

# The Emperor's New Concepts

by

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March 9, 2002

## Abstract

Christopher Peacocke, in *A Study of Concepts*, motivates his account of possession conditions for concepts by means of an alleged parallel with the conditions under which numbers are abstracted to give the numerosity of a predicate. There are, however, logical mistakes in Peacocke's treatment of numbers, which undermine his intended analogy. Nevertheless Peacocke's account of possession conditions for concepts is not rendered inadequate simply by virtue of being deprived of the intended analogy and the motivation it was supposed to afford. His account of concepts deserves still to be considered on its own merits, even if it is more idiosyncratic for being isolated from the paradigm case of numerical abstraction. Peacocke's own account of concepts as abstract objects turns out, though, not to have the logical form that he himself was seeking for it. We show how to re-cast it in an equivalent form of the kind he requires. Then we re-formulate it so as to achieve complete generality. This exercise helps to clarify the central theses in Peacocke's account of concepts. It invites the conclusion that his account of content-determination is rather platitudinous

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\*To appear in *Philosophical Papers*. For helpful comments on an earlier draft of this paper I am grateful to Tom Baldwin, José Luis Bermúdez, Robert Hanna, Robert Kraut, Allan Millar, Hermann Philipse and Ralph Wedgwood.

and unoriginal—except for a claim of ‘doxastic sufficiency’ for content-determination, which emerges at the end of the discussion.

## 1 Introduction

*Are concepts abstract the way numbers are?*

This question is phrased carefully. It is *not* the question ‘Are concepts *abstracted* the way numbers are?’. Concept-abstraction is the psychological or intellectual process whereby a thinker abstracts, from various instances of a kind, what is common to them. I use the word ‘kind’ loosely here; it does not mean ‘natural kind’. Whether the thinker is abstracting to the color-concept GREEN upon seeing various green things, or to the sortal concept DOG upon being acquainted with various dogs, or to the shape-concept SQUARE upon experiencing various square-shaped objects, he is merely homing in (or, in a recently fashionable phrase from Loewer and Rey, ‘locking’<sup>1</sup>) on the appropriate ‘One over Many’. He is latching on to what is saliently common in the various particulars that form the basis for the abstraction.

Ordinary thinkers perform this kind of abstraction all the time. Philosophers who talk about concepts, however, are not ordinary thinkers. They are working at one level up, so to speak. They are inquiring after the nature of concepts themselves, and the role that concepts play in our description of human thought.

That the conceptual abstraction just described is accomplished successfully does not (yet) speak to the question whether the concepts thus ‘abstracted’ are to be thought of *as abstract objects*.

At one extreme there is the naturalizing nominalist, who grants the process of abstraction just described, but who maintains that in so far as there are such ‘things’ as ‘concepts’ in the world, they are nothing more than (types of) neural configurations in thinkers’ brains.<sup>2</sup>

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<sup>1</sup>See Loewer and Rey, ‘Editors’ Introduction’, in *Meaning in Mind: Fodor and his critics*, Blackwell, 1991; at p. xxvii.

<sup>2</sup>Fodor can be read in this way, with ‘mental representation’ in place of ‘neural configuration’. See his *Concepts: Where Cognitive Science Went Wrong*, Oxford University Press, 1998. In so far as a concept is identified with a *type* of mental representation, rather than its tokenings, it is more like a Fregean *Sinn*. The pressing question, then, would be how we are to make these type-identifications. If they are functional type-identities, their Fregean provenance is easier to grant; but if they are neurological or anatomical type-identities, we are giving hostage to empirical fortune.

A more widely-held view is that concepts are *mental* entities: ideas, images or (classificatory) stereotypes.<sup>3</sup> Another widely-held view is that concepts are not really things, but capacities. To ‘have’ a concept is to be able to perform certain intellectual tasks: classifying, sorting, discriminating, ... (for categorial concepts); or drawing certain sorts of inferences (for syncategorematic concepts).<sup>4</sup>

At the other extreme, a modal realist could maintain that it is not in fact at all important to insist on the ‘datum’ of concept-abstraction described earlier. For, according to the modal realist, the world could contain only unthinking brutes, and yet there would be concepts in it for all that. They are logical objects, occupying a ‘third realm’, independently of whether there were thinkers who could grasp them, and whether or not the acquisition of such grasp could correctly be described as some sort of process of abstraction.<sup>5</sup>

One philosophical line of thought holds that concepts (whether or not they are grasped by a process of ‘abstraction’) are best to be understood *as abstract objects*. That is to say, roughly, that expressions of the form ‘the concept *F*’ have the kind of logico-linguistic behavior exhibited by other expressions that paradigmatically stand for abstract objects, such as (say) ‘the direction of line *L*’, ‘the number of *Ps*’, and ‘the color of *x*’. This view respects the Fregean demand for objectivity, inclining towards the modal realist’s view in matters of ontology; while it seeks also to accommodate the Wittgensteinian ‘intellectual capacities’ view, in tying the identity of a concept to the *possession conditions* that a thinker must satisfy in order to be credited with a grasp of it.

## 2 Peacocke’s account of concepts

The most notable recent proponent of this line of thought is Christopher Peacocke. Chapter 4 of his book *A Study of Concepts*<sup>6</sup> is titled ‘The Meta-

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<sup>3</sup>The *locus classicus* is Hume. This is the view that is most vulnerable to Frege’s critique of psychologism.

<sup>4</sup>Variants of such a view have been held by Wittgenstein, Ryle, and Sellars.

<sup>5</sup>One proponent of such a view is George Bealer. See his paper ‘A Theory of Concepts and Concept Possession’ in Enrique Villanueva, ed., *Concepts. Philosophical Issues* vol. 9; Ridgeview Publishing Co., 1998, pp. 261–301.

<sup>6</sup>MIT Press, 1991.

physics of Concepts'.<sup>7</sup> (Page references will be to the book throughout this discussion.) My purpose in this paper is two-fold:

- (i) to show that Peacocke's analogy between contents and concepts, on the one hand, and numbers, on the other, is not well-drawn; and
- (ii) to re-cast Peacocke's theory into an equivalent form which is of the kind he actually requires (but does not himself provide), and to assess its merits upon such reformulation.

This is not a critique conducted at any great remove on any axis provided by metaphysics or epistemology. It is a critique conducted from within, so to speak, and on the very terms in which Peacocke chooses to conceive of both contents and concepts. The reader will be disappointed who expects a Quinean, eliminativist or nominalist critique, or (at the other extreme) a modal realist critique. Rather, I examine the extent to which Peacocke's account could be said to succeed by the internal standards that he sets for his own project, and by widely accepted logical norms for this area of analytical philosophy.

The deficiencies adverted to in (i) and (ii) have, as far as I know, escaped the attention of Peacocke's commentators thus far.<sup>8</sup>

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<sup>7</sup>This chapter is substantially identical to Peacocke's paper of the same title that was published a year earlier in the centennial issue of *Mind*. See S. Blackburn and M. Sainsbury, eds., *Mind and Content*, vol. 100, 1991, pp. 525–546.

<sup>8</sup>See the following reviews of, or critical notices or commentaries on, *A Study of Concepts*: Tyler Burge, *Times Literary Supplement* 4707, June 18, 1993, pp. 14–15; Kirk Ludwig, *Mind and Language* 9, 1994, pp. 469–492; Paul Weirich, *Review of Metaphysics* 48, 1994, pp. 159–160; Herman Philipse, *Inquiry* 37, 1994, pp. 225–252; Jerry Fodor, *London Review of Books* 15 no. 19, 7 Oct. 1993, pp. 14–15; Alan Millar, *Mind* 103, 1994, pp. 73–82; Robert Hanna, *The Philosophical Review* 103, 1994, pp. 541–543; Tim Crane, *European Journal of Philosophy* 2, 1994, pp. 352–356; Andrew Pessin, *Journal of Mind and Behavior* 15, 1994, pp. 295–298; Douglas Glick, *Theory and Psychology* 6, 1996, p. 349; Jane Heal, *Philosophy and Phenomenological Research* 56, 1996, pp. 413–417; Georges Rey, *ibid.*, pp. 419–424; David Papineau, *ibid.*, pp. 425–432. The following are further commentaries on a later paper by Peacocke, 'Implicit Conceptions, Understanding and Rationality', in ed. Villanueva, *op. cit.*, pp. 43–88, in which he further develops his theory of concepts and amends it: Steven Schiffer, *ibid.*, pp. 89–91; Georges Rey, *ibid.*, pp. 93–104; Eric Margolis, *ibid.*, pp. 105–114; and Josefa Toribio, *ibid.*, pp. 115–120.

That the first deficiency ((i) above) escaped critical notice cannot in every case be attributed to shortage of space, and the commentator's attention being directed elsewhere. For both Ludwig and Philipse (at p. 482 and at p. 240 of their respective full-length review article and review discussion) reproduce Peacocke's deficient analysis of numerical abstraction without any critical comment. This, to my mind, shows that it is all the more

Peacocke's approach in Chapter 4, as stated in his Introduction (p. xii), is to

treat the problem of the legitimacy of an ontology of concepts as a special case of the general problem of the application to the empirical world of discourse apparently mentioning abstract objects. Parallels are developed with other forms of discourse in which abstract objects are mentioned in the description of the empirical world. These parallels suggest a positive account of the practice of mentioning concepts in the description of the empirical mental states of thinkers, a positive account that also supports the claim of legitimacy for this practice.

It is not clear from this brief statement whether Peacocke thinks that his positive account in the case of concepts would, by itself and on its own merits, support the claim of legitimacy for the practice of 'concept-application'; or whether the parallel(s) that Peacocke takes himself to have 'developed' (particularly with the case of 'number-application') would themselves be playing an essential role in supporting that claim of legitimacy. If the latter, then any successful undermining of the claimed parallel(s) would detract from Peacocke's 'legitimation' of concept-application. I shall in any event be showing below *both* that the parallel between concept-application and number-application is fundamentally flawed, because of a misdescription of what is going on in the case of number-application; *and* that Peacocke's description of concept-application needs emendation, even by his own lights, whereupon his initially profound-looking theory strikes one as platitudinous or unoriginal.

These are strong claims to be making, after the largely positive critical reaction to *A Study of Concepts*.<sup>9</sup> Such critical claims might, however, be

important to expose the logical mistake buried in the innocuous-seeming regimentation that Peacocke offers. See §3.3 below.

