SOLID MECHANICS

DYNAMICS

TUTORIAL - GYROSCOPES

This work covers elements of the syllabus for the Engineering Council Exam D225 – Dynamics of Mechanical Systems.

This tutorial examines linear and angular motion. The work is then linked with earlier studies of materials and mechanisms to enable you to solve integrated problems.

On completion of this tutorial you should be able to

- Describe a gyroscope.
- Define angular momentum.
- Derive the formula for gyroscopic torque.
- Solve problems involving gyroscopic torque
- Define precession.

It is assumed that the student is already familiar with angular motion, the relationship between angular and linear motion and the way angular quantities may be represented by a vector.

1. GYROSCOPES

A Gyroscope is a spinning disc mounted so that it may pivot in the x, y and z axis.

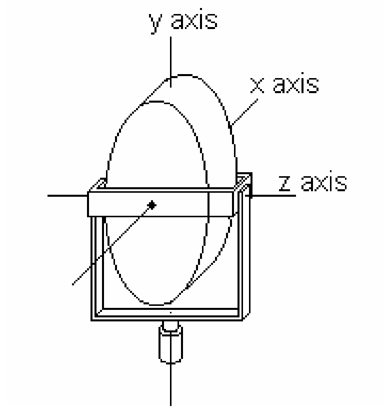

Figure 1

Let's revise the rule for representing angular quantities as vectors. This is the corkscrew rule. Point the index finger of the right hand in the direction of the vector. The angular quantity has a rotation clockwise as if doing up a corkscrew.

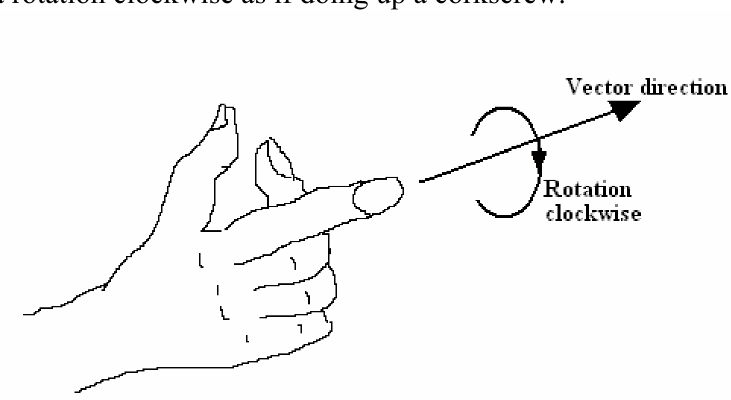

Figure 2

Now consider a disc spinning about the x axis as shown.

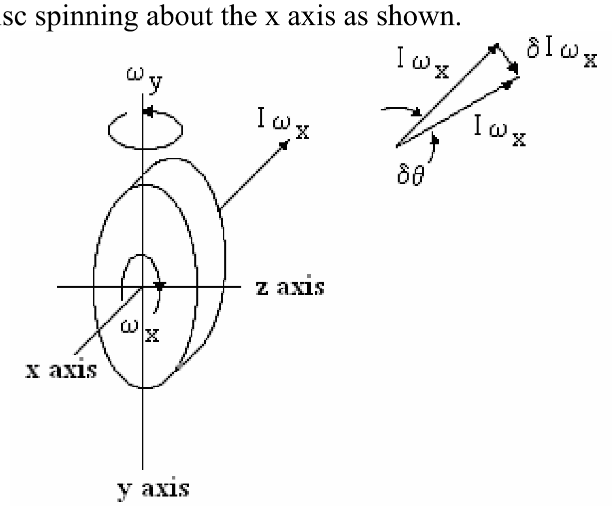

Figure 3

The angular momentum of the disc is $I\omega_X$. This is a vector quantity and the vector is drawn to scale with a direction conforming to the corkscrew rule. This is shown on the diagram.

Suppose the disc also rotates about the y axis as shown through a small angle $\delta\theta$. The vector for $I\omega_X$ changes direction but not magnitude. This produces a change in the angular moment of $\delta I\omega_X$.

