Laser Beams and Resonators: The 1960s

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Abstract—**This paper looks back at how the basic concepts of optical resonators and lensguides emerged during the first decade of the laser era. A subsequent paper will review the continuing developments during the three decades since then.**

Keywords— **Optical resonators, laser beams, history.**

I. Introduction

CLASSICAL optics [1, 2] is rich in distinguished names and associated concepts: the Airy disk, Brewster's angle, Fraunhofer and Fresnel zones, Huygens' integral, Newton's rings, the Sagnac interferometer, and Snell's law, not to mention more exotic examples like the Spot of Arago [3], the Gouy phase shift [4-6], assorted Liouville theorems, the Talbot effect [7], and that mysterious French theorist Jean-Claude Étendue. Operation of the first laser in 1960 [8] brought renewed importance to these older concepts and stimulated the development of new concepts by creative people working in this new field.

This paper looks back at how the fundamental concepts associated with the resonant modes of laser oscillators and the paraxial propagation of laser beams emerged during the first decade of the laser era. A subsequent paper will describe how these concepts have evolved in many further directions in the following three decades. Lasers have of course also had enormous impacts in spectroscopy, nonlinear optics, and the massive advances in fiber optics that are revolutionizing telecommunications today, but these topics are beyond the scope of this review.

In preparing these reviews I have tried to cite important early papers associated with the topics I discuss. I can only hope for partial success in this, however, and must apologize in advance to contributors whom I may have overlooked. I have included every reference dealing with resonator modes known to me in the English literature through 1965, and made special efforts to cite Russian literature and contributions from outside the U.S. Anan'ev's 1992 book [9] is an additional source for such references.

Many earlier reviews are available [10-25], notably the personal reminiscences concerning the earliest optical resonator developments by Kompfner [26]. More details on the technical aspects of optical beams and resonators can be found in the author's lasers text [22]. An updated list of references for both of these papers will also be placed online [27] to permit automated searching.

II. Laser Resonators: The Early 1960s

A. Prehistory

The most important advance in optical resonators came in 1960 with the initial demonstration that open-sided optical resonators really could have distinct transverse as well as axial modes—unique eigenmodes similar though not identical to the microwave waveguide modes that were extensively studied before and during World War II. The existence of such low-loss modes, and the basic idea of using unusually narrow, unusually long optical interferometers with flat or curved mirrors as resonant feedback structures, had precursors during the late 1950s, arising out of interest in beam waveguides for millimeter-wave propagation and growing interest in laser action itself.

Early publications include a laser proposal by Prokhorov [28], early laser and lensguide patents by Dicke [29] and Goubau [30], and the important early laser proposal by Schawlow and Townes [31]. (The Dicke patent, by the way, also gives a clear explanation of something very close to modern laser Q-switching.) Goubau has also noted in a later round-table discussion [32] that parallel-plate resonators were investigated at microwave frequencies in 1954 by Scheiber, King, and Tatsuguchi under a Signal Corp contract at the University of Wisconsin.

(*Note added in proof:* Patent decisions in recent years have also brought increased visibility to an early conception and description of the open-sided laser resonator recorded (along with the concept of Brewster windows for laser tubes) by Gordon Gould in a notebook prepared in November 1957. This notebook has since become the foundation for several extensively litigated and eventually validated U.S. patents. A recent narrative of these events is given by Nick Taylor in *LASER: The Inventor, the Nobel Laureate, and the Thirty Year Patent War*, Simon & Schuster, 2000.)