<sup>9</sup>From the sources cited above, we have the following plaudits: '[The book] provides an attractive combination of sophistication, argumentation, strong ideas and originality.' (Burge, p. 14); '*A Study of Concepts* ... is without doubt Peacocke's best book to date. The argumentation is complex, tight and original, but it is set within an ambitious theoretical framework which lies in the background of the detailed discussions.' (Crane, p. 353); 'Of the impressiveness and the distinctiveness of Peacocke's project there can be no doubt; it is carried through with a rare combination of strategic vision and detailed analysis. If it is defensible it is an achievement on a grand scale.' (Skorupski, p. 144); '[Peacocke] has begun to develop an admirably detailed theory of concepts' (Rey, in *Philosophical Issues*, p. 94); 'Peacocke is not just going over familiar ground but offering new approaches [which]

less surprising to readers who found the book elusive and unclear on certain important points.<sup>10</sup>

A book such as Peacocke's requires unusual patience, if one is to work through all the details in order to assess what they really amount to. This may go some way to explaining the relative dearth of detailed engagements with its main theses and arguments, since the initial flurry of reviews.

In Chapter 4 itself Peacocke provides more detail by way of motivation for his approach. His aim is to

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... require us to stretch and re-adjust our philosophical apparatus. And not content with that, Peacocke then goes on to work out in rigorous detail the consequences of his views' (Heal, p. 417); '[Many of Peacocke's] claims seems [*sic*] to me right, important, and too little appreciated on my side of the Atlantic.' (Rey, *Philosophy and Phenomenological Research*, p. 419); 'Christopher Peacocke's *A Study of Concepts* is a dense and rewarding work.' (Papineau, p. 425); 'Peacocke's aim in this highly original book is to develop a theory of concepts which respects both ... Fregean and Wittgensteinian considerations.' (Millar, p. 74); 'I have not done justice to the wealth, subtlety, and intricacy of argument and allusion in Peacocke's book.' (Ludwig, p. 490); 'There is no doubt that *A Study of Concepts* is a powerful and stimulating book ... and I agree with Steven Schiffer that Peacocke's theory of Concepts is "systematic philosophy in the grand tradition". (Philipse, p. 248); 'Christopher Peacocke's *A Study of Concepts* is about as subtle and sophisticated an elaboration of the idea that concepts are epistemic capacities as you will ever want to read. ... Many's the graduate seminar that will slog its way through, line by line, and will be edified by doing so. ... [Peacocke has spoken] with insight and authority for the Received View.' (Fodor, p. 14).

<sup>10</sup>Fodor took back with the left hand what he had given with the right: '[The book] may, in fact, be a more subtle and sophisticated elaboration of that idea than you will ever want to read. ... The idea that philosophy sets the agenda for psychology, or for any other empirical inquiry, ... strikes me, frankly, as ahistorical and maybe a touch hubristic. The surruration that you hear is legions of cognitive psychologists not holding their breath till their task is fully formulated by philosophers.'

Few philosophers would ever be willing to say in print what they readily confess in private—that the book in question is dense and obscure. (Not altogether tongue-in-cheek question: does 'dense' mean the same in the two contexts 'dense and rewarding' (Papineau, cited above) and 'dense and obscure'?) One bravely willing exception is Pessin: 'It is densely written and argued, in a turgid prose replete with technical terms and neologisms, and sprinkled with symbols.' I do not share Pessin's aversion for sprinklings of symbols. But I am averse (see §3 below) to symbols sprinkled without regard for syntax.

Two other commentators among those listed above have ventured similar misgivings in print, even if not quite so forthrightly. Ludwig wrote of a 'formidable style, and a certain modesty about spelling out sometimes key notions' (p. 490). And Philipse was not so overcome by the powerful stimulation of the book that he could not 'feel that there is a lack of proportion between the impressive apparatus of self-consciously sophisticated techniques and the results obtained. ... Many of Peacocke's results are grammatical truisms, which have been distorted by being dressed up as a theory. And some of his complex arguments contain quite simple fallacies.' (p. 248).

[legitimize] discourse apparently about a domain of abstract objects [i.e. concepts—NT] by giving an account of their empirical application. . . . [Such legitimation by application] involves, first, specifying a kind of statement in which the putative abstract objects are applied to the empirical world—either to its objects or to properties of its objects. Such types of statement will include “the number of  $F$ s =  $n$ ”, “expression token  $t$  is of type  $A$ ”, and, if [my] arguments are correct, “ $x$  believes that  $p$ ”. Instances of these various kinds I call “application statements”. Second, legitimation by application involves specifying, for each application statement, an equivalent that can be understood even by someone who does not possess the concepts of the appropriate pure theory of abstract objects. . . . (That the equivalents can be *understood* without possessing concepts of the appropriate theory of abstract objects does not imply that the equivalents can be *true* without such abstract objects existing.) (pp. 119–120)

### 3 Examination of Peacocke’s proposal in the case of numbers

#### 3.1 The details of Peacocke’s proposal

Let us begin by considering how Peacocke thinks the above proposal goes in the case of numbers (pp. 105 ff.). Paraphrasing for economy, and retaining Peacocke’s numbering of displayed sentences, his account runs as follows. The ‘content of the statement’

- (1) The number of planets is 9

is ‘split up into an entirely empirical component’

- (3) There are nine planets

and a ‘pure, relatively a priori’ component

- (4) 9 is the unique number  $n$  such that necessarily, for any property  $P$ , there are  $n$   $P$ s iff there are nine  $P$ s.<sup>11</sup>

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<sup>11</sup>The numerical quantifiers are intended in the ‘exactly’ sense, not the sense of ‘at least’. Note that Peacocke chose to abbreviate (3) as ‘\_\_planet\_\_’, whence the schematic claim ‘There are nine  $P$ s’ becomes ‘\_\_ $P$ \_\_’. Peacocke then uses the latter on the right-hand side of the biconditional (4). But I have chosen to spell it out in full, because there is no reason not to be that explicit.

As Peacocke rightly observes, (3) ‘does not mention numbers at all’. For, any claim of the schematic form ‘there are nine  $P$ s’, with the word ‘nine’ used adjectivally, has the well-known logical form

$$\exists x_1 \dots \exists x_9 [Px_1 \wedge \dots \wedge Px_9 \wedge x_1 \neq x_2 \wedge \dots \wedge x_8 \neq x_9 \wedge \forall y (Py \rightarrow y = x_1 \vee \dots \vee y = x_9)].$$

Here the quantifications are over the (empirical) things that can enjoy property  $P$ , and there are no singular terms purporting to refer to numbers (as abstract objects). The logical form just given is often abbreviated as  $\exists_9 x P(x)$ . The numerical subscript 9 is of course in adjectival position within this shorter form (which one can read rather directly as ‘there are exactly nine things  $x$  such that  $P(x)$ ’).

Peacocke’s proposal, then, is that we accept as a priori (for the purposes of gaining insight into the kind of abstraction involved), the biconditional

$$(1) \leftrightarrow [(3) \wedge (4)].$$

### 3.2 Any a priori conjunct on one side of an a priori biconditional would be otiose

Now one might understandably wonder why Peacocke bothers with the second conjunct (4). Since (4) is supposed to be a priori,<sup>12</sup> and necessary, its presence really renders the biconditional unnecessarily prolix. It does nothing within the context, except raise the incorrect suspicion that (3) by itself might be insufficient for (1). The reason why that suspicion would be incorrect is that we have the following simple metatheorem, whose proof is given in §7.1 of the Appendix below.

Assume  $\vdash \theta$ . Then  $\varphi \leftrightarrow (\xi \wedge \theta)$  is interdeducible with  $\varphi \leftrightarrow \xi$ .

Thus anyone claiming to provide an (a priori) analysis of a claim  $\varphi$  by asserting its equivalence with the conjunction of a claim  $\xi$  and an *a priori* (hence necessary) claim  $\theta$ , would be well-advised to assert instead just  $\theta$  and the simpler biconditional  $\varphi \leftrightarrow \xi$ . This is a quite general theme that will emerge in this critique. It applies even when the biconditional with an a priori conjunct on its right-hand side is quantified; see, for example, our response below to a suggested defence of Peacocke against the objection about to be developed.

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<sup>12</sup>I do not know why Peacocke writes ‘*relatively* a priori’ (my emphasis). There does not appear to be any relatum only relative to which the claim would be a priori, but not otherwise. It should be a priori, period.



Peacocke's sentence (4) was playing the role of the a priori claim  $\theta$  in the foregoing considerations. But Peacocke's choice of (4) is troublesome for intrinsic reasons. It looks (on a first reading) as though (4) might be a priori and necessary—but this appearance is illusory.

### 3.3 The ill-formedness of Peacocke's a priori conjunct

In fact, (4) is radically ill-formed, and makes no sense at all. To see how this is so, notice that the first occurrence of ' $n$ ' in (4) is substantival, whereas its second occurrence is adjectival. *And one cannot quantify into adjectival position.* More to the point: one cannot quantify over the numerical subscript in the abbreviated form of a numerical quantifier, because (as noted above) that subscript is adjectival.<sup>13</sup> That one is indeed quantifying into the position in question is immediate from the Russellian construal of any sentence of the form ' $t$  is the unique  $x$  such that  $F(x)$ '.

Suppose Jack built exactly one house, and called it Strathallan. Then

Strathallan is the house that Jack built.

Equivalently,

Strathallan is the unique house  $h$  such that Jack built  $h$ .

Equivalently,

Strathallan is the unique  $x$  such that  $x$  is a house and Jack built  $x$ .

As noted earlier, we are here quantifying into the spot marked by the variable ' $x$ ' in the open sentence ' $x$  is a house and Jack built  $x$ '.

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<sup>13</sup>The same mistake was made by David Bostock, in his book *Logic and Arithmetic, Vol. II—Rational and Irrational Numbers*, Clarendon Press, Oxford, 1979. See my review in *Mind* 90, 1981, pp. 473–475. The mistake occurs also in Crispin Wright's monograph *Frege's Conception of Numbers as Objects*, Aberdeen University Press, 1983, at p. 38, where he countenances as well-formed certain 'sentences' that involve quantification over the numerical subscripts used in abbreviations of numerical quantifiers. (Julian Cole drew this to my attention.) Wright wrote in his Preface, at p. x, '... I have derived stimulus and understanding from David Bostock ...', which underscores how strong Oxford traditions are, once they are established. (A similar Oxford story can be told about the quantifier switch fallacy, but I shall spare the reader the details, since space is limited.)

