

Isentropic Flow Through Nozzles

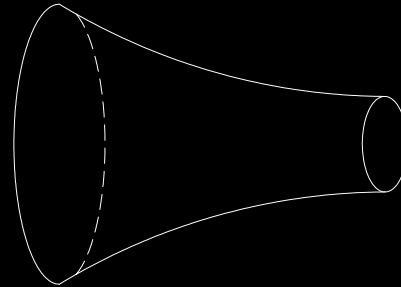
Elton J. Colbert

May 3, 2001

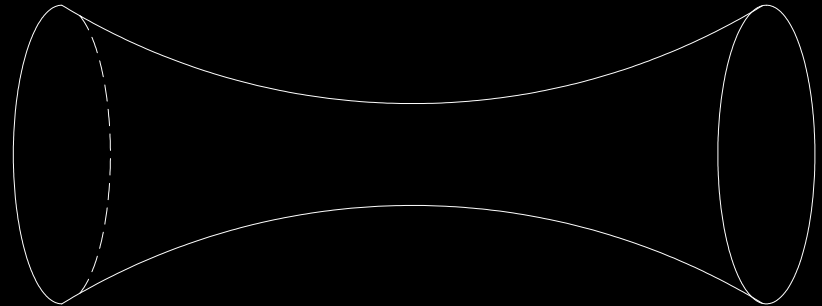


Type of Nozzles

- **Converging:**



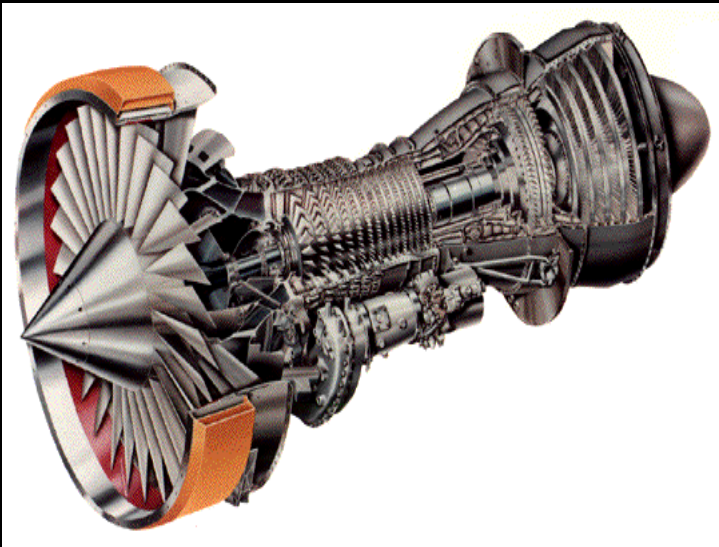
- **Converging-Diverging:**



- **Focus: Converging-Diverging**

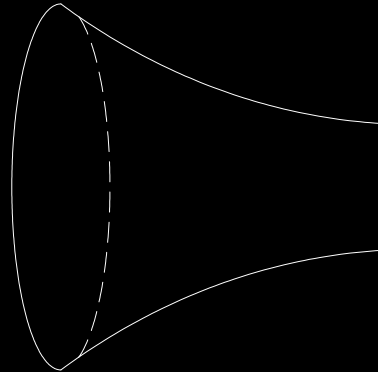
What Makes a Converging-Diverging Nozzle So Special?

- Why not use just an ordinary nozzle? After all an ordinary nozzles are used in turbines, jet engines, and hoses.