B. "Fox and Li"

The modern understanding of optical resonators first emerged, however, in a truly pioneering 1961 paper by Gardner Fox and Tingye Li [33, 34]. Given that Gardner Fox also served for many years as editor of the IEEE Journal of Quantum Electronics, dedicating this review to him seems only appropriate. The best way to summarize the new understanding contributed by this paper may be to simply reproduce its Abstract:

> *"A theoretical investigation has been undertaken to study diffraction of electromagnetic waves in Fabry-Perot interferometers when they are used as resonators in optical masers. An electronic digital computer was programmed to compute the electromagnetic field across the mirrors of the interferometer where [when?] an initial launched wave is reflected back and forth between the mirrors."*

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“It was found that after many reflections a state is reached in which the relative field distribution does not vary from transit to transit and the amplitude of the field decays at an exponential rate. This steady-state field distribution is regarded as a normal mode of the interferometer. Many such normal modes are possible depending upon the initial wave distribution. The lowest-order mode, which has the lowest diffraction loss, has a high intensity at the middle of the mirror and rather low intensities at the edges. Therefore, the diffraction loss is much lower than would be predicted for a uniform plane wave. Curves for field distribution and diffraction loss are given for different mirror geometries and different modes.”

“Since each mode has a characteristic loss and phase shift per transit, a uniform plane wave which can be resolved into many modes cannot, properly speaking, be resonated in an interferometer. In the usual optical interferometers, the resolution is too poor to resolve the individual mode resonances and the uniform plane wave distribution may be maintained approximately. However, in an oscillating maser, the lowest-order mode should dominate if the mirror spacing is correct for resonance.”

“A confocal system has also been investigated and the losses are shown to be orders of magnitude less than for plane mirrors.”

Figure 1, taken from Fig. 5 of this paper, shows the steady-state or self-reproducing amplitude and phase profiles of the lowest-order mode in a typical one-transverse-dimensional or strip resonator some 25 wavelengths wide and 100 wavelengths long. Note how the amplitude profile of this mode drops down to a low value near the mirror edges, and the steady-state phase profile rolls off indicating diffractive spreading in this outer region. Note also the strong diffraction ripples caused by the finite resonator edges in the first transit, and the weaker residual ripples in the final mode profile.

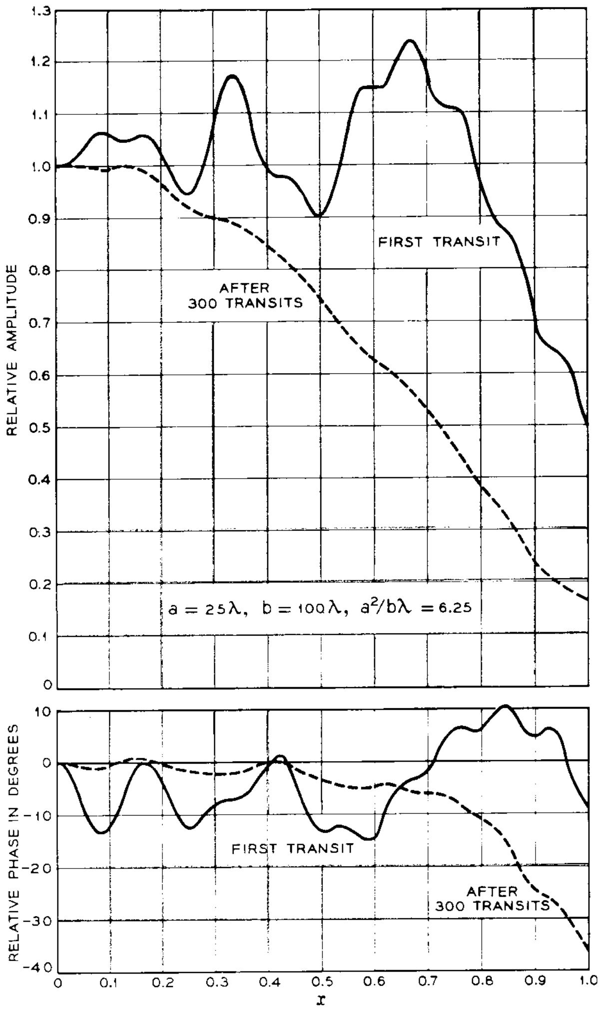

Fig. 1. Transverse amplitude and phase profiles for the lowest-loss mode of a strip resonator with planar mirrors (from Fox and Li, 1961).

