Confinement of Nonneutral Plasmas on Magnetic Surfaces

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The confinement of a nonneutral plasma in a magnetic-surface, or stellarator, configuration is explored. The fluid equilibrium equations are derived and are found to be fundamentally different from previous results. Diocotron modes are predicted to be stable. The collisional confinement time can be very long. Possible applications include positron trapping and confinement of positron-electron plasmas. The basic physics can be addressed experimentally in the simple tabletop stellarator planned for construction at Columbia University.

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INTRODUCTION

Nonneutral single-component and multicomponent plasmas are fundamentally different from quasineutral plasmas, owing to the large space charge electric field [1]. This Letter describes the confinement of nonneutral plasmas confined in a toroidal magnetic surface configuration. The magnetic confinement of nonneutral plasmas in such configurations, which are the focus of magnetic fusion research, has not previously been analyzed.

The equations describing the equilibrium of a pure electron plasma are derived. These equations are fundamentally different from those describing pure electron plasma confined in closed or open field line systems, such as the Penning trap [2], which is the most common confinement configuration for single component plasmas, and the pure toroidal field trap [3]. The equations are also fundamentally different from the equations governing equilibrium of quasineutral plasmas in a magnetic surface configuration. Thus, nonneutral plasmas confined by magnetic surfaces involve new physics that is of fundamental interest. The equilibrium equations show that the equilibrium along the magnetic field constrains the plasma flow to be in the magnetic surfaces. The flow in a Penning trap has no such constraint and unless the trap is perfectly symmetric, the flow may not remain in a spatially bounded region.

Neoclassical confinement times are estimated for a generic magnetic surface configuration, and are found to be very long when the Debye length is small. Experimentally, a stellarator would be the ideal configuration for a study of pure electron plasmas confined by magnetic surfaces. As illustrated in Fig. 1, a suitable configuration can be created by a very simple and elegant coil set.

The use of a stellarator configuration to confine multispecies plasmas, in particular positron-electron plasmas, is also discussed. A stellarator has unique advantages for creating the first confined positron-electron plasmas, including the ability to confine plasmas over the full range of charge, from pure electron to quasineutral, and the ability to confine energetic positrons and electrons at modest magnetic field strengths.

EQUILIBRIUM EQUATIONS IN A MAGNETIC SURFACE CONFIGURATION

Definition and importance of magnetic surfaces.—A magnetic surface is sometimes defined as a surface in which the magnetic field lines lie. We employ a somewhat stricter definition that a field line cover the surface, coming arbitrarily close to every point on the surface. By our definition, neither a simple dipole magnetic field, nor a simple toroidal magnetic field has magnetic surfaces, since each field line closes on itself. The importance of magnetic surfaces has long been recognized in magnetic fusion research. For example, a quasineutral plasma cannot be confined in a pure toroidal field whereas stable, finite pressure equilibria exist in toroidal, nested flux surface configurations such as tokamaks and stellarators. To the best of our knowledge, no one has made a theoretical study of the confinement of nonneutral plasmas using magnetic surfaces.

The magnetic field associated with a set of nested toroidal magnetic surfaces can be written as $\mathbf{B} = \nabla\psi \times \nabla\theta + \iota(\psi)\nabla\varphi \times \nabla\psi$ with φ any toroidal angle, θ a

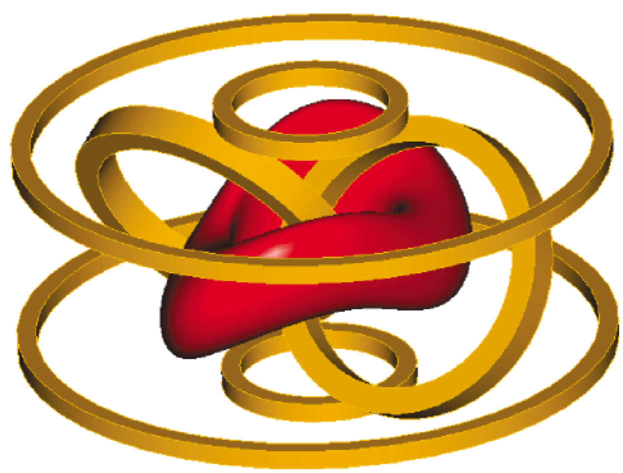

FIG. 1 (color). A stellarator configuration created using only circular coils. The two central coils are interlocking. The figure shows a rendering of the outer magnetic surface created by this coil configuration.

