# RESWITCHING AS A CAUSE OF INSTABILITY OF INTERTEMPORAL EQUILIBRIUM 

Bertram Schefold*<br>Johann Wolfgang Goethe-Universität<br>(October 2002; revised July 2003)


#### Abstract

It is generally recognized that the paradoxes of capital, of which reswitching is the most striking example, are a reason to question the existence of aggregate production functions. It is here shown that they affect intertemporal general equilibrium as well as causes of instabilities.


## 1. INTRODUCTION

This paper establishes one central point. It demonstrates that the stability properties of neoclassical general equilibrium are affected by paradoxes of capital such as reswitching. The reader interested only in the technical aspects may start with section 2 . The introduction gives some hints as to the background. Responding to Mandler (2002), I want to show more explicitly why instability is associated with reswitching in a general equilibrium model, as had been asserted in Schefold (1997, 2000a). Section 2 summarizes the earlier papers. Section 3 discusses Mandler's objections, remarks on an earlier link between paradoxes of capital and stability and defines the stability process here to be analysed. The analysis is carried through in section 4. A special case is considered in greater detail in the Appendix.
'Sraffa's Legacy in Economics' (the title of a recent Symposium in this journal, earlier the title of a collection of essays-Kurz, 2000) was twofold: it consisted of a critique of neoclassical theory and of a 'Revival of Classical Theory' (subtitle of a volume with the proceedings of the Sraffa conference in Florence-Bharadwaj and Schefold, 1990). The critique was directed at the central idea of neoclassical theory: the identification of the problem

[^0]of the distribution of the product and of the employment of the factors of production with the pricing of the factors of production according to their scarcity, while classical theory takes different forms because it attempts to identify the forces which lead to the production and distribution of a surplus in the economy according to specific historical conditions. ${ }^{1}$

A multiplicity of classical approaches and one critique-when I once described this situation to the philosopher Stephen Toulmin as curious-for is Truth not one, and the errors are many?-he replied: there may be many truths and one central mistake.

In this view, the central mistake in neoclassical theory concerns the scarcity explanation of distribution which must lead into difficulties whenever capital, being produced in the form of heterogeneous capital goods, changes its price in a process of substitution, hence its amount as 'capital'. For the substitution causes the relative prices of the individual capital goods to change in a manner inconsistent with the notion of a given amount of aggregate capital. The phenomenon was first discussed and the critique was accepted as valid in the context of the aggregation in the construction of production functions, and it has often been thought to apply only to the aggregate version of neoclassical theory. Bidard wrote as late as 1991 in his remarkable book on classical and neoclassical theory: 'La critique sraffienne, pertinente pour certaines variantes de la théorie marginaliste, est sans impact sur le modèle d'Arrow-Debreu'2 (Bidard, 1991, p. 319). In my contribution to the abovementioned legacy volume, I ended by contrast: '. . . intertemporal equilibrium does not provide a stronghold which could be better defended against the critiques derived from capital theory than the older notions of longperiod neoclassical equilibrium. They stand or fall together' (Schefold, 2000a,

[^1]p. 387). This conclusion seems to have been accepted by the other reviewers of the book in the Symposium, but Mandler (2002) expresses doubts which provide me with a welcome opportunity to clarify the critique. Unfortunately, the technical argument is lengthy and involved; there is no room here to explore the deeper links between the stability problem which is here being uncovered and the validity of neoclassical theory. In fact, it seems premature to attempt to draw far-reaching conclusions, since more research is needed to clarify the nature of the instability which we shall encounter.

Neoclassical theory is complex and the paradoxes of capital accordingly take many forms even in steady-state comparisons. ${ }^{3}$ Several were discussed in Schefold (1997, ch. 18), with proofs of the theorems employed, while a shorter version, augmented by a numerical example, was presented in Schefold (2000a, with an account of the genesis of these papers on p. 364, note). It was shown in both papers how each paradox in the theory of capital could be reformulated as a time-path in intertemporal equilibrium.

Equilibria involving paradoxes of capital therefore exist, but it was argued that they are inherently unstable. The instability argument was developed only briefly in Schefold (1997); however, the 'translation' of the steady-state comparisons into intertemporal equilibria takes much space. I here want to reverse the emphasis and to concentrate on the stability problem, and I shall do so by concentrating on the contrast between two 'scenarios' only, each to be modelled both as a steady-state comparison and as an intertemporal equilibrium. These scenarios are that of 'demechanization' and that of 'immigration with reswitching'. The former is compatible with neoclassical hypotheses while the latter turns out to contradict them. ${ }^{4}$ It is necessary to repeat these scenarios briefly in order that this paper may be read as an independent piece. ${ }^{5}$

Demechanization is the typical neoclassical case: the standard solution to solve problems of unemployment. In the usual representation, there is a well-behaved production function with given amounts of capital $K=\bar{K}$, and labour $L=\bar{L}$. The rate of substitution equals the factor price ratio $-d K / d L=w / r$. If the labour supply suddenly increases, e.g. because of an

[^2]immigration, there will temporarily be unemployment, with a pressure on the real wage rate $w$ which will fall relatively to the rate of profit $r$. The consequent substitution of labour for capital at an unchanged amount of capital decreases the capital-labour ratio so that the additional labour may be employed.