Figure 2, reproduced from the same paper, shows how the wave amplitude at one point on the mirror oscillates on successive round trips as the iterative calculation process nears convergence. The periodic ringing represents interference or transverse mode beating caused by the different round-trip phase shifts or phase velocities of the two lowest-loss modes, while the exponential decay shows how the next higher-loss mode dies out relative to the lowest-loss mode. A hundred or more round trips are required to reach this point in the case at hand, because the losses for still higher-order modes are also small and so the higher-order modes are only slowly stripped out of the computation.

Fox and Li were not sure at the time they initiated this investigation that open-sided resonators would have modes such as these [35], and some of their Bell Labs colleagues are said to have argued that no such modes would exist [26]. But as they state in the final section of their first paper:

“Diffraction calculations carried out on the IBM computer have led to the following conclusions:”

“1. Fabry-Perot interferometers, whether of the plane or concave mirror type, are characterized by a discrete set of normal modes which can be defined on an iterative basis. The dominant mode has a field intensity which falls to low values at the edges of the mirrors, thereby causing the power loss due to diffractive spillover to be much lower than would be predicted on the assumption of uniform plane wave excitation.”

[There is an interesting slip of the pen in this paragraph as well as in the Abstract, since Fox and Li note elsewhere that their modes are not orthogonal and hence not “normal modes” in the usual sense of that term.]

Fox and Li’s numerical approach provides a procedure for finding the resonator eigenmodes that is equivalent to the so-called power method for finding the largest eigenvalue of a complex matrix. Other methods for finding resonator eigenmodes have since been developed, but the Fox and Li method remains widely used even today, although more modern Fast Fourier Transform methods are usually employed for the round-trip propagation. Their approach draws much of its strength from how closely it mimics what really happens in an oscillating laser—that is, buildup of

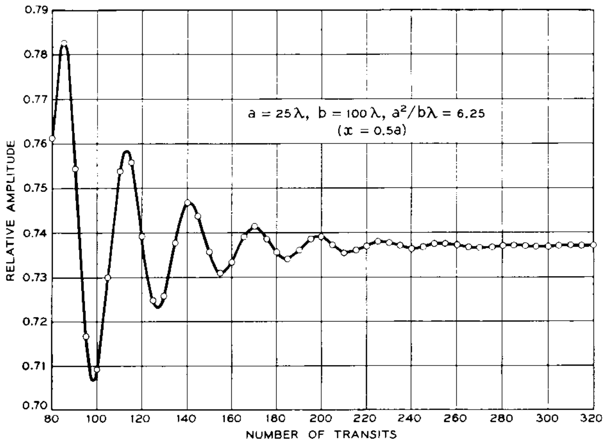

Fig. 2. Convergence of a typical Fox and Li iterative calculation to steady state, showing how the next higher-order mode slowly dies out after many round trips.

the lowest-loss transverse mode or modes, accompanied by the stripping out of higher-order modes as the laser radiation travels around repeatedly within the laser cavity. The fact that many Fox and Li calculations are started from random initial values for the wavefront amplitude at each transverse point even mimics to some extent the fact that real lasers in many cases build up from random spontaneous emission noise.

C. *The Emergence of Gaussian Mode Theory*

The period immediately following the first Fox and Li papers was a time of immensely rapid progress in the understanding of optical resonators. Fox and Li considered both planar and confocal resonators, and pointed out that the modes inside an optical resonator were formally equivalent to the propagation modes of a periodically iterated lensguide or beam waveguide. During the same period Goubau and coworkers were also examining the modal properties of periodic lensguides using rather complex mathematical techniques [36], although these workers do not seem to have discussed their application to resonators.

