ABOUT POSSIBILITY THE VECTOR'S PARABOLIC ANTENNA DP TO IMPROVE RADAR RESOLUTION

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Abstract

The resolution of contemporary parabolic radars as well as radars under development with various scanning patterns are defined by the width of their directionality patterns (DP). Usually these DPs are calculated by the field integration at the parabolic mirror's aperture, and only the real part of DP is taken in consideration. The simple formula $q \approx 70(l/D)$, evaluates the width of such DP when a radiator is at the mirror's focus. But in a real situation, when the radiator is displaced from focus and the classic Gyugens-Kirchhoff approach is used for the field integration, the resulting DP becomes a complex function due to significant phase change. Therefore the directionality pattern of the scanning parabolic antenna should be described by vector function (Vector DP). Vector DP have much larger maximum scanning angle than conventional "amplitude" DP, especially for an asymmetrical parabolic antenna.

In this article several examples of results for calculated and experimental vector DPs are presented and they practically coincide.

The usage of vector DP can vastly improve the parabolic radar resolution. This is especially important for development of large range early warning radars.

Key words: Radar, Parabolic Antenna, Vector DP, and Super-Resolution.

Introduction

This article presents some DP calculation results from the *Calculation of parabolic antennas*[1] published in Russia, in particular, for DP of parabolic antennas with various beam-scanning methods when the radiator is displaced from a parabola's focus. The goal was to reach desired radar specifications, using computer simulation. To solve this problem, one has to determine the relationship between the antenna's location parameters (the scanning angle, side beam's, level

of beam dilation, decrease in amplification) and its technical parameters (the radiator displacement from the focus, the parabolic antenna size to its focus ratio, the required level of illumination of the mirror's edge area).
This article gives a brief description of the two methods for vector DP calculations. The first one is based on integration of the field at the mirror's aperture when the radiator is displaced from focus. The second one is based on the integration of the currents at the mirror surface induced also when the radiator is displaced from focus. The calculations were made for the regions of the main maximum and near side lobes for both symmetrical and asymmetrical antennas.

The first part presents the results for parabolic antennas with scanning angles of no more than 10 degrees. Within this range it is possible to calculate DP by the field integration at the reflector's aperture as is proved by the comparison of the calculation and the experimental results. Besides that, the mutual influence of DP between horizontal and vertical planes may be neglected. This means that the 3-D problem of DP calculation can be reduced to a 2-D problem. In addition, one can get both the amplitude and the phase field components, which are amplitude and phase DP. All these allow characterizing the far-field zone by the Vector DP (VDP), which can much better utilize the antenna's potential. And again, there are a few examples of calculated and experimental amplitude and phase DP and VDP. It is also shown that VDP, with the same displacement from focus, provides much larger scanning angle than conventional amplitude DP.

The second part presents the results of the induced current integration at the mirror surface for cases when the radiator is in and out of focus for both main and parasitic polarizations. For instance, when DP is presented in isolines, one can see field deformation shaped like a comma. All calculations were made in Mathematical Institute of Russian Academy of Science using the computer designed by academician C.A. Lebedev.
The calculation algorithm was elaborated by I. Belova, the coauthor of *Calculation of parabolic antennas*[1].

Part 1: The calculation of a parabolic antenna DP by integrating the field at the reflector's aperture.

The parabolic antenna beam pattern for the main maximum and nearest side lobes can be described in Kirchhoff's approximation by the following formula:

$$g(q) = \int_{x_1}^{x_2} F(x) e^{j[kx \sin q - y(x)]} dx \qquad (1)$$

where q - is the angle that shows the direction to the distant point relative to the antenna axis, normal to the axis x; $x_2 - x_1 = D$ – the parabolic reflector diameter; $k=2p/l$. For the symmetrical reflector $|x_2| = |x_1| = D/2 = x_{\max}$. For asymmetrical, $x_{\max} = x_2$.

The solution for this equation (1) is obtained by the numerical integration for many peculiar cases. This enables to plot the generalized DP for scanning parabolic antennas when the radiator has small offset from the mirror focus.

For these calculations two coordinate systems are used, as shown on Figure 1. Cylindrical coordinates r, α, with polar axis z, going through the focus O_1, and orthogonal coordinates ξ, z with the zero point at the parabolic reflector apex.

