ACHORRIPSIS: A SONIFICATION OF PROBABILITY DISTRIBUTIONS

Edward Childs, Ph.D

Dartmouth College
The Bregman Electronic Music Studio
Electro-Acoustic Music
Department of Music
6187 Hopkins Center
Hanover, NH 03755-3599, USA
Edward.Childs@dartmouth.edu

ABSTRACT

The 1957 musical composition *Achorripsis* by Iannis Xenakis was composed using four different probability distributions, applied over three different organizational domains, during the course of the 7 minute piece. While Xenakis did not have sonification in mind, his artistic choices in rendering mathematical formulations into musical events (time, space, timbre, *glissando* speed) provide useful contributions to the "mapping problem" in three significant ways:

1. He pushes the limit of loading the ear with multiple formulations simultaneously.
2. His mapping of "velocity" to string *glissando* speed provides a useful method of working with a vector quantity with magnitude and direction.
3. His artistic renderings, ie. "musifications" of these distributions, invite the question, in general, as to whether musical/artistic sonifications are more intelligible to the human ear than sonifications prepared without any musical "filtering" or constraints (e.g. that they could be notated and performed by musicians).

1. INTRODUCTION

Xenakis was professionally involved with three distinct disciplines: music, architecture, and science and mathematics. In 1976, he received a "Doctorat d'État" from the Sorbonne [2], for his contributions in these three fields. He studied civil engineering at the Athens Polytechnic and later worked in Paris as an engineering assistant for Le Corbusier, who was so impressed with his work that he delegated architectural projects to him [3]. He went on to design the Phillips Pavilion at the 1958 World's Fair in Brussels. As a composer, Xenakis studied extensively with Olivier Messaien, who encouraged him to use his mathematical and engineering background in composition, and composed full time from 1960 [3]. His science and mathematical work went far beyond civil engineering into the kinetic theory of gases, probability theory and computer science. Many of his compositions were implemented through the use of computer programs. In the 1970's, Xenakis invented the UPIC [5] system which allows the user to create graphical designs on a tablet and have them rendered directly into sound. His last composition *O-Mega* was premiered in November, 1997 [3] and he died on February 4, 2001.

Achorripsis (Greek for "jets of sound"), composed in 1956-57, was first performed in Buenos Aires in August, 1958 under the direction of Herman Scherchen, who, until his death in 1963, championed Xenakis' music [7] [6]. The work received further performances in 1959 in Europe (to mostly scandalous reaction), and in the early 1960's in America under the direction of Gunther Schuller, Lukas Foss and Leonard Bernstein [7]. *Achorripsis* had a major success during the first all-Xenakis festival at the Salle Gaveau in Paris in 1965, performed by the Ensemble de Musique Contemporaine under the direction of Konstantin Simonovitch from which the only extant recording of the piece was made [6].

His inventions and music are controversial. Some critics suggest that his extensive writings on his own musics in *Formalized Music* [1], full of numbers and complex equations, are intentionally obscure.

> As far as Xenakis is concerned, let me emphasize at once that I'd be much more interested in his research if he hadn't set out so obviously to reduce its accessibility and its credibility in a manner which is immediately apparent as soon as you open his book on formal musics. *Pierre Schaeffer* [4]

Xenakis is regarded in some circles as "sloppy" in the practice of applying his mathematical expressions to the actual notes in his scores. In order to verify that Xenakis' procedures for *Achorripsis* were "faithful" to the statistical formulations, the author has examined two sections of the score in detail and compared them to Xenakis' own documentation in *Formalized Music*. The details are too lengthy to present here, but the conclusions will be presented in Section 2 of this paper. In Section 3, some excerpts from the musical score will be examined which illustrate how Xenakis "musified" the raw distributions. His compositional process could be likened to a "filtering" of data. Several MIDI sound examples have been prepared, as the basis for listening tests [11]. Finally, the use of string *glissando* speed is examined as a potentially rich mapping tool for vector quantities such as velocity which have both magnitude and direction.

