Relating decision under uncertainty and multicriteria decision making models*

D. Dubois*, M. Grabisch**,† F. Modave*** and H.Prade*
* IRIT-CNRS Université P.Sabatier, 118 route de Narbonne,
31062 Toulouse Cedex, France
email: {dubois , prade }@irit.fr
**LIP6, Université de Paris VI, 4 Place Jussieu, 75252 Paris, France
email: Michel.Grabisch@lip6.fr
***KVO Laboratory, School of Information Technology
Griffith University, Gold Coast Campus, Australia
email f.modave@gu.edu.au

Abstract

This short overview paper points out the striking similarity between decision under uncertainty and multicriteria decision making problems, two areas which have been developed in almost completely independent ways until now. This comparison pertains both to additive and non-additive (including qualitative) approaches existing for the two decision paradigms. It leads to emphasize the remarkable formal equivalence between postulates underlying these approaches (like between the "sure-thing principle" and mutual preferential independence of criteria). This analogy is exploited by surveying classical results as well as very recent advances. This unified view should be fruitful for a better understanding of the postulates underlying the approaches, for cross-fertilization, and for adapting AI uncertainty representation frameworks to preference modelling.

1 Introduction

For a long time, Artificial Intelligence had not been much concerned by decision issues. However, many reasoning tasks are more or less oriented towards decision or involve decision steps. During the last five years, decision under uncertainty has become a topic of interest in AI. The application of classical expected utility theory to planning under uncertainty and the algorithmic issues raised by its implementation have been specially investigated, as well as a search for more qualitative models [3],[4] , or the use of game theoretic models [29].

Besides, AI brings also new, more qualitative, frameworks for describing the incomplete and uncertain knowledge about the world, via nonmonotonic logics for instance, or for representing the decision maker's preference. It should be noted that uncertainty and preferences are not then directly assessed by means of probability or utility functions, but are rather expressed through collections of pieces of information which implicitly constrain such functions.

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†On leave from Thomson-CSF, Central Research Laboratory, Domaine de Corbeville, 91404 Orsay Cedex, France

Interestingly enough, AI research has not much considered multicriteria decision making (MCDM) until now. One reason may be that the two topics, decision under uncertainty and MCDM, have been studied by two different schools, in completely independent ways. However, we may observe that the two standard models of these two paradigms, namely the expected utility and the weighted average of the degrees of satisfaction of the criteria respectively, which are used in practice for ranking the possible decisions, are formally very similar. It suggests some analogy between the two types of problems. It raises the question of the similarity of the postulates which lead to these additive models, as well as the one of the parallel between non-standard, i.e non-additive models which have been developed more recently for both decision under uncertainty and MCDM.

Besides, the multicriteria decision problem is usually viewed in these models as the joint satisfaction of the set of criteria, with or without compensation between the levels of satisfaction, taking into account the levels of importance of the criteria. The aggregation has thus a "conjunctive" flavor, even if the conjunction is not really of a logical type when compensation is allowed. However, the subjective evaluation, of a possible decision, of an object, according to several criteria, is often in practice, not as simple as such a conjunctive attitude. One criterion, or a set of criteria may have a veto effect if they are not all satisfied, or on the contrary, they may not be enough for favoring a high evaluation (whatever the level of satisfaction of the other criteria is) in case it or they are satisfied to a sufficient level. The description of an evaluation procedure may also involve conditional statements, such as, for instance, "if the criterion is satisfied, then it is better to have this other criterion also satisfied". This procedure exhibits some form of non monotonicity if, for instance, when a criterion cannot be satisfied, the alternatives are evaluated according to some other criterion not considered in the general situation where the first criterion is satisfied. The modelling of such aggregation attitudes may then benefit by taking advantage of models developed in Artificial Intelligence.

The paper is organized as follows. Section 2 makes a parallel between decision under uncertainty and multicriteria decision and introduces some notations. Section 3 presents theoretical results for each of the two decision problems, some classical ones pertaining to additive settings, and some newer ones in non-additive or qualitative frameworks. Beyond the striking similarity of the evaluations in decision under uncertainty and MCDM, some similarities between the underlying postulates are pointed out and open questions are raised. Section 4 briefly pursues the parallel between decision under uncertainty and MCDM in the logical setting.