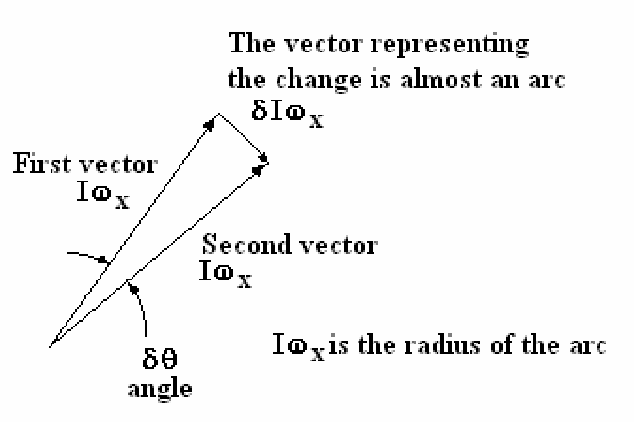

Figure 4

The vector diagram conforms to the vector rule, first vector + change = final vector. The change is the arrow going from the tip of the first to the tip of the second so the direction is as shown.

The vector representing the change is almost an arc of radius $I\omega_X$ and angle $\delta\theta$. The length of the arc is the product of radius and angle. Taking the radius as $I\omega_X$ and the angle as $\delta\theta$ the change is $\delta I\omega_X = I\omega_X\,\delta\theta$.

Newton's second law of motion applied to rotating bodies tells us that the change in momentum can only be brought about by applying a torque.

Torque = rate of change of angular momentum.

If the rotation occurred in time δt seconds, the rate of change of momentum is

$$\text{Change in momentum per second} = I\omega_x \frac{\delta\theta}{\delta t} \text{ and if the change is at a constant rate}$$

$$\frac{\delta\theta}{\delta t} \text{ is the angular velocity about the y axis } \omega_y$$

$$\text{Change in momentum per second} = I\omega_x \omega_y \omega_y$$

Hence the torque required to produce the change in direction is $\mathbf{T = I\,\omega_x\,\omega_y}$

This is the torque that must be applied to produce the change in angle and the direction of the vector is the same as the change in momentum. The applied torque may hence be deduced in magnitude and direction. Examining the vector for this torque we can deduce that it applied about the z axis.

If the torque is applied about the z axis, the result will be rotation about the y axis and this is called precession.

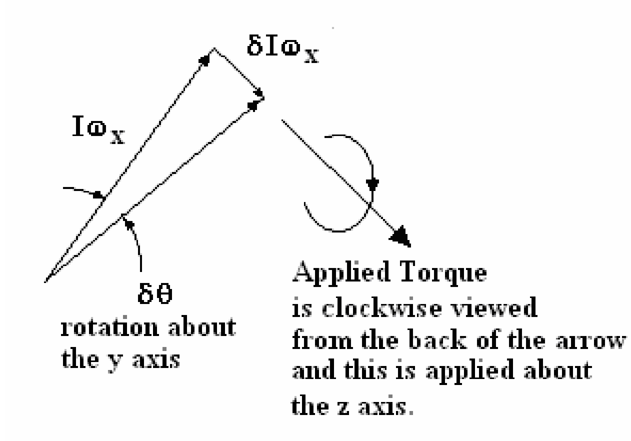

Figure 5

If the torque is not applied and the rotation is made to happen (applied), a reaction torque will be produced (Newton's 3 rd. law) and the disc will respond to the reaction torque.

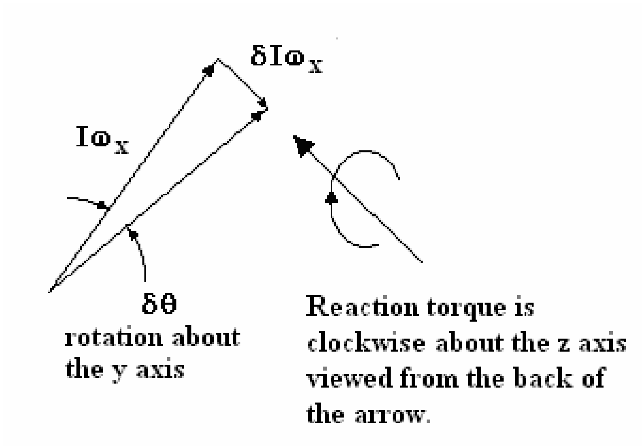

Figure 6

A gyroscopic torque may occur in any machine with rotating parts if a change in the direction of the x axis occurs. Examples are aeroplanes, ships and vehicles where a gyroscopic torque is produced by the engines when a change is made in the course.