2. ANALYSIS

2.1. Top Level Organization

The overall scheme for *Achorripsis* is shown in Fig. 1, and consists of a matrix of 28 columns (representing time blocks) and 7 rows (representing timbral classes of instruments). These timbral classes are:

1. *Flute* (Piccolo, E♭ Clarinet, Bass Clarinet)
2. *Oboe* (Oboe, Bassoon, Contrabassoon)
3. *String glissando* (Violin, Cello, Bass)
4. *Percussion* (Xylophone, Wood Block, Bass Drum)
5. *Pizzicato* (Violin, Cello, Bass)
6. *Brass* (2 Trumpets, Trombone)
7. *String arco* (Violin, Cello, Bass)

where the *italicized* entries are the **names** of the timbral classes and the parenthesized instruments are those that make up that class. In all, there are 3 Violins, 3 Cellos and 3 Basses, and all strings move back and forth over the course of the piece between *glissando*, *pizzicato* and *arco* passages.

The total length of the piece is set to be 7 minutes, which means that each of the 28 columns lasts 15 seconds. Each of the 28 time blocks of 15 seconds is set to be 6.5 measures in length, in which the time signature is $\frac{2}{2}$ with half note = MM 52. Thus each measure (two half notes) lasts $\frac{120}{52}$ seconds, and there are 182 measures in the score.

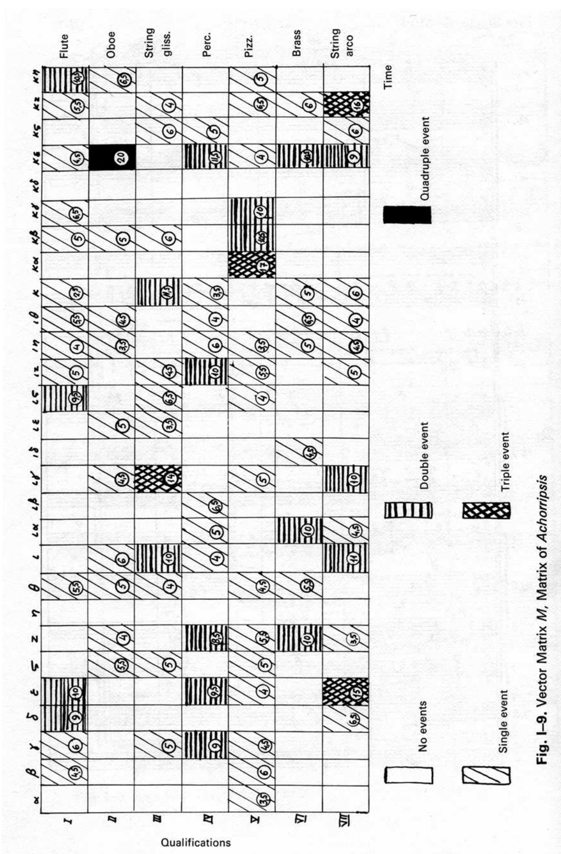

Figure 1: *The Matrix of Achorripsis* [1]

Xenakis decides how to allocate musical events to the $7\times 28 = 196$ cells of the matrix. To do this, he starts with the assumption that the average number of events/cell $\lambda = 0.6$.

He invokes the Poisson probability distribution, which is used for situations in which one wants to estimate how many instances of a particular event will occur in a given time or space.

In the case of *Achorripsis*, given the (artistic choice) that the average number of events per cell is 0.6, what is the probability that in any given cell there will be 0, 1, 2, 3, 4 or 5 events occurring?

This may be estimated using Poisson's formula:

$$P_k = \frac{\lambda^k}{k!}e^{-\lambda} \tag{1}$$

where k is the number of events ($k = 0, 1, 2, 3, 4, 5$ in this situation), e is the base of natural logarithms ($e = 2.71828\ldots$) and $k!$ (k factorial) for $k! = 5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$. Equation 1 is valid as long as $\lambda < 7$ [9]. By definition $0! = 1$. For example, the probability of 0 events occuring in a cell is:

$$P_0 = \frac{0.6^0}{0!}e^{-0.6} = 0.5488 \tag{2}$$

from which we see that in slightly over half of the cells, no events will be occurring: $196 \times 0.5488 = 107$. Applying this same procedure for $k = 1, \ldots, 5$, we find that the number of cells n_k in which k events occur is:

$$\begin{aligned} n_1 = P_1 \times N &= 0.3293 \times 196 = 65, \\ n_2 = P_2 \times N &= 0.0988 \times 196 = 19, \\ n_3 = P_3 \times N &= 0.0198 \times 196 = 4, \\ n_4 = P_4 \times N &= 0.0030 \times 196 = 1, \\ n_5 = P_5 \times N &= 0.0004 \times 196 = 0. \end{aligned} \tag{3}$$

where N is the total number of cells in the matrix, ie. $N = 196$.

2.2. Time Block Organization

Xenakis imposes an additional constraint on the distribution of the various event classes among his 196 cells. He decrees that the frequencies of zero, single, double, triple and quadruple events be statistically distributed amongst the 28 time blocks in accordance with Poisson's Law. Thus, the new "unit" or "cell" is now the time block.

Since there are a total of 65 single events distributed over 28 cells, the average number of single events per cell is now $65/28 = 2.32$, which becomes the new λ in the reapplication of Poisson's Law, so that the probability P_0 of no single events occurring in a cell (time block) is:

$$P_0 = \frac{2.32^0}{0!}e^{-2.32} = 0.09827 \tag{4}$$

so since there are 28 time blocks, the number $t_{0,single}$ in which no single events occur is $28 \times 0.09827 = 3$. We may now calculate the number of time blocks $t_{k,single}$ in which k single events occurs, $k = 1, \ldots, 7$:

$$\begin{aligned} t_{1,single} = P_1 \times T &= 0.22799 \times 28 = 6, \\ t_{2,single} = P_2 \times T &= 0.26447 \times 28 = 8, \\ t_{3,single} = P_3 \times T &= 0.20453 \times 28 = 5, \\ t_{4,single} = P_4 \times T &= 0.11862 \times 28 = 3, \\ t_{5,single} = P_5 \times T &= 0.05504 \times 28 = 2, \\ t_{6,single} = P_6 \times T &= 0.02128 \times 28 = 1, \\ t_{7,single} = P_7 \times T &= 0.00705 \times 28 = 0. \end{aligned} \tag{5}$$

where $T = 28$ is the total number of time blocks.

A similar procedure is used to distribute the 19 double events, 4 triple events and single quadruple events over the 28 time blocks. The "Top Level" and "Time Block" organizational levels have recently been analyzed in the context of Game Theory [12].

2.3. Cell Level Organization

At this stage, Xenakis turns his focus to the generation of events at the note level in each of the 196 cells of *Achorripsis*. He states the theoretical basis for the calculation of:

1. The time between successive events (ie. notes).
2. The interval between successive pitches.
3. The "speed" of the *glissandi* in the string *glissando* cells.

Note that he does not address other aspects of the score such as:

1. The starting pitches for each instrument in each cell.
2. The duration of each note.
3. Dynamics.
4. Articulation. (String *arco* passages have some *staccato* notes, brass and woodwind have no articulation, and there are no accents).
5. The timbral choices.

Xenakis chooses the following statistical distributions:

1. The *exponential distribution* is used to govern the time between successive events.
2. The *linear distribution* is used to govern the intervals between successive pitches.
3. The *normal distribution* is used to govern *glissando* "speed".

2.3.1. Exponential Distribution

Squibbs has provided an excellent overview of Xenakis' general use of these statistical distibutions in his Ph.D. thesis [8], and has simplified some of the notation found in Xenakis [1]. Squibbs' versions will be used in this paper. The distribution for the time between successive notes is then:

$$P_i = e^{-\delta iv}(1 - e^{-\delta v}) \tag{6}$$

for $i = 0, 1, 2, \ldots$, v is the size of the time range and P_i is the probability that the time between events will fall within the given time range iv. Xenakis chooses a time range of 0.1 measure, which would be $\frac{12}{52}$ seconds. δ is the average number of sounds per measure, and corresponds to the circled numbers in each cell in Fig. 1. In the scheme of *Achorripsis*, a δ at or near 5.0 corresponds to a "single event", $\delta = 10$, a double event, etc.

