A View on Full-Diversity Modulus-Preserving Rate-One Linear Space-Time Block Codes*

Shengli Zhou[1], Xiaoli Ma[2], and Krishna Pattipati[1]

Paper Number:	**SIGPRO-D-05-00038**
Associate Editor:	Prof. Visa Koivunen
Submitted:	April 11, 2005
Revised:	September 8, 2005, Oct. 4, 2005
Accepted:	October 12, 2005

* X. Ma is partially supported by the U.S. Army Research Laboratory and the U.S. Army Research Office under grant number W911NF-04-1-0338.

[1] S. Zhou and K. Pattipati are with the Department of Electrical and Computer Engineering, University of Connecticut, 371 Fairfield Road U-2157, Storrs, Connecticut 06269, USA (email: shengli@engr.uconn.edu; krishna@engr.uconn.edu).

[2] X. Ma is with the Department of Electrical and Computer Engineering, Auburn University, AL 36849, USA (email: xiaoli@auburn.edu).

Abstract

In this paper, we consider the class of linear space time block codes (STBCs) that possess the following properties: i) enabling full antenna diversity, ii) preserving the modulus of information symbols, and iii) achieving the maximum rate of one symbol per channel use under i) and ii). We first provide an alternative construction for full-diversity rate-one STBCs that are based on linear constellation precoding (LCP). In sharp contrast to existing LCP-based designs, the proposed construction is modulus-preserving, thanks to an explicit unitary diagonal precoding on symbol blocks combined with an implicit fast-Fourier-transform (FFT) precoding via circularly shifted transmissions over multiple antennas. Our proposed construction also allows flexible choices on the precoder size, a feature not available in the original design. We then demonstrate that quasi-orthogonal (QO) STBCs can be constructed by embedding modulus-preserving LCP designs into orthogonal structures. Interestingly, QO-STBCs can be interpreted as space-time-frequency (STF) block codes, that were originally developed for multi-antenna multi-carrier systems, up to FFT transformations.

Index Terms

Space time block coding, linear constellation precoding, circular delay diversity, quasi-orthogonal, full antenna diversity, modulus-preserving.

I. Introduction

Space-time coding (STC) has by now been well documented as an attractive means of achieving high data rate transmission with robust performance over fading channels. Space time coding amounts to a two-dimensional coding that maps information bits into parallel transmitted symbols across multiple antennas ("space") over a certain time duration ("time"). In this paper we focus on one class of STC, where the coded transmission is linear with respect to information symbols (the real and imaginary parts) in both space and time. Let N_t and N_r denote the number of transmit- and receive-antennas in the system. Collect P information symbols into a vector $\mathbf{s} = [s_1, \ldots, s_P]^T$. Based on the information symbol block $\mathbf{s}$, a linear space time block code (STBC) constructs a space time transmission matrix as [9], [20]

$$\mathbf{X} = \sum_{p=1}^{P} (s_p \mathbf{A}_p + s_p^* \mathbf{B}_p), \tag{1}$$

where $\mathbf{A}_p$ and $\mathbf{B}_p$ are complex $T \times N_t$ matrices. The N_t columns of $\mathbf{X}$ are transmitted through N_t antennas simultaneously in T symbol intervals.

Design of linear STBCs boils down to the construction of matrices $\{\mathbf{A}_p, \mathbf{B}_p\}_{p=1}^{P}$. The quality of the design could be judged from various perspectives, as summarized in the following.

- *Diversity order*. Diversity order measures the slope of the block-error-rate (or, the bit-error-rate) versus signal-to-noise-ratio (SNR) on a log-log scale, at the *high SNR* region. Over flat-fading rich-scattering channels, the maximum achievable diversity is $N_t N_r$. A STBC that achieves a diversity order of $N_t N_r$ is called a full-diversity code.
- *Rate*. The transmission rate of (1) is $R = P/T$ symbols per channel use. High rate STBC is desirable.
- *Delay optimality*. For the maximum diversity of $N_t N_r$ to be achieved, it is known that the minimum possible decoding delay T is equal to N_t. Schemes that achieve maximum diversity with the minimum delay N_t are called delay optimal [7].
- *Modulus*. To alleviate possible distortions due to amplifier non-linearity, it is desirable to have small amplitude variations on the transmitted symbols. A STBC is termed "modulus-preserving", if each entry of $\mathbf{X}$ has the same modulus as the original information symbol. This means that each entry of $\mathbf{X}$ can be either 0, $e^{j\phi}s_p$, or $e^{-j\phi}s_p^*$, where $e^{j\phi}$ is a constellation rotation factor.
- *Decoding complexity*. Collecting full antenna diversity requires maximum likelihood (ML) or near-ML receivers. Decoding complexity decreases if different parts of $\mathbf{s}$ can be decoded separately.

Nice properties on all aspects mentioned above cannot be possessed simultaneously by one particular linear STBC design. In this paper, we emphasize *i) full-diversity*, and *ii) modulus-preserving* properties. Notice that to achieve full antenna diversity, each symbol has to appear at least once per column of $\mathbf{X}$; hence at least N_t times in $\mathbf{X}$. On the other hand, the modulus preserving property dictates that TN_t entries of $\mathbf{X}$ can host TN_t (rotated) symbols at most. Combining these two, we must have $PN_t \leq TN_t$ and $R \leq 1$. Thus the transmission rate is no more than one, if both i) and ii) are satisfied. Under i) and ii), we focus on the *maximum-rate* designs with $R = 1$ symbol per channel use.