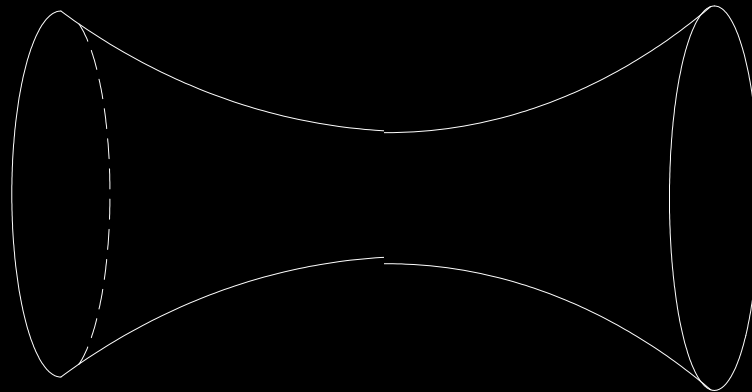
Limitations

- $M=1$ for



- $M>1$ for

- Standard on all supersonic aircraft

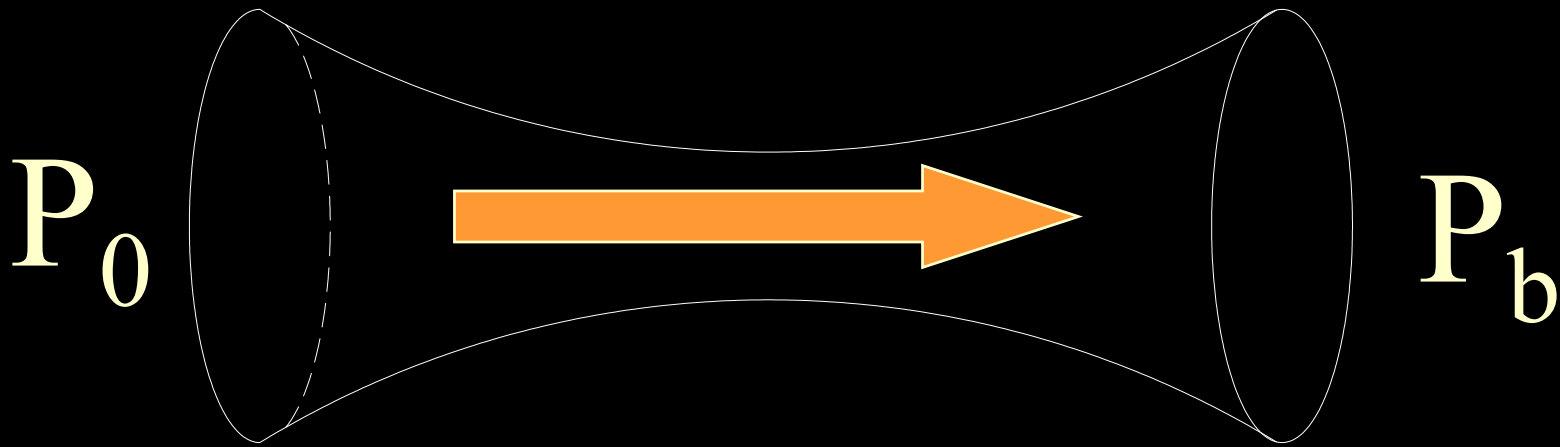


The Resulting Combined Flow Section

- Is a converging-diverging nozzle which is standard equipment on all supersonic aircraft.

This Is Not a Guarantee

- If the back pressure is not in the correct range the fluid might decelerate instead of accelerate.