2 A parallel between the two decision paradigms

Let us first consider the decision under uncertainty problem. Let S be a set of possible states of the world, and Ω be a set of possible consequences ω. An act, or a decision d is then viewed, following [Savage 72] as a mapping from S to Ω, which leads to a consequence $\omega = d(s)$ when performed in state s. Starting from postulates that a relation of preference in Ω^S, the set of potential acts, should fulfil, Savage has shown, when S is a continuum, that choosing between acts amounts to choose the one(s) maximizing an expected utility of the form (in a finite setting $S = \{s_1, \ldots, s_n\}$):

$$U(d) = \sum_i p(s_i) u(d(s_i)) \quad (1)$$

where p is a probability distribution over S and u a real-valued utility function over Ω. More recently, other types of integrals have been justified (under other postulates) for ranking the acts (see section 3) using a purely ordinal setting, (where scales for uncertainty and preferences are sets of linearly ordered levels), the following estimate has been proposed for ranking acts

$$U_\pi(d) = \min_i [u(d(s_i)) \vee (1 - \pi(s_i))] \tag{2}$$

where the utility function u and the possibility distribution π have been mapped to the same ordinal scale (commensurability assumption of the preference and of the uncertainty scales). $1 - (\cdot)$ is just here to denote the order-reversing map on the scale. Expression (2) has been justified from an axiomatic point of view [14], in a way which parallels Von Neumann and Morgenstern justification of expected utility, in terms of preference relations between lotteries, i.e. uncertain consequences. Then the following counterpart of (2) is obtained

$$U'_\pi(d) = \min_\omega [u(\omega) \vee (1 - \pi_d(\omega))] \tag{3}$$

where $\pi_d(\omega)$ estimates to what extent it is possible that decision d leads to consequence ω. Clearly (2) or (3) favors decisions for which there does not exist a state which is both highly possible and leads to poor consequences.

Let us turn now towards multiple criteria decision. Let $C_1, \ldots, C_n$ be a finite set of criteria. We furthermore assume that each decision d can be extended, according to C_i, by means of a function u_i from the set D of decisions to the same evaluation scale. Then, examples of multicriteria evaluations are

$$\mathcal{E}(d) = \sum_i \alpha_i u_i(d), \text{ with } \sum_i \alpha_i = 1 \tag{4}$$

$$\mathcal{E}(d) = \min_i [(1 - \alpha_i) \vee u_i(d)], \text{ with } \vee_i \, \alpha_i = 1 \tag{5}$$

where (4) is the classical weighted average aggregation, while (5) is a weighted conjunction (if the level of importance α_i is 0, the bottom element of the scale of C_i is not taken into account, while if for all i, $\alpha_i = 1$, the top element in the scale, we recover the logical, conjunctive, min aggregation). See Section 3 for a justification of (4) and (5). Note that (5) can be viewed as the degree of inclusion of the fuzzy set of important criteria into the set of more or less satisfied ones.

The resemblance between (1) and (4), and between (2) and (5) is striking. *Each criterion* in (4) or (5) *corresponds to a state* in (1) or (2), and the levels of importance α_i are analogues of the probability (or possibility) distributions over S. Dubois and Prade [13] have pointed out that (4) and (5) can be viewed respectively as the probability and the necessity of a fuzzy event made by the set of criteria satisfied by d, viewing the α_i's as a (probability or possibility) distribution. Then, α_i can be viewed as the level of probability (or possibility) to be in a state s_i where the criteria C_i is used to evaluate act d. A multiple criteria evaluation problem is then equated to the problem of satisfying an uncertain criterion. It leads to interpret $\mathcal{E}(d)$ in (4) as the expected utility to satisfy the "right" criteria, to read (5) as the uncertainty of satisfying the "right" criteria. In the latter case, it amounts to prefering the acts d which satisfy all the criteria which have a high possibility to be the right one (i.e. all the important/highly possible criteria, according to the chosen interpretation, should be satisfied as much as possible). The next section investigates the correspondence between multicriteria decision and decision under uncertainty problems, in a broader and more theoretical way.