Now let us briefly comment on existing linear STBC designs. The first category is the orthogonal space time block code (OSTBC) [1], [7], [20], [22]. OSTBC enables full diversity. The most distinct feature of OSTBC is that the optimal receiver relies only on simple linear processing for decoding *each* information symbol. When $N_t = 2$, the Alamouti code [1] achieves the maximum rate $R = 1$. However, when $N_t > 2$, OSTBC suffers from rate loss [7], [20], [22], which is a major drawback.

The second category is quasi-orthogonal (QO) STBC [11], [12], [17], [21] that enables higher rate than OSTBC, but only possesses partial orthogonality among information symbols. Original QO-STBCs do not collect full antenna diversity [12], [21], which is then recovered through judicious constellation rotation [17]–[19]. QO-STBC usually preserves the symbol modulus [2], [12], [18], [19].

The third category is the STBC based on linear constellation precoding (LCP), see e.g., [3], [23] and references therein. The LCP framework provides a systematic way to construct full-diversity rate-one delay-optimal STBC [23]. The main criticism of LCP-based STBC is that it does not preserve the modulus of information symbols due to linear precoding, which leads to a large peak-to-average power ratio in the transmitted sequence.

The fourth category is high-rate STBC, whose rate is larger than one, and could be as high as N_t symbols per channel use. Examples include diagonal BLAST [4], vertical BLAST [5], linear dispersion (LD) code [9], and full-diversity full rate (FDFR) designs [6], [14]. Since high rate STBCs cannot achieve full diversity while preserving the symbol modulus at the same time, we will not focus on them hereafter.

At the outset, let us reiterate that we will consider full-diversity modulus-preserving rate-one linear STBCs. Our contributions are as follows.

- In Section II, we develop an alternative construction for LCP-based STBCs, that preserves the symbol modulus. The key idea is to apply the unitary precoding of [23] in two steps: *explicit* unitary diagonal precoding on information blocks and *implicit* FFT precoding via circularly shifted transmissions (corresponding to the circular delay diversity in [15]) through multiple antennas. This construction also provides additional flexibility on the choices of the precoder size, which enables an interesting tradeoff between performance and complexity/delay-optimality.
- In Section III, we demonstrate that QO-STBCs can be constructed by embedding modulus-preserving LCP designs into orthogonal STBCs, and that they are equivalent to the space-time-frequency (STF) block codes, originally developed in [13] for multi-antenna orthogonal-frequency-division-multiplexing (OFDM) systems, up to FFT transformations. We thus offer a unified view that reveals the hidden links among various existing techniques.

Notation: Bold upper and lower letters denote matrices and column vectors, respectively; $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^{\mathcal{H}}$ denote transpose, conjugate, and Hermitian transpose, respectively; $\otimes$ stands for Kronecker product; $\mathbf{I}_N$ is the $N \times N$ identity matrix; $\mathbf{0}_{M\times N}$ denotes an all-zero matrix of size $M \times N$, and $\mathbf{F}_N$ denotes a unitary $N \times N$ FFT matrix with the $(p+1, q+1)^{\mathrm{st}}$ entry as $\frac{1}{\sqrt{N}}e^{-j\frac{2\pi}{N}pq}$; The matrix $\mathbf{D}(\mathbf{a})$ is diagonal with diagonal elements from the vector $\mathbf{a}$.

II. Modulus-Preserving LCP Design

For brevity, we assume one receive antenna in the system, and denote $\mathbf{h} = [h_1, \ldots, h_{N_t}]^T$ as the channel vector between N_t transmit antennas and the receive antenna. Denote $\mathbf{X}$ as the space time codeword of dimensionality $P \times N_t$, that is transmitted through N_t antennas in P time slots.

The proposed space-time code is constructed as follows. Denote α as a real constant, whose value will be specified later. We set the length $P \geq N_t$, and define two matrices as:

$$\mathbf{\Lambda}(\alpha) = \mathbf{D}\left([1, e^{-j\alpha}, \ldots, e^{-j\alpha(P-1)}]\right), \quad \mathbf{J} = \begin{bmatrix} \mathbf{0}_{1\times(P-1)} & 1 \\ \mathbf{I}_{P-1} & \mathbf{0}_{(P-1)\times 1} \end{bmatrix}. \tag{2}$$

When multiplying a vector $\mathbf{a}$, the matrix $\mathbf{\Lambda}(\alpha)$ performs successive phase rotations on elements of $\mathbf{a}$, while $\mathbf{J}$ performs a circular downshift on $\mathbf{a}$. We construct $\mathbf{X}$ via the following steps:

1) collect P information symbols $\{s_p\}_{p=1}^{P}$ into a vector, denoted as $\mathbf{s} = [s_1, \ldots s_P]^T$.
2) apply the diagonal precoding (or, successive phase rotation) on $\mathbf{s}$ to obtain: $\tilde{\mathbf{s}} = \mathbf{\Lambda}(\alpha)\mathbf{s}$.
3) each antenna transmits a circularly shifted version of $\tilde{\mathbf{s}}$, with the mth antenna shifting $\tilde{\mathbf{s}}$ by $(m-1)$ times. The $P \times N_t$ space-time matrix so constructed is:

$$\mathbf{X} = [\tilde{\mathbf{s}}, \mathbf{J}\tilde{\mathbf{s}}, \ldots, \mathbf{J}^{N_t-1}\tilde{\mathbf{s}}], \tag{3}$$

with the mth column transmitted through the mth antenna.