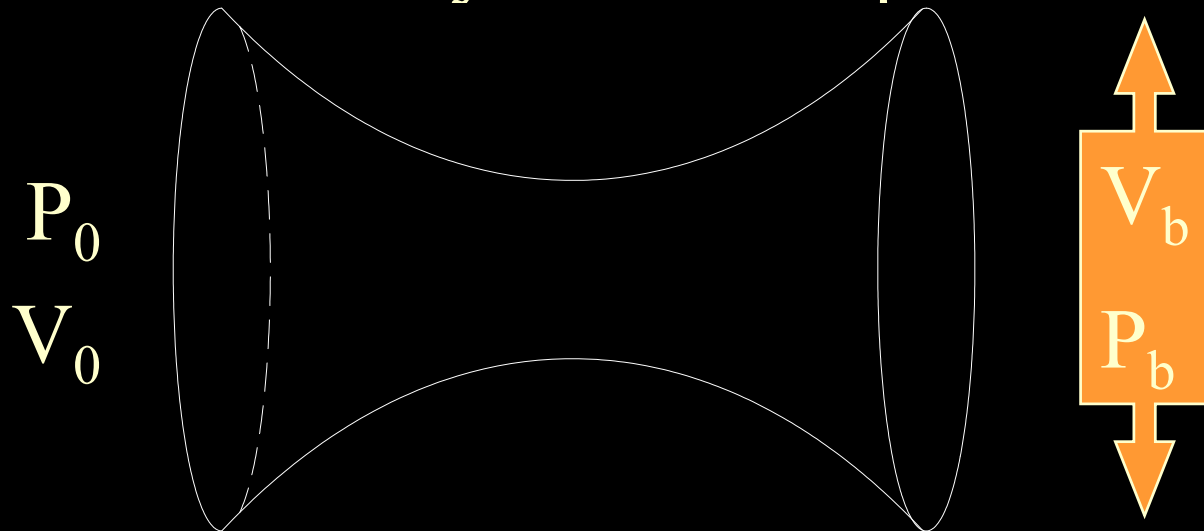


P_0 = Initial Pressure

P_b = Back Pressure

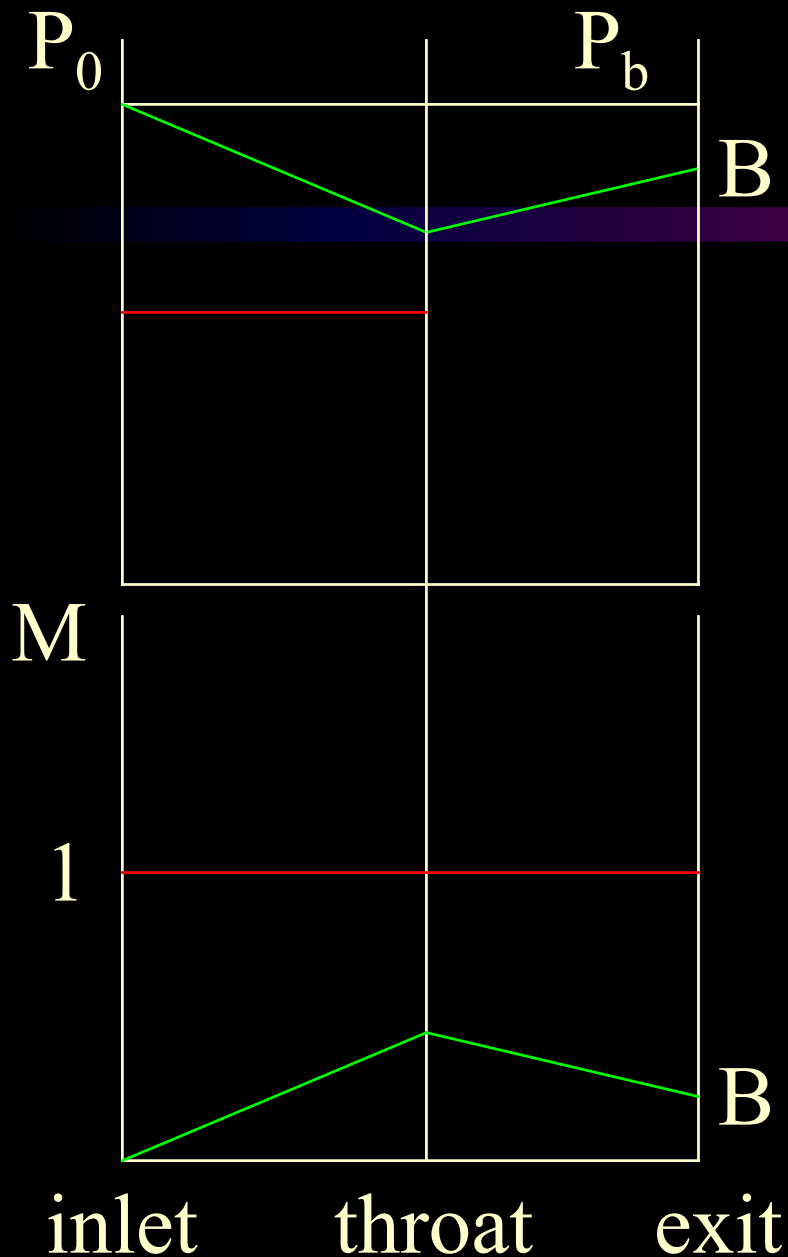
Let's Begin

- As before we begin by changing the nozzle to increase velocity and lower pressure.



For $P_b = P_0$ there is no flow.

When $P_b = P_B$



Flow Begins

Velocity Rises

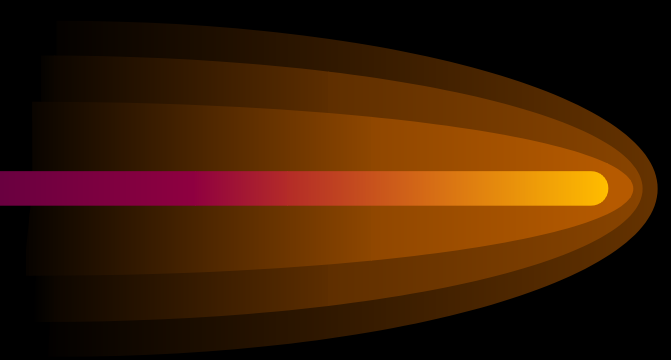
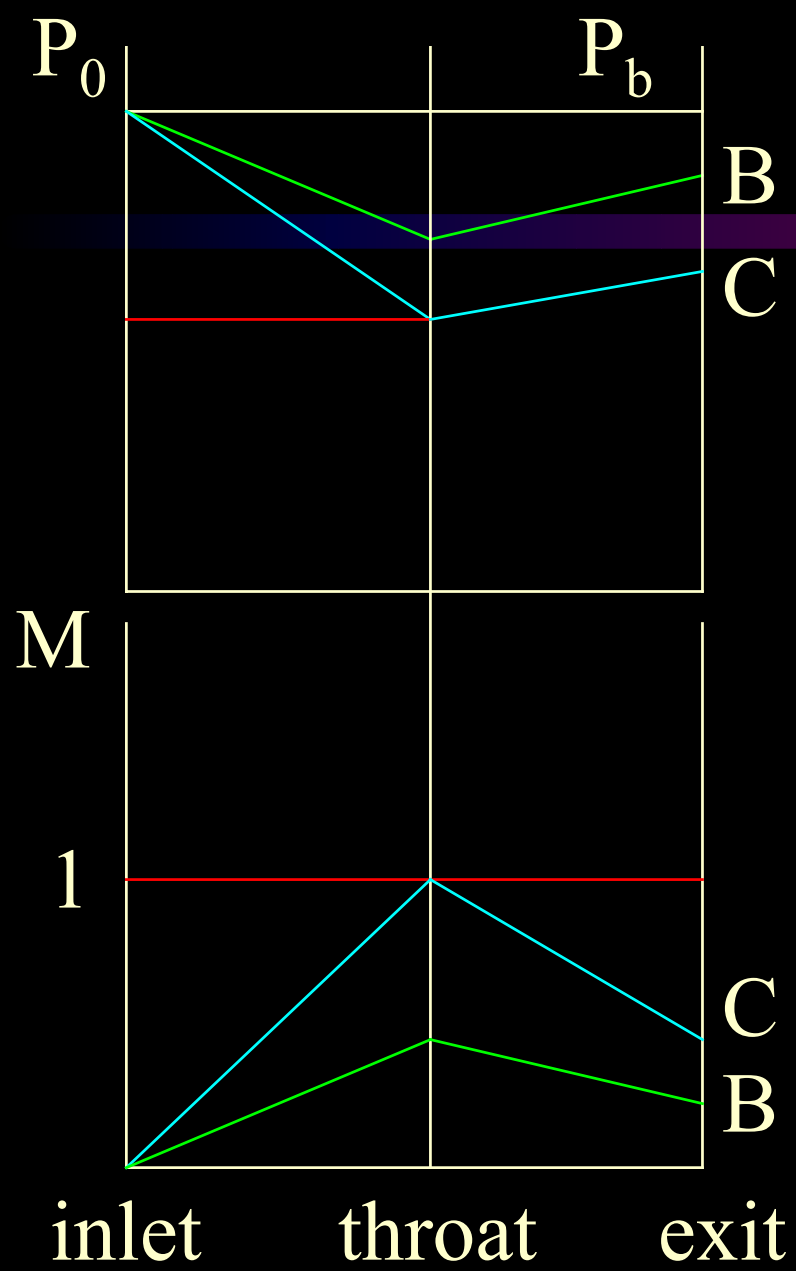
Pressure Drops