2.3.2. Linear Distribution

Squibbs [8] provides a simplified version of Xenakis' formulation of the linear distribution, which governs the intervals between successive pitches in *Achorripsis*. This type of distribution is often applied to non-temporal situations.

To constrain the size the intervals between the pitches of successive entrances, Xenakis uses (in the Squibbs simplification, [8], p. 86):

$$P_i = \frac{2}{n+1}(1 - \frac{i}{n}) \tag{7}$$

where $i = 0, 1, 2, \ldots, n$. $n = \frac{g}{v}$, where g is maximum interval size and v is the interval increment to be used in preparing the table of probabilities P_i.

2.3.3. Normal Distribution

Xenakis' use of *glissando* strings occurred first in *Metastasis* (*arco*) and next in *Pithoprakta* (*pizzicato*). In his analysis of *Pithoprakta*, he relates the distribution of "speeds" of the *glissandi* (change in pitch df divided by time increment dt, $\frac{df}{dt}$) to the distribution "speeds" of gas molecules, as derived by Maxwell/Boltzmann. He carries over the analogy to *Achorripsis*, thereby, in a sense, "mapping" the concept of velocity to string *glissandi*. It turns out that the distribution of speeds in a gas follows the Gaussian or Normal distribution, which is slightly more complicated mathematically than the Poisson, Exponential or Linear distributions. First, the probability density function $f(v)$ for the existence of a speed v is:

$$f(v) = \frac{2}{\alpha\sqrt{\pi}} e^{\frac{-v^2}{\alpha^2}}, \tag{8}$$

where α is defined as the "quadratic mean of all possible values of v" [1], p. 32, and is related to the temperature of the gas. Equation 8 does not yield the value of the probability directly. The area bounded by the x-axis, $f(v)$, vertical lines $x = v_1$ and $x = v_2$ is the probability $P(\lambda)$ that a given velocity v will fall within the range v_1 to v_2 ($v_2 > v_1$). The numerical value for this area may be obtained by integrating Equation 8 between the limits 0 and λ_1, and then again between 0 and λ_2, and subtracting the first value from the second. $\lambda_1 = \frac{v_1}{\alpha}$, $\lambda_2 = \frac{v_2}{\alpha}$:

$$P(\lambda) = \theta(\lambda_2) - \theta(\lambda_1), \tag{9}$$

where

$$\theta(\lambda) = \frac{2}{\sqrt{\pi}} \int_0^{\lambda} e^{-\lambda^2} d\lambda. \tag{10}$$

2.4. Analysis Conclusions

A thorough analysis and comparison of theory with the score was carried out in two cells (Cell III ιz and Cell V α), but the results are too lengthy to reproduce here. The general conclusions are:

1. Xenakis was most rigorous is applying the exponential distribution to the time between events, and less so in applying the linear distribution to the intervals between pitches and the the normal distribution to *glissando* speeds.
2. Clearly his intent was to compose music, so some artistic adjustments were made to his distribution results. However, he followed them closely enough so that they can be usefully examined from the standpoint of sonification.
3. The preparation of the score was a remarkable feat considering that he worked without the help of a computer, but calculated all distributions, and their musical implementation, by hand.

3. PERCEPTION

Achorripsis could be perceived as a sonification of events occurring in real time. Each of the timbral classes could be considered as a separate data stream (for a total of seven). The data could represent any phenomenon governed by the probablility distributions similar to those chosen by Xenakis. In this scenario, every musical event would represent one data point or event. Each 15 second cell would then represent a group of data points which conform to a specific statistical distribution (for timing, pitch change, and (when specified) *glissando* speed in the strings.

It is probably easiest to first focus on the timing of the events, which is governed in *Achorripsis* by the exponential distribution, see Equation 6, and the associated explanation. It is critical to realize that while Equation 6 provides for the timing of events to fall within a continuum, the Xenakis realization uses only discrete values, based on the durations of notes which he chooses to use in the score, see Fig. 2. The basic note values, and their durations, are expressed as the numerator of a fraction with 52 as the denominator (to correspond to the tempo of MM = 52). To realize an exponential distribution of time between events, Xenakis draws from a rhythmic "palette" of 2 against 3 against 4 against 5. The smallest time between events is $\frac{3}{52}$ seconds, which occurs between successive entrances of an eighth note and a quintuplet eighth note.