WORKED EXAMPLE No.1

A cycle takes a right hand bend at a velocity of v m/s and radius R. Show that the cyclist must lean into the bend in order to go around it.

SOLUTION

First remember that the velocity of the edge of the wheel must be the same as the velocity of the bike so $\omega_x = v/r$ where r is the radius of the wheel.

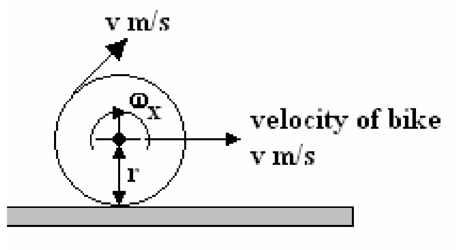

Figure 7

The angular velocity of the bike about the centre of the bend is $\omega_y = v/R$

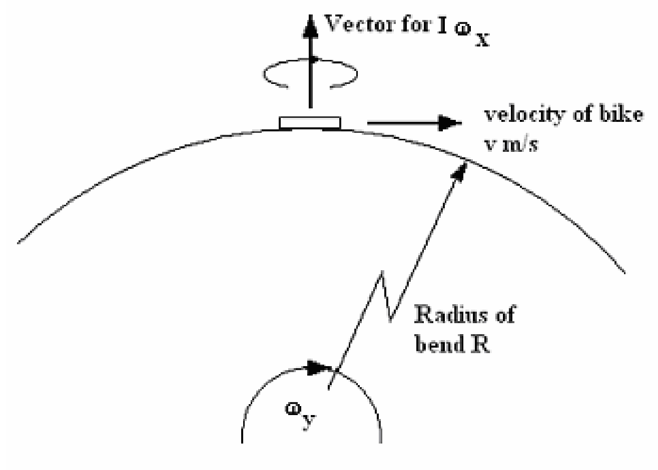

Figure 8

Now draw the vector diagrams to determine the change. As the wheel goes around a right hand bend the direction of the vector for ω_x changes as shown. The applied torque is a vector in the same direction as the change so we deduce that the torque must act clockwise viewed from behind the cyclist. This torque may be produced be leaning into the bend and letting gravity do the job. By applying this torque, the wheels must precess in the correct direction to go round the bend.

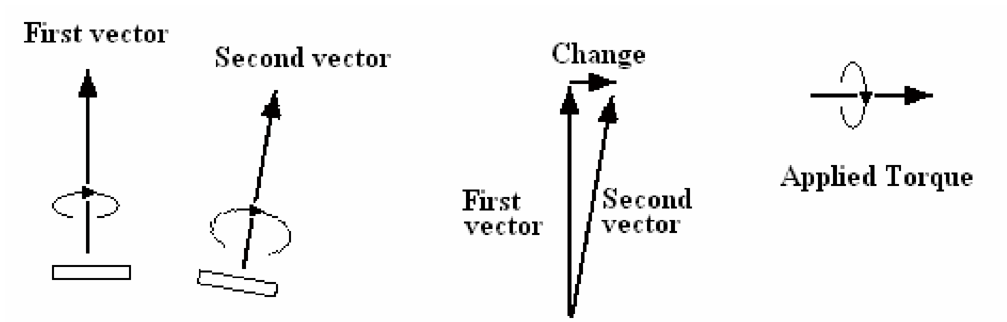

Figure 9

If the cyclist steers around the bend by turning the handlebars, then the reaction torque would throw him over outwards (anticlockwise as viewed from the back).

WORKED EXAMPLE No.2

A ship has its turbine engine mounted with its axis of rotation lengthways in the ship. The engine rotates clockwise at 6000 rev/min when viewed from the back. The effective rotating mass of the engine is 900 kg with a radius of gyration of 0.5 m.

Calculate the magnitude of the gyroscopic couple produced when the ship turns right on a radius of 300 m with a velocity of 2.2 m/s. Explain clearly the effect of the couple on the ships motion.