Diffuser Kicks In

Velocity Drops

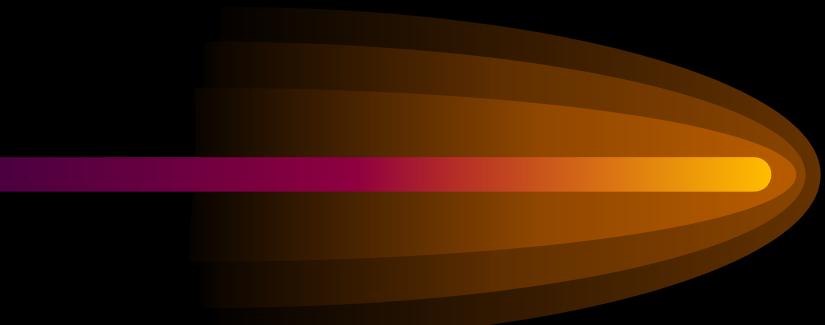
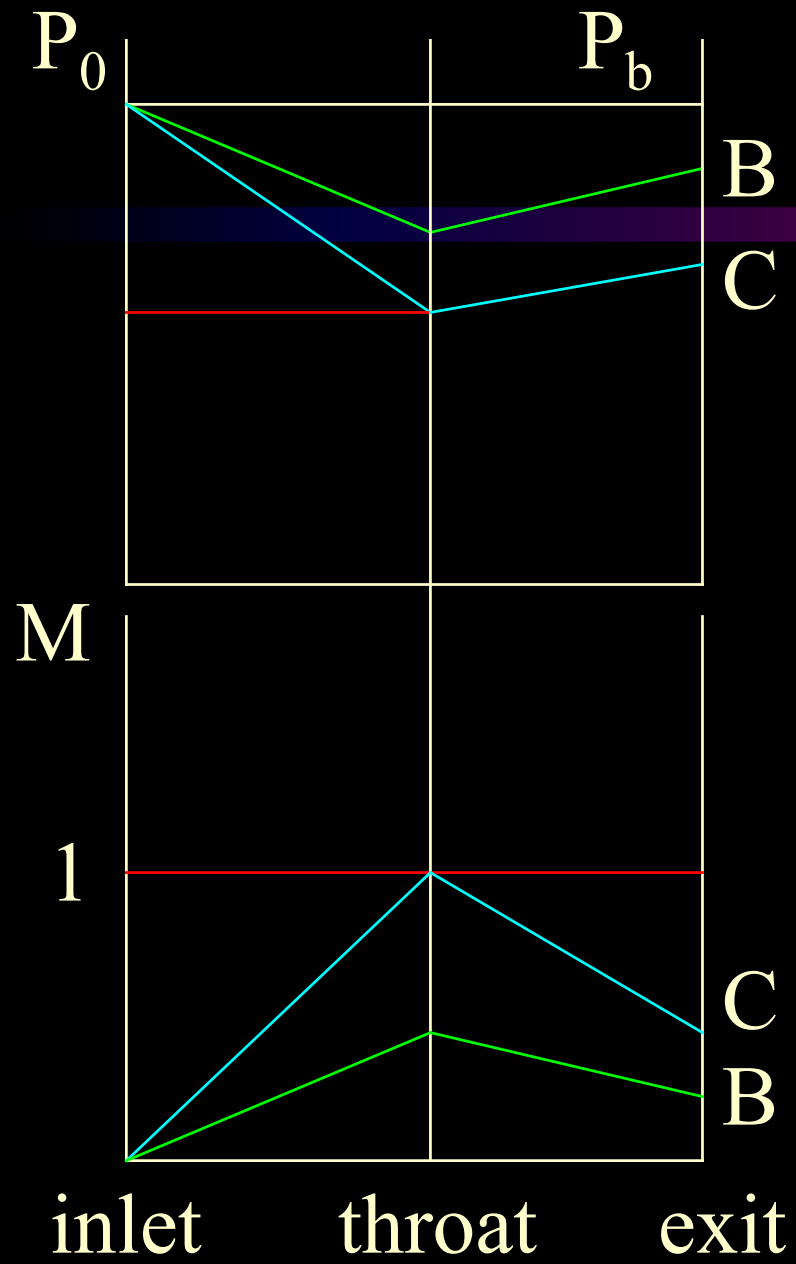
Pressure Rises