These equivalences have their parallel in the following reformulation of Peacocke's (4) above. If (but only if) (4) is well-formed, it is equivalent to the following:

9 is the unique  $x$  such that  $x$  is a number and necessarily, for any property  $P$ , there are  $x$   $P$ s iff there are nine  $P$ s.

This brings out the problem that I alleged earlier. The would-be 'subformula'

there are  $x$   $P$ s

cannot be written out in full *as a formula with  $x$  occurring in it as a free variable*. The variable  $x$  is allowed only to occupy substantival positions within the English sentence being regimented; yet here it is trying to occupy an adjectival position, and the result is ill-formed and non-sensical.

For particular adjectival substituends for  $x$  (such as 'two', 'three', etc.) one can write out a *sentence* (i.e. a closed formula, with no free variables) meaning 'there are *that many*  $P$ s'. For example, we can write out a sentence meaning 'there are two  $P$ s':

$$\exists x \exists y [Px \wedge Py \wedge x \neq y \wedge \forall z (Pz \rightarrow z = x \vee z = y)];$$

and then write out another sentence meaning 'there are three  $P$ s':

$$\exists x \exists y \exists w [Px \wedge Py \wedge Pw \wedge x \neq y \wedge y \neq w \wedge x \neq w \wedge \forall z (Pz \rightarrow z = x \vee z = y \vee z = w)];$$

and so on. But note that none of the sentences in this recursively definable series has any free variables; nor are they cut to a common pattern that can be exhibited, syntactically, as

there are [this particular number of]  $P$ s.

Thus Peacocke's use, in (4), of the impostor fragment

there are  $n$   $P$ s

is illegitimate. He clearly thinks, mistakenly, that this fragment corresponds to an open sentence with the sortal (numerical) variable  $n$  free.

So Peacocke's ill-formed (4) ought to be abandoned. The question now arises: can we avoid this kind of syntactic error yet still recover an account of numerical abstraction that would help Peacocke set up a useful analogy with the supposedly parallel case of conceptual abstraction?

Peacocke tried to account for (1) by holding it equivalent to the conjunction of (3) and (4):

$$(1) \leftrightarrow [(3) \wedge (4)].$$

### 3.4 A leaner account of numbers

The only revised account that I can think of is the leaner one that would hold (1) equivalent to (3), period:

$$(1) \leftrightarrow (3).$$

Thus:

the number of planets is 9 if and only if there are nine planets.

Note that

1. ‘the number of planets’ is a singular term formed from the predicate ‘\_\_ is a planet’ by applying the variable-binding term-forming operator ‘ $\#x\Phi(x)$ ’, which means ‘the number of  $\Phi$ s’;
2. ‘9’ is a special kind of singular term, formulated in some canonical notation,<sup>14</sup> and is called a *numeral*; and
3. the right-hand side, by itself, carries no commitment to numbers.

Generalizing from the substantival ‘9’ and the adjectival ‘nine’, we can assert every instance of the following *Schema N*.

Schema *N*: the number of *Ps* is *n* if and only if there are *n Ps*.

An instance of the schema is obtained by choosing a natural number *k* and then:

1. replacing ‘*P*’ by any predicate;
2. replacing ‘*n*’ by the numeral for *k*; and
3. replacing ‘there are *n Ps*’ by the appropriate (i.e.  $(k + 1)$ -th) regimentation from the list of regimentations of the claims ‘There are no *Ps*’, ‘There is exactly one *P*’, ‘There are exactly two *Ps*’, ‘There are exactly three *Ps*’, ... .

Examples of instances of Schema *N* are:

$$\#xP(x)=0 \leftrightarrow \neg\exists xP(x);$$

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<sup>14</sup>The usual canonical notation employs the name 0 and the successor symbol *s*. Thus 9 would be written in this canonical notation as *ssssssss0*.

$$\#xP(x)=s0 \leftrightarrow \exists x(P(x) \wedge \forall y(P(y) \rightarrow y=x));$$

$$\#xP(x)=ss0 \leftrightarrow \exists x\exists y(P(x) \wedge P(y) \wedge x \neq y \wedge \forall z(P(z) \rightarrow z=x \vee z=y)).$$

As remarked above, the right-hand sides of these biconditionals are not cut to a common pattern, even though it is easy to give a recursive definition of the sequence of such right-hand sides: of sentences that respectively say, of each natural number  $n$ , that there are exactly  $n$  things that are  $P$ .

Not only would we want to assert each such instance of Schema  $N$  (of which we have given the first three above); we would also want to be able to *derive* each such instance within an a priori, necessarily true theory of the application-of-numbers. This is exactly analogous to the way Tarski wanted to be able, within a theory of truth, to *derive* each instance of the  $T$ -schema

$s$  is  $T$  in language  $L$  if and only if  $p$ ,

where an instance is obtained by replacing ' $s$ ' by some structural-descriptive name of a sentence of the object-language  $L$ , and replacing ' $p$ ' by a translation of that sentence into the language of the theory of truth-for- $L$ . The task of so deriving all instances of Schema  $N$  was set out as an adequacy condition on a logicist theory of the natural numbers, in my book *Anti-Realism and Logic*.<sup>15</sup>

The suggestion, therefore, is that Peacocke ought to have been seeking the foregoing adequacy condition on a theory of the-application-of-numbers. The theory vouchsafes as theorems<sup>16</sup> every instance of the schema

the number of  $Ps$  is  $\underline{n}$  if and only if there are  $n$   $Ps$ .

Thus we may assert, for each number  $k$ , (modally, and at second order)

necessarily, for all  $P$ , the number of  $Ps$  is  $\underline{k}$  if and only if there are  $k$   $Ps$ .

This, I submit, is more sensible than Peacocke's attempt to assert,<sup>17</sup> for each number  $k$ ,

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<sup>15</sup>Clarendon Press, Oxford, 1987. See especially chapter 20, 'Constructive Logicism: An Adequate Theory of Number', and chapter 25, 'On Deriving the Basic Laws of Arithmetic: or, how to Frege-Wright a Dedekind-Peano'. Peacocke could have had 'off the shelf' exactly the theory he needed in order to show that numbers are 'legitimated by application'.

<sup>16</sup>For details, see my *Anti-Realism and Logic*, *loc. cit.*.

<sup>17</sup>At p. 105 Peacocke wrote

We can imagine someone asking the following metaphysical question about numbers: how it is possible for us to mention numbers, those abstract ob-

the number of  $Q$ s is  $\underline{k}$

if and only if

there are  $k$  things that are  $Q$  and  $\underline{k}$  is the unique number  $n$  such that necessarily, for every property  $P$ , there are [exactly]  $n$  things that are  $P$  iff [there are exactly  $k$  things that are  $P$ ],

with its supposedly ‘empirical’ first conjunct (on the right-hand side) and its supposedly ‘a priori and necessary’ second conjunct. We have seen that that second conjunct is ill-formed; and, if well-formed, would be otiose anyway.

### 3.5 One cannot make the adjectival position substantival

A defender of Peacocke might suggest<sup>18</sup> that we have made too much of Peacocke’s ill-formed (4), since he could easily replace (4) with (4’) below, in which all occurrences of  $n$  are substantival:<sup>19</sup>

(4’) 9 is the unique number  $n$  such that necessarily, for any property  $P$ , the number of  $P$ s is  $n$  if and only if there are nine  $P$ s:

$$9 = \iota n \Box \forall P (\#xP(x) = \underline{n} \leftrightarrow \exists 9xP(x)).$$

But (4’) is equivalent to

(4’’) Necessarily, for any property  $P$ , the number of  $P$ s is 9 if and only if there are nine  $P$ s:

$$\Box \forall P (\#xP(x) = 9 \leftrightarrow \exists 9xP(x)).$$

(The implication from (4’) to (4’’) is straightforward, being a simple application of iota-elimination. But the converse implication, though valid, is not so straightforward to prove. See §7.2 of the Appendix below for details.)

So the suggestion, on Peacocke’s behalf, is that we should assert the revised biconditional

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jects, in describing the empirical world? For the restricted case of natural numbers and sentences like (1) [The number of planets is nine], the logical equivalence of (1) to the conjunction of (3) [there are nine planets] and (4) [Nine is the unique number  $n$  such that ...] supplies a satisfying answer to that question.

I am arguing that this answer is far from satisfying. If I am right, then we will have to inquire whether any similar answer in the case of concepts (as abstract objects) can satisfy the corresponding metaphysical question about concepts.

<sup>18</sup>Alan Millar made this suggestion in private correspondence.

<sup>19</sup>I supply precise logical forms after first stating each claim in logician’s English.

$$(1) \leftrightarrow [(3) \wedge (4'')].$$

This suffers an interesting collapse, since  $(4'')$  is of the form

Necessarily, for any property  $P$ ,  $(1) \leftrightarrow (3)$ :

$$\Box \forall P (\#xP(x) = 9 \leftrightarrow \exists_9 xP(x)).$$

Moreover, the revised biconditional is intended to be both general in  $P$  and a priori; whence what is being asserted is

Necessarily, for any property  $P$ ,

$(1) \leftrightarrow [(3) \wedge \text{Necessarily, for any property } P, (1) \leftrightarrow (3)]:$

$$\Box \forall P (\#xP(x) = 9 \leftrightarrow [\exists_9 xP(x) \wedge \Box \forall P (\#xP(x) = 9 \leftrightarrow \exists_9 xP(x))])$$

which collapses to  $(4'')$ .

The collapse is actually overdetermined, in a rather interesting way. For we have the following metatheorem:<sup>20</sup>

If  $\vdash \theta$  (whence  $\theta$  contains no free occurrences of the variable  $P$ )<sup>21</sup> then  $\Box \forall P (\Phi(P) \leftrightarrow (\Psi(P) \wedge \Box \theta))$  is interdeducible with  $\Box \forall P (\Phi(P) \leftrightarrow \Psi(P))$ .

