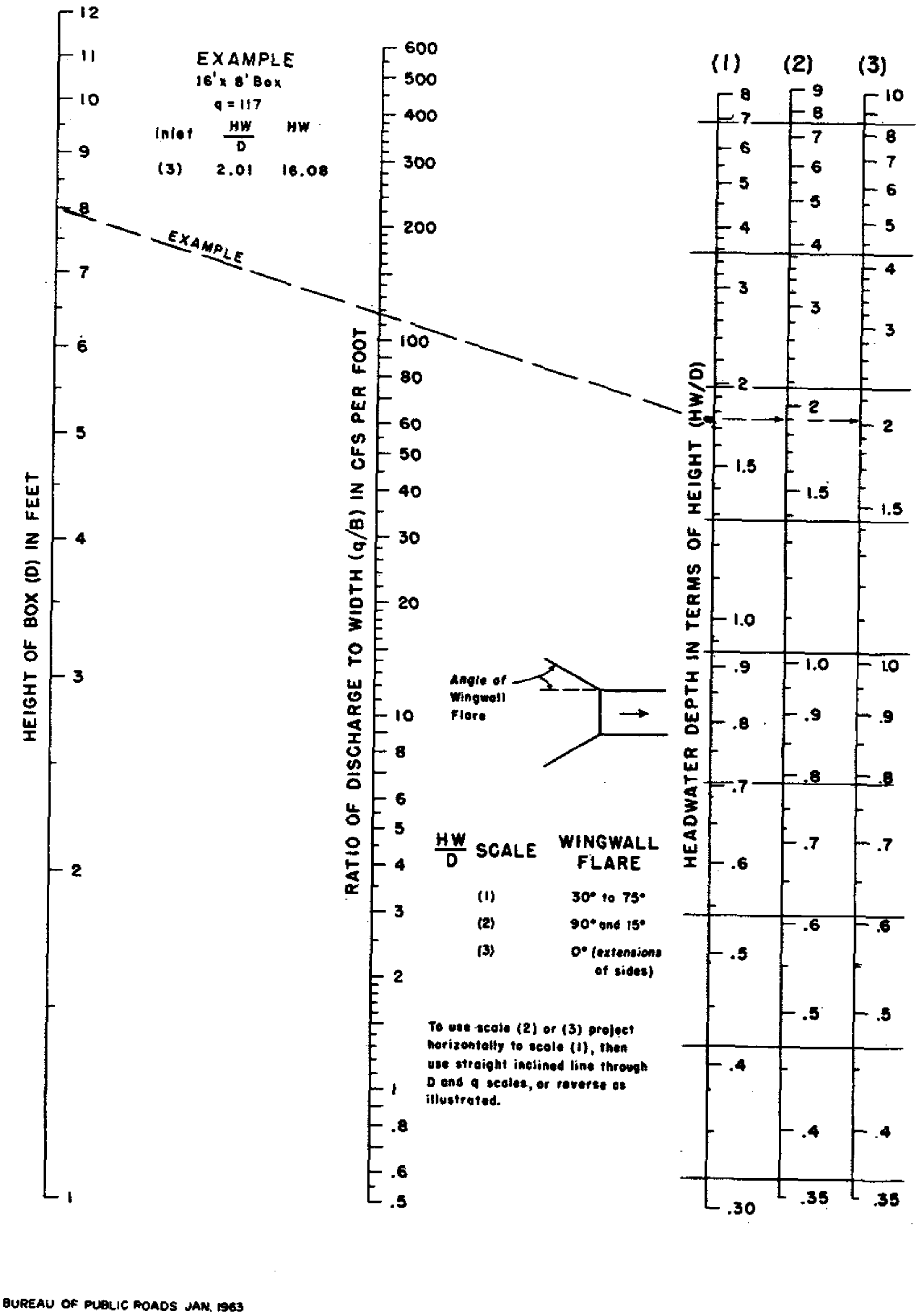


Exhibit 14-6. Headwater depth for box culverts with inlet control.

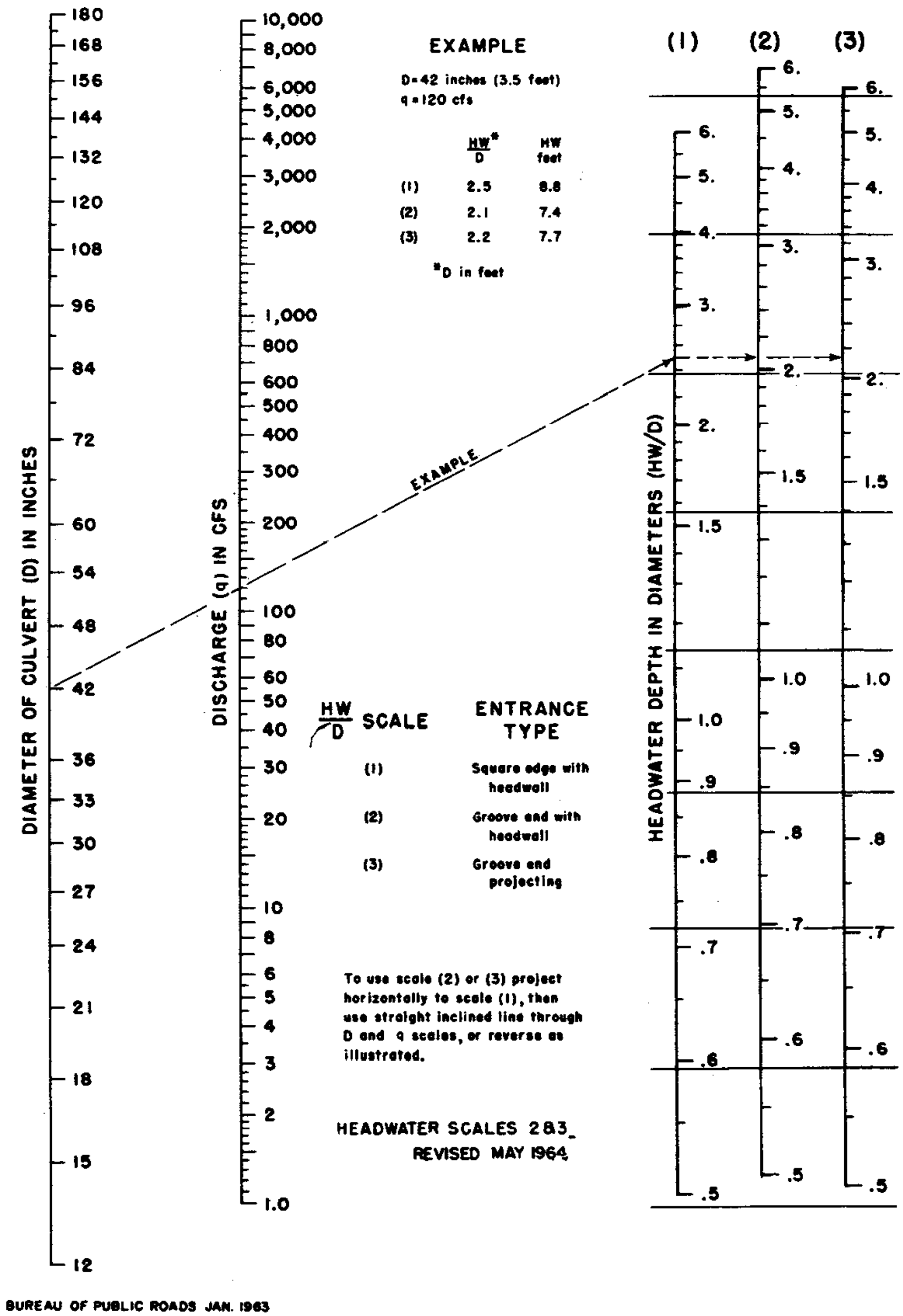


Exhibit 14-7. Headwater depth for concrete pipe culverts with inlet control.