When $P_b = P_C$



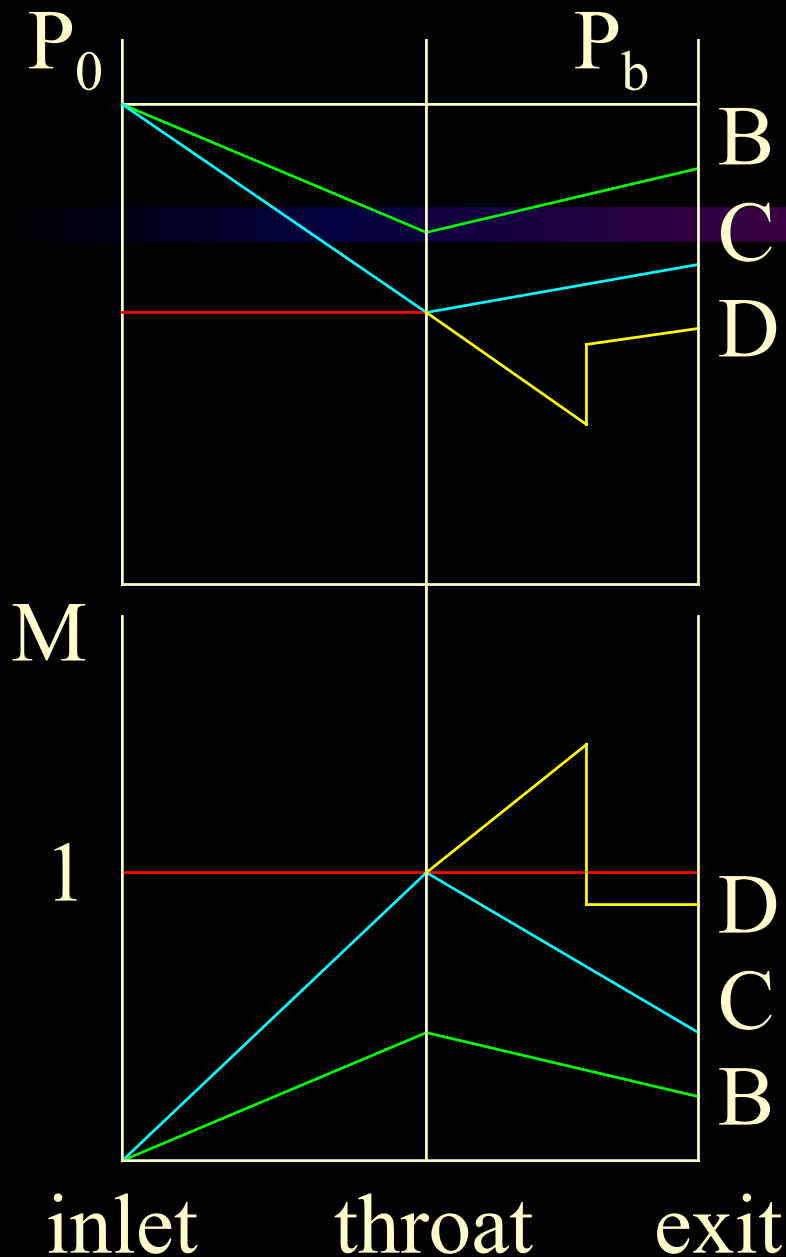
- Sonic Velocity
- Diffuser Kicks In
- Velocity Drops
- Pressure Rises

Remember When...



Nozzle at throat:
Lowest pressure
Velocity max at sonic
Reduce pressure more means nothing to me.
But it does in diverging