A second very significant early paper on optical resonators by Gary Boyd and James P. Gordon [37] appeared in same March 1961 issue of the Bell System Technical Journal as the Fox and Li paper. In this paper Boyd and Gordon showed that the exact eigenmodes of a confocal resonator with square mirrors are prolate spheroidal wavefunctions whose mathematical properties had previously been considered by their Bell Labs colleagues Slepian and Pollack [38]. The exactly confocal case with square mirrors is in fact one of the few common optical resonators for which an exact solution, including losses and mode profiles, can be given in closed form.

Boyd and Gordon then showed that these prolate spheroidal eigensolutions became Hermite-gaussian modes in the practical limit of vanishingly small losses or large enough mirrors. They introduced the notations of w for the gaussian spot size and b for the confocal parameter (although this parameter is now more commonly replaced by the Rayleigh range $z_R = b/2$ [39-41] and described how the spot size $w(z)$ and wavefront curvature $R(z)$ of the Hermite-gaussian beams varied with distance inside and outside the resonator. They also extended their analysis to symmetric concave spherical reflectors separated by any distance up to twice their common radius of curvature and derived the spot sizes, resonant frequencies, and mode degeneracy properties for what they labelled as the TEM_{nmq} modes in these resonators.

(Many of the same results are also given in a contemporaneous analysis of optical lensguides by another Bell Labs colleague John R. Pierce [42] who in turn references similar work by Goubau and Schwering [36].)

This approach was extended the following year by Boyd and Herwig Kogelnik [43] who considered gaussian modes in resonators with unequal radii of curvature and arbitrary mirror spacings, and interpreted their results using what we now call the resonator stability diagram (which they attributed to Fox and Li). They also described the Laguerre-gaussian modes appropriate to mirrors with circular cross section. These two papers together put forth most of the now well-known basic properties of gaussian modes in stable two-mirror resonators.

III. LASER RESONATORS: THE MID 1960S

A. *Additional Resonator Mode Analyses*

With these conceptual foundations firmly in place, Fox and Li and many others continued the exploration of resonator properties, extending their results to tilted mirrors [44-46], other mirror shapes [47], hole coupling [48], and the effects of gain saturation and mode deformation [49-51]. Streiffer showed that the Fox and Li formulation could also be efficiently solved by matrix diagonalization methods [52-55], and Heurtley showed that the exact prolate spheroidal solutions for square confocal mirrors could be extended to hyperspheroidal solutions for confocal circular-mirror resonators [56].

Despite the simplicity of the approximate gaussian solutions, the exact mathematical properties of the modes of open-sided optical resonators are complex (and remain so to this day). The operator that describes the eigenmodes of such resonators is not hermitian, and the resulting eigenmodes are in general not orthogonal, i.e. are not a set of "normal modes" in the usual sense. In fact the existence of a complete set of eigenmodes is not even guaranteed, as discussed by a number of authors [57, 58]. Variational and Schmidt approaches to the eigenmode problem were attempted by a number of authors [52, 54, 59-62], but criticized on fundamental grounds by Morgan [63].

A very different and apparently mathematically solid approach based on a sophisticated waveguide analysis was introduced by Vainshtein (or Weinstein) [64, 65] and nicely summarized by Toraldo di Francia in a 1964 round-table discussion [66], although it does not seem to have found much practical application since then. A perturbation method valid for small mirrors or large cavity losses was also put forth by Bergstein and Schachter [67-69]. Additional Russian and other resonator calculations are presented in the following references [47, 70-74].

B. Other Early Ideas and Experiments

This theoretical ferment was of course accompanied by rapid development of practical laser devices and structures which I will not attempt to review here, except to cite a few early observations of higher-order Hermite-gaussian and Laguerre-gaussian modes [75-80] and transverse mode beats and mode degeneracies in oscillating lasers [79, 81]. Transverse mode discrimination and the suppression of higher-order and parasitic modes was studied theoretically by Li [82, 83] and explored experimentally by many workers, especially on high-gain ruby lasers [84-87].