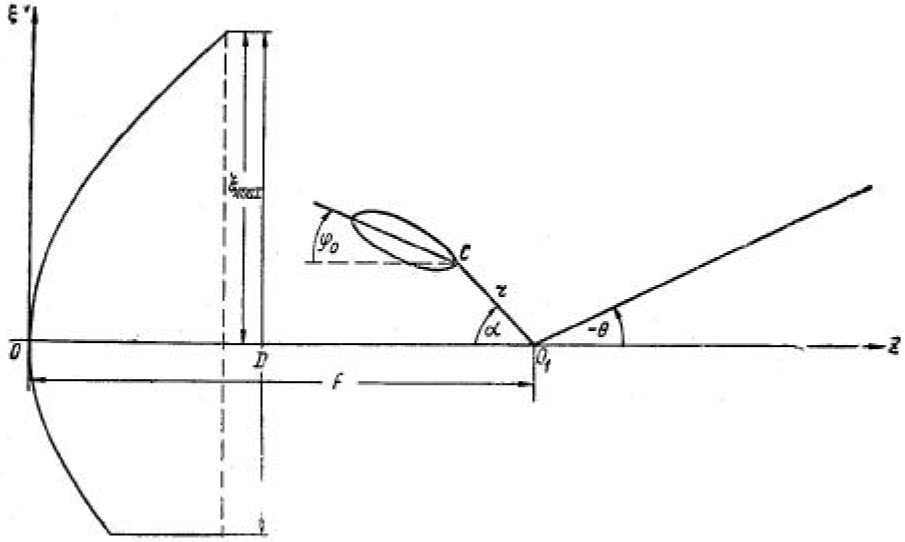


Figure 1: The coordinate systems were used in calculations

Let us put the radiator at the point C, with the beam pattern maximum direction under the angle j_0 to the axis z. Let us put in dimensionless values: $r/F=a$; $x_{max}/F=b$, new variable $?=x/\xi_{\max}$ and denote $u/p = (D/l)\sin q$. Then the integral (1) can be written as:

$$g(u/p) = \int_{x_i = \frac{x_1}{x_{\max}}}^{1} F(x) e^{j[ux - y(x)]} dx \qquad (2)$$

For numerical integration, the integral (2) is presented by:

$$g(u/p) = A(u/p) e^{j\Phi\left(\frac{u}{p}\right)} \Delta x$$

Then the far-field amplitude according *Calculation of parabolic antennas*[1] is equal to:

$$A(u/p) = \Delta x \sqrt{\left\{\sum_{i=1}^{n} F(x_i)\cos[ux_i - y(x_i)]\right\}^2 + \left\{\sum_{i=1}^{n} F(x_i)\sin[ux_i - y(x_i)]\right\}^2} \quad (3)$$

and the far-field phase:

$$\Phi(u/p) = \arctan \frac{\sum_{i=1}^{n} F(x_i)\sin[ux_i - y(x_i)]}{\sum_{i=1}^{n} F(x_i)\cos[ux_i - y(x_i)]} \quad (3a)$$

Here: $x_1 = x_1/x_{\max}$; $x_i = x_{i-1} + Dx$, … $x_n = 1$; $n = D/Dx$ – the number of the point of summing.
It was shown that for the DP calculation at different n, at the main maximum, and at nearby side lobes for $D \approx 100l$, it is sufficient to calculate $A(u/p)$ and $\Phi(u/p)$ with intervals $Dx \cong 2{,}5l$. For smaller antenna calculation intervals Dx needs to be smaller to provide an $n \geq 40$. For the determination of $F(x)$ and $y(x)$ let us use the Figure 2.