3 Theoretical issues

3.1 Non-additive measures

It has been known for a long time that additive set functions (e.g probability measures) were not well-adapted to represent all the facets of human behavior. We will see further that the additivity of a set function representing a preference relation is linked to the so-called sure-thing principle. In Ellsberg paradox, this principle is violated, leading some subjects not to maximize the expected utility [16], [5]. Therefore, we are naturally led to introduce non-additive set functions when dealing with uncertainty. We will also see the advantage of using non-additive set functions in multicriteria decision making.
In this section Ω denotes a finite set and $\mathcal{P}(\Omega)$ the set of subsets of Ω.

Definition 1 *A non-additive measure (also called fuzzy measure) on* $(\Omega, \mathcal{P}(\Omega))$ *is a set function* $\mu : \mathcal{P}(\Omega)) \to [0, +\infty]$ *such that*
$\mu(\emptyset) = 0$ *and*
if $A, B \subset \Omega$, $A \subset B$, *then* $\mu(A) \leq \mu(B)$, *that is,* μ *is a non decreasing set function.*

Note that this definition encompasses the notions of probability measures, possibility and necessity measures, the belief functions of Shafer [27] that were already known and used in the AI community. Let us now give the definitions of the main integrals of non-additive measure theory. See [6] and [30] for the original articles.

Definition 2 *Let* μ *be a non-additive measure on* $(\Omega, \mathcal{P}(\Omega))$ *and an application* $f : \Omega \to [0, +\infty]$. *The Choquet integral of* f *w.r.t* μ *is defined by:*
$(C) \int_B f d\mu = \int_0^{+\infty} \mu(\{x : f(x) > t\}) dt$
which reduces to
$(C) \int_B f d\mu = \sum_{i=1}^{n} (f(x_{(i)}) - f(x_{(i-1)}))\mu(A_{(i)})$ *with* $\Omega = \{x_1, \cdots, x_n\}$
The subscript $(.)$ *indicates that the indices have been permuted in order to have* $f(x_{(1)}) \leq \cdots \leq f(x_{(n)})$, $A_{(i)} = \{x_{(1)}, \cdots, x_{(n)}\}$ *and* $f(x_{(0)}) = 0$ *by convention.*

Definition 3 *Let* μ *be a non-additive measure on* $(\Omega, \mathcal{P}(\Omega))$ *and an application* $f : \Omega \to [0, +\infty]$. *The Sugeno integral of* f *w.r.t* μ *is defined by:*
$(S) \int_B f \circ \mu = sup_{\alpha \geq 0} \{\alpha \wedge \mu(\{f \geq \alpha\})\}$
which reduces to
$(S) \int_B f \circ \mu = \bigvee_{i=1}^{n} (f(x_{(i)}) \wedge \mu(A_{(i)}))$
and where $\vee$ *is the supremum,* $\wedge$ *is the infimum and with the same notations and conventions as above.*

From a formal point of view, the Choquet integral and the Sugeno integral differ only by the operators used in their definition; respectively $+$, $\times$ and $\vee$, $\wedge$. Nevertheless, they are very different in essence, since the Choquet integral is more adapted to numerical problems and the Sugeno integral is better suited for qualitative problems.

3.2 Additive measures in decision theory

We briefly recall the main results in decision theory when the preference relation is represented in an additive way.
Decision under uncertainty: The first result is due to Savage [25]. He has shown that under a set of seven conditions on the preference relation $\succeq$ on the acts, there exist a unique probability measure P on the (infinite) set S of the states of the world and a unique utility function $u : \Omega \to \mathbb{R}$ that represent the preference relation in the sense that: $f \succeq g \Leftrightarrow \int_S u(f) dP \geq \int_S u(g) dP$. Since we are mostly interested in the additivity property, we present only the second condition of the seven that is responsible for the additivity of P.