The "Collins chart" for describing gaussian beam propagation in a graphical fashion was developed by Stuart Collins and extended by others during this period [88-92]. The concept of gaussian mode matching also emerged [93, 94] and Gordon and Kogelnik developed generalized equivalence relations among stable gaussian resonators [95, 96]. Robert Pole and H. Wieder demonstrated a particularly clever and interesting conjugate-concentric resonator with concentric spherical end surfaces on the internal laser rod [97].

This era also saw the invention of the confocal scanning interferometer [98], a modern implementation of the Connes interferometer [99-101], which has since become of great importance as a tunable filter and optical spectrum analyzer. A small flurry of interest in passive multibounce stable resonators as optical delay lines and long-path-length spectroscopic absorption cells also led to some particularly clever experiments [102, 103].

C. The Unstable Resonator

In 1965 I introduced the so-called unstable optical resonator which has very different physical and mathematical properties as compared to gaussian stable resonators [104]. Earlier papers by Fox and Li [44, 45] had calculated a few unstable resonator cases as an aside, without recognizing their quite different basic character, and there had been a predecessors to this idea in the form of "diffraction coupling" from Fabry-Perot resonators [105, 106].

My initial paper on this topic led to a number of additional calculations [107-116]. The concept of the equivalent Fresnel number N_{eq} for the unstable resonator was introduced by Ray Arrathoon [114], and the importance of the confocal unstable resonator, shown in Fig. 3, and the distinction between positive-branch and negative-branch unstable resonators emerged from important early experiments by William Krupke and Walt Sooy [117].

Significant work in the Soviet Union by Anan'ev and coworkers [73, 118-121] included introduction of the astigmatic stable-unstable resonator, and the important concept of edge tapering of the output coupler in order to reduce or eliminate mode-crossing effects characteristic of hard-edged unstable resonators. A very different and powerful asymptotic analysis of unstable resonators was developed in 1973 by Horwitz [122] and later given a physical interpretation in terms of scattering of the magnified mode pattern on each round trip into Keller edge waves [123-126] at the hard edges of the output coupler. This has led to the so-called virtual source approach [127-131] which greatly simplifies accurate calculation of the complicated unstable resonator mode profiles [132]. The special mathematical difficlties associated with the unstable resonator were also explored in the 1970s by Landau [133, 134].

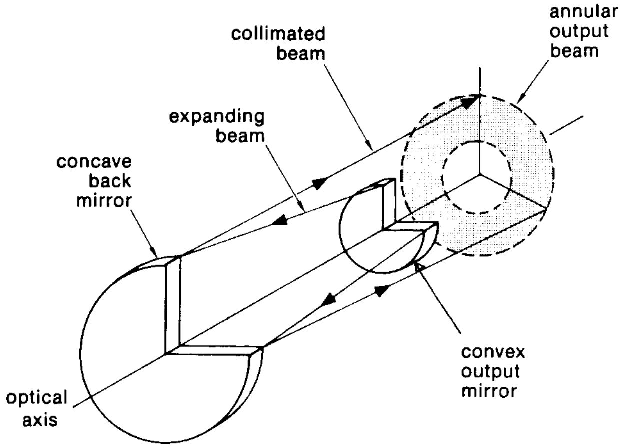

Fig. 3. Schematic drawing of a confocal hard-edged unstable resonator (from Krupke and Sooy, 1969).

Much of this initial development is summarized in reviews by Anan'ev [15] and Siegman [17, 135]. Geometrically unstable resonators, particularly when supplemented by variable reflectivity mirror (VRM) concepts, have since become of some importance for obtaining high power or high energy outputs with good beam quality from large-volume laser media. Most recently a group in Leiden has made the interesting discovery that the mode profiles of unstable resonators appear in fact to be fractal in character [136, 137].