When $P = N_t$, $\mathbf{X}$ in (3) is square. For example, when $P = 3, 4, 5$, the space time matrices are

$$\mathbf{X}_{3\times 3} = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_3 & \tilde{s}_2 \\ \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_3 \\ \tilde{s}_3 & \tilde{s}_2 & \tilde{s}_1 \end{bmatrix}, \quad \mathbf{X}_{4\times 4} = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_4 & \tilde{s}_3 & \tilde{s}_2 \\ \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_4 & \tilde{s}_3 \\ \tilde{s}_3 & \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_4 \\ \tilde{s}_4 & \tilde{s}_3 & \tilde{s}_2 & \tilde{s}_1 \end{bmatrix}, \quad \mathbf{X}_{5\times 5} = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_5 & \tilde{s}_4 & \tilde{s}_3 & \tilde{s}_2 \\ \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_5 & \tilde{s}_4 & \tilde{s}_3 \\ \tilde{s}_3 & \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_5 & \tilde{s}_4 \\ \tilde{s}_4 & \tilde{s}_3 & \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_5 \\ \tilde{s}_5 & \tilde{s}_4 & \tilde{s}_3 & \tilde{s}_2 & \tilde{s}_1 \end{bmatrix}. \tag{4}$$

When $P > N_t$, $\mathbf{X}$ in (3) is a tall matrix. It can be obtained by keeping only the first N_t columns of a square matrix $\mathbf{X}_{P\times P}$.

Next we show how the transmission in (3) is related to the LCP framework in [23]. The vector containing received symbols in P time slots is:

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{w} = \underbrace{(h_1\mathbf{I} + h_2\mathbf{J} + \ldots + h_{N_t}\mathbf{J}^{N_t-1})}_{:=\tilde{\mathbf{H}}}\tilde{\mathbf{s}} + \mathbf{w} = \tilde{\mathbf{H}}\tilde{\mathbf{s}} + \mathbf{w}, \tag{5}$$

where $\mathbf{w}$ is the additive white Gaussian noise. The matrix $\tilde{\mathbf{H}} = \sum_{m=1}^{N_t} h_m \mathbf{J}^{m-1}$ is circulant, and thus can be decomposed as $\tilde{\mathbf{H}} = \mathbf{F}_P^{\mathcal{H}}\mathbf{D}(\tilde{\mathbf{h}})\mathbf{F}_P$, where $\tilde{\mathbf{h}} = [H(0), \ldots H(P-1)]^T$ with $H(p) := \sum_{m=1}^{N_t} h_m e^{-j\frac{2\pi}{P}mp}$ [8, p. 202]. Performing FFT on $\mathbf{y}$, we obtain:

$$\mathbf{z} = \mathbf{F}_P\mathbf{y} = \mathbf{D}(\tilde{\mathbf{h}})\underbrace{\mathbf{F}_P\mathbf{\Lambda}(\alpha)}_{:=\mathbf{\Theta}}\mathbf{s} + \underbrace{\mathbf{F}_P\mathbf{w}}_{:=\mathbf{n}} = \mathbf{D}(\tilde{\mathbf{h}})\mathbf{\Theta}\mathbf{s} + \mathbf{n}, \tag{6}$$

where the resulting noise $\mathbf{n}$ is still white.

Eq. (6) reveals that the proposed transmission falls into the LCP framework of [23]. Indeed, the information block $\mathbf{s}$ is precoded by a matrix $\mathbf{\Theta}$, and the symbols in the precoded block $\mathbf{\Theta s}$ go through diagonal channels contained in $\mathbf{D}(\tilde{\mathbf{h}})$ sequentially. The *difference* is that the proposed method diagonally transmits precoded symbols in the *frequency* domain, while the original approach in [23] is in the *time* domain. Our frequency domain LCP is achieved by time-domain *explicit* diagonal precoding followed by *implicit* FFT precoding via circularly shifted transmissions. Note that our proposed construction *does not* change the modulus of the transmitted symbols. This is in sharp contrast to the original LCP construction.

Now, we specify how to choose P and α.

- When $N_t = 2^d, d = 1, 2, \ldots,$ we set $P = N_t$ and $\alpha = \frac{\pi}{2P}$. For constellations carved from a square lattice, e.g., quadrature-amplitude-modulation (QAM), $\mathbf{\Theta} = \mathbf{F}_P \mathbf{\Lambda}(\alpha)$ is optimal in terms of maximizing the coding gain among all possible linear precoders [23]. Therefore, the proposed transmission achieves the optimal performance as the original LCP design, meaning that modulus-preserving property is obtained with *no cost* at all.
- When $N_t \neq 2^d$, we have two choices. One choice is to set $P = N_t$, and find α through some heuristic rules [23]. The precoder $\mathbf{\Theta} = \mathbf{F}_{N_t} \mathbf{\Lambda}(\alpha)$, being unitary, may achieve less coding gain than some non-unitary alternatives [23]. In this setup, the desirable modulus property is obtained at the expense of some possible performance loss.