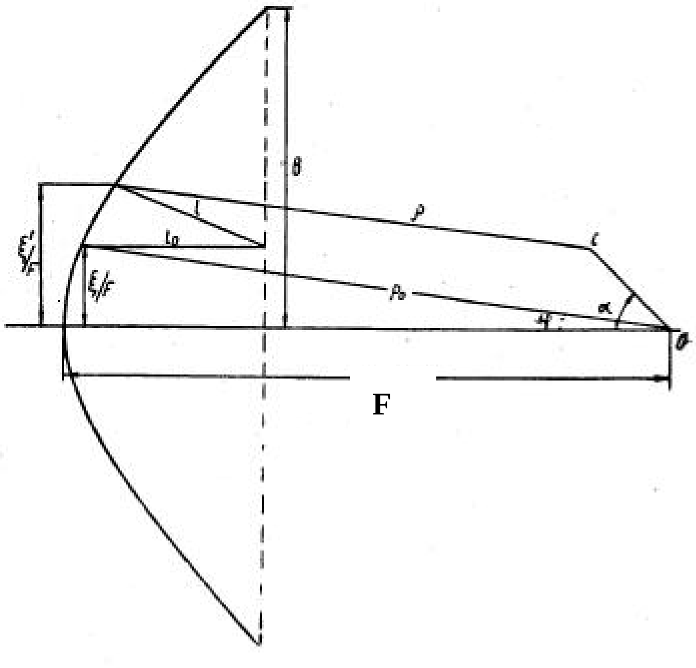


Figure 2: The process of field formation at the mirror's aperture.

Figure 2 shows the incident ($r_?$ and r) and reflected (l and l_o) beams, which came at the same aperture point, when the radiator is in focus or at some arbitrary point C respectively. This figure was used for determination the field distribution at the antenna aperture. In the general case, the equation for the calculation of $y(x)$ is rather cumbersome. But in our case, with the radiator having a small displacement from the focus, it was assumed that:

$$x' \cong x \text{ and } l_o \cong l. \tag{4}$$

then, according *Calculation of parabolic antennas*[1]:

$$y(x)=\frac{pD}{bl}\cdot\frac{a\left\{bx\sin a+\left[1-\left(\frac{bx}{2}\right)^2\right]\cos a\right\}-\frac{a^2}{2}}{1+\left(\frac{bx}{2}\right)^2} \tag{5}$$

where $x = x/F$, $a=r/F$ - is the radiator displacement from the focus, and $b=x_{\max}/F$ Now, let us determine the expression for calculating the value $F(x)$ in the formula (3). Let us assume that the real radiator DP (for power) is described by function $f(x)$. Then, the field distribution along the aperture, with condition (4), can be described as:

$$F(x)=\frac{\sqrt{f(x)}}{r}$$

here r is convenient to write as:

$$r=\sqrt{(bx-a\sin a)^2+\left[1-\left(\frac{bx}{2}\right)^2-a\cos a\right]^2} \tag{6}$$

Absolute value of $F(x)$ depends on the value r, i.?. on the mirror dimension and on the radiator displacement from the focus. It is more convenient in the equation (3) to use the normalizing value of the amplitude at the mirror aperture i.e. $F_n(x)=F(x)/F(x)_{\max}$. If the radiator's DP is normalized, then one can determine the radiator position at the focus and the direction of its maximum to the parabola's apex, $r_?=1$? $f_?(x)=1$, when the linear dimensions are presented through the focal distance. In the general case, when the radiator's DP maximum is turned from the optical axis on some angle φ_0 (as in Figure 1), or in the case of asymmetrical reflectors, when $x_1 \neq x_2$, the radiator's DP maximum has to be directed approximately to the mirror's center.

By failing to get the simple formula for the DP calculation of real horn-type antenna, the determination of *f(x)* can be done quite well by the following approximation:

$$f(x) = e^{-[n(j - j_o)]^2} \tag{7}$$

Here j - the angle between straight line r and the reflector optical axis z, equal to:

$$j = \arcsin \frac{bx - a \sin a}{r}$$

The relation between j and x, for the case when the radiator is at the parabolic focus, is expressed by:

$$j = 2 \arctan \frac{bx}{2} \tag{8}$$

The value j_0 in (7) is the angle of radiator's beam rotating with respect to the reflector optical axis. The function (7) is a good approximation of real radiator DP in the vicinity of reflector's aperture, as was shown by the calculation. In the limit case of the uniform field distribution in the antenna's aperture $f(x) = r^2$.