When $P_b = P_D$



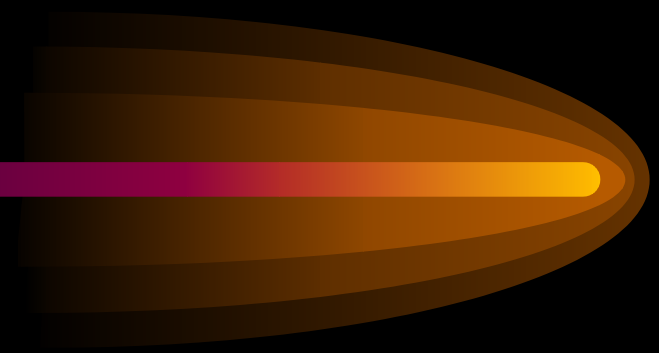
Pressure drops

Velocity supersonic

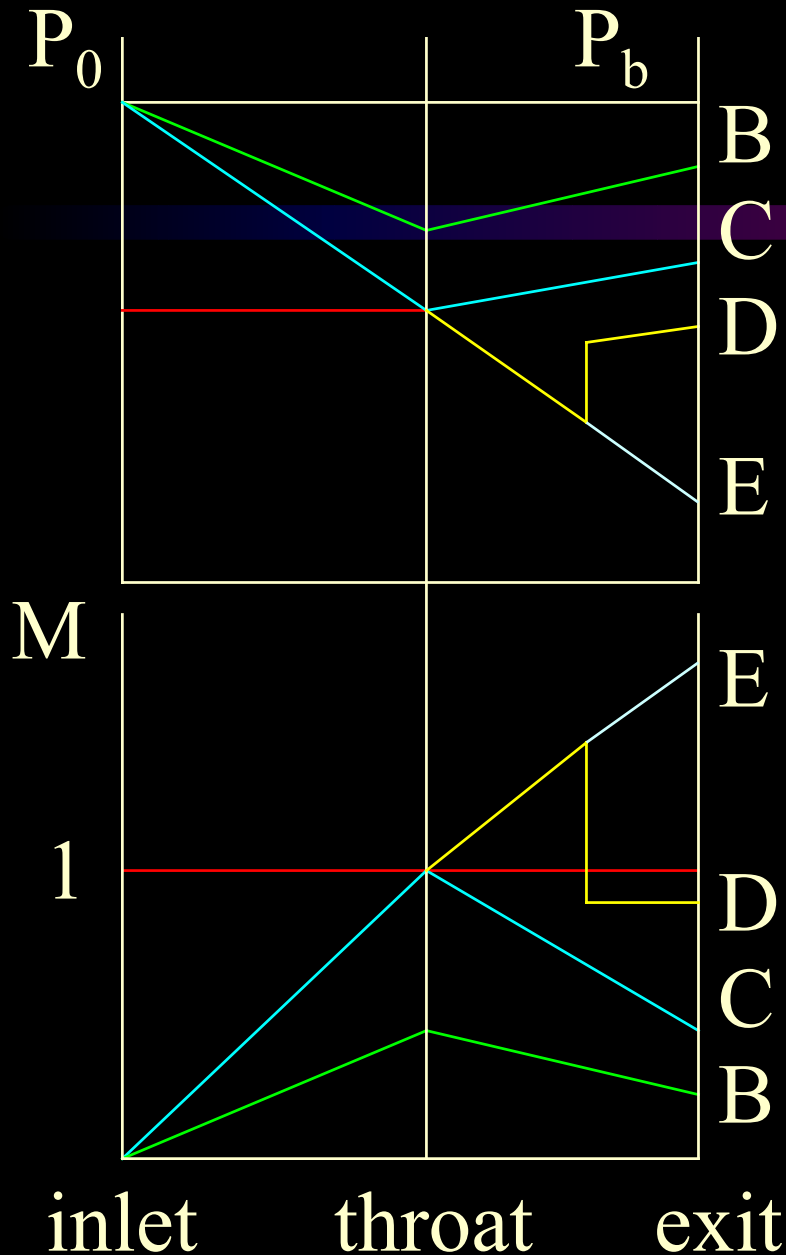
Normal Shock STOPS!