It is taught today as a matter of course that full employment will thus be achieved through a process of substitution under competitive conditions, but earlier economists were very sceptical in this regard. The expression 'demechanization' has here been chosen in order to emphasize that, in a process of accumulation with technical change, full employment is, if necessary, achieved by falling back on 'old' methods of production. When the full employment mechanism was proposed by Böhm-Bawerk, Arthur Salz, an Austrian economist who, for all his life, kept a wavering position between Austrian economists, the classical school and the historical school, ridiculed this solution to the problem of employment by interpreting it as a justification of the Luddites. He summarized Böhm-Bawerk ironically: 'Where there are too many workers, for whatever reason, it is only necessary to shorten the period of production accordingly and all evil ends. That is what Malthus and his successors should have learnt in the treatment of the wage question and all the talk about a redundant supply of labour would have been superfluous. And those conditions in England in the beginning of the 19th century were superfluous, too. In order to feed the workers, the entrepreneurs should simply have shortened the period of production, in other words, they should simply have allowed them to wreck the machines . . .' (my translation from Salz, 1905, pp. 180-1).

The process is a reversal of Ricardo's mechanization as a form of technical progress where the adoption of more mechanized techniques allows to save labour at the cost of using more capital. ${ }^{6}$ Authors between the historical school and the classical tradition, challenged by the emergent neoclassical theory, sought to characterize the technical conditions and the social forces which might favour a long upswing at high level of employment, but

[^3]the neoclassical approach prevailed and the entire process is regarded as controlled by prices. This presupposes a flexibility in substitution which we do not always observe in reality. Usually, the lack of flexibility of wages is being blamed. But if technology is represented as a linear spectrum of activities, the appropriate activities to preserve full employment may simply not be available or, if they are, be available only if factor prices move in a counterintuitive direction.

It is this latter possibility that we here explore, and we intend to demonstrate that the counter-intuitive movements of factor prices lead to a source of instability, if represented in an intertemporal equilibrium context.

## 2. THE MODEL

In order to make the point, it is first necessary to translate the scenario of 'demechanization' into an intertemporal equilibrium involving linear activities. We keep the notation of Schefold (2000a). We start from a long-period equilibrium where $n$ commodities are being produced and used as means of production in $n$ activities, represented by a square input-output matrix $\mathbf{A}_{\alpha}$ and by a vector of uniform labour inputs $\mathbf{l}^{\alpha}$ so that the following equations for prices in the long period obtain:

$$
\left(1+r_{\alpha}\right) \mathbf{A}_{\alpha} \mathbf{p}^{\alpha}+w_{\alpha} \mathbf{I}^{\alpha}=\mathbf{p}^{\alpha}
$$

The matrix $\mathbf{A}_{\alpha}$ is productive and indecomposable. We are in a steady state. The vector of net output is being produced by means of a vector of activity levels $\mathbf{q}$ such that $\mathbf{q}\left(\mathbf{I}-\mathbf{A}_{\alpha}\right)=\mathbf{d}$. In order to have a notion of essentially only one intensity of capital, $K / L$, associated with this technique, it is convenient to assume that the wage curve, $w_{\alpha}(r)$, is approximately linear. Employment is equal to $\mathbf{q l}^{\alpha}=L_{\alpha}$. If employment must increase, i.e. if $L$ goes up to $L_{\beta}$, the additional labour is in this case of demechanization absorbed not because activity levels $\mathbf{q}$ increase but because labour requirements $\mathbf{l}$ increase in a steady-state comparison. We therefore assume that there is a second technique with a higher maximum rate of profit $R_{\beta}$, given by a second input-output matrix $\mathbf{A}_{\beta}$, and a second labour vector $\mathbf{l}^{\beta}$. This technique is different from the first because one process has been changed. If the change represents demechanization, the labour coefficient of the first method of production, $l_{1}$, is represented by a new labour coefficient $l_{0}>l_{1}$. More labour is used but less 'capital', hence the vector of inputs $\mathbf{a}_{1}$ is represented by another vector of inputs $\mathbf{a}_{0}<\mathbf{a}_{1}$-the idea of demechanization and of a use of less capital is unambiguous if less is used of all inputs in the process where the

[^4]substitution takes place. In order to render the idea of demechanization clear, we assume that the wage curve of $\beta$ also is approximately linear (although it is not possible for both wage curves simultaneously to be strictly linear). ${ }^{7}$

The adoption of the less mechanized technique is therefore largely independent of the vectors of relative prices, but dependent on distribution: if the wage rate falls and the rate of profit increases, the less mechanized technique will get the cost advantage according to figure 1 .


Figure 1. Move from a more mechanized technique $\alpha$ to a less mechanized technique $\beta$ in a steady-state comparison by increasing the rate of profit from $r_{\alpha}$ to $r_{\beta}$.