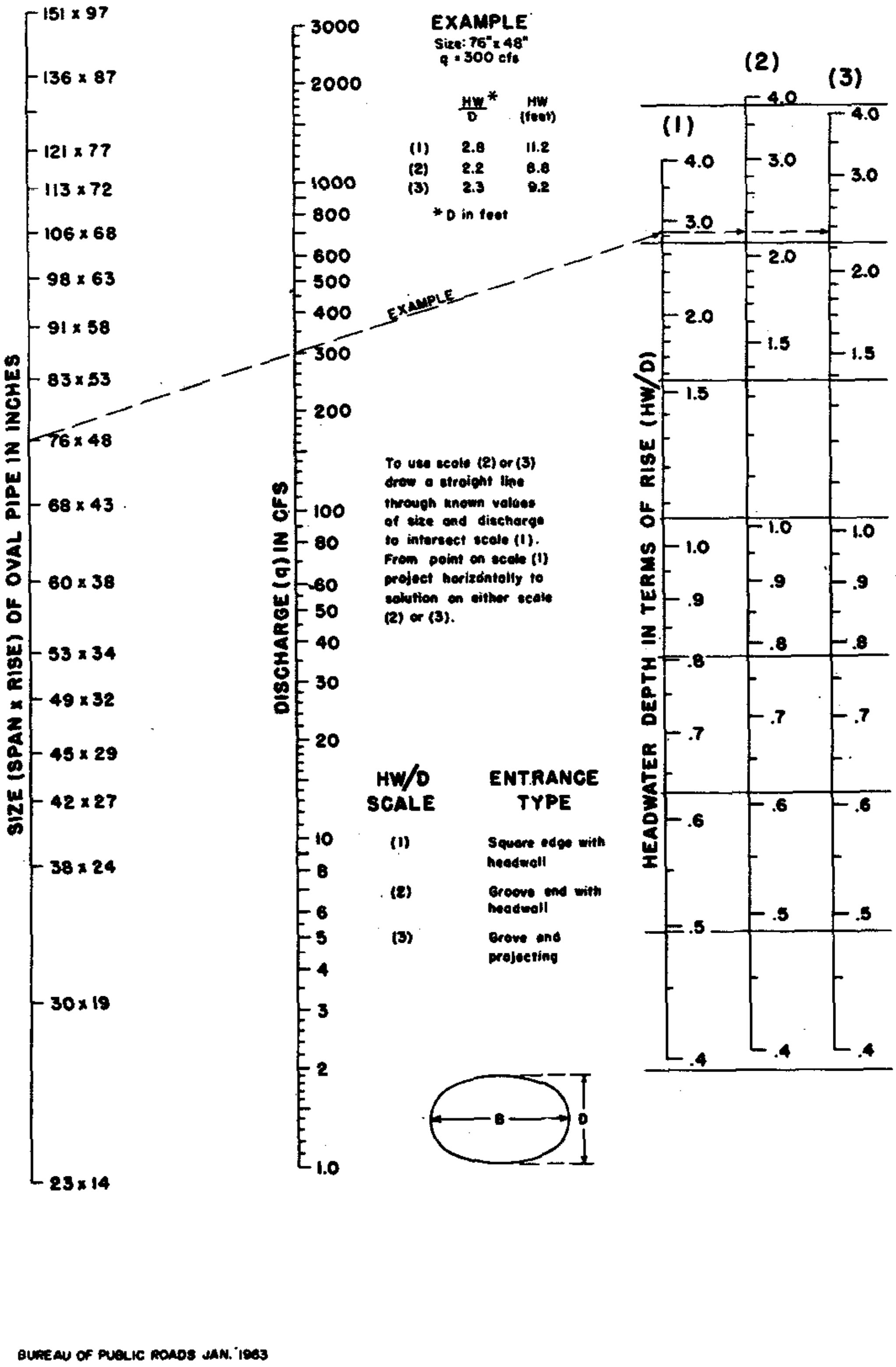


Exhibit 14-8. Headwater depth for oval concrete pipe culverts long axis horizontal with inlet control.

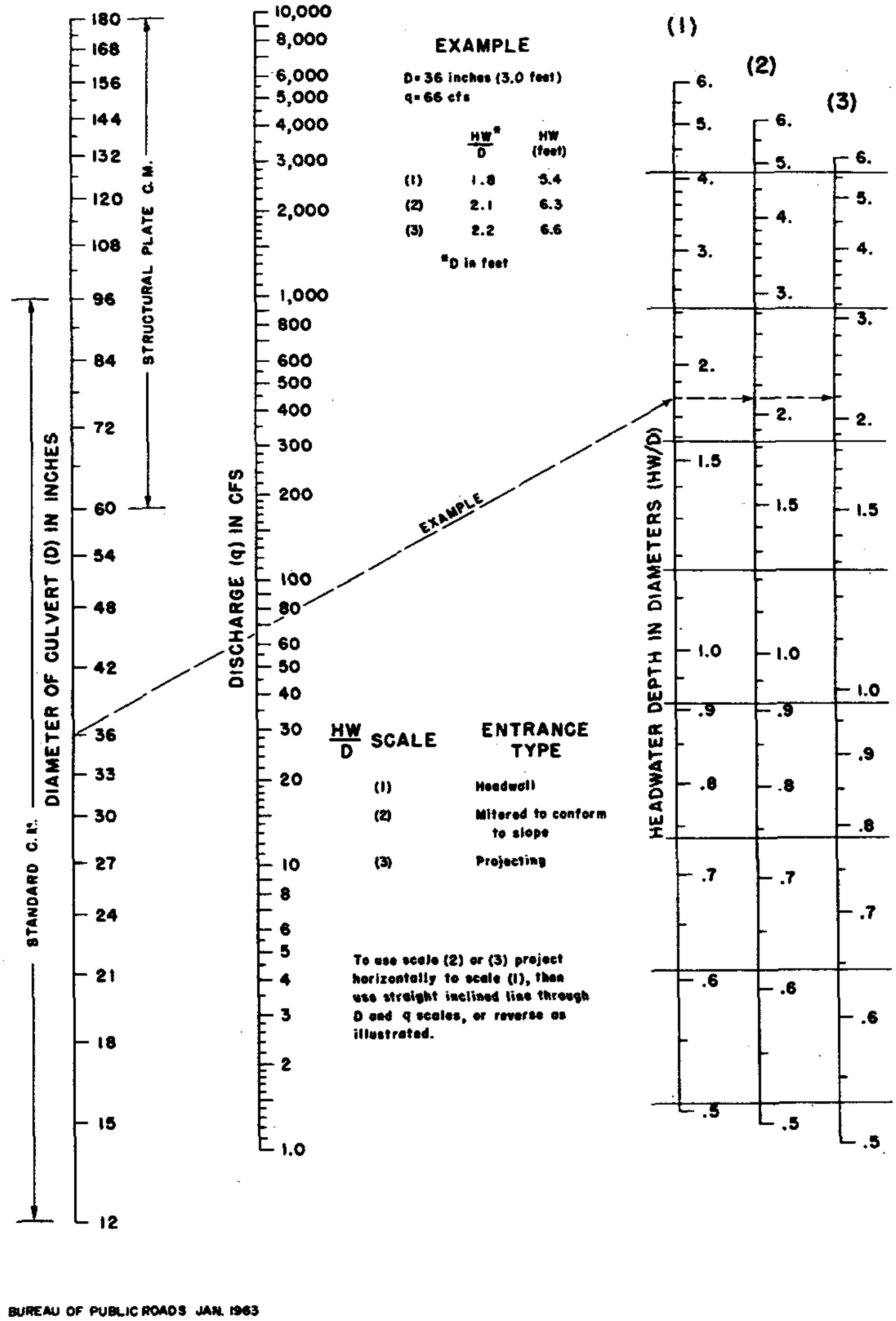


Exhibit 14-9. Headwater depth for C. M. pipe culverts with inlet control.