SOLUTION

The essential quantities are N = 6000 rev/min
M = 900 kg   k = 0.5 m   v = 2.2 m/s R = 300 m
First calculate the angular velocity about the x axis.

$$\omega_x = \frac{2\pi N}{60} = \frac{2\pi \text{ x } 6000}{60} = 628.3 \text{ rad/s}$$

Next calculate the angular velocity of the precession.

This is found from the linear velocity.

$$\omega_y = \frac{\text{linear velocity}}{\text{radius}} = \frac{2.2}{300} = 7.333 \text{ x } 10^{-3} \text{ rad/s}$$

Next find the moment of inertia.

$$I = Mk^2 = 900 \text{ x } 0.5^2 = 225 \text{ kg m/s}$$

Now find the gyroscopic torque.

$$T = I\omega_x\omega_y = 225 \text{ x } 628.3 \text{ x } 7.333 \text{ x } 10^{-3} = 1036.7 \text{ Nm}$$

Now sketch the motion of the ship taking a right turn. Draw the vectors for the angular momentum using the corkscrew rule. The rotation is clockwise viewed from the back so the vector must point forward as shown.

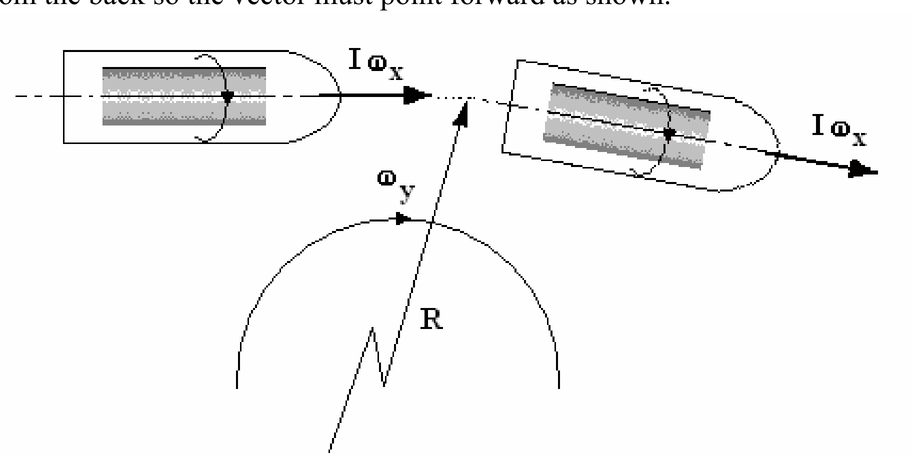

Figure 10

Now deduce the effect of the torque.

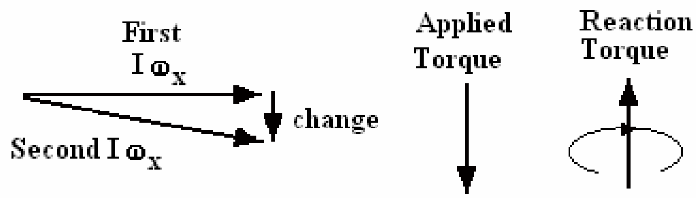

Figure 11

The vectors for the angular momentum are drawn as shown and the direction of the change is deduced (from the tip of the first to the tip of the second). The reaction torque is the opposite direction as shown (up on the diagram). From this we apply the corkscrew rule and deduce that the torque produces a rotation such that the bow of the ship dips down and the stern comes up. The magnitude of the torque is 1037.7 Nm.

WORKED EXAMPLE No.3

The engine of an aeroplane rotates clockwise when viewed from the front. The moment of inertia of all the rotating parts is 300 kg m^2. The engine rotates at 1200 rev/min.

Determine the magnitude and effect of the gyroscopic action resulting when the aeroplane makes a right hand bend of radius 5000m at a speed of 1500 km/h.

SOLUTION

The essential data is as follows.