Definition 4 *We say that $\succeq$ verifies the independence with respect to equal subalternatives iff for every acts f, g, f', g' such that, on $B \subset S$ we have $f = f'$ and $g = g'$ and on B^c we have $f = g$ and $f' = g'$ then $g \succeq f$ implies $g' \succeq f'$.*

This property is known as the sure-thing principle. Anscombe and Aumann [1] have proposed a less general but simpler result where the set of consequences considered is only the set of money lotteries.

Definition 5 *A preference relation $\succeq$ is said to be independent iff for all acts f, g, h, and every real $\alpha \in (0,1)$, $f \succ g \Rightarrow \alpha f + (1-\alpha)h \succ \alpha g + (1-\alpha)h$.*

Then, there exist a unique probability measure P on S and a unique utility function u that represent $\succeq$ iff, $\succeq$ is an independent weak order that verifies some continuity and monotonicity properties.

Multicriteria decision making: A very important notion in multicriteria decision making is that of preferential independence. In fact, it is the equivalent in (MCDM) of the sure-thing principle.

In the sequel, we will use the following notations. $X_J = \times_{j \in J} X_j$ and x_J denotes an element of X_J.

Definition 6 *If $(X = X_1 \times \cdots \times X_n, \succeq)$ is a multicriteria decision making problem where $I = \{1, \cdots, n\}$ and if $J \subset I$ then J is said to be preferentially independent of J^c iff for every $x_J, y_J \in X_J$, for every $x_{J^c}, y_{J^c} \in X_{J^c}$ we have $(x_J, x_{J^c}) \succeq (y_J, x_{J^c}) \Leftrightarrow (x_J, y_{J^c}) \succeq (y_J, y_{J^c})$ that is, the preference on the attributes of J is not influenced by the other attributes. If every $J \subset I$ is preferentially independent, the attributes are said to be mutually preferentially independent.*

If $(X, \succeq)$ is a multicriteria decision making problem and if it has a utility function $u : X \to \mathbb{R}$ that is represented by a Choquet integral w.r.t a non-additive measure μ i.e $u(x) = (C)\int u(x) d\mu$, Murofushi [24] has shown the following equivalence.

If there are at least three attributes for which the preference relation is non-trivial (we say that the attributes are essential), the fuzzy measure μ is additive iff the attributes are mutually preferentially independent.

In fact, Debreu [7] had given a first result concerning additive representations of preference relations in multicriteria decision making before Murofushi that was not restricted to a Choquet integral representation but had the drawback of having topological assumptions difficult to verify in practice.

3.3 Non-additive measure in decision making

We now come to the non-additive refinement of the previous results and see how non-additive measures can avoid the problems encountered in the additive case like Ellsberg or Allais paradoxes. Using the previous result of Savage, we know that representing the preference relation in a non-additive way requires a weakening of the independence hypothesis, that is, the sure-thing principle. In the additive case, even in the less general approach of Anscombe and Aumann, the independence hypothesis is not always verified. Schmeidler [26] has proposed a non-additive version of Anscombe-Aumann theorem. With the same continuity and monotony hypotheses, the result holds if independence is replaced by co-monotonic independence that is independence restricted to co-monotonic acts (acts inducing the same ordering on the states of the world), and probability measure is replaced by non-additive measure. This result is the starting point of a non-additive expected utility theory that gives the opportunity to have a better representation and a better understanding of the processes involved in decision problem. It has been refined by Wakker [32], who has weakened the independence hypothesis of co-monotonicity to what he called max-min independence.

Definition 7 *Acts f and g are said to be max-min related iff either of the following conditions are verified:*

$$\forall s_i \in S, (\forall s_j \in S, f(s_i) \succeq f(s_j)) \textit{ or } (\forall s_j \in S, g(s_i) \preceq g(s_j))$$
$$\forall s_i \in S, (\forall s_j \in S, g(s_i) \succeq g(s_j)) \textit{ or } (\forall s_j \in S, f(s_i) \preceq f(s_j))$$

Definition 8 *A preference relation $\succeq$ is said to satisfy max-min independence iff for all max-min related f, g, h and $\alpha \in (0,1)$, $f \succ g \Rightarrow \alpha f + (1-\alpha)h \succ \alpha g + (1-\alpha)h$.*

Then there exists a unique non-additive measure and a unique utility function (up to a positive affine transformation) representing $\succeq$, iff $\succeq$ verifies the same continuity and monotony hypotheses as above and $\succeq$ is max-min independent.