D. Hole-Coupled and Roof Resonators

During the first decade of the laser era there was also considerable interest in roof resonators with end mirrors consisting of prisms or of flat surfaces intersecting at dihedral angles. These reflectors were easy to fabricate, and offered a certain degree of alignment insensitivity in Q-switched lasers employing a spinning end mirror. Despite the considerable literature that developed during the early 1960s (cf., e.g., [138]), these resonators seem to have been a technological dead end and do not seem to have led to any particularly interesting new concepts.

Hole coupling through a small central hole in stable cavity resonators [48, 139] was also considered during this period. The idea of hole coupling seems to arise periodically in the laser field—although it is almost always a fundamentally bad idea.

E. Distributed Feedback Lasers

The concept of distributed feedback, in which the laser mirrors are replaced by distributed feedback from some form of Bragg grating distributed continuously along the length of a laser medium, was introduced in the early 1970s by Kogelnik and Shank [140-142] and has become of some practical importance for semiconductor lasers. I have not reviewed the extensive literature on this subject because the interesting questions focus on the longitudinal coupled-wave and laser feedback aspects rather than on transverse mode or beam propagation aspects.

F. Twisted Mode Resonators and Axial Mode Selection

Spatial hole burning associated with the axial nulls in a conventional standing-wave laser cavity favors laser operation in two or more axial modes, especially in wide-line solid-state lasers. One early and effective method for obtaining single-axial mode operation was the unidirectional ring laser [143]. Another early solution was the "twisted mode resonator" [144] in which quarter-wave plates at each end of the resonator produce axial modes in the form of twisted ribbons with constant intensity along the cavity length. This concept has subsequently found a number of other applications [145-149].

Other ingenious methods for obtaining single axial mode oscillation in wide-line lasers were developed during the first decade of the laser era, including multiple-mirror cavities, vernier-Michelson interferometers, Fox-Smith interferometers, White interferometers, Sagnac interferometers, and intracavity etalons. An excellent review of transverse and axial mode selection technology as of 1972 is given by Smith [150]

IV. ALSO DURING THE 1960S: OPTICAL LENSGUIDES

A. Lensguides for Information Transmission

Fox and Li's initial paper noted that the transverse modes of laser resonators were equivalent to the modes of periodic focusing systems or optical lensguides. The concepts of stable and unstable rays in periodic focusing systems had been developed some time earlier in connection with the focusing of electron beams [151]; and the transverse modes of beam waveguides for millimeter-wave or optical wavelengths had been the subject of considerable research along a parallel but almost independent track in the late 1950s and early 1960s because of their potential for high-capacity telecommunications through underground pipes, sometimes referred to as "optical pipelines" [152].

Early references on beam waveguides include the pioneering work of Goubau and others [30, 36, 42, 152-157]. Experimental tests of kilometer-long optical waveguides in the mid-1960s are described by Goubau [158] and Gloge [159, 160], and reviews of this technology as of 1968 are given by Goubau [161, 162]. The emerging promise of optical fibers beginning around 1966 as pointed out by Kao in Britain [163] followed by the first demonstration of truly low-loss fibers at Corning in the early 1970s [164, 165], however led to the rapid demise of beam waveguides as a communications medium.

B. Gas Lenses

There is, however, one intriguing even if dead-end technology from the beam waveguide era that seems worth mentioning. To eliminate the unacceptable losses associated with glass lenses and their imperfect optical coatings, researchers at Bell Laboratories developed "gas lenses" consisting of meter long, centimeter diameter heated pipes or other more complex structures with cold gas flowing axially thorough the center, as shown in Figs. 4 and 5 [166-171].