I = 300 kg m^2
N = 1200 rev/min. Convert to rev/s, N = 1200/60 = 20 rev/s
v = 1500 km/h, convert to m/s v = 1500 000/3600 = 416.7 m/s
R = 5000 m

$\omega_x = 2\pi N = 2\pi \times 20 = 125.67$ rad/s
$\omega_y = v/R = 416.7/5000 = 0.08334$ rad/s
$T = I\, \omega_x\, \omega_y = 300 \times 125.67 \times 0.08334 = 3142$ Nm

Now sketch the motion of the aeroplane making a right turn. Draw the vectors for the angular momentum using the corkscrew rule. The rotation is clockwise viewed from the front so the vector must point backwards as shown.

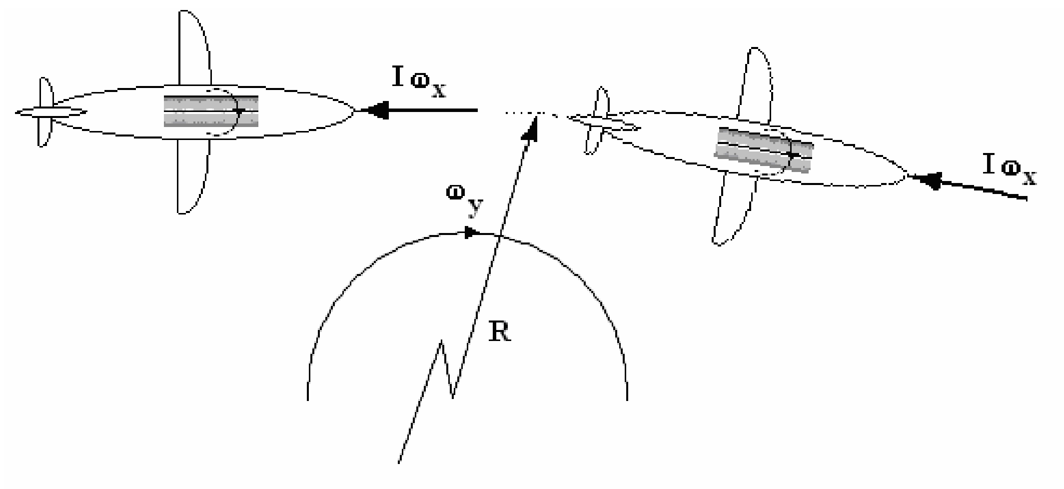

Figure 12

Now deduce the effect of the gyroscopic torque.

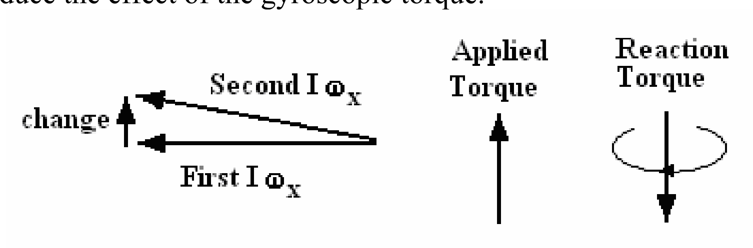

Figure 13

The change in angular momentum is a vector vertical as drawn above. The applied torque is a vector in the same direction and the reaction torque is the opposite (down on the diagram). Applying the corkscrew rule, the reaction torque would tend to lift the nose and depress the tail (pitch). The magnitude is 3142 N m.

Note that if a torque was applied so that the nose pitched down, the aeroplane would precess and make a right turn.

SELF ASSESSMENT EXERCISE No.1

1. A ship has an engine that rotates clockwise when viewed from the front. It has an effective moment of inertia of 250 kg m^2 and rotates at 400 rev/min. Calculate the magnitude and effect of the gyroscopic torque when the ship pitches bow (front) down at a rate of 0.02 rad/s.
   (Answer 209.4 Nm causes the boat to yaw to the left.)

2. A motorcycle travels at 80 km/h around a left bend of radius 30 m. The wheels have an outer diameter of 0.5 m and a radius of gyration of 240 mm. Calculate the following.
   i. The angular velocity of the wheels. (88.89 rad/s)
   ii. The moment of inertia of each wheel. (0.161 kg m^2)
   iii. The magnitude of the gyroscopic torque produced on the bike. (21.22 Nm)

   Predict the affect of this torque.
   (It tends to tip the bike over to the outside of the bend)

3. Explain with the aid of vector diagrams why a motorcyclist going around a left bend must lean into it in order to go round it.