so-called magnetic poloidal angle, and $\iota(\psi)$ the rotational transform. Let $\theta_c = \theta - \iota(\psi)\varphi$, then

$$\mathbf{B} = \nabla\psi \times \nabla\theta_c \,. \tag{1}$$

The rotational transform $\iota(\psi)$ gives the twist of a magnetic field line about a toroidal surface containing magnetic flux ψ.

If the rotational transform is a rational number $\iota = N/M$ with N and M integers, the $\psi = constant$ surface is called a rational flux surface, and each magnetic field line on the surface closes on itself after M circuits of the torus toroidally and N circuits poloidally. On irrational flux surfaces no integers N and M exist such that $\iota = N/M$, and a field line approaches each point on the surface arbitrarily closely. Rational flux surfaces are not true magnetic surfaces, whereas irrational flux surfaces are. However, the distinction between the two is of practical importance only for low order rational numbers, i.e., when M and N are small. High order rational surfaces are effectively good magnetic surfaces due to finite Larmor radius effects.

Electron force balance.—The fundamental theoretical issue for the confinement of pure electron plasmas on nested toroidal surfaces is the equilibrium. The force equation for a pure electron plasma is

$$m_e n_e\left(\frac{\partial \mathbf{v}_e}{\partial t} + \mathbf{v}_e \cdot \nabla\mathbf{v}_e\right) = -en_e(\mathbf{v}_e \times \mathbf{B} - \nabla\phi) - \nabla p \,. \tag{2}$$

In equilibrium, the time derivative vanishes. If the electron density is far below the Brillouin limit [4], defined as

$$n_B = \frac{\epsilon_0 B^2}{2m_e} \,, \tag{3}$$

then the $\mathbf{v}_e \cdot \nabla\mathbf{v}_e$ term is negligible compared to the dominant terms and force balance reduces to

$$\nabla p = -en_e\mathbf{v}_e \times \mathbf{B} + en_e\nabla\phi \,. \tag{4}$$

The ratio of the pressure to the electrostatic term is $(\lambda_D/a)^2$ where λ_D is the Debye length

$$\lambda_D^2 = \epsilon_0 T_e/(e^2 n_e) \,, \tag{5}$$

and a is the minor radius of the plasma. The limit of small Debye length means the plasma is cold, $|T_e/e\phi| \approx (\lambda_D/a)^2$, and has negligible pressure.

Cold plasma equilibrium.—The equilibrium equation for a cold pure electron plasma, $\lambda_D/a \to 0$, is

$$en_e\nabla\phi = en_e\mathbf{v}_e \times \mathbf{B} \,. \tag{6}$$

This implies $\mathbf{B} \cdot \nabla\phi = 0$, so the electric potential must be constant along each magnetic field line. If each magnetic field line covers a toroidal surface containing flux ψ, then $\mathbf{B} \cdot \nabla\phi = 0$ implies that the electric potential is a function of ψ alone, $\phi(\psi)$. The plasma flows with a velocity

$$\mathbf{v}_e = -\frac{d\phi}{d\psi}\nabla\psi \times \mathbf{B}/B^2 + \frac{v_\parallel}{B}\mathbf{B} \,. \tag{7}$$

The parallel component of the flow must be consistent with the steady-state constraint $\nabla \cdot \Gamma = 0$ with $\Gamma = n_e\mathbf{v}_e$. This constraint leaves a net parallel flux of the plasma undetermined, $\Gamma_n = \gamma(\psi)\mathbf{B}$, which adjusts to minimize dissipation.