The proof of this metatheorem, for which we assume that  $\theta$  is an arbitrary theorem, is made even easier if in addition it is assumed that  $\theta$  is the claim  $\forall P (\Phi(P) \leftrightarrow \Psi(P))$  itself. For then the theoremhood of  $\theta$  means that  $\Box \forall P (\Phi(P) \leftrightarrow \Psi(P))$  will be deducible from the empty set of premisses, and one will not need to use  $\Box \forall P (\Phi(P) \leftrightarrow (\Psi(P) \wedge \Box \theta))$  as a premiss. And that is precisely the situation we are in after following the current suggestion in this attempted defence of Peacocke. That is why I said earlier that the collapse of  $(1) \leftrightarrow [(3) \wedge (4'')]$  to  $(4'')$  is overdetermined.

Now note that  $(4'')$  is the instance, for  $n = 9$ , of Schema  $N$ . So the suggestion, on Peacocke's behalf, that we should use  $(4')$  rather than the ill-formed  $(4)$  reduces to the recommendation of the leaner theory that appeals more simply to the instances of Schema  $N$ .

<sup>20</sup>The proof will be left as an exercise to the reader, since it is so similar to the proof in §7.3 of the Appendix.

<sup>21</sup>We follow the proof-theoretic convention whereby variables, intending for binding by quantifiers and term-forming operators, never occur free in any sentences involved in a proof. Instantiations of quantified sentences within proofs are by means of parametric terms, which do not contain any free variables. See my *Natural Logic* for details concerning the first-order case. The second-order case is handled in an exactly similar fashion.

### 3.6 The best account is via Schema N

What the leaner theory does is link a numerical identity statement on the left-hand side of Schema *N* with a number-free truth-condition expressed on the right-hand side. *This* is the essentially Fregean move in abstraction. Fregean abstraction, importantly, exploits a priori equivalences between claims that *can themselves be contingent*. Fregean abstraction involves an identity statement, conventionally on the left-hand side of the equivalence, involving reference to the ‘newly existing’ abstract objects of the kind being introduced; while on the right-hand side are given the truth-conditions for that identity claim, using only concepts that apply to such things as ‘already’ exist. Thus, for example, the identity claim

the number of planets is nine

is (because of its equivalence to it) just as contingent as the claim that

there are nine planets,

even though the former involves commitment to the existence of the abstract object 9 while the latter involves commitment only to the existence of planets. Moreover, the identity statement in our preferred treatment of the natural numbers as abstract objects involves two very different kinds of singular term: an *abstractive term* (‘the number of *Ps*’) and a *numeral*. By means of the abstractive terms we apply numbers to count collections; and by means of the numerals we refer to the numbers as the objects of our ‘pure’ mathematics. Finally, note that because of the provability of all the instances of Schema *N*, we are entitled simultaneously to read the variable-binding term-forming operator ‘ $\#x\Phi(x)$ ’ as meaning ‘the number of  $\Phi$ s’, and to read the successor symbol *s* (which is used in the formation of the canonical numerals) as meaning ‘the next number after’. Because of what the right-hand sides of those biconditionals antecedently mean, these interpretations are conferred upon the symbols  $\#$  and *s* in the left-hand sides. (To see this, note how it is the very provability of those instances that rules out deviant interpretations of ‘ $\#x\Phi(x)$ ’ such as ‘the set of all  $\Phi$ s’, or ‘the unique  $\Phi$ ’, or ‘the stuff of which the  $\Phi$ s are composed’, etc. Only the numerical interpretation fits in with the provability, hence truth, of the biconditionals that instantiate Schema *N*.)<sup>22</sup>

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<sup>22</sup>Compare my conjecture about the simultaneous fixing of the concepts of truth and translation via Tarski’s *T*-Schema, in ‘Game Theory and Convention T’, *Nordic Journal of Philosophical Logic*, 6, no. 1, 2001, pp. 3–20.

The question now arises whether a theory of numbers as abstract objects (such as the theory presented in *Anti-Realism and Logic*) would provide a good enough number-theoretic analogue to lend some measure of plausibility by philosophical precedent to a treatment (such as Peacocke's) of concepts as abstract objects.

## 4 Examination of Peacocke's proposal in the case of concepts

Peacocke's main claim in the *numerical* case was

$$(1) \leftrightarrow [(3) \wedge (4)].$$

In the *conceptual*<sup>23</sup> case Peacocke wanted to claim, in parallel fashion, that

$$(2) \leftrightarrow [(5) \wedge (6)],$$

where the sentences involved were (pp. 105–6):

- (2) John believes that Lincoln Plaza is square
- (5) John is in some state **S** that has the relational property **R**
- (6) The content that Lincoln Plaza is square is the unique content *p* such that necessarily for any state **S**, **S** is a belief that *p* iff **S** has the relational property **R**.

Peacocke wrote (p. 106)

The proposal I want to explore is that something structurally similar [to the numerical analysis he had given—NT] holds for thoughts. According to this proposal, there is some relational

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<sup>23</sup>I say 'conceptual' here, even though, strictly speaking, I should say 'contentual'. Throughout chapter 4, Peacocke is concerned with the question of how *contents* can be regarded as abstract objects, by analogy with the way numbers are. His eventual aim is to extract concepts from contents of which they are constituents. But he had also, in chapter 1, given the general form of a theory of possession-conditions for *concepts*. Presumably the two approaches have to mesh. One could characterize certain concepts by the method of chapter 1, and then combine them to form a content, say *p*. Alternatively, one could try to characterize the whole content *p* more directly (in terms of belief-states) by the method of chapter 4. The two methods would have to deliver the same content. In this paper we are concerned, as Peacocke was in chapter 4, with the latter characterization of whole contents.



property **R** with two characteristics. First, the relational property **R** can be specified by mentioning no relationships to concepts or thoughts but only relations to other empirical things and states. Second, [(2)] is equivalent to the conjunction of (5) and (6). ... Let us take it for the moment that we have fixed on one particular mode of presentation of that plaza and on one particular mode of presentation of the property of being square. The proposal, then, is that (2), so understood, stands to the conjunction of (5) and (6) as (1) stands to the conjunction of (3) and (4). Reference to a content in describing a mental state serves as a means of encoding a condition on the mental state, a condition that can be formulated without making reference to contents. Under the proposal I am developing, concepts in turn pull their weight in the description of the empirical world by making a systematic contribution to the condition a mental state must satisfy if it is to be a state with a given propositional content.

The considerations in the previous section show that there is no point in trying to force a parallel between the equivalences

$$(1) \leftrightarrow [(3) \wedge (4)]$$

and

$$(2) \leftrightarrow [(5) \wedge (6)].$$

If the latter equivalence, concerning the case of concepts, happens to be true, it will have to be argued on its own merits to be a correct account of concepts as abstract objects. Such an argument should not rely on a forced and false analogy with the former equivalence, concerning the case of numbers.

It behooves us, then, to look more closely at the precise logical forms of (5) and (6), and indeed at that of the equivalence  $(2) \leftrightarrow [(5) \wedge (6)]$  itself. To this end, we shall introduce some abbreviations, whose use will make it easier to see at a glance what the logical forms of Peacocke's claims are. We shall employ the following expressions on the left to mean what is furnished on the right, or in accordance with the convention stated on the right:

$x$	variable ranging over thinkers
$S, \psi, \theta$	variables ranging over states of a thinker

$R[\psi]$	state $\psi$ has relational property $R$
$\psi[x]$	thinker $x$ is in state $\psi$
$p, \pi$	variables ranging over propositions
$\sigma$	variable ranging over sentences of English
$\gamma(\sigma)$	the proposition that $\sigma$
$\iota p F(p)$	the proposition $p$ such that $F(p)$
$\mathcal{B}(\psi, p)$	$\psi$ is a state of belief with content $p$
$B(x, p)$	$x$ has a belief with content $p$
$\text{Bel}(x, \sigma)$	$x$ believes that $\sigma$

Peacocke writes ‘...is a belief that ...’ rather than ‘...is a state of belief with content ...’. We are trying here to be scrupulous in not conflating sentences with propositions. If  $p$  is already understood as (a variable for) a *proposition*, then the singular term ‘that  $p$ ’ is really ill-formed. In any singular term of the form ‘that  $\varphi$ ’ denoting a proposition,  $\varphi$  must be a placeholder for a *sentence*. The claims  $B(x, \gamma(\sigma))$  and  $\text{Bel}(x, \sigma)$  are analytically equivalent from the vantage point of the theorist committed to the existence of propositions.

Our terminological conventions above enable us from now on to suppress the nouns or noun-phrases ‘thinker’, ‘proposition’, ‘belief’, ‘state’ and ‘relational property’. We shall also be achieving a measure of generality, by replacing Peacocke’s sentence ‘Lincoln Plaza is square’ with the simple letter  $\sigma$ , which will do duty for declarative sentences in general; and by replacing the name ‘John’ with the variable  $x$  for thinkers in general.

Properly regimented, then, (5) becomes, in its more general form,

$$(5a) \exists S \exists R (S[x] \wedge R[S])$$

and (6) likewise becomes

$$(6a) \gamma(\sigma) = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi])$$

What Peacocke thinks is a conjunction of the form (5) $\wedge$ (6)—yielding his account of ‘ $x$  believes that  $\sigma$ ’ ( $\text{Bel}(x, \sigma)$ )—is really an existentially quantified conjunction:

#### **Peacocke’s analysis of a simple belief-attribution**

The simple belief-attribution  $\text{Bel}(x, \sigma)$  is to be analyzed as

$$\exists S \exists R ( (S[x] \wedge R[S]) \wedge \gamma(\sigma) = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi]) ).$$

This is clear from the fact that in Peacocke's sentence (6) (as stated on p. 106) he uses his variable **R** with anaphoric back-reference to its occurrence in (5).

We are not told what a 'relational property' of a thinker's state might be, except in so far as it may *not* be specified 'by mentioning [any] relations to concepts or thoughts. It may be specified by mentioning 'only relations to other empirical things and states'. (p. 106) Nor are we told whether these states (of a thinker) are to be characterized in purely physicalistic terms, or whether they may be specified by means of psychological, functional or intensional vocabulary. The 'bold view', according to Peacocke, is that purely physicalistic characterization is possible; the 'unassuming view' is that one might have to resort to using psychological, functional or intensional vocabulary.