  The other choice is to let $P > N_t$, where P is a power of two that leads to an optimal $\alpha = \frac{\pi}{2P}$. With $P > N_t$, the space time code is not delay-optimal. Increasing P also leads to a slight complexity increase, as the receiver needs to decode P (rather than N_t) symbols jointly. Interestingly, this choice always has better performance than $P = N_t$, as evidenced by our numerical results. The *flexibility* on the selection of the precoder size is not realized in [23], where P equals N_t throughout.

III. Embedding LCP into OSTBC and the Space-Time-Frequency Interpretation

QO-STBC has been extensively studied in the literature [11], [12], [17], [19], [21]. A very general *formulation* for QO-STBC is provided in [18]. On code construction, a recursive method is developed in [18], while another method is provided in [2] based on linear Hadamard codes. We next present another construction of QO-OSTBC by embedding the modulus-preserving LCP designs into orthogonal structure. Thus, our construction provides a different realization of the general formulation in [18]. Although different QO-STBC constructions are equivalent performance-wise, our approach clearly demonstrates how modulus-preserving LCP, OSTBC, and QO-STBC are related. And more importantly, our approach reveals a space-time-frequency interpretation for QO-STBC, which is not available based on other constructions. Instead of a general (yet less reader-friendly) presentation, which relies on the general matrix representation of OSTBC [20], we just specify several important cases in the following.

A. Embedding LCP into the Alamouti Code — Rate-one Quasi-Orthogonal Designs

We first consider embedding LCP into the 2×2 Alamouti code [1]. Without loss of generality, we choose an even P and define $K = P/2$. We split the symbol vector $\mathbf{s}$ into two equi-length sub-blocks $\mathbf{s} = [\mathbf{s}_a^T, \mathbf{s}_b^T]^T$. Based on $\mathbf{s}_a$ and $\mathbf{s}_b$, we construct the LCP codewords $\mathbf{X}_a$ and $\mathbf{X}_b$ from (3). Embedding $\mathbf{X}_a$ and $\mathbf{X}_b$ into the Alamouti structure, the overall space-time codeword is:

$$\mathbf{X}_{\rm ab} = \begin{bmatrix} \mathbf{X}_a & \mathbf{X}_b \\ -\mathbf{X}_b^* & \mathbf{X}_a^* \end{bmatrix}. \tag{7}$$

We list the example codes for $P = 4$ and $P = 8$ as:

$$\mathbf{X}_4^{\rm qo} = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_2 & \tilde{s}_3 & \tilde{s}_4 \\ \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_4 & \tilde{s}_3 \\ -\tilde{s}_3^* & -\tilde{s}_4^* & \tilde{s}_1^* & \tilde{s}_2^* \\ -\tilde{s}_4^* & -\tilde{s}_3^* & \tilde{s}_2^* & \tilde{s}_1^* \end{bmatrix}, \quad \mathbf{X}_8^{\rm qo} = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_4 & \tilde{s}_3 & \tilde{s}_2 & \tilde{s}_5 & \tilde{s}_8 & \tilde{s}_7 & \tilde{s}_6 \\ \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_4 & \tilde{s}_3 & \tilde{s}_6 & \tilde{s}_5 & \tilde{s}_8 & \tilde{s}_7 \\ \tilde{s}_3 & \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_4 & \tilde{s}_7 & \tilde{s}_6 & \tilde{s}_5 & \tilde{s}_8 \\ \tilde{s}_4 & \tilde{s}_3 & \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_8 & \tilde{s}_7 & \tilde{s}_6 & \tilde{s}_5 \\ -\tilde{s}_5^* & -\tilde{s}_8^* & -\tilde{s}_7^* & -\tilde{s}_6^* & \tilde{s}_1^* & \tilde{s}_4^* & \tilde{s}_3^* & \tilde{s}_2^* \\ -\tilde{s}_6^* & -\tilde{s}_5^* & -\tilde{s}_8^* & -\tilde{s}_7^* & \tilde{s}_2^* & \tilde{s}_1^* & \tilde{s}_4^* & \tilde{s}_3^* \\ -\tilde{s}_7^* & -\tilde{s}_6^* & -\tilde{s}_5^* & -\tilde{s}_8^* & \tilde{s}_3^* & \tilde{s}_2^* & \tilde{s}_1^* & \tilde{s}_4^* \\ -\tilde{s}_8^* & -\tilde{s}_7^* & -\tilde{s}_6^* & -\tilde{s}_5^* & \tilde{s}_4^* & \tilde{s}_3^* & \tilde{s}_2^* & \tilde{s}_1^* \end{bmatrix}. \tag{8}$$

It is easy to verify that the $\mathbf{X}_4^{\rm qo}$ in (8) is equivalent to the 4×4 quasi-orthogonal design in [21, Eq. (6)] by row and column permutations as well as relabelling the symbols. It can be also obtained from the design in [12, Eq. (5)] by row/column multiplication with -1, row/column permutation, and symbol relabelling. We will show in Sec. III-C that $\mathbf{X}$ in (7) will enable separate decodings on $\mathbf{s}_a$ and $\mathbf{s}_b$; hence, it is a quasi-orthogonal design. For this reason, we used the notation $\mathbf{X}^{\rm qo}$. For $N_t = 3, 4$, any N_t columns of $\mathbf{X}_4^{\rm qo}$ can be used as a space-time codeword. For $N_t = 5, 7, 8$, any N_t columns of $\mathbf{X}_8^{\rm qo}$ can be used as a codeword. For $N_t = 6$, one can either use 6 columns of $\mathbf{X}_8^{\rm qo}$, which is then not delay-optimal, or one can directly construct $\mathbf{X}_6^{\rm qo}$ that is delay-optimal. This delay-optimal $\mathbf{X}_6^{\rm qo}$ cannot be obtained using the recursive method in [18] or the Hadamard-code based method in [2].