The formulas (5) – (7) permit the parabolic antenna DP calculation by the usage of equations (3). However, in order to investigate the influence of specific parameters on the antenna's DP such as for instance $b = x_{max}/F$, it is necessary to keep aperture field distribution the same, while changing F. From (8) one can see that at the same x and at different b the angles j and j_1 are related by the equation: $\tan(j_1/2) = (b_1/b)\tan(j/2)$. For this case following condition[1] provides the equal field distribution at the mirror's aperture:

$$f(x) = f_1(x)\left\{\frac{(1+\cos j_1)\cdot(1+\cos j_0)}{(1+\cos j)\cdot(1+\cos j_{01})}\right\}^2 \tag{9}$$

Here the real radiator DP is approximated by function:

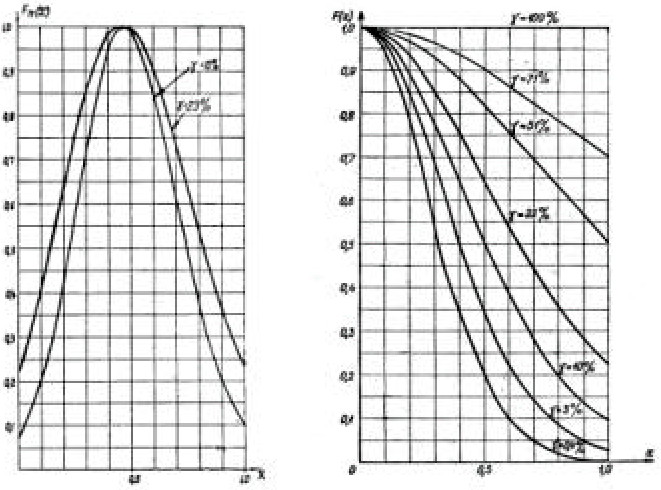
$$f_1(x) = e^{[n(j - j_0)]^2} \tag{10}$$

The condition (9) permits one to move into the calculation the series of universal distribution *F(x)* which, for the case of radiator's disposition in the focus, can be calculated by the formula:

$$F(x) = \frac{\sqrt{f(x)}}{r} \tag{11}$$

By using formula (11), such series of amplitude distributions becomes invariant for any b values.

As a rule $j_?$ (at Figure 1) is chosen from the condition of symmetrical illumination at the antenna's aperture. For asymmetrical reflectors at $x_1 \neq x_2$ the maximum radiator's DP, $\varphi_?$ is directing approximately to the center of reflector, Figure 3 shows the series of amplitude distribution *F(x)* for asymmetrical (a) and symmetrical (b) reflectors. The value γ is the field level at the reflector's corner in % and is determined by the formula (10) for different *n*.



a **b**

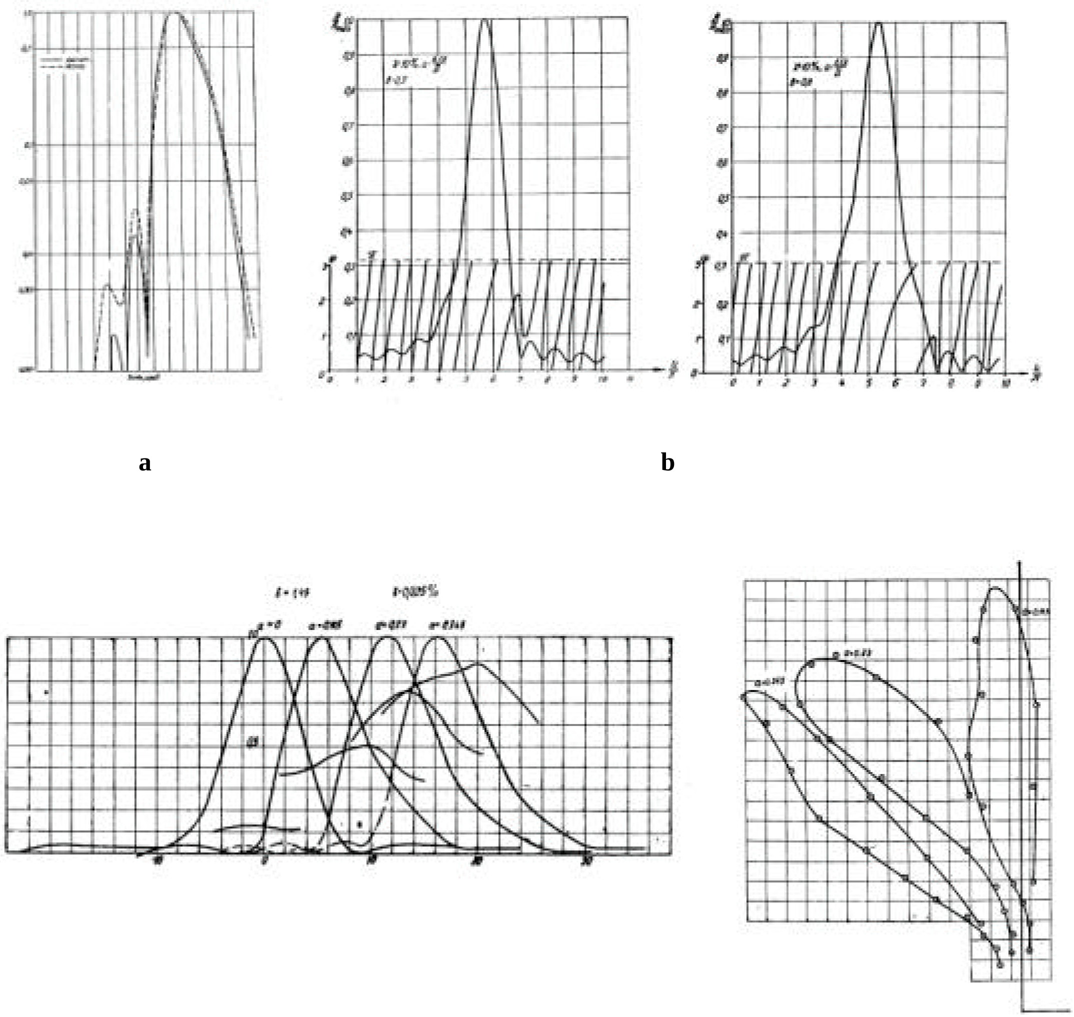
Figure 3: (a) - aperture field for asymmetrical reflector,
(b) - for symmetrical reflector and various *g*