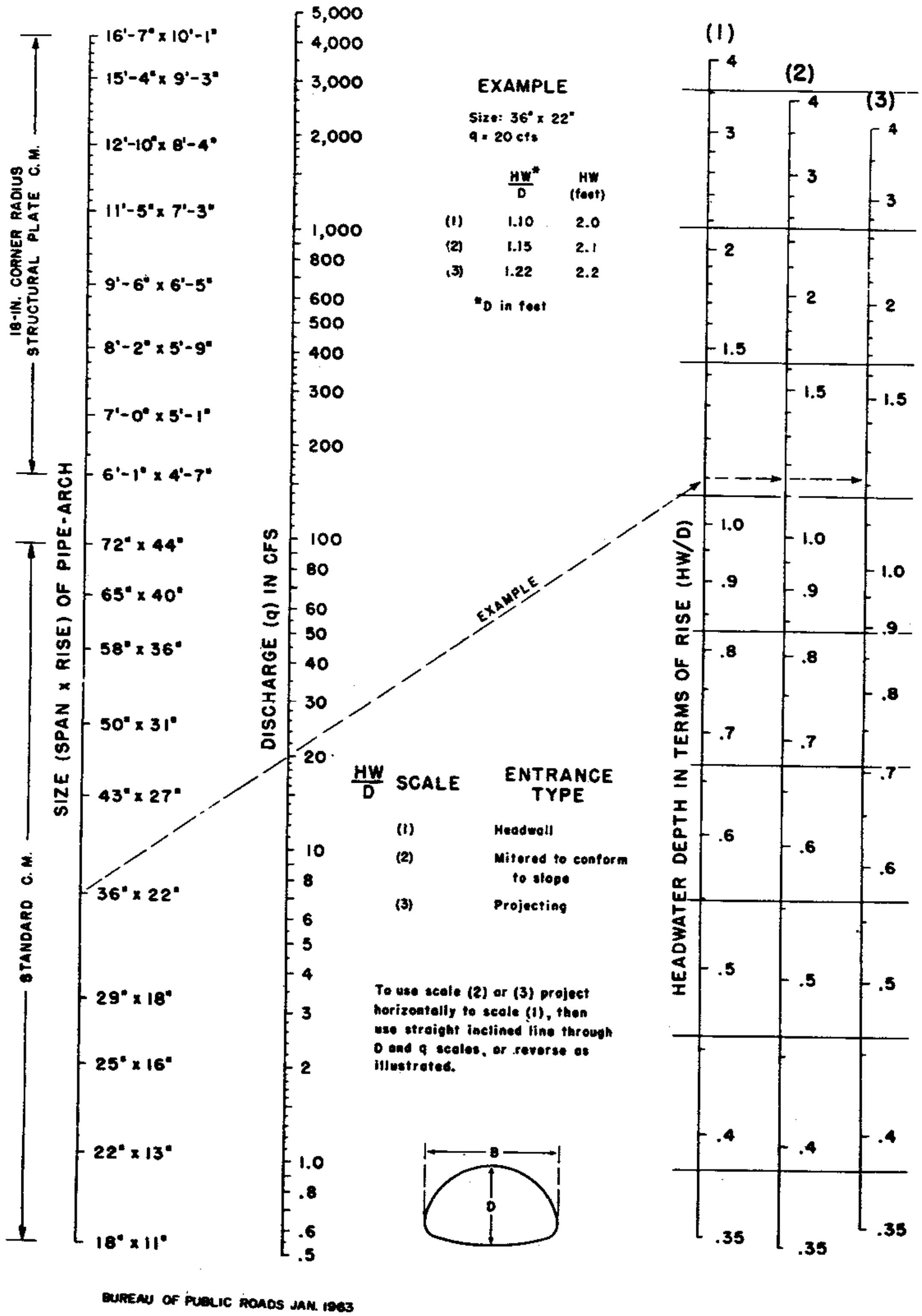


Exhibit 14-10. Headwater depth for C.M. pipe-arch culverts with inlet control.

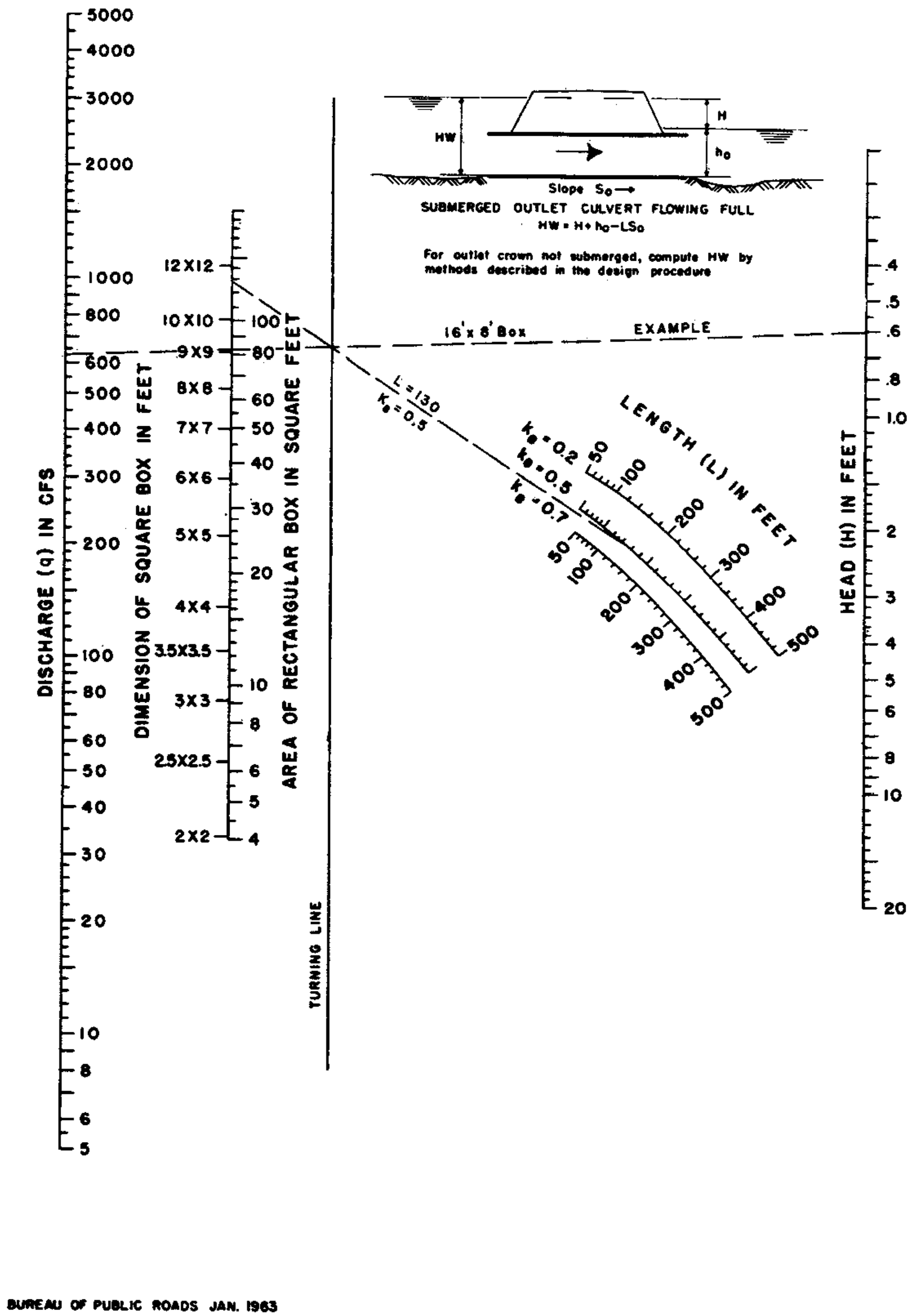


Exhibit 14-11. Head for concrete box culverts flowing full n = 0.012.