The $\mathbf{X}$ in (7) is one design of *full-diversity modulus-preserving rate-one quasi-orthogonal* code.

B. Embedding 2×2 LCPs into OSTBC — The ABBA Quasi-Orthogonal Designs

We now consider embedding LCPs into general OSTBCs other than the Alamouti code, where rate-one designs are not possible. We only consider 2×2 LCPs, and will not include LCPs with larger size. With an OSTBC that consists of N_s complex symbols $\{x_1, \ldots, x_{N_s}\}$, we now replace each x_i by a 2×2 LCP in (3) based on $\mathbf{s}_i := [s_{2i-1}, s_{2i}]^T$.

Let us illustrate the construction with the rate 3/4 STBC with $N_s = 3$. We pick $P = 2N_s = 6$ symbols, and divide the vector $\mathbf{s} = [s_1, \ldots, s_6]^T$ into three 2×1 sub-vectors $\mathbf{s}_1$, $\mathbf{s}_2$, and $\mathbf{s}_3$. Generating the 2×2 LCP matrices from $\{\mathbf{s}_i\}_{i=1}^3$, and then embedding them into the orthogonal STBC, we obtain:

$$\tilde{\mathbf{X}}_8^{\mathrm{qo}} = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_2 & \tilde{s}_3 & \tilde{s}_4 & \tilde{s}_5 & \tilde{s}_6 & 0 & 0 \\ \tilde{s}_2 & \tilde{s}_1 & \tilde{s}_4 & \tilde{s}_3 & \tilde{s}_6 & \tilde{s}_5 & 0 & 0 \\ -\tilde{s}_3^* & -\tilde{s}_4^* & \tilde{s}_1^* & \tilde{s}_2^* & 0 & 0 & \tilde{s}_5 & \tilde{s}_6 \\ -\tilde{s}_4^* & -\tilde{s}_3^* & \tilde{s}_2^* & \tilde{s}_1^* & 0 & 0 & \tilde{s}_6 & \tilde{s}_5 \\ -\tilde{s}_5^* & -\tilde{s}_6^* & 0 & 0 & \tilde{s}_1^* & \tilde{s}_2^* & -\tilde{s}_3 & -\tilde{s}_4 \\ -\tilde{s}_6^* & -\tilde{s}_5^* & 0 & 0 & \tilde{s}_2^* & \tilde{s}_1^* & -\tilde{s}_4 & -\tilde{s}_3 \\ 0 & 0 & -\tilde{s}_5^* & -\tilde{s}_6^* & \tilde{s}_3^* & \tilde{s}_4^* & \tilde{s}_1 & \tilde{s}_2 \\ 0 & 0 & -\tilde{s}_6^* & -\tilde{s}_5^* & \tilde{s}_4^* & \tilde{s}_3^* & \tilde{s}_2 & \tilde{s}_1 \end{bmatrix}. \tag{9}$$

It is easy to see that $\tilde{\mathbf{X}}_8^{\mathrm{qo}}$ is equivalent to [19, eq. (30)] up to row and column permutations. This is not surprising, since the quasi-orthogonal codes in [19] are the ABBA codes of [10], [21]. The ABBA code has the form $\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix}$, where $\mathbf{A}$ and $\mathbf{B}$ are two OSTBCs based on two different sets of information symbols. Hence, the ABBA code embeds OSTBCs into a 2×2 LCP design. By row and column permutation, it is equivalent to the embedding of multiple 2×2 LCPs into an OSTBC that we presented here.

C. Separable Decoding and the Optimal Constellation Rotation

We illustrate the receiver processing with the codeword based on the Alamouti structure (7). Denote $\mathbf{y}_a$ and $\mathbf{y}_b$ as the received blocks corresponding to the first and second halves of the transmission. Suppose that we have P antennas, and we collect the channel coefficients into two $K \times 1$ vectors $\mathbf{h}_a$, and $\mathbf{h}_b$. With $\mathbf{w}_a$ and $\mathbf{w}_b$ denoting the additive noise, we have

$$\begin{bmatrix} \mathbf{y}_a \\ \mathbf{y}_b \end{bmatrix} = \begin{bmatrix} \mathbf{X}_a & \mathbf{X}_b \\ -\mathbf{X}_b^* & \mathbf{X}_a^* \end{bmatrix} \begin{bmatrix} \mathbf{h}_a \\ \mathbf{h}_b \end{bmatrix} + \begin{bmatrix} \mathbf{w}_a \\ \mathbf{w}_b \end{bmatrix}. \tag{10}$$