The greatly reduced freedom in the choice of the electric potential in systems with magnetic surfaces removes a primary source for instabilities. Equation (7) implies the flow of the plasma can never cross the ψ surfaces, $\mathbf{v}_e \cdot \nabla\psi = 0$. This prevents diocotron modes, which rearrange the electrostatic potential in the plane perpendicular to the magnetic field, while maintaining its constancy along the field lines. Diocotron modes could potentially couple to parallel plasma oscillations, hence breaking force balance along the field lines. Such hybrid modes can be stabilized either by Landau damping [provided $R/(\iota\lambda_D)\sqrt{n/n_B} \ll 1$, with R the major radius], or by magnetic shear [5].

The electrical current produced by the flowing electrons produces a negligible change in the magnetic field:

$$\delta B/B \approx \left(\frac{n_e}{n_B}\right)^2\left(\frac{a}{c/\omega_c}\right)^2. \tag{8}$$

The speed of light divided by the cyclotron frequency is $c/\omega_c = 1.70$ mm per tesla. As an example, the Columbia Nonneutral Torus is designed to have $a = 120$ mm, $B = 0.1$ T, and $n_e/n_B \approx 2 \times 10^{-3}$, so $\delta B/B \approx 2 \times 10^{-4}$.

If the rotational transform is a rational number, so the field lines close on themselves, and the flux surface is not a true magnetic surface, the electrostatic potential can satisfy $\mathbf{B} \cdot \nabla\phi = 0$ and be a function of ψ and θ_c, $\phi = \phi(\psi, \theta_c)$, of Eq. (1). In the case of an axisymmetric toroidal field, the rotational transform is zero, so the electric potential can have an arbitrary dependence on the poloidal angle.

Even though the electrostatic potential is constant along field lines, the electron density is not. It must be consistent with the electric potential, that is, it has to satisfy Poisson's equation, $en_e = \epsilon_0\nabla^2\phi$, which in a nontrivial geometry (such as that of a stellarator), must be a function of all three coordinates.

Warm plasma equilibria.—Interesting effects are expected in pure-electron plasmas over a broad range of the ratio of the Debye length to the system size λ_D/a. Even when λ_D/a is small, the pressure changes the mathematical form of the equilibrium equations. Even when λ_D/a is substantially larger than one, the self-consistent electric field modifies the motion of the particles and must be retained.

Dotting $\mathbf{B}$ with Eq. (4), one finds that $\mathbf{B} \cdot \nabla p = en_e\mathbf{B} \cdot \nabla\phi$. The electron temperature tends to be constant along the magnetic field, $\mathbf{B} \cdot \nabla T_e = 0$. When this is the case and magnetic surfaces exist, the electron density must have the form

$$n_e = N(\psi)\exp\frac{e\phi}{T_e(\psi)} \tag{9}$$

as well as be consistent with the Poisson equation. Therefore, the fundamental equilibrium equation for a pure electron plasma confined on magnetic surfaces is

$$\nabla^2 \phi = \frac{e}{\epsilon_0} N(\psi) \exp \frac{e\phi}{T_e(\psi)} . \tag{10}$$

It contains two functions of ψ alone, $N(\psi)$ and $T_e(\psi)$. These functions will be determined by the experimental sources of electrons and energy as well as the transport processes. Choosing $N(\psi)$ and $T(\psi)$ constant corresponds to a global thermodynamic equilibrium. The magnetic field plays no role in this equilibrium, and electrostatic repulsion localizes the plasma to the boundary region in a sheath a few Debye lengths thick [6].