One feature of Peacocke's account is striking: in his sentence (6), there is no mention of John. Correspondingly, in the regimented analysans there is no occurrence of the variable  $x$  (for the thinker) in the second conjunct, the embedded propositional-identity claim. Thus what it is about the state  $S$  of the thinker that makes  $S$  a belief with content  $p$  is to be spelled out *without any reference to the thinker in question*.

Now the original analysandum (or explicandum) was

$x$  believes that  $\sigma$  ( $\text{Bel}(x, \sigma)$ );

and its truth is to consist, according to Peacocke, in  $x$ 's being in a certain state with a certain relational property—a relational property which, necessarily, makes any state that has it a belief that  $\sigma$ , and which the belief that  $\sigma$  has. The relational property must therefore exist as a function of (and only of) the English sentence  $\sigma$  involved in the original belief-attribution.

## 5 A re-formulation of Peacocke's proposal in the case of concepts

### 5.1 Generality about propositions

Note that Peacocke's account, as it stands, has the form

$x$  believes that  $\sigma \leftrightarrow \exists S \exists R ((S[x] \wedge R[S]) \wedge \gamma(\sigma) = \iota p \Phi(R, p))$ .

In this form we have the symbol  $\sigma$  as a placeholder for an English sentence. The operation represented by the function symbol  $\gamma$  is that of determining

the proposition expressed by the sentence  $\sigma$ . But Peacocke explicitly maintains that his account applies to propositions quite generally, whether or not they are expressed by sentences in a language mastered by the thinker. Peacocke disavows any commitment to the ‘linguistic priority’ thesis.<sup>24</sup> The linguistic priority thesis is that one can grasp a proposition only by understanding a sentence (of some language) that expresses it. Hence also, one can grasp a concept only by understanding some linguistic expression of it. Not to be committed to this thesis is to maintain the possibility, then, that there might be some proposition that some thinker might be able to grasp *without* necessarily having any way of expressing it in a (public) language. This possibility, however, is compatible with the claim that every proposition is nevertheless linguistically expressible. Language may well be adequate unto propositions; while thinkers might not always need language in order to grasp them. Thus we have to be cautious before inferring from Peacocke’s disavowal of the linguistic priority thesis that he would countenance any *linguistically inexpressible* propositions. He might suffer no loss of generality from taking every proposition to be within the range of the function  $\gamma$  that maps English sentences to their contents. Still, it is reasonable to assume, given what Peacocke says about perceptual contents that might be linguistically inexpressible,<sup>25</sup> that he would be happier to deal with propositions in general rather than with propositions that happen to be contents of English sentences.

So we could say, on Peacocke’s behalf, that his account is also to the effect that

$$x \text{ believes } \pi \leftrightarrow \exists S \exists R ((S[x] \wedge R[S]) \wedge \pi = \iota p \Phi(R, p)),$$

where the symbol  $\pi$  now ranges over propositions, whether or not they are linguistically expressed. Thus far we have preserved the essential form of the analysis that Peacocke actually gave. The right-hand side of the foregoing conditional, written out in full, we shall call Peacocke’s analysans  $\alpha(x, \pi)$ :

$$\alpha(x, \pi) : \exists S \exists R ((S[x] \wedge R[S]) \wedge \pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi])).$$

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<sup>24</sup>He writes (p. 118): ‘This treatment can assist the theorist of thought in meeting the objection that the structure of thoughts is but a reflection of the structure of sentences. ... [My] account ... gives an explanation of what it is for a thought to have a certain structure and constituents that does not immediately appeal to the notion of a sentence and its constituents.’

<sup>25</sup>I am assuming that all linguistically expressible contents are conceptual contents. Hence Peacocke’s invocation of non-conceptual contents (at p. 63) in his treatment of perceptual concepts would appear to entail their linguistic non-expressibility.

## 5.2 Finding two genuine conjuncts

Peacocke, however, had been seeking an analysis of the form

$$x \text{ believes } \pi \leftrightarrow (A(x, \pi) \wedge D(\pi)),$$

where  $A(x, \pi)$  would be a contingent claim and  $D(\pi)$  would be an a priori claim. We therefore need to find, on his behalf, two sentences  $A(x, \pi)$  and  $D(\pi)$  that fit the bill: their conjunction must be equivalent to Peacocke's own analysans  $\alpha(x, \pi)$ . Such sentences are as follows.

$$\begin{aligned} A(x, \pi) &: \exists \theta (\theta[x] \wedge \mathcal{B}(\theta, \pi)); \\ D(\pi) &: \exists R (\pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi])). \end{aligned}$$

The sentence  $A(x, \pi)$  says that  $x$  is in some state which is a belief that  $\pi$ —an acceptable circumlocution for ‘ $x$  believes that  $\pi$ ’.  $A(x, \pi)$  is a consequence of Peacocke's analysans  $\alpha(x, \pi)$ —as will be proved in due course—so he ought to find  $A(x, \pi)$  acceptable.

Similarly with the sentence  $D(\pi)$ . The sentence  $D(\pi)$  says that there is a relational property  $R$  such that  $\pi$  is the proposition for which, necessarily, belief-states (with that proposition as content) are precisely the states enjoying relational property  $R$ .

Note that Peacocke's analysans

$$\exists S \exists R ( (S[x] \wedge R[S]) \wedge \pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi]) )$$

has as a rather easy consequence<sup>26</sup> the following existential quantification of its second embedded conjunct:

$$\exists R \pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi]).$$

And this is our sentence  $D(\pi)$ .

## 5.3 Our re-formulated proposal is equivalent to Peacocke's

It is a straightforward exercise to show that the conjunction of our two sentences  $A(x, \sigma)$  and  $D(\pi)$  is equivalent to Peacocke's account  $\alpha(x, \pi)$  above.

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<sup>26</sup>The inference in question is of the form

$$\frac{\exists S \exists R (\Phi(R, S) \wedge \Psi(R))}{\exists R \Psi(R)}$$

Its proof takes four steps, and will be left to the reader.

We have just noted how  $D(\pi)$  follows from  $\alpha(x, \pi)$ . We shall give in §7.3 of the Appendix below two proofs, which we shall call  $\Pi$  and  $\Sigma$ , and which effect transitions from the indicated premiss(es) to the indicated conclusions, respectively:

$$\begin{array}{ccc} \alpha(x, \pi) & & \underbrace{A(x, \pi), D(\pi)} \\ \Pi & & \Sigma \\ A(x, \pi) & & \alpha(x, \pi) \end{array}$$

The proof  $\Sigma$  (in the Appendix below) shows that  $A(x, \pi) \wedge D(\pi)$  implies  $\alpha(x, \pi)$ . The converse implication is secured by the proof  $\Pi$  (in the Appendix) and the fact, already noted, that  $D(\pi)$  follows very easily from  $\alpha(x, \pi)$ :

$$\begin{array}{ccc} \alpha(x, \pi) & \alpha(x, \pi) & \\ \Pi & \vdots \text{ easy inference} & \\ A(x, \sigma) & D(\pi) & \\ \hline A(x, \sigma) \wedge D(\pi) & & \end{array}$$

Recall that  $\alpha(x, \pi)$  was Peacocke's actual analysis of ' $x$  believes  $\pi$ ', which unfortunately was not of the form that he wanted. He wanted a genuine conjunction, not an existentially quantified conjunction. In  $A(x, \pi) \wedge D(\pi)$  we have found a genuine conjunction equivalent to  $\alpha(x, \pi)$ . (For the fully detailed formal proofs that establish this equivalence, see §7.3 of the Appendix.)

#### 5.4 What it is for John to believe that Lincoln Plaza is square

Here, then, is our amended proposal on Peacocke's behalf. Let  $\pi$  be the proposition THAT LINCOLN PLAZA IS SQUARE. (After each sentence we give its logical form.) The claim

John believes that Lincoln Plaza is square:

$$B(j, \text{THAT LINCOLN PLAZA IS SQUARE})$$

is equivalent to the (now genuine!) conjunction of the contingent claim

John is in a state which is a belief with the content that Lincoln Plaza is square:

$$\exists \theta (\theta[j] \wedge \mathcal{B}(\theta, \text{THAT LINCOLN PLAZA IS SQUARE}))$$

and the a priori claim

There is some relational property making THAT LINCOLN PLAZA IS SQUARE the unique content  $p$  such that necessarily, the states with that relational property are states of belief with content  $p$ :  
 $\exists R(\text{THAT LINCOLN PLAZA IS SQUARE} = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi]))$ .

### 5.5 Generalizing the amended proposal

Clearly matters can be stated with greater generality, as Peacocke no doubt intended. He did not take himself to be dealing only with the proposition THAT LINCOLN PLAZA IS SQUARE. Nor did he take himself to be dealing only with propositions of the form

PREDICATIONAL CONCEPT(REFERENTIAL CONCEPT).

Rather, we ought to be able, in the above analysis, to replace THAT LINCOLN PLAZA IS SQUARE with the variable  $\pi$ , standing for an arbitrary proposition, of any degree of complexity, and with any manner of primitive constituents (provided only that they are properly combined).

By the same token, Peacocke did not intend his theory to deal only with one fortunate individual John. Whatever is said about John should be sayable about thinkers in general. Let us, therefore, re-formulate matters so that we achieve the required generality on both thinker and proposition:

For all thinkers  $x$  and for all propositions  $\pi$ :

$x$  believes  $\pi$ :  
 $B(x, \pi)$

is equivalent to the conjunction of the contingent claim

$x$  is in a state which is a belief with the content  $\pi$ :  
 $\exists \theta (\theta[x] \wedge \mathcal{B}(\theta, \pi))$

and the a priori claim

There is some relational property making  $\pi$  the unique content such that necessarily, the states with that relational property are states of belief with content  $\pi$ :  
 $\exists R(\pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi]))$ .

What we have, then, is an account of the form

$$(F) : \forall x \forall \pi (B(x, \pi) \leftrightarrow (A(x, \pi) \wedge D(\pi))).$$

For any instantiations of  $x$  and of  $\pi$ , the first conjunct  $A(x, \pi)$  on the right-hand side will be contingent, while the second conjunct  $D(\pi)$  will be a priori—or so Peacocke maintains.