Taking the FFT of $\mathbf{y}_a$ and $\mathbf{y}_b$, and conjugating $\mathbf{F}_K\mathbf{y}_b$, we obtain:

$$\begin{bmatrix} \mathbf{F}_K\mathbf{y}_a \\ (\mathbf{F}_K\mathbf{y}_b)^* \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{D}(\tilde{\mathbf{h}}_a) & \mathbf{D}(\tilde{\mathbf{h}}_b) \\ -\mathbf{D}(\tilde{\mathbf{h}}_b^*) & \mathbf{D}(\tilde{\mathbf{h}}_a^*) \end{bmatrix}}_{:=\boldsymbol{\mathcal{D}}_{ab}} \begin{bmatrix} \boldsymbol{\Theta}_K\mathbf{s}_a \\ \boldsymbol{\Theta}_K\mathbf{s}_b \end{bmatrix} + \begin{bmatrix} \mathbf{F}_K\mathbf{w}_a \\ (\mathbf{F}_K\mathbf{w}_b)^* \end{bmatrix}, \tag{11}$$

where $\boldsymbol{\Theta}_K = \mathbf{F}_K\mathbf{D}(\alpha)$ from the results in Sec. III. The matrix $\boldsymbol{\mathcal{D}}_{ab}$ has orthogonal columns: $\boldsymbol{\mathcal{D}}_{ab}^{\mathcal{H}}\boldsymbol{\mathcal{D}}_{ab} = \mathbf{I}_2 \otimes \boldsymbol{\Lambda}_{\text{equ}}^2$, where

$$\boldsymbol{\Lambda}_{\text{equ}} = \left[\mathbf{D}^{\mathcal{H}}(\tilde{\mathbf{h}}_a)\mathbf{D}(\tilde{\mathbf{h}}_a) + \mathbf{D}^{\mathcal{H}}(\tilde{\mathbf{h}}_b)\mathbf{D}(\tilde{\mathbf{h}}_b)\right]^{\frac{1}{2}}. \tag{12}$$

Left-multiplying eq. (11) by a unitary matrix, we have:

$$\begin{bmatrix} \mathbf{z}_a \\ \mathbf{z}_b \end{bmatrix} = \underbrace{(\boldsymbol{\mathcal{D}}_{ab}^{\mathcal{H}}\boldsymbol{\mathcal{D}}_{ab})^{-\frac{1}{2}}\boldsymbol{\mathcal{D}}_{ab}^{\mathcal{H}}}_{\text{unitary matrix}} \begin{bmatrix} \mathbf{F}_K\mathbf{y}_a \\ (\mathbf{F}_K\mathbf{y}_b)^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Lambda}_{\text{equ}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_{\text{equ}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Theta}_K\mathbf{s}_a \\ \boldsymbol{\Theta}_K\mathbf{s}_b \end{bmatrix} + \begin{bmatrix} \mathbf{n}_a \\ \mathbf{n}_b \end{bmatrix}, \tag{13}$$

where the post-processing noise $[\mathbf{n}_a^T, \mathbf{n}_b^T]^T$ remains white. Hence, without loss of optimality, $\mathbf{s}_a$ and $\mathbf{s}_b$ can be decoded separately from

$$\mathbf{z}_a = \boldsymbol{\Lambda}_{\text{equ}}\boldsymbol{\Theta}_K\mathbf{s}_a + \mathbf{n}_a, \quad \mathbf{z}_b = \boldsymbol{\Lambda}_{\text{equ}}\boldsymbol{\Theta}_K\mathbf{s}_b + \mathbf{n}_b. \tag{14}$$

The orthogonality in the Alamouti structure enables the receiver to perform two separate decodings, each one having a reduced size.

Notice that the precoders are now applied on blocks of size $K = P/2$. Let us stick to the case $K = 2^d$, then the optimal α should be $\alpha = \pi/(2K)$. For the $\mathbf{X}_8^{\text{qo}}$, we have $K = 4$, $\alpha = \pi/8$.

Now, let us talk about the ABBA case in Section III-B of embedding 2×2 LCPs into general orthogonal STBCs. The receiver can be similarly constructed, leading to parallel decodings on multiple 2×1 blocks. In this case, $K = 2$, the optimal $\alpha = \pi/4$. Hence, after diagonal precoding, half of the symbols are drawn from the original constellation $\mathcal{A}$, and the other half from the rotated constellation $e^{-j\frac{\pi}{4}}\mathcal{A}$. This result is consistent with [19]. Hence, based on the links of QO-STBCs and embedded LCPs, the optimal design in [19] coincides with [23] with $K = 2$, when QAM constellations are considered.

D. The Space-Time-Frequency Interpretation

The system input-output relationship in (11) is identical to the counterpart when an STF block code is transmitted in a multi-antenna OFDM system [13]. Therefore, surprisingly, the QO-STBC is equivalent to an STF block code, although they both have appeared for a while in different application scenarios.

To be more specific, let us analyze the code in (7). It is easy to verify that

$$(\mathbf{I}_2 \otimes \mathbf{F}_K)\mathbf{X}_{\text{ab}}(\mathbf{I}_2 \otimes \mathbf{F}_K^{\mathcal{H}}) = \sqrt{N_t} \begin{bmatrix} \mathbf{D}(\boldsymbol{\Theta}_K\mathbf{s}_a) & \mathbf{D}(\boldsymbol{\Theta}_K\mathbf{s}_b) \\ -\mathbf{D}^*(\boldsymbol{\Theta}_K\mathbf{s}_b) & \mathbf{D}^*(\boldsymbol{\Theta}_K\mathbf{s}_a) \end{bmatrix}. \tag{15}$$

The right hand side of (15) is an STF block code with LCP precoding across K diagonal subchannels and Alamouti coding on each subchannel [13]. Essentially, the QO-STBC in (7) divides the P antennas into two groups. Each group of K antennas are converted into one virtual antenna with K channel taps through the circular delay diversity [15]. With FFT operation, each virtual antenna generates K flat channels in the frequency domain. The transmitter applies LCP across frequencies and Alamouti coding in space and time.