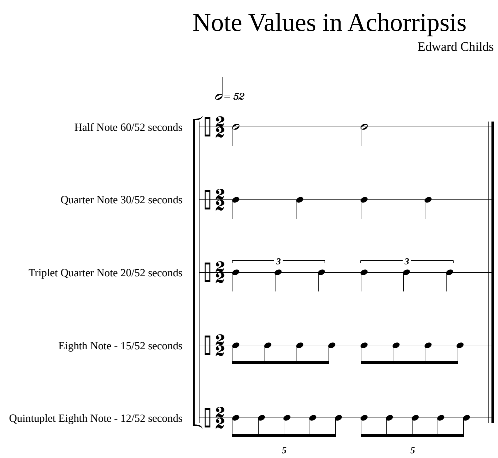

Figure 2: *Note Values in Achorripsis*

The theoretical and actual event timings for Cell Vα are shown in Table 1. The corresponding section of the score is shown as Fig. 3.

Duration	Δt	Score	Exp. Distribution
0 - 12	4, 10, 12, 12, 12, 12	6	6
12 - 24	20, 20, 20, 24	4	4
24 - 36	30, 30	2	3
36 - 48	42, 48, 48	3	2
48 - 60	60, 60	2	2
60 - 72	72	1	1
72 - 84	80	1	1
84 - 96	96	1	1
	Totals	21	21

Table 1: Comparison of Theory with Data: Δt, Cell Vα

In the column of Table 1 labeled Δt, is a list of all the "times between events" which occur in Cell Vα (21 in total), tabulated by duration range. So, e.g., in the first row, the exponential distribution calls for 6 Δt's in the range 0 - 12. Examining the score, and calculating the time between entrances of the 22 notes, we find six discrete values of Δt in the range 0 - 12: 4, 10, 12, 12, 12, 12, where, e.g., 4 implies $\frac{4}{52} = 0.0769$ secs. This value is the time between the entrance of the $D\sharp$ in the Contrabass, and the $C\sharp$ in the Violoncello, in the first measure (third and fourth entrances in that measure).

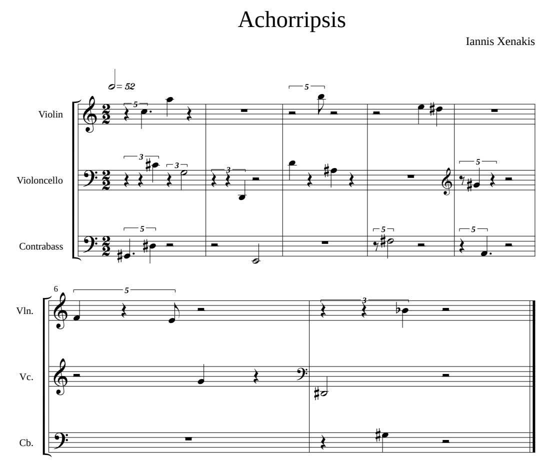

Figure 3: *Cell Vα*

If the musical constraints were absent, any 6 duration values in the (continuous) range 0 - 12 would still satisfy the exponential distribution. The use of discrete values, drawn from the "palette" of note values (Fig. 2), results in a discernible rhythmic signature, which corresponds to the density $\delta = 3.5$ (average number of events per measure) chosen for this cell (see Fig. 1).

Since all 89 "active" cells each have a distinct value for the density δ (the circled numbers in Fig. 1) it is tempting to speculate that the use of discrete note values makes it easier (or at least more enjoyable) for the listener to recognize, from the rhythm of the cell, what its density is. To test this speculation, a "game" will be set up, as a preliminary listening test, at the author's web site [11] in which the player will attempt to associate soundfiles from individual cells in *Achorripsis* with the displayed matrix cells. Soundfiles which satisfy the exponential distribution but use a continuous range of Δt, will be available for comparison.