### 5.5.1 Disanalogies with Fregean abstraction

Two comments are now in order, which detract from the Fregean credentials of Peacocke's enterprise. First, note that the foregoing universally quantified biconditional ( $F$ ) has, on both its sides, occurrences of a variable ranging over propositions. So we cannot have from it what a Fregean abstraction succeeds in providing. That is, we cannot claim an elucidation of the truth-conditions of a canonical claim (usually in the form of an identity) about the newly abstracted objects *in terms that manifestly carry no commitment to the existence of such objects*.

Moreover, if Peacocke tries to meet this objection by resorting to the form of analysis in which the analysandum is a relation between a thinker and an *English sentence*  $\sigma$ , rather than a proposition—

$$\forall x \forall \sigma (\text{Bel}(x, \sigma) \leftrightarrow (A(x, \gamma(\sigma)) \wedge D(\gamma(\sigma))))$$

—thereby claiming commitment to propositions only by virtue of their being invoked on the right-hand side, he faces further objections. For he is now restricted to beliefs that are linguistically expressible. Moreover, for the Fregean, the claim involving the new abstract objects is taken to be *the analysandum*—whereas for Peacocke it would appear to be *the analysans* that involves the new abstract objects.

Secondly, we noted above, in the numerical case, that having an a priori (hence necessary)<sup>27</sup> conjunct  $\theta$  in an analytic biconditional

$$\varphi \leftrightarrow (\xi \wedge \theta)$$

would be rather fruitless. Given its a priori status,  $\theta$  should simply be asserted as an a priori claim, unconditioned by  $\varphi$ ; and the biconditional should be informatively strengthened to

$$\varphi \leftrightarrow \xi.$$

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<sup>27</sup>The a priori claims in question are definitely not contingent; for they are quite unlike the Kripke example 'The standard meter rod is one meter long.'



### 5.5.2 Two general axioms

What this suggests for our re-formulation of Peacocke's account is that all the severally assertable a priori claims

$$D(\pi_1), D(\pi_2), D(\pi_3) \dots$$

should be brought within the scope of a single (a priori) generalization:

$$(D) : \quad \forall \pi D(\pi),$$

while the informatively strengthened biconditionals resulting from separating out their a priori components should be gathered into another generalization:

$$(E) : \quad \forall x \forall \pi (B(x, \pi) \leftrightarrow A(x, \pi)).$$

Peacocke's actual account, on its intended general reading, was

$$\begin{aligned} & \forall x \forall \pi (B(x, \pi) \leftrightarrow \\ & \exists S \exists R ( (S[x] \wedge R[S]) \wedge \pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi]) ), \end{aligned}$$

or, using another abbreviation that was introduced earlier,

$$\forall x \forall \pi (B(x, \pi) \leftrightarrow \alpha(x, \pi)).$$

We have established, by means of the proofs in the Appendix, that

$$\alpha(x, \pi) \leftrightarrow (A(x, \pi) \wedge D(\pi)).$$

Hence by substitution of logical equivalents we reached our initial re-formulation of Peacocke's account as

$$(F) : \quad \forall x \forall \pi (B(x, \pi) \leftrightarrow (A(x, \pi) \wedge D(\pi))).$$

In §7.4 of the Appendix below, we show the following:

If  $D(b)$  is a priori, that is, if there is a proof of  $D(b)$  from no assumptions, parametric in  $b$ , then  $(E)$  and  $(F)$  are interdeducible.

This provides the logical justification for our final re-casting of Peacocke's theory as the closure of the axioms  $(D)$  and  $(E)$ .

Expanding the abbreviations  $A(x, \pi)$  and  $D(\pi)$  therein we therefore obtain the following axiomatization of Peacocke's theory:

$$\begin{aligned} (E) : & \quad \forall x \forall \pi (B(x, \pi) \leftrightarrow \exists \theta (\theta[x] \wedge \mathcal{B}(\theta, \pi))); \\ (D) : & \quad \forall \pi \exists R (\pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi])). \end{aligned}$$

## 6 Critical assessment of the re-formulated proposal

Our re-formulation of Peacocke's account via the axioms  $(E)$  and  $(D)$  just given is logically equivalent to his. Thus whatever we can learn from our re-formulation will apply to his original account. And what we have learned is that Peacocke's account, *by his own sought lights*, amounts to the following. First, we have a prolix way (given by  $(E)$ ) of re-stating any belief-attribution, which involves *postulating a state of the thinker*. The re-statement of belief-attributions provided by  $(E)$  is platitudinous; *of course* to have a belief is to be in a certain state.

Secondly, we have the general a priori claim  $(D)$  to the effect that any content is individuated by there being an empirical relational property holding of exactly those states that are beliefs with that content.

### 6.1 The a priori status of $(D)$

If  $(D)$  is a priori, how is it to be established? What train of armchair-bound reasoning will deliver  $(D)$  as a conclusion unconditioned by any assumptions? Given the logical form of  $(D)$ , what is clearly needed is a method which, applied to an arbitrary given content  $\pi$ , will yield a suitable (empirical!) relational property  $R$  with the claimed feature: namely, that

$$\pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi]).$$

Peacocke believes that this can be done by starting with the possession conditions of the constituent concepts of  $\pi$ , and 'multiplying them out' in order to attain the required specification of  $R$ . This is quite a tall order, given that the process of 'multiplying out' was illustrated only for the degenerate example of a singular predication ('Lincoln Plaza is square'). Nowhere does Peacocke explain, with the required degree of generality, how this 'multiplying out' should proceed with contents of arbitrary compositional pedigrees and with all manner of primitive embedded concepts—and the great variety that can be anticipated in the detailed statements of their possession conditions. But that is Peacocke's problem, not mine. My task here has been simply to show, first, that Peacocke's analysis  $\alpha(x, \pi)$  is provably equivalent to our conjunction  $A(x, \pi) \wedge D(\pi)$ ; and secondly, that  $(F)$  is equivalent to the conjunction of  $(D)$  and  $(E)$ , if  $(D)$  is indeed a priori. For that twofold task we do not need to know whether a satisfactory explanation can be given for the claimed a priori status of  $(D)$ , or indeed of any of its instances  $D(\pi)$ .

The reader exercised by this question, however, will no doubt be asking exactly what sort of necessity is to be understood by the modal operator  $\Box$  in  $(D)$  and in  $D(\pi)$  (and in  $\alpha(x, \pi)$ ). Is it epistemic? Is it metaphysical? The stress on  $R$  being *empirical* inclines one to think that the modality must be that of metaphysical necessity; for how could the empirical facts that constitute a thinker's state as one of believing a certain content be determinable *a priori*? Fortunately, given the dialectical structure of this discussion, this is a problem that can be left for Peacocke's followers to solve.

The *a priori* claim made by  $(D)$  strikes one at first glance as a rather obvious principle of conceptual-role semantics or functionalist philosophy of mind—hence not at all original.

But this judgment may be too harsh. Peacocke's cannot be a straightforward version of functionalism (in the tradition of Putnam), since he says nothing about the states' essential role being played out in a program mediating between perceptual input and motor output. He treats of belief in a theoretically insulated way, using the notion 'state of belief' as a primitive, and concerning himself only with the relation between such states and their contents—also taken as theoretically primitive, albeit composed, ultimately, out of constituent concepts. The wider 'embedding', as it were, of these states within the mental and behavioral ecology of a properly functioning mind seems to be neglected. So, to the extent that Peacocke's version of conceptual role semantics is original, a factor that detracts from its originality is that it seems not to address the wider issues that provided the *raison d'être* of the traditional version of such a semantics.

## 6.2 Extensionality of the relation $\mathcal{B}$

There is another respect in which Peacocke's principle is unusual. If so, then it is not a respect that he himself has stressed, and it is not one on which any of his earlier commentators has remarked.

It is this: Peacocke thinks it is possible to determine contents *without considering any propositional attitudes apart from belief*. How is this so?

Let me point out, first, that in all the logical manipulations in the first two proofs given in §7.3 of the Appendix below, the modal operator  $\Box$  is otiose. One could expunge it at all its occurrences, and the two proofs in question would still go through, relieved now of the need for the occasional step of  $\Box$ -elimination. (We do not perform any  $\Box$ -introductions in these proofs.) Moreover, if we venture to drop the box, then whatever purportedly *a priori* claims we make—without the box in formerly dominant position in

positive subformulas—will be *all the easier* to defend as a priori.

That having been done, we can then consider the de-modalized claims in the same relations of logical implication. In particular, the general a priori claim that we called (D) above will have become the claim

$$(D1) \forall \pi \exists R (\pi = \iota p \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi])).$$

Let us for the moment ignore Peacocke's own stipulation that the relational property  $R$  'can be specified by mentioning no relationships to concepts or thoughts but only relations to other empirical things and states.' For such a stipulation cannot avert the fact that (D1) has the consequence

$$(D2) \forall p \forall \pi (\forall \psi (\mathcal{B}(\psi, p) \leftrightarrow \mathcal{B}(\psi, \pi)) \rightarrow p = \pi),$$

a principle that involves no mention of the relational property  $R$ . (For the formal proof that (D1) implies (D2), see §7.5 of the Appendix.)

The principle (D2) is a familiar way of stating 'extensionality' of the relation  $\mathcal{B}$ .<sup>28</sup> (Think of this relation for the time being simply as a binary relation in first-order logic, without the earlier connotations of belief.) The most familiar instance of this extensionality principle is of course in set theory, where we have

$$\forall x \forall y (\forall z (z \in x \leftrightarrow z \in y) \rightarrow x = y).$$

Interestingly, we also have such a principle in the theory of linear orderings:

$$\forall x \forall y (\forall z (z < x \leftrightarrow z < y) \rightarrow x = y).$$

The extensionality principle for any binary relation says, simply, that no two distinct things can be borne the relation by exactly the same things. (Thus a good counterexample to extensionality would be the relation ' $x$  loves  $y$ '. One person can be the sole lover of two others.)

Peacocke's general a priori principle (D2) about content-identity is to the same effect: it says that no two distinct contents could be contents of exactly the same states that count as believings. One presumes Peacocke would be taking into consideration not just actual states of thinkers, but *all possible* states of thinkers. (See the closing considerations below, concerning a modalized form of the extensionality principle.)