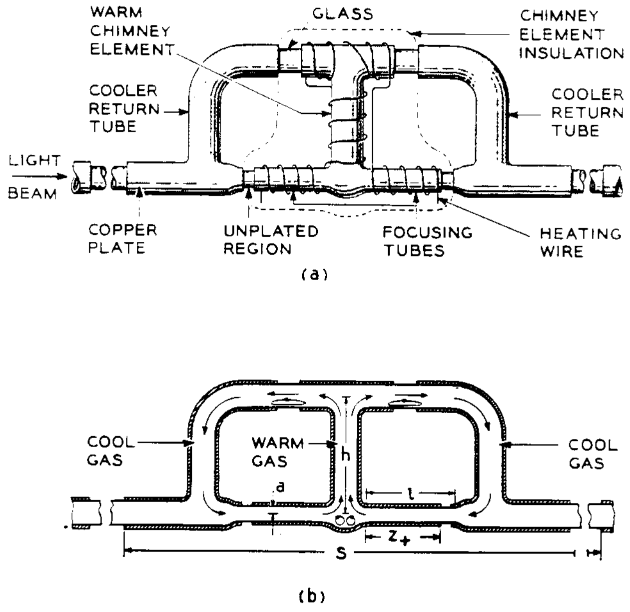

Fig. 4. Schematic drawing of a flowing gas lens (from Berreman, 1965).

The radially varying refractive index produced by colder gas on axis with heated gas surrounding it created a weak but very low loss lens which could be employed in an optical waveguide. The need to supply electrical power (or hot water) to heat the surrounding structure led to the intriguing figure of merit for these lenses of "diopters per kilowatt".

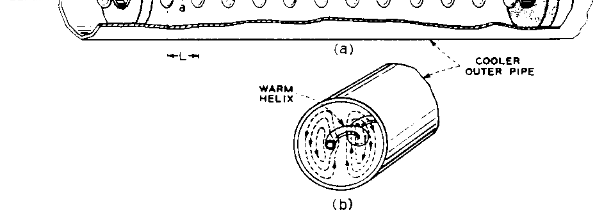

Fig. 5. Helical convective gas lens (also from Berreman, 1965).

V. Summary

All the basic concepts associated with real stable resonators and real gaussian beams were in place by the middle 1960s, as documented in Kogelnik and Li's classic review [10] and in the revised (1965) edition of Ramo, Whinnery and van Duzer's classic text [172]. Real ray matrices, already known in standard optics texts [173], had been applied to optical resonators [70, 174-179] and converted to the now standard ABCD notation [102, 180-182]. Kogelnik in particular [180] had identified the bilinear transformation of the complex $\tilde{q}$ parameter through any paraxial optical system using the ABCD matrix. The historical connection between the differential phase shifts of gaussian modes and the Gouy phase shift of the 19th century [4-6, 183, 184] was recognized; and Baues [185, 186] and Collins [187] made the important observation that Huygens' integral through a cascaded sequence of paraxial optical elements could be written as a single integral involving only the cascaded ABCD matrix of the system.

Development of new concepts and still further knowledge in optical resonators and paraxial beam propagation nonetheless continued at a rapid pace during the following decades and continues even today. These advances have been motivated by new types of lasers, by requirements for more complex and multielement laser resonators, by advances in laser beam propagation for practical applications, and by a desire for increased basic understanding of optical beams and resonators. The most notable of the further developments in these areas occurring after the mid to late 1960s are described in an immediately following paper [188].

Anthony E. Siegman retired at the end of 1998 from the McMurtry Chair of Engineering and Applied Physics at Stanford University. During his career at Stanford he made many contributions to microwave electronics, lasers, and optical resonators, as reported in some 40 dissertations, 250 scientific articles, and three books including the widely used textbook LASERS (University Science Books, 1986). He was elected to the National Academy of Engineering and National Academy of Science, and awarded the Wood Prize and Ives Medal of the Optical Society of America, the Quantum Electronics Award of IEEE LEOS, and the Schawlow Medal of the Laser Institute of America. He served as President of the Optical Society of America during 1999, and is currently engaged in technical writing, teaching, and consulting.

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