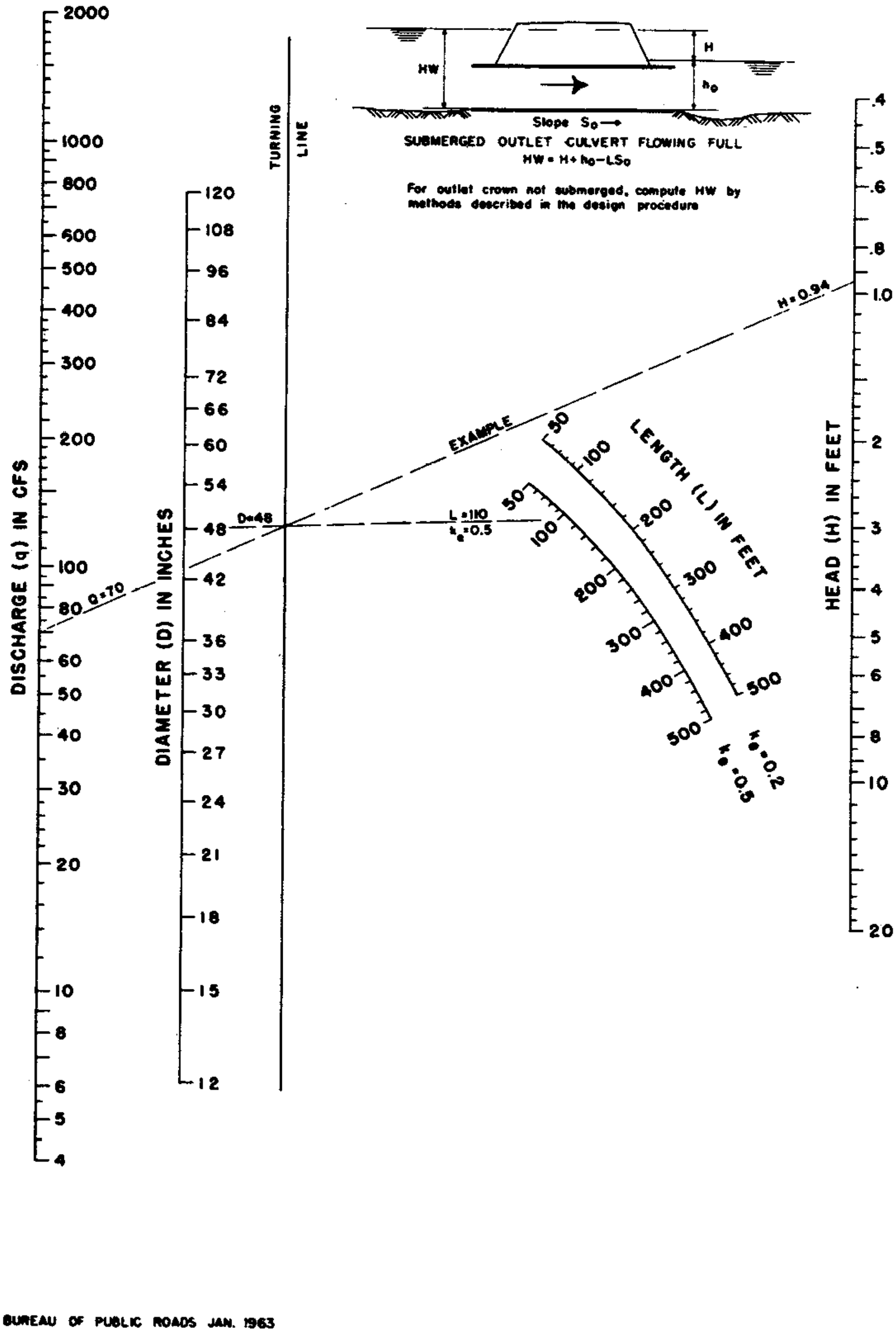


Exhibit 14-12. Head for concrete pipe culverts flowing full n = 0.012.

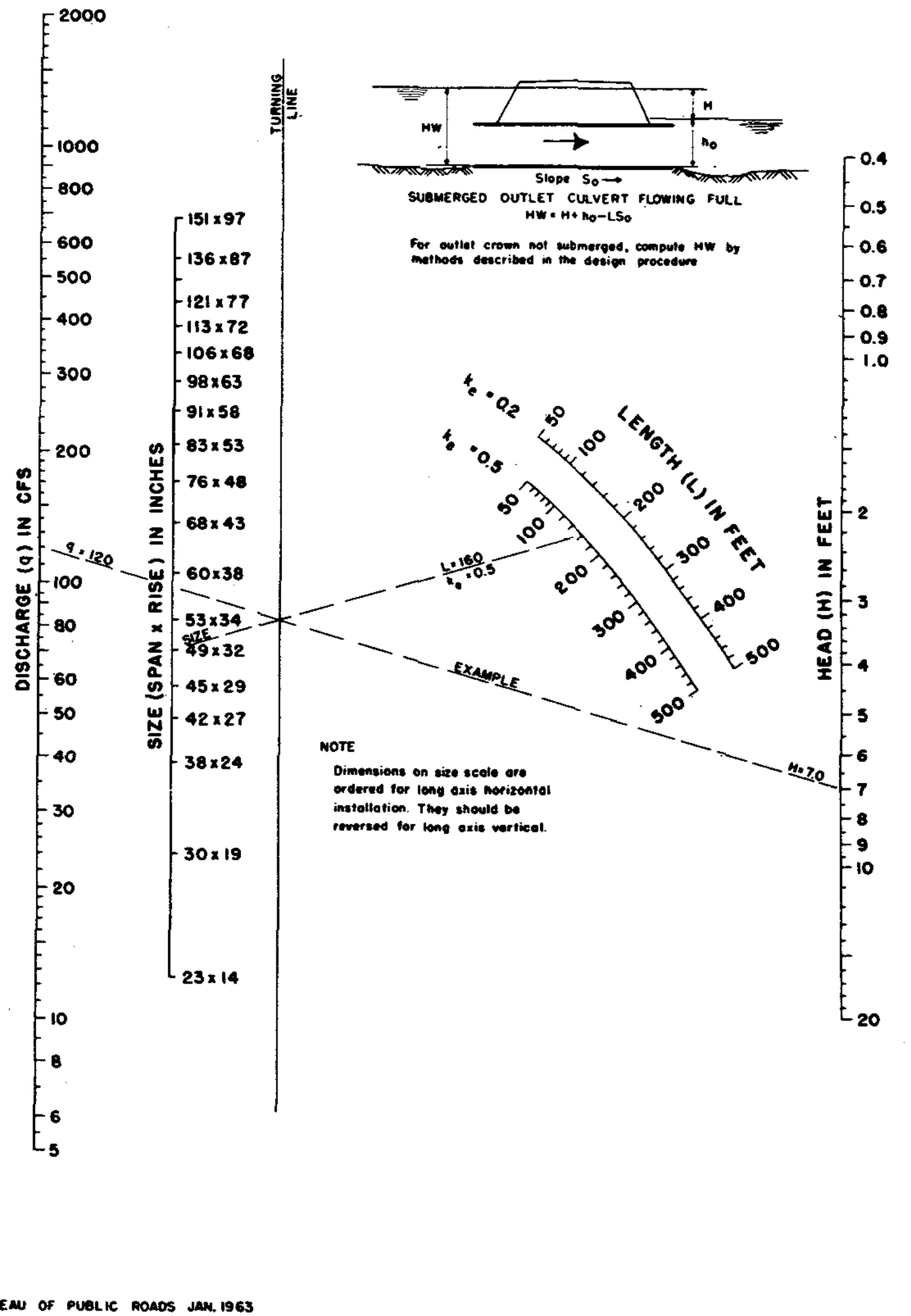


Exhibit 14-13. Head for oval concrete pipe culverts long axis horizontal or vertical flowing full n = 0.012.