In *Calculation of parabolic antennas* [1] it was shown that for the case when the radiation comes from the focus and j is taken as an argument, the aperture field distribution is similar to the Gaussian curve described by the formula (10), but with a different index of exponent. Herewith, both indexes, for reflector and radiator, are linear to each other.
Formula (3) was used for the calculation of the DP series. To determine the range of *u/p* variation, i.e. the range of angle q, one can use well-known empirical formula for determining the q_{max}

$$(u/p)_{max} = D/l \ \sin q_{max} \approx (D/l)\ (\text{a} \sin a)/(1+ b^2/8)$$

The range for u/p is determined to be 3-4 DP's width, which can be approximately calculated, by the known formula $q_{p/2} = l/D$ in radians.
The algorithm, based on the mentioned formulas, allows us to calculate the values: *A(u/p* and *? (u/p)* i.e. amplitude and phase DP.
For testing the proposed method, calculations of DP were provided for real antennas, with different radiator offset from the reflector focus. The real radiator DP was approximated by the formula (10).

Figure 4 shows the examples of calculated and experimental DP.



a b

c d

Figure 4: The examples of calculated and experimental DP:

(a) - calculated and experimental amplitude DPs;

(?) - calculated amplitudes and phase DPs;

(?) - experimental amplitude and phase DPs;

(d) - vector DPs plotted using the amplitude and phase DPs from (c).