In summary, *QO-STBC can be viewed as the embedding of modulus-preserving LCP into OSTBC, and it can be interpreted as a space-time-frequency block code after FFT transformations*. This novel viewpoint builds explicit links among various existing approaches. This viewpoint is not available before. Our approach has the additional benefit that we clearly see how the decodings of individual blocks are separated via linear processing [c.f. (10)-(14)].

IV. Numerical Results

Recall that in the case of $N_t = 2^d$, our new construction is always preferred over the original LCP transmission, preserving the symbol modulus at no cost. We next present some numerical results with $N_t \neq 2^d$. We adopt QPSK constellations and use bit error rate (BER) as the figure of merit.

We use the sphere decoder in [16] on the receiver side. For $N_t = 3$, Fig. 1 compares the performance of the following four setups: i) the original diagonal LCP transmission with the optimized 3×3 precoder (listed in Sec. VI of [23]); ii) the modulus-preserving delay-optimal $\mathbf{X}_{3\times 3}$ in (4) with $\alpha = \pi/9$; iii) the modulus-preserving delay-non-optimal $\mathbf{X}_{4\times 3}$ (the first three columns of $\mathbf{X}_{4\times 4}$ in (4)); and iv) the first three columns of the QO-STBC $\mathbf{X}_4^{\mathrm{qo}}$ in (8). For $N_t = 5$, Fig. 2 compares the performance of the following four setups: i) the original diagonal LCP transmission with the 5×5 Vandermonde precoder constructed from the roots of $x^5 = 1 + j$ [23]; ii) the modulus-preserving delay-optimal $\mathbf{X}_{5\times 5}$ in (4), with $\alpha = 2\pi/35$ from [23]; iii) the first five columns of the modulus-preserving $\mathbf{X}_{8\times 8}$ (delay non-optimal); and iv) the columns $\{$1,2,3,5,6$\}$ of the quasi-orthogonal code $\mathbf{X}_8^{\mathrm{qo}}$ in (8). From Figs. 1-2, we observe:

- In the case of $N_t \neq 2^d$, if we keep the delay optimality with $P = N_t$, our alternative LCP transmission with modulus-preserving property could render slight performance loss relative to the original LCP transmission, as evidenced in Figs. 1 and 2.
- In the case of $N_t \neq 2^d$, it is beneficial to adopt our alternative LCP transmission with $P > N_t$, where P is a power of 2. This new transmission preserves the modulus and improves the performance, at the price of a slight increase in decoding complexity and delay.

- The quasi-orthogonal codes (with optimal constellation design) have better performance than the STBC based on LCP alone. The QO-STBC exploits the orthogonality in Alamouti structure as much as possible, so that the LCP encoding and decoding are applied to blocks with halved sizes. Both complexity reduction and performance improvement are achieved.

V. Conclusions

In this paper, we investigated full-diversity modulus-preserving rate-one linear STBC. We first presented an alternative construction for the linear constellation precoding based STBC, where we applied linear precoding judiciously in two steps: explicit unitary diagonal precoding on symbol blocks and implicit FFT precoding via circularly shifted transmissions over multiple antennas. Compared with the original LCP design, our proposed construction possesses two distinct features: i) it preserves the symbol modulus, and ii) it is flexible on the choices of the precode size, opening room for different performance tradeoffs. We then demonstrated that quasi-orthogonal STBCs can be constructed by embedding modulus-preserving LCP designs into orthogonal structures. And interestingly, QO-STBC can be interpreted as a space-time-frequency block code, which was originally developed for multi-antenna OFDM systems, up to FFT transformations. We thus offer a unified view that provides new insights into various existing techniques.

References

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[2] A. Boariu and D. M. Ionescu, "A class of nonorthogonal rate-one space-time block codes with controlled interference," *IEEE Transactions on Wireless Communications*, vol. 2, no. 2, pp. 270–276, Mar. 2003.

[3] M. O. Damen, K. Abed-Meraim, and J.-C. Belfiore, "Diagonal algebraic space-time block codes," *IEEE Transactions on Information Theory*, vol. 48, no. 3, pp. 628–636, Mar. 2002.

[4] G. J. Foschini and M. J. Gans, "Layered space-time architecture for wireless communication in a fading environment with using multiple antennas," *Bell Labs. Syst. Tech. J.*, vol. 2, pp. 41–59, Autumn 1996.

[5] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, "Simplified processing for high spectral efficiency wireless communication employing multi-element arrays," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 11, pp. 1841–1852, Nov. 1999.

[6] H. El Gamal and M. O. Damen, "Universal space-time coding," *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1097–1119, May 2003.

[7] G. Ganesan and P. Stoica, "Space-time block codes: a maximum SNR approach," *IEEE Transactions on Information Theory*, vol. 47, no. 4, pp. 1650–1656, May 2001.

[8] G. H. Golub and C. F. Van Loan, *Matrix Computations*, Johns Hopkins University Press, 3rd edition, 1996.