3.4 Non-additive measure in multicriteria decision

Non-additive measures and integrals have been used as well in multicriteria decision making (MCDM) [17], giving much more flexibility than additive approaches. Indeed examples where additive models are not sufficient for representation purposes are easily found. For instance, take two criteria and three candidates A,B,C, among whom we want to express preferences; suppose A,B,C have the following respective scores on the two criteria $(5, 15)$, $(15, 5)$, $(11, 11)$. Then there is no way to find weights α and $1-\alpha$ for the two criteria in such a way that $\alpha \times 11 + (1-\alpha) \times 11 = 11$ be both ranked before $5 \times \alpha + 15 \times (1-\alpha)$ and $15 \times \alpha + 5 \times (1-\alpha)$. However if we require that C is both better than A and B, this can be represented by a Choquet integral.

Unfortunately, there is no corresponding theorem to the one of Murofushi, giving a characterization of non-additive measures, or even particular cases such as belief functions, and the Choquet integral in MCDM. However, using the parallel between decision under uncertainty and MCDM that we are advocating in this paper, it has been recently possible to borrow a result from Wakker [31] in decision under uncertainty and to translate it in the MCDM framework. The result provides a very general condition under which a preference ordering between vectors of multiple criteria evaluations (on commensurate scales) can be represented by a Choquet integral, namely the preference ordering should be a continuous order, that does not reveal comonotonic contradictory tradeoffs; see [23, 21, 22] for details on this very technical condition.

Moreover, recent results and formalisms about non-additive measures have shed some light on their interpretation in MCDM [18, 20]. We develop this point below. Let us denote X a set of n criteria. By analogy with cooperative game theory, a non additive measure μ defined on X assigns to every *coalition* $A \subset X$ of criteria a number in $[0,1]$ giving the importance of the coalition A for the decision problem in consideration. A well-known concept in cooperative game theory is the one of Shapley value of a game [28], expressing the average importance of each player in the game. Murofushi has applied this concept to MCDM, obtaining a convenient and theoretically well founded notion of importance of criteria.

A second key concept here is the one of interaction. Although it was felt from a long time that non additive measures can model some kind of interaction between criteria, this was not formalized until Murofushi proposed a definition of an interaction index I_{ij} for a pair of criteria i,j, borrowing concepts from multiattribute utility theory:

$$I_{ij} = \sum_{K \subset X \setminus \{i,j\}} \xi(|K|)[\mu(K \cup \{i,j\}) - \mu(K \cup i) - \mu(K \cup j) + \mu(\{i,j\})] \quad (6)$$

with $\xi(k) = \frac{(n-k-2)!k!}{(n-1)!}$. Later, Grabisch extended this definition to any number of criteria, leading to what was called an *interaction representation* of non additive measures, encompassing the Shapley value and I_{ij} [8]. This representation through interaction indices happens to be much closer to the decision maker's mind than the usual measure representation.

It has been shown that if two criteria i, j have a positive interaction I_{ij}, then it means that they act in a *conjunctive* way, i.e. *both* of them have to be satisfied in order to have some impact on the decision (complementary criteria). By contrast, if I_{ij} is negative, criteria i, j act in a *disjunctive* way, which means that one of the two is sufficient (redundant or substitutive criteria). Another effect which can be modelled by interaction indices is the *veto* effect. A criterion i is said to be a veto if the utility function can be written as follows:

$$u(x_1, \ldots, x_n) = u_i(x_i) \wedge u'(x_1, \ldots, x_n)$$

It means that the utility u cannot exceed the one of criterion i. The dual effect, where $\wedge$ is replaced by $\vee$, is called *the favor* effect. If u is expressed under the form of a Choquet integral (as in the theorem of Murofushi above (...)), it is possible to model these two effects by a suitable choice of the non additive measure μ. It has been shown that non-additive measures whose corresponding I_{ij} are positive (resp. negative) for any j model a veto effect (resp. a favor effect) on criterion i.