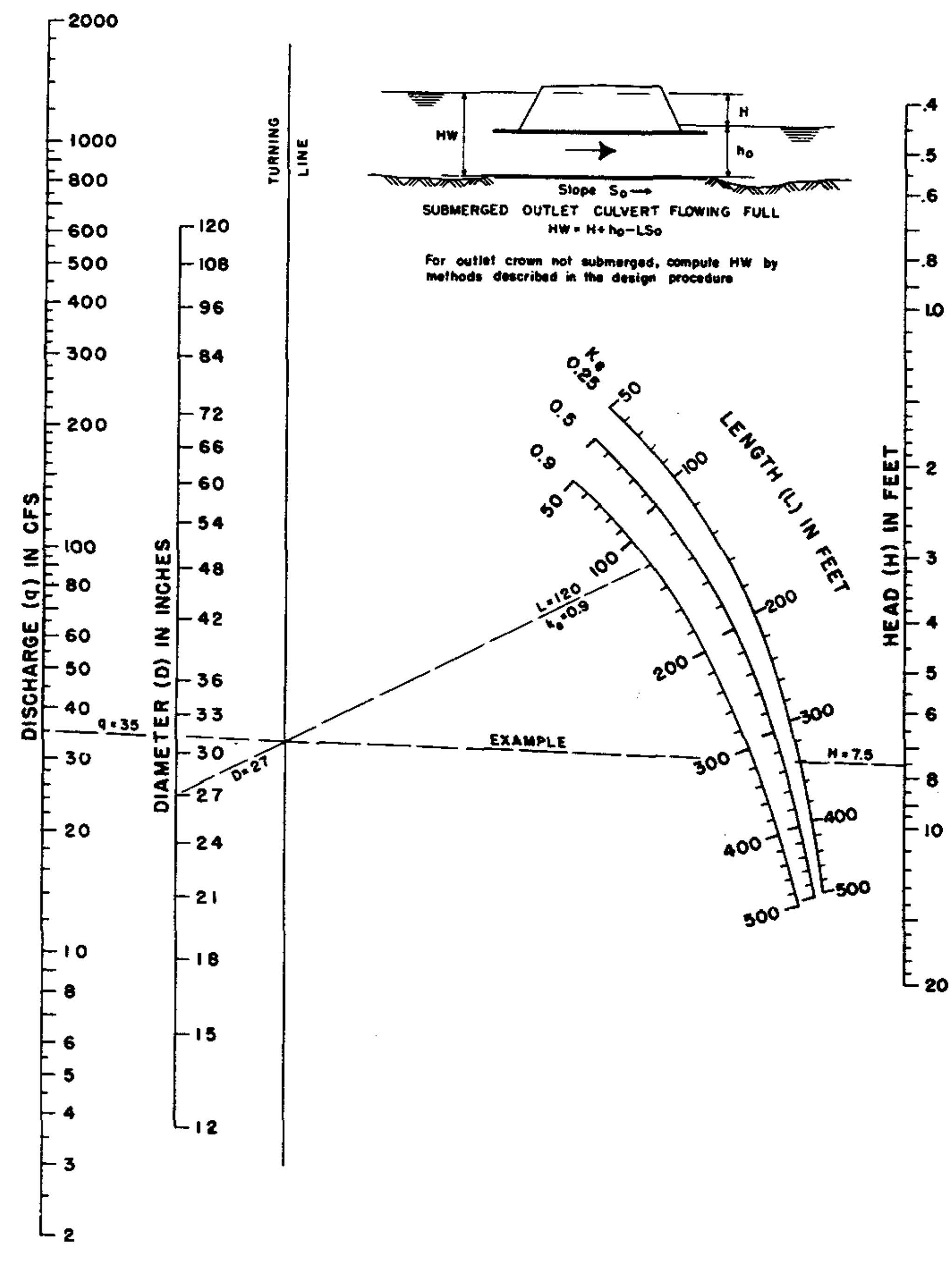


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Exhibit 14-14. Head for standard C. M. pipe culverts flowing full $n = 0.024$.

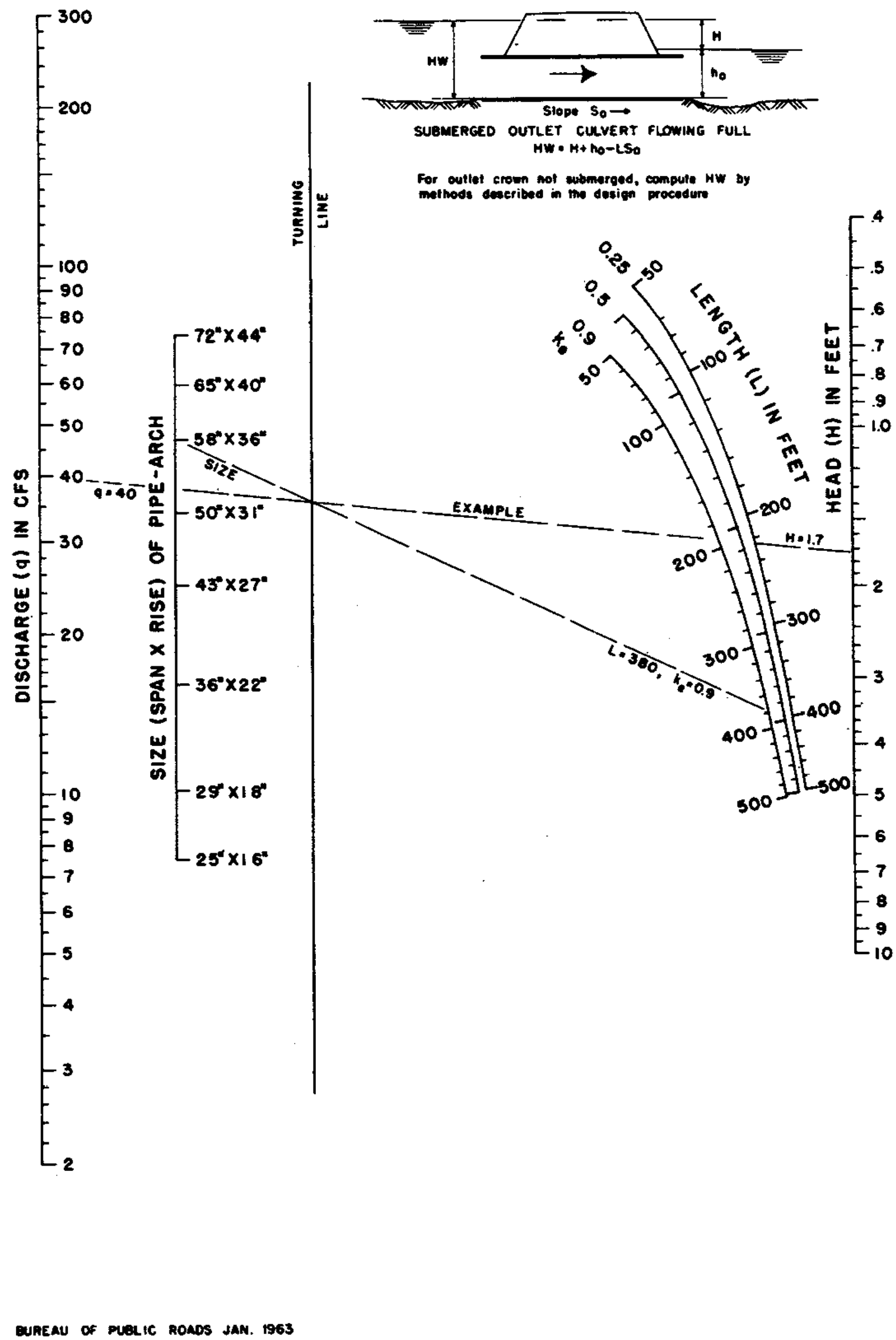
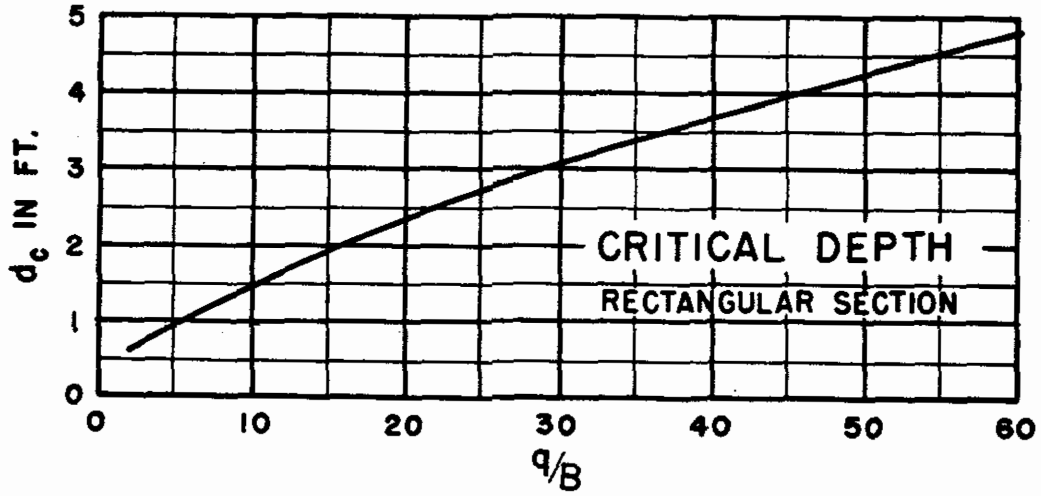


Exhibit 14-15. Head for standard C. M. pipe-arch culverts flowing full n = 0.024.