[9] B. Hassibi and B. M. Hochwald, "High-rate codes that are linear in space and time," *IEEE Transactions on Information Theory*, vol. 48, pp. 1804–1824, July 2002.

[10] A. Hottinen, O. Tirkkonen, and R. Wichman, *Multi-antenna Transceiver Techniques for 3G and Beyond*, John Wiley & Sons, 2003.

[11] J. Hou, M. Ho Lee, and J. Y. Park, "Matrices analysis of quasi-orthogonal space-time block codes," *IEEE Communications Letters*, vol. 7, no. 8, pp. 385–387, Aug. 2003.

[12] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Commun. Letters*, vol. 49, no. 1, pp. 1–4, Jan. 2001.

[13] Z. Liu, Y. Xin, and G. B. Giannakis, "Space-Time-Frequency Coded OFDM over Frequency-Selective Fading Channels," *IEEE Transactions on Signal Processing*, vol. 50, no. 10, pp. 2465–2476, Oct. 2002.

[14] X. Ma and G. B. Giannakis, "Full-diversity full-rate complex-field space-time coding," *IEEE Transactions on Signal Processing*, vol. 51, no. 11, pp. 2917–2930, Nov. 2003.

[15] X. Ma and G. B. Giannakis, "Space-Time-Multipath Coding using Digital Phase Sweeping or Block Circular Delay Diversity," *IEEE Transactions on Signal Processing*, vol. 53, no. 3, pp. 1121–1131, Mar. 2005.

[16] D. Pham, K. R. Pattipati, P. K. Willett, and J. Luo, "An improved complex sphere decoder for V-BLAST systems," *IEEE Signal Processing Letters*, vol. 11, no. 9, pp. 748–751, Sept. 2004.

[17] N. Sharma and C. B. Papadias, "Improved quasi-orthogonal codes through constellation rotation," *IEEE Transactions on Communications*, vol. 51, no. 3, pp. 332–335, Mar. 2003.

[18] N. Sharma and C. B. Papadias, "Full rate full diversity linear quasi-orthogonal space-time codes for any transmit antennas," *EURASIP J. on Applied Signal Processing*, no. 9, pp. 1246–1256, Aug. 2004.

[19] W. Su and X.-G. Xia, "Signal constellations for quasi-orthogonal space time block codes with full diversity," *IEEE Transactions on Information Theory*, vol. 50, no. 10, pp. 2331–2347, Oct. 2004.

[20] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.

[21] O. Tirkkonen, A. Boariu, and A. Hottinen, "Minimal nonorthogonality rate 1 space-time block code for 3+ tx antennas," in *Proc. IEEE 6th Int. Symp. Spread-Spectrum Techniques and Applications (ISSSTA)*, Sept. 2000, pp. 429–432.

[22] O. Tirkkonen and A. Hottinen, "Square-matrix embeddable space-time block codes for complex signal constellations," *IEEE Transactions on Information Theory*, vol. 48, no. 2, pp. 384–395, Feb. 2002.

[23] Y. Xin, Z. Wang, and G. B. Giannakis, "Space-time diversity systems based on linear constellation precoding," *IEEE Transactions on Wireless Communications*, vol. 2, no. 2, pp. 294–309, Mar. 2003.

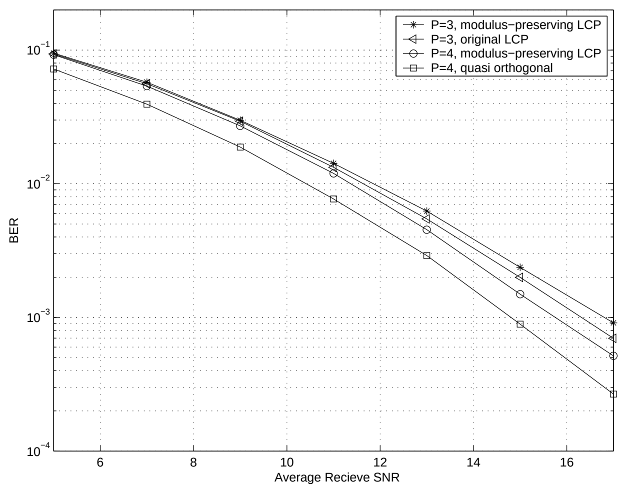

Fig. 1. The comparison results with $N_t = 3$

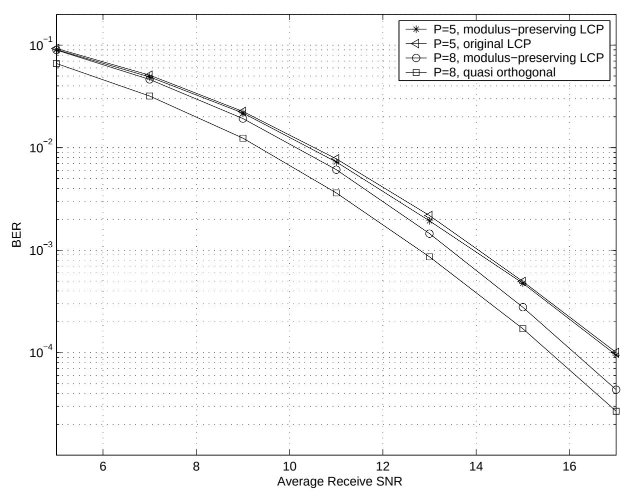

Fig. 2. The comparison results with $N_t = 5$