These results, along with others related to the interaction representation, lay the foundations to a comprehensive use of non-additive measures in MCDM.

A question arises about the interpretation of such results in decision under uncertainty, namely: *What is the meaning of interaction in the framework of decision under uncertainty?* We already know that the Shapley value corresponds to the pignistic transformation of belief functions proposed by Smets. It remains to interpret the corresponding transformation for I_{ij} and higher order indices.

3.5 Qualitative decision

We now turn to decision making where only qualitative (ordinal) information is available. In decision making under uncertainty, a qualitative equivalent to the Von Neuman Morgenstern (VNM) utility model has been proposed by Dubois and Prade [14]. We present it briefly, in order to clarify its analogy with qualitative MCDM.

We consider the set S of states of the world, Ω the set of consequences, and a given act d, thus a mapping from S to Ω. The original probabilistic VNM approach tends to model the preference of the decision maker on the different probability distributions on Ω (called "lotteries"). We deal here with "possibilistic lotteries" on S, taking values on a ordinal scale L. Thus for any π in L^S, $\pi(s)$ expresses the belief that the true state of the world is s. We denote by $\succ$ the preference relation of the decision maker on the different possibilistic lotteries, and we consider that utility functions on the consequences are also qualitative, taking values on an ordinal scale T.

Under a set of 5 conditions on $\succ$, which can be said to be the qualitative counterpart of the VNM model, Dubois *et al.* [10], simplifying the initial proposal in [14], have shown that there exists a utility function $u : \Omega \rightarrow T$ which represents the preference of the decision maker, i.e.

$$\pi_1 \succ \pi_2 \Leftrightarrow U(\pi_1) > U(\pi_2)$$

with

$$U(\pi) = \min_{\omega \in \Omega} [(1 - \Pi(d^{-1}(\omega))) \vee u(\omega)] \quad (7)$$

where Π is the possibility measure generated by π. It is easy to show that U can be written as well as a minimum on the set of states:

$$U(\pi) = \min_{s \in S} [(1 - \pi(s)) \vee u(d(s))] \quad (8)$$

Let us turn now to qualitative MCDM. We consider as before that a non additive measure μ, defined on S the set of criteria, is able to model the importance of coalitions of criteria on a qualitative scale T. We consider an act d, described by a vector of consequences $[\omega_i]_{i\in S}$ on $\times_{i\in S}\Omega_i$, and qualitative utility functions $u_i : \Omega_i \longrightarrow T$, $i \in S$. Of course, the Choquet integral is of no use here, but we can use instead the Sugeno integral, where medians substitute to means, which can be said to be the qualitative counterpart of the Choquet integral, possessing maxivity and minivity properties on co-monotonic acts [15]:

$$U(d) = \max_{i\in S}[u_i(\omega_{(i)}) \wedge \mu(A_{(i)})]$$

with notations of section 3. It can be shown that if μ is a necessity measure generated by a possibility distribution π, then

$$U(d) = \min_{i\in S}[u_i(\omega_i) \vee (1 - \pi(i))], \qquad (9)$$

where u_i depends on d, an expression which is the same as (7) in the case of decision under uncertainty. Thus again, the parallel has been established between multicriteria decision making and decision making under uncertainty. Note that the counterpart of (8) can be written as well, but its interpretation in the framework of MCDM is not clear

$$U(d) = \min_{y\in Y}[y \vee (1 - \Pi(d^{-1}(u^{-1}(y))))]$$

An important question in the qualitative approach is how to define the Shapley value and the interaction indices, since original definitions are obviously suitable only for the quantitative case. Based on the qualitative counterpart of the founding axioms of the Shapley value, Grabisch [19] has recently shown that the only reasonable definition of the Shapley value seems to be

$$s_\vee(i) := \bigvee_{A\subset X\setminus i} [\mu(A\cup i) \neg_\vee \mu(A)].$$

where $a \neg_\vee b = a$ if $a > b$, and 0 otherwise. This permits to define as in the numerical case interaction indices.

Lastly, we mention that veto and favor effects, which could be modelled by the Choquet integral w.r.t. some suitable non additive measure, can be modelled as well by the Sugeno integral w.r.t. the *same* measures.