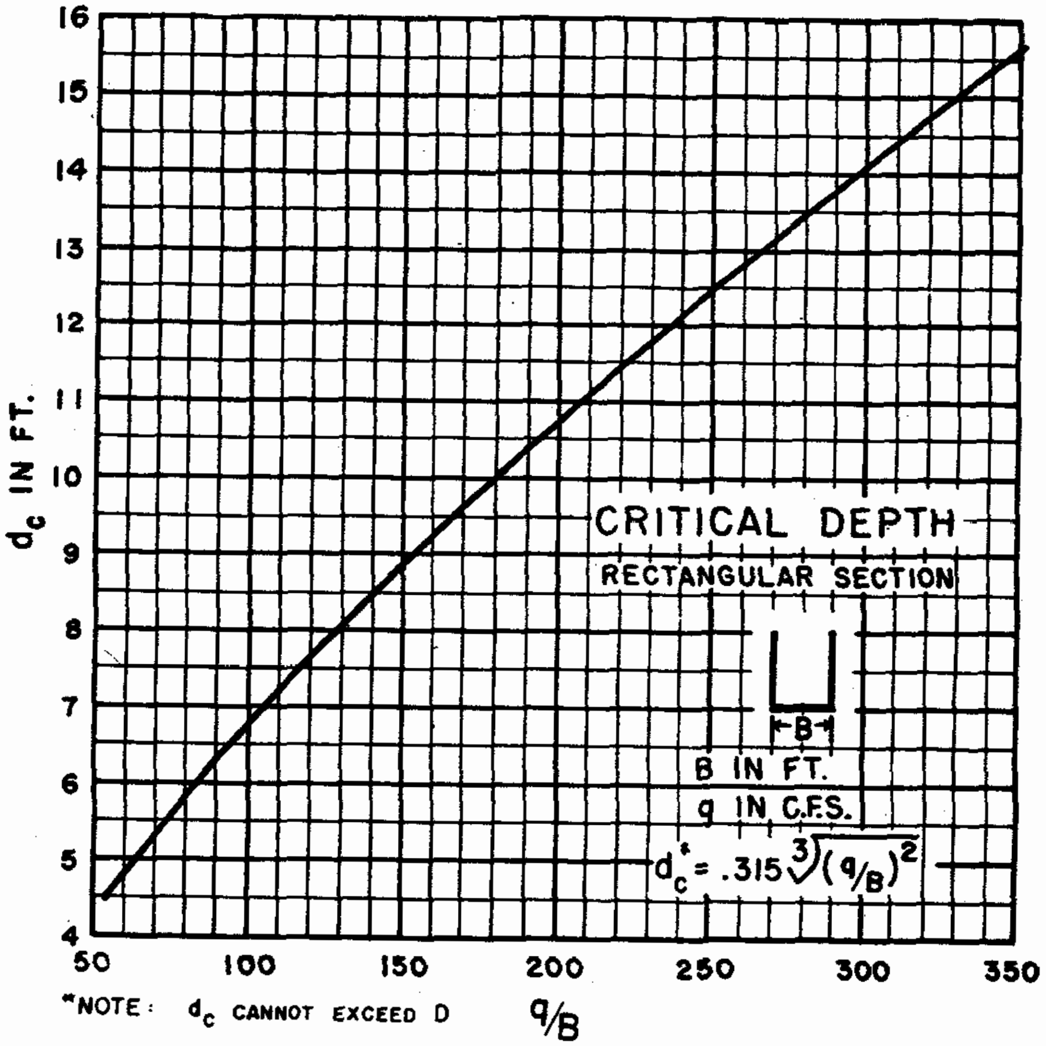


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Exhibit 14-16. Critical depths-rectangular section.