Although the electric potential is not constant on the magnetic surfaces when the pressure is nonzero, the electron flow cannot cross the magnetic surfaces, $\mathbf{v}_e \cdot \nabla\psi = 0$. The equilibrium equation, Eq. (4), implies

$$\mathbf{v}_e = \left(\frac{\nabla p}{en} - \nabla\phi \right) \times \mathbf{B}/B^2 + \frac{v_\parallel}{B} \mathbf{B} . \tag{11}$$

The pressure, $p = T(\psi)N(\psi)\exp[e\phi/T(\psi)]$, is a function of two variables only, ψ and ϕ. Thus,

$$\begin{aligned} \nabla p &= \frac{\partial p(\psi,\phi)}{\partial\psi} \nabla\psi + \frac{\partial p(\psi,\phi)}{\partial\phi} \nabla\phi \\ &= \frac{\partial p(\psi,\phi)}{\partial\psi} \nabla\psi + n_e e \nabla\phi . \end{aligned} \tag{12}$$

Combining Eqs. (11) and (12), the $\nabla\phi$ terms cancel, and

$$\mathbf{v}_e = \frac{\partial p(\psi,\phi)}{\partial\psi} \frac{\nabla\psi \times \mathbf{B}}{e n_e B^2} + \frac{v_\parallel}{B} \mathbf{B} , \tag{13}$$

which demonstrates $\mathbf{v}_e \cdot \nabla\psi = 0$.

It is remarkable that $\partial p(\phi,\psi)/\partial\psi$ does not vanish when p vanishes (in the zero temperature limit). A simple application of Eq. (12) gives $\frac{\partial p}{\partial\psi}\nabla\psi = \nabla p - n_e e \nabla\phi$, so as ∇p vanishes, $\frac{\partial p}{\partial\psi}\nabla\psi$ becomes $-e n_e \phi$. Thus, Eq. (13) reduces to Eq. (7). Ultimately, this apparent paradox comes from the fact that the pressure depends exponentially on the factor $e\phi/T_e \propto (a/\lambda_D)^2$ which goes to infinity in the zero pressure limit.

ESTIMATE OF COLLISIONAL TRANSPORT

Transport processes in pure electron plasmas confined on magnetic surfaces is determined by the deviation of the electron trajectories from a constant pressure surface. At zero temperature, the motion of individual electrons is determined by the $\mathbf{E} \times \mathbf{B}$ drift, which is within the magnetic surfaces. Consequently, the confinement must be excellent at zero temperature.

At nonzero temperature, two effects give transport by causing the electron drift trajectories to deviate by a distance δ from the magnetic surfaces: the variation of the electric potential and the variation of the magnetic field strength on the surfaces. The variation of the electric potential on a magnetic surface, $\delta\phi/\phi$, is a geometric factor times $(\lambda_D/a)^2$. The ratio of the drift speed caused by the variation of the magnetic field strength to the $\mathbf{E} \times \mathbf{B}$ speed is also a geometric factor times $(\lambda_D/a)^2$. Consequently, the relative deviations in the electron trajectories from the magnetic surfaces, δ/a, due to each of these effects scale as $(\lambda_D/a)^2$ times geometric factors that are determined by the shapes of the magnetic surfaces and can be made small. The deviation of the drift orbits from the magnetic surfaces leads to radial transport if either $N(\psi)$ or $T_e(\psi)$ is nonconstant. The confinement time is expected to scale as $(a/\delta)^2\tau_e$ with δ the deviation of drift orbits from the magnetic surfaces and τ_e the electron collision time. Consequently, the confinement time is expected to scale as

$$\tau_p \approx (a/\lambda_D)^4 \tau_e . \tag{14}$$

Previous theoretical work in toroidal nonneutral plasmas has focused on investigations of equilibrium and stability of a single component plasma in an axisymmetric toroidal magnetic field (thus, no magnetic surfaces). The existence of a stable equilibrium solution in this case has been demonstrated by a number of authors, using somewhat different theoretical approaches [7,8]. Magnetic pumping causes radial diffusion in such systems, and a confinement time of [9]:

$$\tau_{tp} \approx \left(\frac{R}{\lambda_D} \right)^2 \tau_e . \tag{15}$$

Here, R is the major radius of the plasma. The confinement is a factor of $(a/R)^2(a/\lambda_D)^2$ longer in a magnetic surface geometry, Eq. (14), than in the pure toroidal geometry, Eq. (15). For example, design parameters for the proposed experiment at Columbia University are $R/a = 2.9$ and $a/\lambda_D \approx 100$, so its confinement time is expected to be at least 1000 times longer than a similar size purely toroidal field confinement device.