4 Logical setting

It has been recently advocated [2] that decision systems can benefit from being handled in a logical setting, since decision machineries may need to be equipped with reasoning and explanation capabilities.

More particularly it has been pointed out [9] that the qualitative decision process recalled in Section 3.5 can be encoded in a possibilistic logic framework. It is based on the fact that a possibility distribution or a fuzzy set can be viewed as the semantic counterpart of a possibilistic logic base [12]. Namely, the semantics of a possibilistic logic base $K = \{(p_i, \alpha_i); i = 1, \cdots, n\}$, where p_i is a classical proposition and α_i, a level in a totally ordered scale V, is given by the function from the set of interpretations Ω to V, defined by

$$\mu_K(\omega) = \min_{i=1,\cdots,n} \max(v_\omega(p_i), 1 - \alpha_i). \qquad (10)$$

where $v_\omega(p_i) = 1$ if ω is an interpretation which makes p_i true and $v_\omega(p_i) = 0$ if ω makes p_i false.

Then the qualitative decision under uncertainty problem can be viewed in the following way. Let K be a possibilistic logic base which gathers the available pieces of information (with their certainty levels) expressing what is known about the state of the world. Let P be another possibilistic logic base gathering a set of goals to be reached with their levels of priority. P expresses preferences. Let K_α (resp. $P_{\bar{\beta}}$) be the set of formulas in K (resp. P) with a level at least equal to α (resp. strictly greater than β).

Then it has been established that it is the same

- i) to look for the greatest α such that

$$(K_d)_\alpha \vdash P_{\overline{1-\alpha}} \tag{11}$$

  where K_d is what is known about the world, once decision d is chosen and applied (this expresses that d is all the better as only the most certain part of K_d, namely $(K_d)_\alpha$, is enough for ensuring that the goals in P, even those with a rather low degree of priority, namely those in $P_{\overline{1-\alpha}}$ are satisfied);

- ii) to maximize (8) where $\Pi(d^{-1}(.))$ is taken as the associated semantics of K defined by(10), and u is taken as the semantics of P (in the sense of (10)).

Then it is natural to wonder about the multiple criteria counterpart of this logical view of decision under uncertainty. However this is not straightforward, first since in (8) and (9) decision d does not appear at the same place. Clearly the fuzzy sets of important criteria defined by the $\pi(i)$ in (9) and the fuzzy set of well satisfied criteria defined by the u_i's can be easily interpreted in terms of α-level cuts: Namely the set of criteria whose importance is greater or equal to α and the set of criteria which are satisfied at least at the level α for decision d. Formally speaking, a finite fuzzy set F can be always viewed as a set of possibilistic formulas $([F_{\alpha_i}], 1 - \alpha_{i+1})$ for $i = 1, m$ with $1 = \alpha_1 > \alpha_2 \ldots > \alpha_{m+1} = 0$ and $[F_{\alpha_i}]$ is the proposition whose extension is F_{α_i} the α_i-level cut of F. However the interpretation of the α-level cuts of those two fuzzy sets in propositional terms, meaningful for a user, does not seem to be obvious in practice, since the set of logical interpretations ω (in(10)) is here just the set of criteria. Thus, using a counterpart of (11) in MCDM does not seem very useful.

A more natural logical view of the multiple criteria decision making problem appears to view each criterion as a fuzzy set represented as a set of prioritized goals, and then to perform the (weighted) combination of the criteria at the syntactic level in agreement with the aggregation at the semantic level [2]. Then the syntactic level provides a way for analyzing conflicts between criteria, and the best decision(s) d is/are the one(s) which maximize(s) the fuzzy set resulting from the semantical aggregation and which is the semantical counterpart of the possibilistic logic base obtained by combination at the syntactic level.

5 Conclusion

This paper has provided a brief structured overview of numerical and qualitative models both in decision under uncertainty and in MCDM, emphasizing the parallel between these two paradigms. Some recent developments or new lines of research suggested by this comparison, as well as by the parallel between numerical and qualitative models, have been pointed out.

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