Velocity drops

Pressure rises



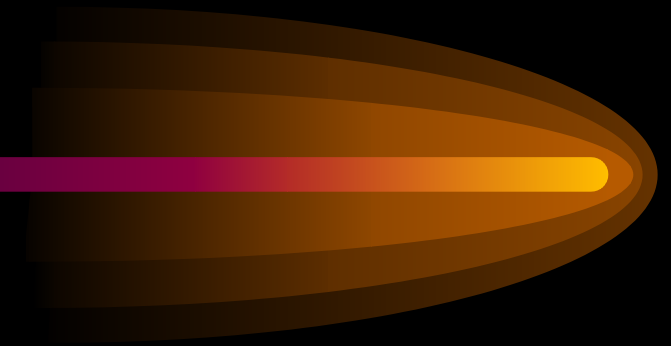
When $P_b = P_E$

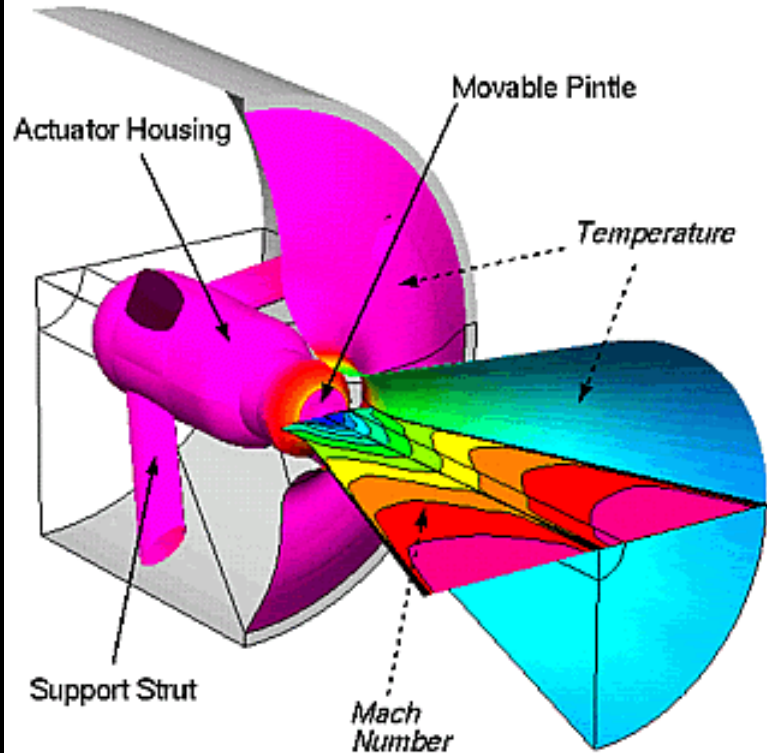


Pressure drops

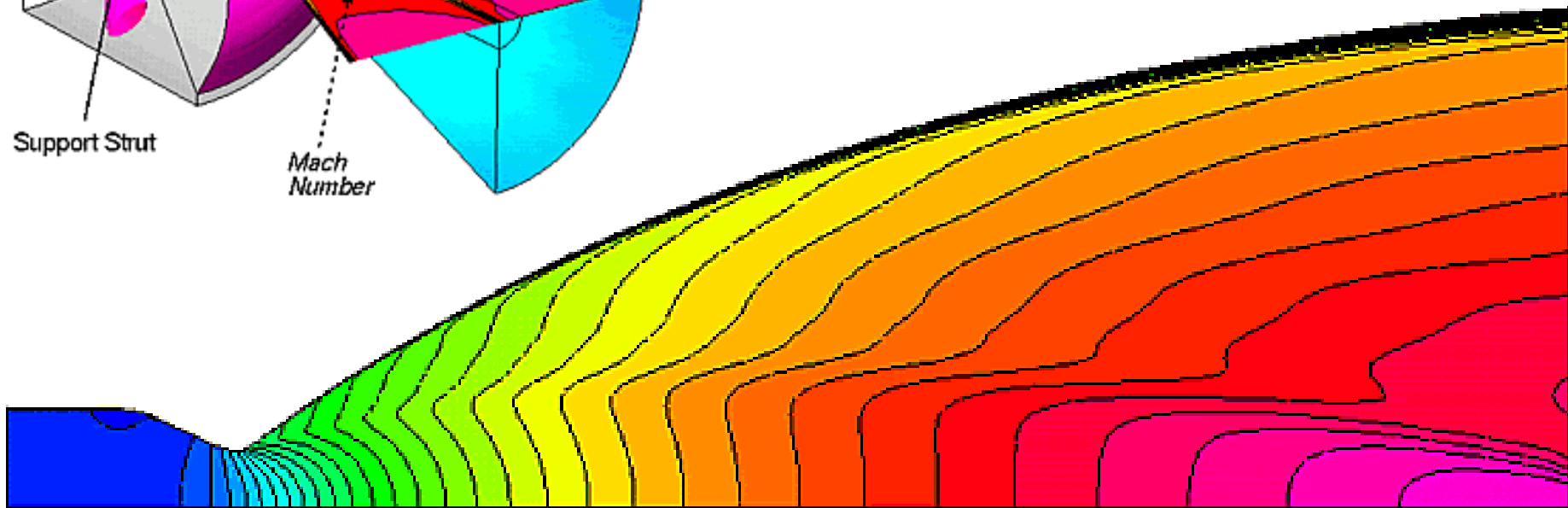
Normal Shock moves away

Supersonic all the way

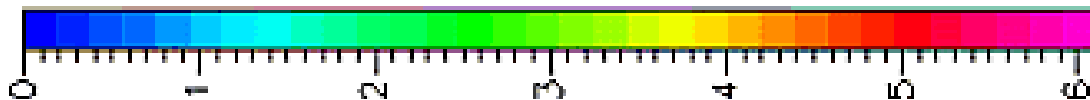




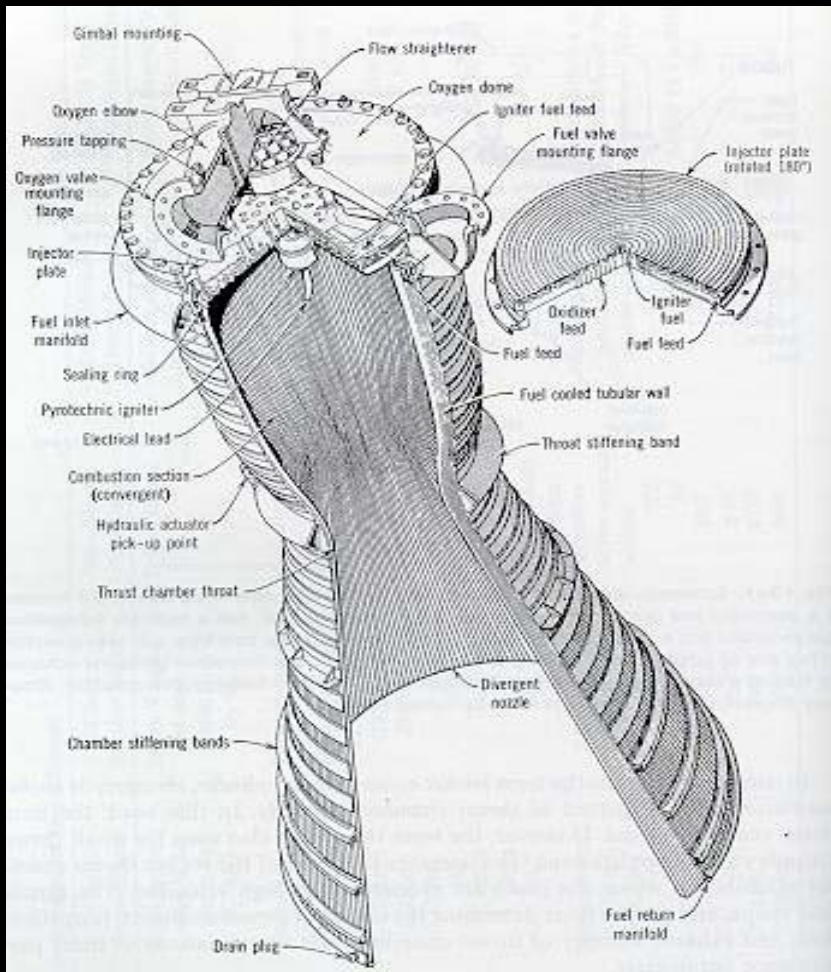
Example



MACH



Example



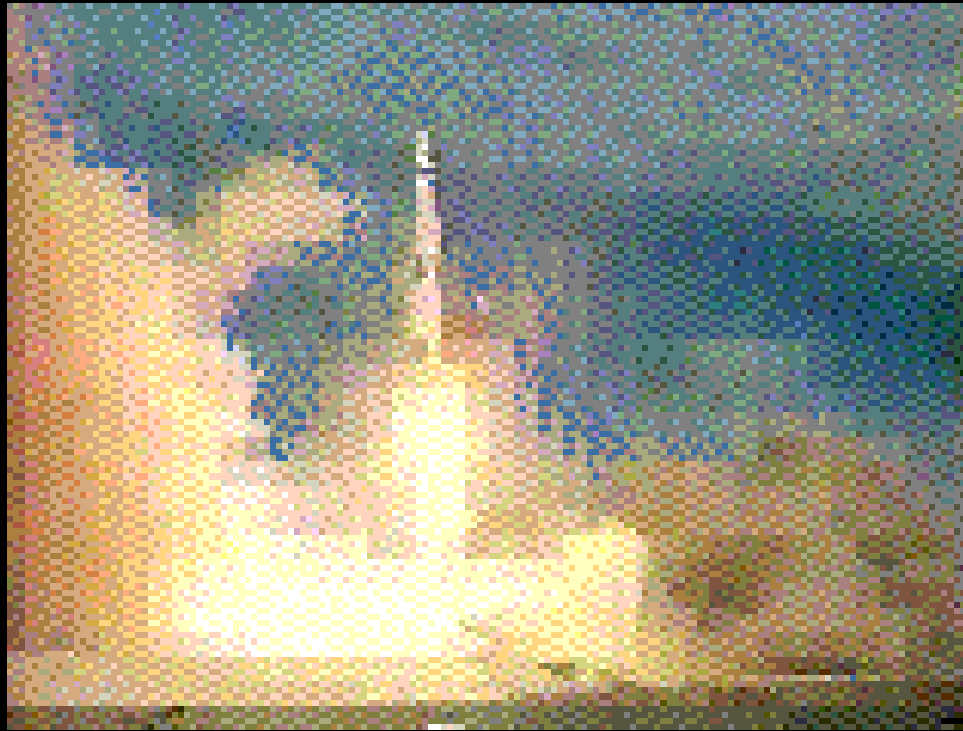
<http://www.engapplets.vt.edu/fluids/CDnozzle/cdinfo.html>

Check It



gemini[1]

Check It Out



`aplaunch[1]`

Yo Yo Check It Out



shuttle[1]

Table A-15

- A relation for :
- flow area A vs. throat area A^*
- pressure P vs. initial pressure P_0
- density ρ vs. initial density ρ_0
- temperature T vs. initial temperature T_0
- as a function of the Mach number.

V^* = Throat Velocity

A^* = Throat Area

P^* = Throat Pressure

T^* = Throat Temperature

ρ^* = Throat Density

V_e = Exit Velocity

A_e = Exit Area

P_e = Exit Pressure

T_e = Exit Temperature

ρ_e = Exit Density

Variables:

\dot{m} = Mass Flow Rate

k = Specific Heat Ratio

R = Gas Constant

M = Mach Number

P_0 = Initial Pressure

T_0 = Initial Temperature

ρ_0 = Initial Density

Equations:

- $P = \rho TR$
- $V = M\sqrt{kRT}$
- $\dot{m} = \rho AV$

Problem:

- $\dot{m} = 13.111 \text{ kg/sec}$
- $k = 1.4$
- $R = 0.287 \text{ kJ/(kg K)}$
- $P_0 = 1.0 \text{ MPa}$
- $T_0 = 880 \text{ K}$