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<sup>28</sup>The now outmoded terminology 'internal relation' was introduced by Andrzej Mostowski, 'An undecidable arithmetical statement', *Fundamenta Mathematicae* 36, 1949, pp. 143–164; at p. 146. Logicians nowadays use 'extensional' rather than 'internal'.

### 6.3 Belief and desire

Note that there is no mention, in Peacocke's account (nor, therefore, in our equivalent re-formulation of it) of any other propositional attitude, such as desire. Yet belief makes no sense except in so far as it dovetails with desire to produce intentional action. Belief and desire are inextricably linked concepts, each needing the other in order to make sense. There could be no such agent as a 'pure believer', capable only of beliefs but of no other propositional attitudes. For on what basis could one possibly attribute a belief to such an agent? Certainly not by interrogating it—for whatever response one might thereby obtain would have to be explained by appeal, at least in part, to certain desires that must have prompted the agent to reply to the interrogation.<sup>29</sup> It might be the desire to satisfy us with an answer, or the desire to avoid being tortured for being non-responsive. But it will at least be a non-trivial desire—however unrelated its content might be to the content(s) with respect to which one might be trying to elicit expressions of belief or disbelief.

The mutual necessitation of belief-states and desire-states is evident also in the way that one has to increase the complexity and discrimination of the respective contents on each of these two dimensions *together*, and not only on one of them. Unless a creature is capable of, say, perceptually based conceptual discriminations that are fine-grained enough, it makes no sense to postulate its being in a fine-grained desire-state. For, attributing a desire involves commitment to an appropriate conception of what, for the agent in question, would satisfy the desire—namely, the eventual (and detectable) obtaining of the state of affairs represented by the desire-content in question. The agent needs, as it were, to be able to tell that it has finally got what it wants, in order for any wanting not to be logically wanting. *Mutatis mutandis*: unless the creature can entertain sufficiently finely grained desires, there is no reason to attribute to it a capacity to form correspondingly finely-grained beliefs. We can call this phenomenon the *analytic equilibrium* of the capacity for beliefs and the capacity for desires. They have the same range of potential contents largely because they require a matching fine-grainedness of their respective contents.

So, given that belief calls for desire (just as much as desire calls for belief) it is an interesting claim on Peacocke's part<sup>30</sup> that, within a fuller

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<sup>29</sup>This is why a computer's passing a Turing test for intelligence would not reveal the computer to be capable of having beliefs—no matter what desires it might *claim* to have.

<sup>30</sup>The claim in question is one that we have derived by pure logic from Peacocke's own

populational psychology, so to speak, one could focus just on the belief-states in order to determine the identities of thought-contents.<sup>31</sup>

#### 6.4 The thesis of doxastic sufficiency for determination of content

If there is anything unusual about Peacocke's account, it seems to me that it is this claim (*D2*) of 'doxastic sufficiency' for the determination of content. It is not my concern here to argue for or against this claim; it is enough to have uncovered it as a general a priori claim at work in Peacocke's account. For what has emerged is that Peacocke's 'Principle of Dependence':

There can be nothing more to the nature of a concept than is determined by a correct account of the capacity of a thinker who has mastered the concept to have *propositional attitudes* to contents containing that concept [emphasis added]

is really the following stronger principle:

There can be nothing more to the nature of a concept than is determined by a correct account of the capacity of a thinker who has mastered the concept to have *beliefs*<sup>32</sup> involving contents containing that concept.

There is a strong echo here of the way that one might take the *introduction rule* as constitutive of the sense of the logical operator that it governs, and take the corresponding *elimination rule* as merely explicative of that sense; or, conversely, take the *elimination rule* as constitutive and the *introduction rule* as explicative. The logical operator could not be in play while being governed *solely* by its introduction rule or *solely* by its elimination rule. Both rules have to be in force (used and respected by competent reasoners) in order for the logical expression in question to signify the logical operation in question (or: to determine the logical concept in question). Even

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claims. I am not aware of any textual evidence that Peacocke knew that he had committed himself to (*D2*)—or that he had *succeeded* in so doing via his choice of 'axiom(s)', should he find (*D2*) an acceptable principle.

<sup>31</sup>One could also focus on just the desires—a point I owe to Julian Cole. In all the principles under discussion, simply replace  $\mathcal{B}$  with  $\mathcal{D}$ , and they should enjoy unchanged status as true and/or a priori. But are desires subject to constraints of rationality as demanding as those for beliefs?

<sup>32</sup>Respectively: *desires*—see the previous footnote.

though the principle of harmony conjures up the elimination rule as the required ‘balancer’ of the introduction rule (and *vice versa*), each of these rules suffices to *fix the sense* of the logical operator in question.

Similarly, Peacocke seems to be maintaining that a content’s role in belief-states suffices to fix its sense; and perhaps also that its role in states of desire likewise suffices to do so. Yet each attitude (belief and desire) conjures up the other, in necessary conceptual tandem. One cannot be capable of either one of them without being capable of the other.

## 6.5 The need for normativity in the primitive relation $\mathcal{B}$

The principle of doxastic sufficiency for determination of content calls for further comment. It strikes me as misguided, to say the least, that one might regard as a licit ‘belief-state’  $\psi$  on the part of a thinker—that is, a state such that  $\mathcal{B}(\psi, p)$  for some propositional content  $p$ —any state that a thinker is purportedly ‘in’ when making what would strike one as irrational protestations of belief. Not just any old occurrence of believing-because-the-thinker-claims-it can figure in the scope of the quantification over belief-states that help to determine content. If I enter the room wild-eyed and raving, claiming that there is a pink elephant mounting a purple giraffe in the corner, that ‘belief-state’ is rightly regarded as *ineligible* to help determine content in the way provided for by the doxastic sufficiency principle. I am able genuinely to believe (albeit in an obviously hallucinatory way) *that content* only to the extent that I possess the constituent concepts. The relevant possession claims would have to be grounded in exercises of conceptual mastery in *other* situations where my professed beliefs have been reasonable in the light of the available evidence.

It therefore behooves us to read the expression  $\mathcal{B}(\psi, p)$  as ‘ $\psi$  is a state of *reasonable* belief with content  $p$ ’, in so far as we are concerned to characterize the constitution conditions for contents correctly. Indeed, it seems as though one must read it this way in order to solve the problem of normativity when attacking the problem in Peacocke’s direction.<sup>33</sup> For Peacocke’s strategy is to locate propositional contents first in conceptual space, so to speak, and then to carve from them their constituent concepts. That seems to be his ‘order of explanation’ (to borrow a phrase from Dummett)<sup>34</sup> Yet Peacocke also seems to exploit the reverse order, from the (possession conditions of) the constituent concepts in a thought to the determination of the

<sup>33</sup>I owe this point to Tyler Hower.

<sup>34</sup>See M. A. E. Dummett, *Frege: Philosophy of Language*, Duckworth, 1973, at p. 4.

relational property  $R$  so crucial to characterizing it (the thought) as the very content that it is. To be sure, his characterization of possession conditions for concepts has them (potentially) shot through with normativity, given the provisos that Peacocke builds in about normal perceptual conditions, proper functioning of sensory organs, primitively compelling argument forms etc.;<sup>35</sup> whence his procedure of ‘multiplying out’ the various clauses from possession conditions for constituent concepts can yield normative constraints on the content composed out of those concepts. This, however, corresponds to what Dummett (*loc. cit.*) called the *order of recognition*. For the story of content *constitution* that is to be told in the contrasting order of explanation, we would appear to need some way of locating normativity in the whole contents themselves. Such normativity would then be able to be inherited by conceptual constituents. One cannot obtain normative concepts by carving up non-normative contents; though of course one can obtain normative contents by composing them out of normative concepts.

## 6.6 Modalized extensionality

Earlier I explained why one could suppress occurrences of the modal operator  $\Box$  in the regimentation of Peacocke’s account. The discovery that the extensionality principle is implied by that account requires us, however, to re-visit the issue of the modal operator before closing. Peacocke had originally placed the modal operator in the second embedded conjunct of his analysans  $\alpha(x, \pi)$ , as follows:

$$\alpha(x, \pi) : \exists S \exists R ((S[x] \wedge R[S]) \wedge \gamma(\sigma) = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi])).$$

In our provably equivalent re-formulation of Peacocke’s account, the same modal operator found its way into the principle we called (D):

$$(D) \quad \forall \pi \exists R (\pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi])).$$

We then suppressed the modal operator, to obtain the principle we called (D1):

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<sup>35</sup>Skorupski (*loc. cit.*) claims that Peacocke’s actual possession conditions for the concepts RED and AND fail to imbue them with the required normativity. This is obviously a criticism orthogonal to the one that I am making. If Skorupski is right, then there is a double failure on Peacocke’s part: a failure to capture the required normativity of concepts with an appropriate formulation of their possession conditions; and a failure to invest whole contents with normativity via the determination principle that treats of their being potential objects of *reasonable* beliefs.



$$(D1) \quad \forall \pi \exists R (\pi = \iota p \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi])).$$

The proof in §7.5 of the Appendix, in a free second-order logic without modal operators (but with iota, the descriptive operator, primitive), shows that (D1) implies the extensionality principle (D2):

$$(D2) \quad \forall p \forall \pi (\forall \psi (\mathcal{B}(\psi, p) \leftrightarrow \mathcal{B}(\psi, \pi)) \rightarrow p = \pi).$$

If we were to restore the modal operator, we would obtain the modalized extensionality principle (D2□):

$$(D2\Box) \quad \forall p \forall \pi (\Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow \mathcal{B}(\psi, \pi)) \rightarrow p = \pi).$$

There is a sound system of inferential rules for free second-order modal logic with iota primitive, in which one can deduce (D2□) from (D). We shall not detain the reader with the details here. The proof has basically the same structure as the proof in §7.5 of the Appendix, with the extra steps needed for the introduction and elimination (in S4-like fashion) of the modal operator □. All the usual introduction and elimination rules—for connectives, quantifiers, the description operator and the modal operator—need to be re-formulated carefully so that when acting in concert they produce a sound system. This is a phenomenon familiar to modal logicians, and its exploration (in order to justify this brief remark about deducibility in the modal system) would take us too far afield, given the scope of our concerns in this paper. This closing consideration does mean, however, that (on Peacocke’s behalf) we can say that the extensionality principle implicit in his account can be taken to be the modalized one—(D2□)—thereby justifying the presumption that Peacocke is taking into account *all possible* states of thinkers when stating a sufficient yet ‘extensional’ condition for content-determination.<sup>36</sup>

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<sup>36</sup>The reader is reminded that by ‘extensional’ here we mean the property of a relation  $R$  expressed by the sentence

$$\forall x \forall y (\forall z (R(z, x) \leftrightarrow R(z, y)) \rightarrow x = y).$$

Thus, ‘ $R$  is extensional’ is a claim like ‘ $R$  is symmetric’—this being captured by the familiar condition

$$\forall x \forall y (R(x, y) \rightarrow R(y, x))$$

—or like ‘ $R$  is transitive’—for which the condition is

$$\forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Rxz)).$$

That  $R$  itself might involve the modal operator □ (and indeed in dominant position) in no way tells against this established usage of the term ‘extensionality’. The sense of ‘extensionality’ which is usually contrasted with *intensionality* is somewhat different.