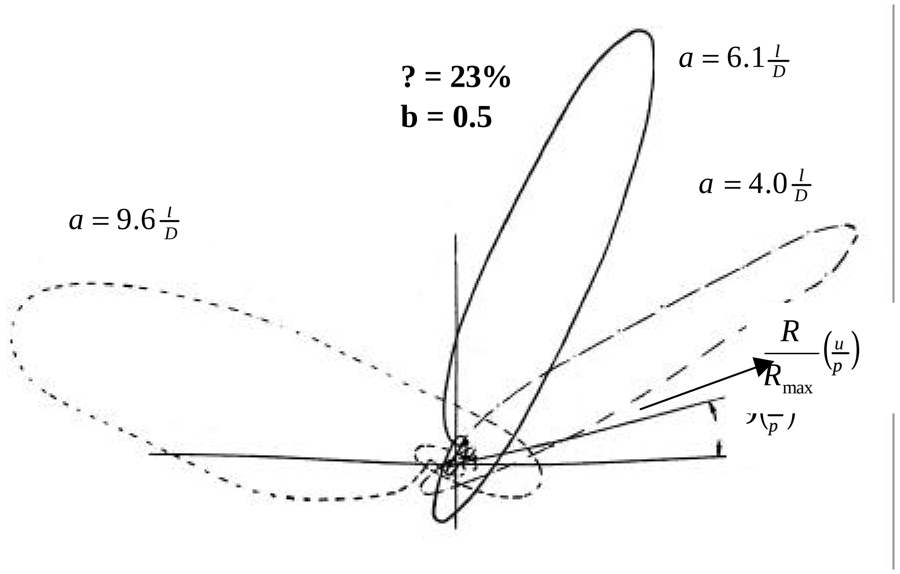


Figure 5: An example of calculated vector's DPs of the antenna with the symmetrical reflector

Figure 4 and Figure 5 show that the vector DP maximum, with the same rotation of phase front at the reflector's aperture, scan with a much greater angle than the traditional antennas, which only use the amplitude DP.

Obviously, the vector DP (VDP) can vastly improve the locator resolution, particularly for distant detection.

In *Calculation of parabolic antennas* [1] the numerous examples of amplitudes and phases DP were calculated for different generalized parameters: $a = r/F$ – relative offset radiator from reflector focus; b = x_{max}/F, where ξ_{max} (for a symmetric reflector), is equal to $D/2$, where D – reflector diameter, g - the field level at the reflector's edge relative to maximum value in %.

Part 2: Some results of DP calculation for parabolic antennas by integration of the current on the reflector's surface.

It is known, that electrical and magnetic currents generated by arbitrary radiator at the surface S produce far-field **E** and **H** at the distance *R*. These fields are described by following expressions *Antennas of centimeter's and decimeter's waves* [2]:

$$\mathbf{E} = \frac{k}{4pj}\cdot\frac{e^{-jk|R|}}{|R|}\left[\left(\sqrt{\frac{m}{e}}F_{\Theta} + F_{mj}\right)\cdot i_{\Theta} + \left(\sqrt{\frac{m}{e}}F_{j} - F_{m\Theta}\right)\cdot i_{j}\right] \qquad (12)$$

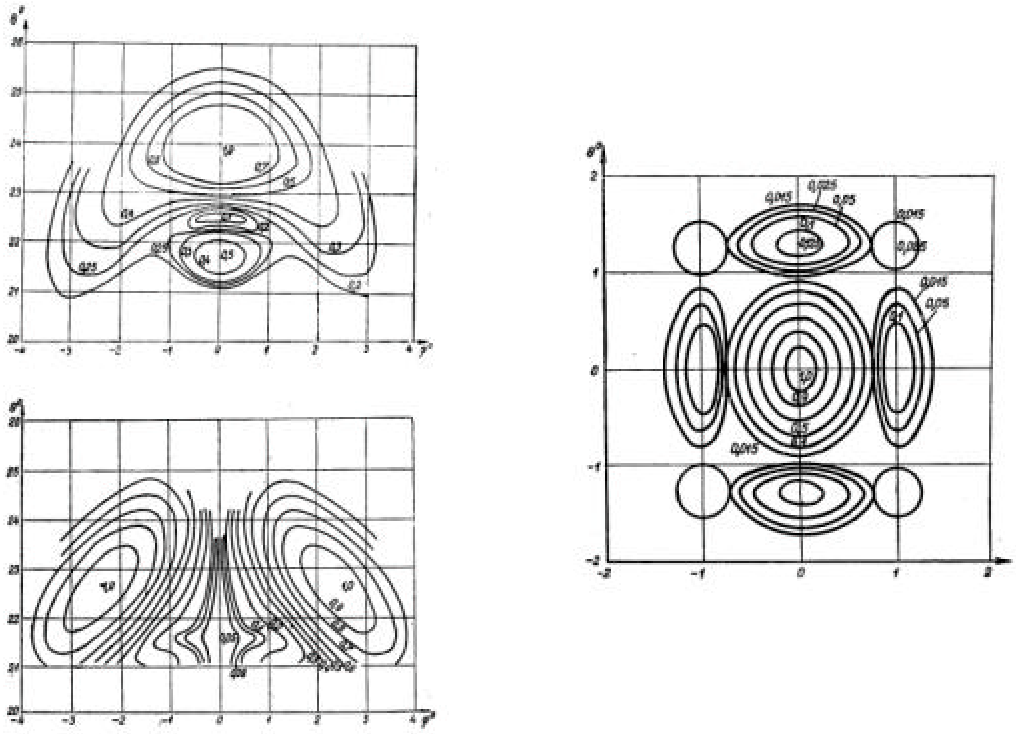
$$\mathbf{H} = \sqrt{\frac{e}{m}}\,[\,i_R\,\mathbf{E}]$$