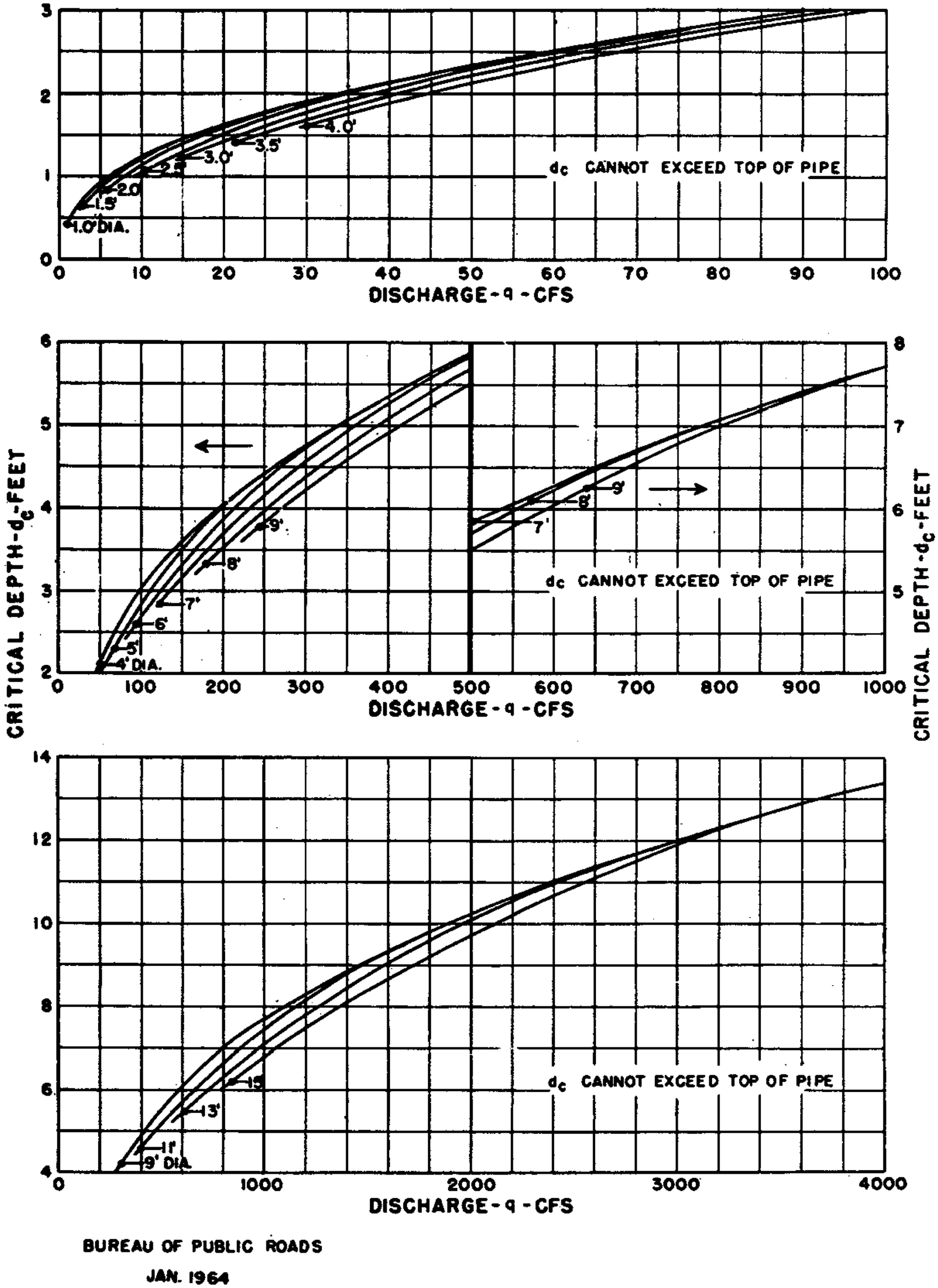


Exhibit 14-17. Critical depth. Circular pipe

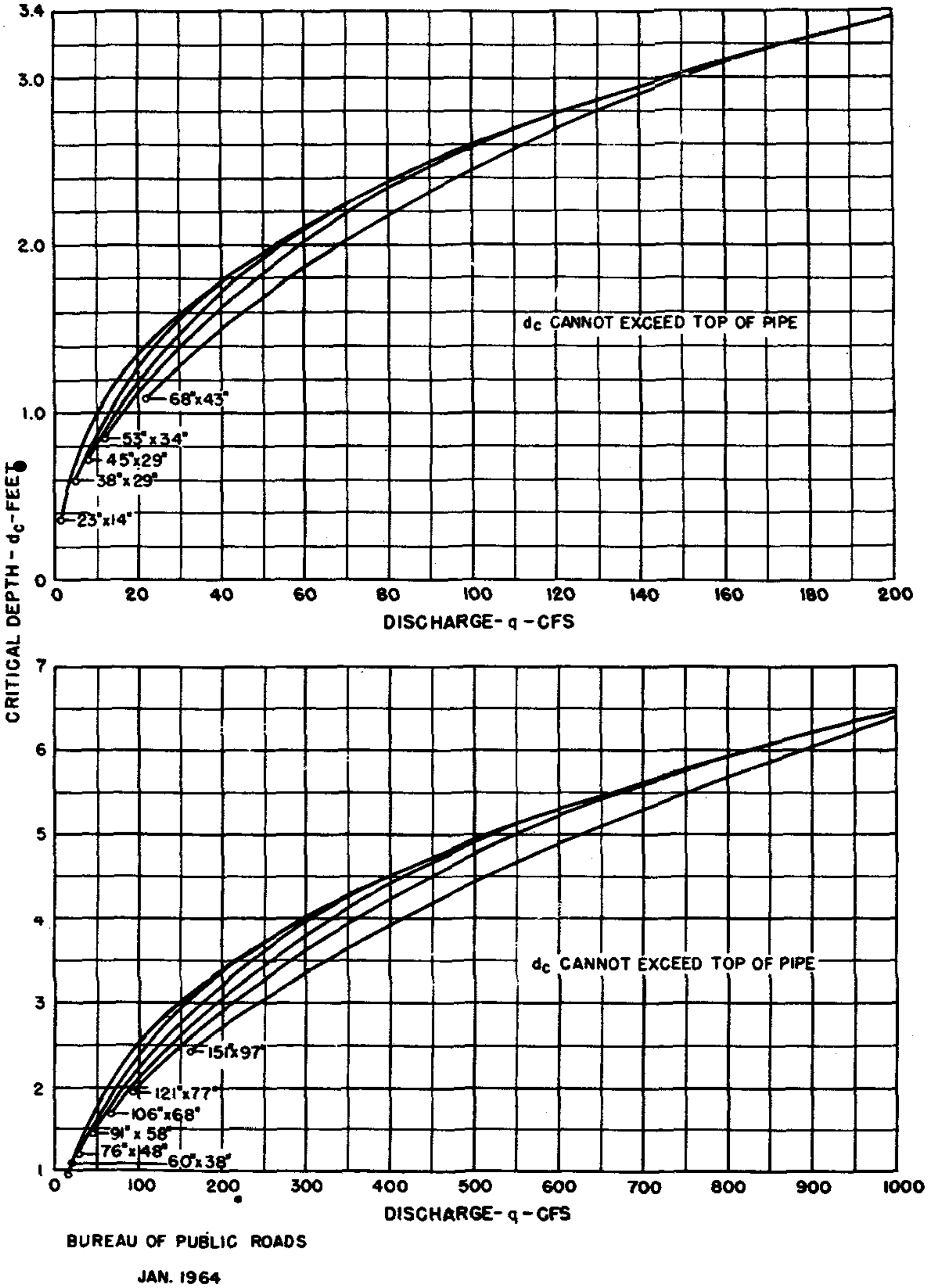
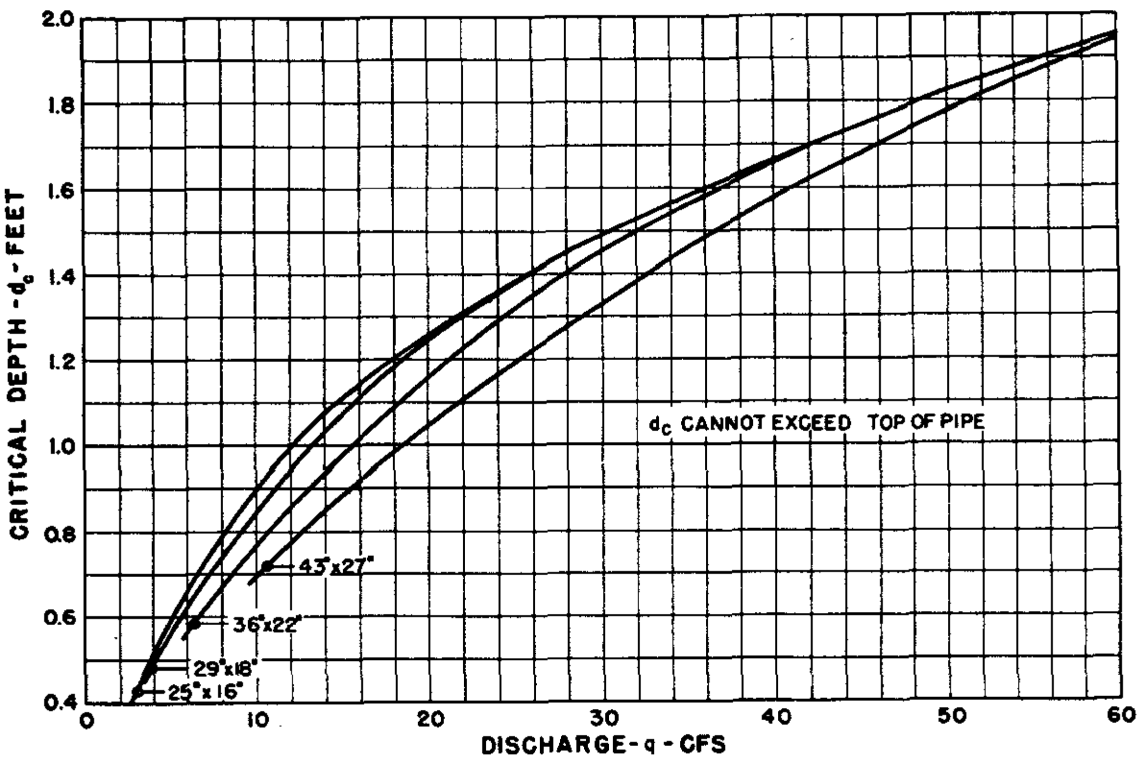


Exhibit 14-18. Critical depth. Oval concrete pipe. Long axis horizontal.