EXPERIMENTAL CONSIDERATIONS

Pure electron plasma confinement in a simple toroidal field (a closed field line system without magnetic surfaces) has been tested in several experiments [10–12]. Experimental studies of single component plasma in an axisymmetric magnetic surface configuration with a levitated current carrying ring have only recently begun [13]. A simple stellarator nonneutral experiment is currently being designed at Columbia University together with W. Reiersen, F. Dahlgren, A. Brooks, and N. Pomphrey as part of the Princeton Plasma Physics Laboratory University Support Program. This nonneutral plasma experiment will be unique in that the magnetic surfaces are created entirely by external coils, the coils of a stellarator. Unlike an axisymmetric magnetic surface configuration, a stellarator requires no levitated conductors and can be steady state.

The stellarator magnetic field topology usually has to be carefully tailored to avoid large neoclassical losses. This can lead to complicated and expensive external coil sets. However, a basic stellarator experiment to explore equilibrium, stability, and confinement of pure electron plasmas could be an inexpensive, simple tabletop device with a small number of planar, circular coils, Fig. 1. This particular configuration has excellent magnetic surface quality, significant rotational transform, and a low aspect ratio. The configuration has not been optimized to prevent energetic particles from having large excursions from the magnetic field lines. However, the electrostatic potential actually reduces the drift orbit excursions, an effect which is included in Eq. (14). As long as there are many Debye lengths in the device, neoclassical confinement times will be very large. Thus, neoclassical transport may not play a significant role for single component plasmas even in a stellarator not optimized for transport.

CONFINEMENT OF MULTISPECIES NONNEUTRAL PLASMAS

Magnetic surface configurations can confine plasmas at any level of neutrality, from pure electron to quasineutral. This is in contrast to the Penning traps, which can confine either positive or negative particles, depending on the electrostatic biasing, but not both species at the same time.

One of the most important applications of magnetic surface configurations would be the creation of the first confined laboratory positron-electron plasmas. Electron beam-positron plasma interactions have been observed in a laboratory [14], but confined positron-electron plasmas have not yet been created. Positron-electron plasmas are predicted to be fundamentally different from electron-ion plasmas in a number of important ways. For example, in an isothermal positron-electron plasma, electrostatic drift waves and acoustic waves are absent [15]. An experimental test of the dynamics of such plasmas is still lacking though, but it would be an important benchmark of basic plasma theory. Configurations with magnetic surfaces have the added advantage that, owing to the small mass, energetic positrons or electrons can be confined at rather modest magnetic field strengths. Penning traps require that the external electrostatic potential be greater than the particle energy, and this is not technically feasible at the near-relativistic energies that positrons are born at.

A nonneutral multispecies plasma can be created by injection of positive ions or positrons into an initially pure electron plasma, or by injection of neutral gas, which will then subsequently ionize in a warm electron plasma. The positive species (ions or positrons) will be confined not only by the magnetic field, but by the electrostatic well created by the electron space charge. Diffusion of the positive species will be directed inward towards the plasma center, where the positive species will accumulate, even if injected from the edge. Thus, injection and accumulation of positive species into an electron plasma should be easily achieved in a magnetic surface configuration.

Simultaneous confinement of positrons and antiprotons in a stellarator, which presumably is similar to hydrogen plasma confinement, could lead to production of large amounts of antihydrogen. Antihydrogen production is currently being attempted in a nested Penning trap device [16].

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