Calculation of parabolic antennas [1] describes the method for calculation of the spatial DP of the parabolic antenna based on the calculation of the current at the reflector surface S, excited by the radiator arranged at the reflector's focus or somewhere near it, and on the subsequent calculation of the spatial distribution of the far-field components **E** and **H**, i.e. the spatial DP.

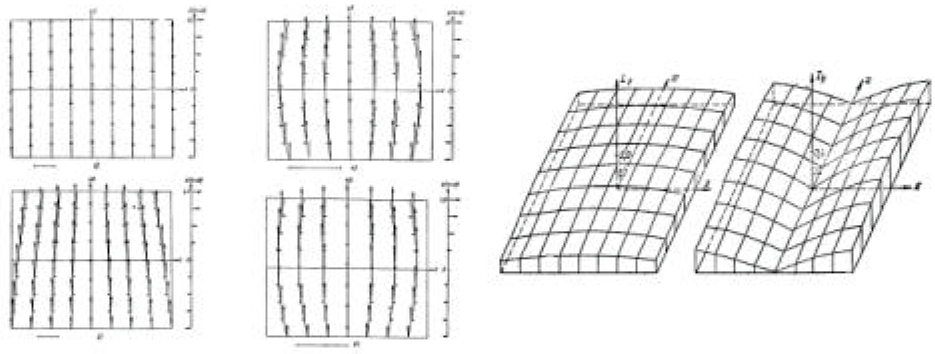
The calculations were based on the following assumption: the reflector's conductance is infinite and, therefore, on the reflector surface there exist only the current distribution $\mathbf{k} = -2[\mathbf{n}\mathbf{H}_\mathbf{s}]$, where $\mathbf{H}_\mathbf{s}$ **–** the field vector of radiator near the reflector surface and **n** – unit vector normal to the reflector surface, directed to the radiator. Magnetic currents induced on the reflector's surface generate far-field $\mathbf{E}(R)$ and $\mathbf{H}(R)$, which can be calculated by the formula (12) with:

$$\mathbf{F} = \int_S [H_S \cdot n]\, e^{jk|r|\cos d} dS$$

Here H_S is the vector of radiator's field at the reflector surface S. Figure 6 shows some examples of calculations. (Note: in (12) the value $F_\mu = 0$ due to reflector's infinite conductance).



a,b **c**



d e

Figure 6: Examples of calculated far-field zones, when the radiator is displaced from the reflector focus for *z* = 18 *l* and *z*=0.

(a) - for primary and (b) - for parasitic polarization respectively at *z* = 18 *l*
(c) - for primary polarization at *z* = 0 and with emphasized level (up to 85%) of illumination of the reflector's outer boundary
(d) - the pattern of current directions at the reflector surface
left: up – for electrical and magnetic dipoles (the horn analog) in the reflector focus,
down – dislocated from the focus
right: up - electrical dipole in the focus,
down - dislocated from the focus
(e) - current distribution at the mirror surface when electrical dipole is at the focus

Conclusion

From a brief description of this research, which is described in detail in *Calculation of parabolic antennas* [1], one can make the following conclusion:

The actual DPs of parabolic antennas with the radiator displaced from reflector's focus, are described by the complex function: $g(q) = A\ e^{i\ j}$ where ? – is "amplitude" and j - "phase" DPs .

With an increase of the radiator displacement from the parabola's focus, especially with asymmetrical reflectors, n phase chops multiple to p take place inside the "amplitude" DP, i.e. the resulting vector DP (VDP) contains several maximums.

Use of VDP i.e. measuring amplitude and phase of received signals, allows significant improvement in resolution of parabolic locators. From the big series[1] of amplitude and phase DP calculations, it follows that the biggest advantage of VDP occurs with an asymmetrical parabolic reflectors because of their sharper phase DP.

Literature

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