## 7 Appendix

Some of the proofs below occasionally use rules for (the elimination of) the description operator. These rules, which are part of a sound and complete set of inferential rules for a free logic with the description operator primitive, can be found in my *Natural Logic*, Edinburgh University Press, 1978. For the record here, the two elimination rules that we use below are

$$\frac{t = \iota x F(x)}{F(t)} \quad \frac{\exists! u \quad F(u) \quad t = \iota x F(x)}{u = t}$$

Informally expressed, the first rule says that from the premiss that  $t$  is (identical to) the  $F$ , one may infer that  $t$  has property  $F$ . The second rule says that from premisses to the effect that the existent  $u$  has property  $F$ , and  $t$  is the  $F$ , one may infer that  $u$  is identical to  $t$ .

### 7.1 Proofs showing that if $\vdash \theta$ , then $\varphi \leftrightarrow \xi$ is interdeducible with $\varphi \leftrightarrow (\xi \wedge \theta)$

$$\begin{array}{c} \begin{array}{c} (1) \text{---} \\ \varphi \quad \varphi \leftrightarrow (\xi \wedge \theta) \\ \hline \xi \wedge \theta \\ \hline \xi \end{array} \quad \begin{array}{c} (1) \text{---} \quad \nabla \\ \xi \quad \theta \\ \hline \xi \wedge \theta \end{array} \quad \varphi \leftrightarrow (\xi \wedge \theta) \\ \hline \varphi_{(1)} \\ \hline \varphi \leftrightarrow \xi \end{array}$$
  

$$\begin{array}{c} \begin{array}{c} (1) \text{---} \\ \varphi \quad \varphi \leftrightarrow \xi \\ \hline \xi \end{array} \quad \begin{array}{c} \nabla \\ \theta \end{array} \quad \begin{array}{c} \text{---} (1) \\ \xi \wedge \theta \\ \hline \xi \end{array} \quad \varphi \leftrightarrow \xi \\ \hline \xi \wedge \theta \\ \hline \varphi_{(1)} \\ \hline \varphi \leftrightarrow (\xi \wedge \theta) \end{array}$$

### 7.2 Proof showing that 9, if it is so, is *unique* in being, necessarily, for any property $P$ , the number of $P$ s just in case there are nine $P$ s

Let us for a moment introduce the abbreviation  $\Phi(y)$  for the property expressed by

$$\Box \forall P (\#x P(x) = y \leftrightarrow \exists_9 P(x)).$$

Our premiss (called (4'') in the text) is  $\Phi(9)$ . In order to conclude that 9 is the *unique* individual with the property  $\Phi$  one has to establish in addition

that (i) 9 exists, and (ii) for any  $z$ , if  $\Phi(z)$  then  $z = 9$ . (i) can be proved independently, if somewhat drearily. Some ingenuity, however, is needed for the proof of (ii). Let  $Q(x)$  be the property  $(N(x) \wedge x < 9)$ , i.e. ‘ $x$  is a natural number preceding 9’. Alternatively, take the property

$$x = 0 \vee x = 1 \vee x = 2 \vee x = 3 \vee x = 4 \vee x = 5 \vee x = 6 \vee x = 6 \vee x = 8.$$

One can prove, as a theorem,  $\exists_9 x Q(x)$ . Remember that 9 here is in adjectival position! So let us abbreviate this theorem as  $\text{NINE}(Q)$ , to avoid any temptation to treat 9 substantivally therein. The proof of  $\text{NINE}(Q)$  is registered by the inverted triangle in the proof-scheme below. Given our choice of  $Q(x)$ , this is how to prove (ii):

$$\begin{array}{c}
 \begin{array}{cc}
 \begin{array}{c}
 (1) \frac{\frac{\frac{\square \forall P(\#xP(x) = a \leftrightarrow \text{NINE}(P))}{\forall P(\#xP(x) = a \leftrightarrow \text{NINE}(P))}}{\text{NINE}(Q) \quad \#xQ(x) = a \leftrightarrow \text{NINE}(Q)}} \\
 \#xQ(x) = a
 \end{array}
 &
 \begin{array}{c}
 (4'') : \\
 \frac{\frac{\frac{\square \forall P(\#xP(x) = 9 \leftrightarrow \text{NINE}(P))}{\forall P(\#xP(x) = 9 \leftrightarrow \text{NINE}(P))}}{\text{NINE}(Q) \quad \#xQ(x) = 9 \leftrightarrow \text{NINE}(Q)}} \\
 \#xQ(x) = 9
 \end{array}
 \end{array} \\
 \hline
 \frac{a = 9}{\square \forall P(\#xP(x) = a \leftrightarrow \text{NINE}(P)) \rightarrow a = 9} (1) \\
 \hline
 \frac{\square \forall P(\#xP(x) = a \leftrightarrow \text{NINE}(P)) \rightarrow a = 9}{\forall z(\square \forall P(\#xP(x) = z \leftrightarrow \text{NINE}(P)) \rightarrow z = 9)} \\
 \text{i.e. (ii)}
 \end{array}$$

### 7.3 Proofs showing that $\alpha(x, \pi)$ is equivalent to $A(x, \pi) \wedge D(\pi)$

The proof  $\Pi$  of our sentence  $A(x, \pi)$  from Peacocke’s  $\alpha(x, \pi)$  is as follows. It employs  $P$  as a parameter for elimination of the existential  $\exists S$  (there exists a state  $S \dots$ ), and employs  $L$  as a parameter for elimination of the existential  $\exists R$  (there exists a relational property  $R \dots$ ). The last step of the proof, to save space, is a double existential elimination.

$$\begin{array}{c}
\frac{(1) \frac{\pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow L[\psi])}{\Box \forall \psi (\mathcal{B}(\psi, \pi) \leftrightarrow L[\psi])}}{\forall \psi (\mathcal{B}(\psi, \pi) \leftrightarrow L[\psi])} \\
\frac{(1) \frac{L[P]}{\mathcal{B}(P, \pi) \leftrightarrow L[P]}}{\mathcal{B}(P, \pi)} \\
\frac{(2) \frac{(P[x] \wedge L[P]) \wedge \pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow L[\psi])}{P[x] \wedge \mathcal{B}(P, \pi)} \quad \frac{P[x]}{\mathcal{B}(P, \pi)}}{\frac{P[x] \wedge \mathcal{B}(P, \pi)}{\exists \theta (\theta[x] \wedge \mathcal{B}(\theta, \pi))}} \\
\alpha : \frac{\exists S \exists R ((S[x] \wedge R[S]) \wedge \pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi])) \quad \frac{P[x] \wedge \mathcal{B}(P, \pi)}{\exists \theta (\theta[x] \wedge \mathcal{B}(\theta, \pi))}}{\exists \theta (\theta[x] \wedge \mathcal{B}(\theta, \pi))} \quad (2) \\
\text{i.e. } A(x, \pi)
\end{array}$$

So we have shown that our sentence  $A(x, \pi)$  follows from Peacocke's  $\alpha(x, \pi)$ .  
The proof  $\Sigma$  of  $\alpha(x, \pi)$  from our sentences  $A(x, \pi)$  and  $D(\pi)$  is as follows:

$$\begin{array}{c}
\frac{(1) \frac{\pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow L[\psi])}{\Box \forall \psi (\mathcal{B}(\psi, \pi) \leftrightarrow L[\psi])}}{\forall \psi (\mathcal{B}(\psi, \pi) \leftrightarrow L[\psi])} \\
\frac{(2) \frac{Q[x] \wedge \mathcal{B}(Q, \pi)}{Q[x]} \quad \frac{\forall \psi (\mathcal{B}(\psi, \pi) \leftrightarrow L[\psi])}{\mathcal{B}(Q, \pi) \leftrightarrow L[Q]} \quad \frac{Q[x] \wedge \mathcal{B}(Q, \pi)}{\mathcal{B}(Q, \pi)}}{\frac{Q[x]}{L[Q]}} \quad (1) \\
\frac{Q[x] \wedge L[Q]}{\pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow L[\psi])} \\
D(\pi) : \frac{\exists R (\pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi])) \quad \frac{(Q[x] \wedge L[Q]) \wedge \pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow L[\psi])}{\exists S \exists R ((S[x] \wedge R[S]) \wedge \pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi]))}}{\exists S \exists R ((S[x] \wedge R[S]) \wedge \pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi]))} \quad (1) \\
A(x, \pi) : \frac{\exists \theta (\theta[x] \wedge \mathcal{B}(\theta, \pi)) \quad \text{i.e. } \alpha}{\alpha} \quad (2) \\
\alpha
\end{array}$$

#### 7.4 Proofs showing that if $D(b)$ is provable then $(E)$ and $(F)$ are interdeducible

The following two proofs show that if there is a proof of  $D(b)$  from no assumptions, parametric in  $b$ , then  $(E)$  and  $(F)$  are interdeducible:



As remarked at the end of the preceding section, the foregoing proof has a modal analogue which establishes that the modalized extensionality principle

$$(D2\Box) \quad \forall p \forall \pi (\Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow \mathcal{B}(\psi, \pi)) \rightarrow p = \pi)$$

follows from the axiom

$$(D) \quad \forall \pi \exists R (\pi = \iota p \Box \forall \psi (\mathcal{B}(\psi, p) \leftrightarrow R[\psi]))$$

of our re-formulation of Peacocke's theory.