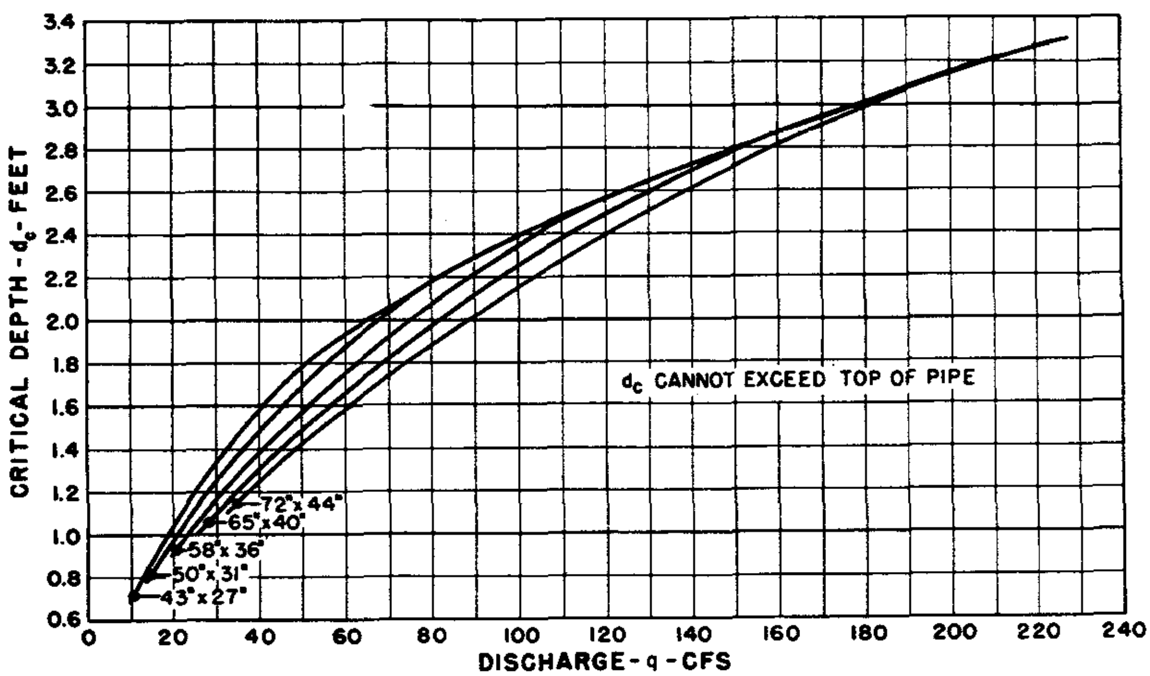


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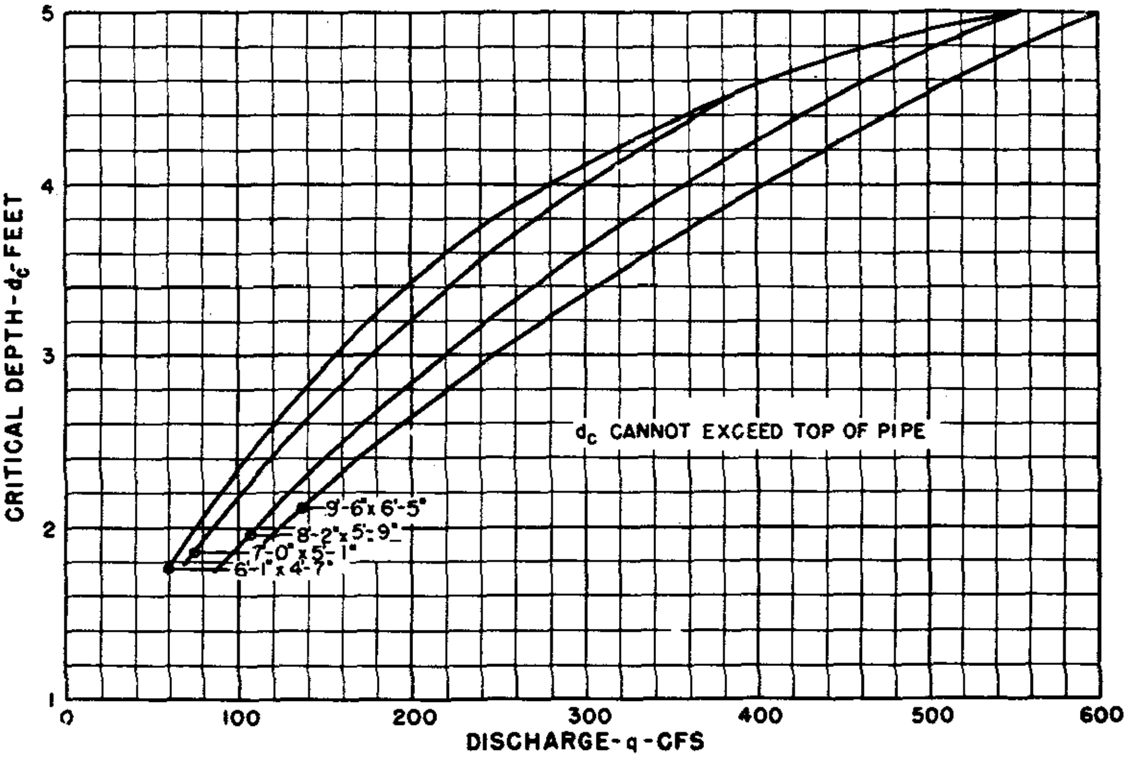
Exhibit 14-19. Critical depth. Standard C.M. pipe-arch.



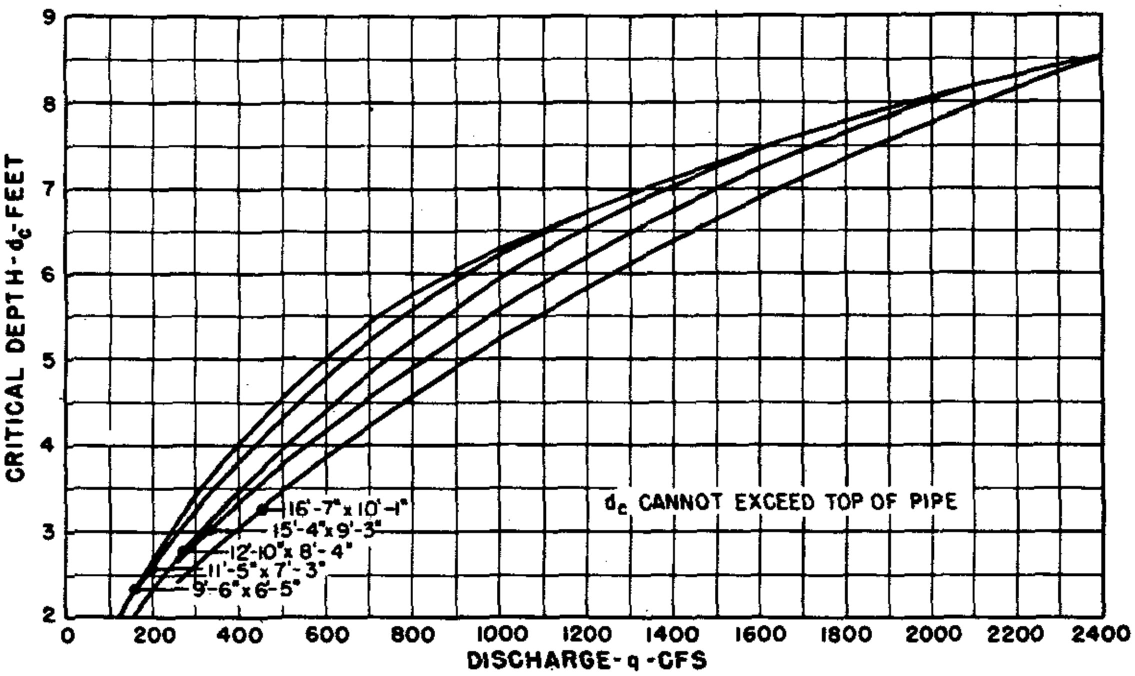


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Exhibit 14-20. Critical depth. Structural plate. C.M. pipe-arch.

Exhibit 14-21. Entrance loss coefficients.

Coefficient k_e to apply to velocity head $\frac{V^2}{2g}$ for determination of head loss at entrance to a structure, such as a culvert or conduit, operating full or partly full with control at the outlet.

$$\text{Entrance head loss } H_e = k_e \frac{V^2}{2g}$$

Type of Structure and Design of Entrance	Coefficient k_e
Pipe, Concrete	
Projecting from fill, socket end (groove-end)	0.2
Projecting from fill, sq. cut end	0.5
Headwall or headwall and wingwalls	
Socket end of pipe (groove-end)	0.2
Square-edge	0.5
Rounded (radius = 1/12D)	0.2
Mitered to conform to fill slope	0.7
End-Section conforming to fill slope	0.5
Pipe, or Pipe-Arch, Corrugated Metal	
Projecting from fill (no headwall)	0.9
Headwall or headwall and wingwalls	
Square-edge	0.5
Mitered to conform to fill slope	0.7
End-Section conforming to fill slope	0.5
Box, Reinforced Concrete	
Headwall parallel to embankment (no wingwalls)	
Square-edged on 3 edges	0.5
Rounded on 3 edges to radius of 1/12 barrel dimension	0.2
Wingwalls at 30° to 75° to barrel	
Square-edged at crown	0.4
Crown edge rounded to radius of 1/12 barrel dimension	0.2
Wingwalls at 10° to 25° to barrel	
Square-edged at crown	0.5
Wingwalls parallel (extension of sides)	
Square-edged at crown	0.7