A follow up to these tests would be to determine if it is easier for a listener to track two or more streams more easily with the discrete rhythmic configuration (Xenakis has, at most, six going simultaneously). How many different streams could the listener be expected to track? From a musical standpoint, *Achorripsis* can be performed under a good conductor, who is able to hear all of these events and determine whether or not a mistake has been made.

Xenakis' use of the linear distribution for the intervals between successive pitches is more difficult to correlate with the actual notes in the score. Furthermore, these distributions do not appear to be affected by parameters which change from one cell to the other. That is, other than constraints imposed by the ranges of the instruments, this linear distribution appears to be the same in all cells, and thus has a "neutral" influence, providing more of a vehicle for experiencing the rhythmic events.

Xenakis makes a useful contribution to the "mapping problem" by choosing to map the concept of velocity to *glissando*. In this mapping, the rate of change of the pitch (in, say, semitones per measure) is used to represent some sort of velocity (in, say, meters per second). This choice leaves open the possibility of representing a vector with magnitude and direction. The *glissando* speed represents the magnitude of the vector. A direction to the "right" could be represented by an upward *glissando*, to the "left", downward. "Up" and "down" could be represented by exponential shaping of the *glissando*. Once this has been done, the actual starting pitch of the glissando, the timbre and the register are still "free" to convey additional information.

4. CONCLUSIONS

The score of *Achorripsis* adheres closely enough to the composer's stated statistical distributions, especially in the time domain to be considered a useful contribution to the "mapping problem" in sonification. The work is challenging in that a lot of information is being conveyed in each cell, and so probably pushes the loading of sound to the limit. His musical rendering of these distributions may make them easier to grasp. Listening tests are needed to test this thesis. The mapping of *glissando* speed to velocity has useful properties for conveying vector quantities with both magnitude and direction, for example in the sonification of computational fluid dynamics [10].

5. REFERENCES

[1] Iannis Xenakis, *Formalized Music: Thought and Mathematics in Music*, Pendragon Press, Hillsdale, NY, 1992.

[2] Iannis Xenakis, tr. Sharon Kanach, *Arts/Sciences: Alloys, The Thesis Defense of Iannis Xenakis*, Pendragon Press, New York, NY, 1985.

[3] James Harley, "Iannis Xenakis Bibliography and Discography" *LEONARDO on-line*, http://mitpress2.mit.edu/e-journals/Leonardo/isast /spec.projects/Xenakisbib.html

[4] Pierre Schaeffer, "Music and Computers", in *Music and Technology, Stockholm Meeting June 8-12, 1970, organized by UNESCO*, La Revue Musical, Paris, 1970.

[5] CCMIX - Center for Composition Iannis Xenakis, *formerly Les Ateliers UPIC*, http://www.ccmix.com

[6] Nouritza Matossian, *Xenakis*, Kahn & Averill, London, Pro/Am Music Resources Inc., White Plains, New York, 1991.

[7] Bálint András Varga *Conversations with Iannis Xenakis* faber and faber, 3 Queen Square London WC1N 3AU, 1996.

[8] Ronald J. Squibbs, *An Analytical Approach to the Music of Iannis Xenakis: Studies of Recent Works* A Dissertation Presented to the Faculty of the Graduate School of Yale University in Candidacy for the Degree of Doctor of Philosophy, November 1996, UMI Microform 9714309, 300 North Zeeb Road, Ann Arbor, MI 48103.

[9] William Mendenhall and Robert J. Beaver, *Introduction to Probability and Statistics*, Duxbury Press, Belmont, California, 1994.

[10] Edward Childs, "The Sonification of Numerical Fluid Flow Simulations", in *Proceedings of the 2001 International Conference on Auditory Display, Espoo, Finland, July 29 - August 1, 2002*

[11] http://eamusic.dartmouth.edu/ ed/research.html

[12] Linda Arsenault, "Iannis Xenakis's *Achorripsis*: The Matrix Game", *Computer Music Journal*, Vol. 26, No. 1 (Spring 2002), 58-72.