- $A^* = 20 \text{ cm}^2$
- $M_e = 2$
- Flow is steady, one-dimensional, isentropic.

Determine:



- Throat conditions.
- Exit plane conditions, including the exit area.
- Mass flow rate through the nozzle.

Analysis:

- Exit Mach number = 2:
 - sonic at the throat
 - supersonic in the diverging
- Inlet velocity is negligible:
 - throat pressure = inlet pressure
 - throat temperature = inlet temperature.

Solution to throat conditions:

- Using $P = \rho TR$ and solving for

$$\rho_0 = 4.355 \text{ kg/m}^3$$

- From Table A-15 for $M = 1$, $P^*/P_0 = 0.5283$,
 $T^*/T_0 = 0.8333$, $\rho^*/\rho_0 = 0.6339$.

$$P^* = 0.5283 \text{ Mpa}$$

$$T^* = 666.6 \text{ K}$$

$$\rho^* = 2.761 \text{ kg/m}^3$$

- Using $V^* = M^* \sqrt{(kRT^*)}$

$$V^* = 517.5 \text{ m/s}$$

Please calculate exit conditions:

- From Table A-15 for $M=2$, $P_e/P_0 = 0.1278$, $T_e/T_0 = 0.5556$, $\rho_e/\rho_0 = 0.2301$, $A_e/A^* = 1.6875$.

$$P_e = 0.1275 \text{ Mpa}$$

$$T_e = 444.5 \text{ K}$$

$$\rho_e = 1.002 \text{ kg/m}^3$$

$$A^* = 33.75 \text{ cm}^2$$

- Using $V_e = M_e \sqrt{(kRT_e)}$

$$V_e = 845.2 \text{ m/s}$$

Please calculate \dot{m}

- Steady flow:
 - \dot{m} is the same at all sections of the nozzle
 - calculated by using properties at any cross section of the nozzle.
- Using $\dot{m} = \rho AV$

$$\dot{m} = 2.858 \text{ kg/s}$$

Questions