[^5]To leave the amount of capital unchanged in the transition would mean, strictly speaking, that a transition is made from activity levels $\mathbf{q}^{\alpha}$ to activity levels $\mathbf{q}^{\beta}$ such that $\mathbf{q}^{\alpha} \mathbf{A}_{\alpha} \mathbf{p}^{\alpha}=\mathbf{q}^{\beta} \mathbf{A}_{\beta} \mathbf{p}^{\beta}$ where prices are made compatible by means of a numéraire $\mathbf{s}$ with $\mathbf{s p}^{\alpha}=\mathbf{s p}^{\beta}$. The value of capital thus would be equal in both situations but the form of capital would have to change and, to preserve full employment, we should assume $\mathbf{q}^{\alpha} \mathbf{1}^{\alpha}=L_{\alpha}<\mathbf{q}^{\beta} \mathbf{1}^{\beta}=L_{\beta}$.

However, it is simpler to establish the comparison on the basis of the assumption that gross outputs remain constant (keeping net output constant has been discussed in Schefold, 1997, 489-90). With gross output kept constant, consumption changes from $\mathbf{q}\left(\mathbf{I}-\mathbf{A}_{\alpha}\right)$ to $\mathbf{q}\left(\mathbf{I}-\mathbf{A}_{\beta}\right)$. It increases because $\mathbf{A}_{\beta} \leq \mathbf{A}_{\alpha}$, but employment increases as well, since $\mathbf{q} \mathbf{l}^{\beta}>\mathbf{q} \mathbf{l}^{\alpha}$, so that consumption per unit of labour (consumption measured in long-term prices) falls, as can be read from the $w$-axis. In fact, $\mathbf{a}_{0}<\mathbf{a}_{1}$ is sufficient to rule out reswitching, and we have $w_{\alpha}(0)>w_{\beta}(0)$. The consumer therefore enjoys higher consumption on path $\beta$, but not per unit of labour.

The process of substitution is here mainly governed by the change in factor prices because we unambiguously have $\mathbf{a}_{0}<\mathbf{a}_{1}$ (less 'capital' in $\beta$ independently of prices) and get at $r_{\alpha}$

$$
\left(1+r_{\alpha}\right) \mathbf{A}_{\alpha} \mathbf{p}^{\alpha}+w_{\alpha} \mathbf{l}^{\alpha} \leq\left(1+r_{\alpha}\right) \mathbf{A}_{\beta} \mathbf{p}^{\alpha}+w_{\alpha} \mathbf{l}^{\beta}
$$

while we have at $r_{\beta}$

$$
\left(1+r_{\beta}\right) \mathbf{A}_{\beta} \mathbf{p}^{\beta}+w_{\beta} \mathbf{I}^{\beta} \leq\left(1+r_{\beta}\right) \mathbf{A}_{\alpha} \mathbf{p}^{\beta}+w_{\beta} \mathbf{l}^{\alpha}
$$

The higher capital cost at the lower rate of profit is more than compensated by the lower labour requirement, which is decisive at high wages.

In order to represent the transition from one state to the other as an intertemporal equilibrium, two steps are required:
(1) Movements of prices and quantities must be dated.
(2) It has to be shown that there are preferences such that the transition is desired. This reversal of the usual procedure-usually, preferences are given and the corresponding equilibrium path is being sought-is useful to construct examples which, once they are presented in a consistent form, can be interpreted as conventional neoclassical equilibria: the path that has been constructed may be regarded as the solution to the preferences that are now regarded as given.

We therefore assume that there are $T$ periods, with activity levels $\mathbf{q}^{t}$ during each period; $t=1, \ldots, T$. During each period $t$, techniques, $\mathbf{A}_{t}, \mathbf{I}^{t}$ are being

[^6]used so that, with single production, gross output (which is equal to $\mathbf{q}^{t}$ ) divides into consumption and investment according to
$$
\mathbf{q}^{t-1}=\mathbf{c}^{t-1}+\mathbf{q}^{t} \mathbf{A}_{t} ; t=1 \ldots, T-1
$$

It is useful to postulate that a terminal stock $\mathbf{f}$ is left over in the last period so that

$$
\mathbf{q}^{T}=\mathbf{c}^{T}+\mathbf{f}
$$

Stationary states thus do not end with a sudden rise of consumption if $\mathbf{f}$ is equal to the stock which would be necessary if the stationary state were to be continued. $\mathbf{f}=\mathbf{0}$, the more usual assumption, is not ruled out here. Initial stocks are represented by a vector of given endowments $\mathbf{q}^{0}$. In our case, gross output levels are constant: $\mathbf{q}^{t}=\overline{\mathbf{q}}$ for all $t$. Consumption in the stationary state then is equal to $\overline{\mathbf{c}}^{t}=\overline{\mathbf{q}}\left(I-\mathbf{A}_{\alpha}\right) ; t=0, \ldots, t^{\prime}-1$; and employment is equal to; $\mathbf{q}^{t} \mathbf{1}^{\alpha}=L_{\alpha} ; t=1 \ldots, t^{\prime}$ where $L_{\alpha}$ is the amount of labour available prior to 'immigration'. In $t^{\prime}+1$, the amount of labour available is $L_{\beta}>L_{\alpha}$ so that $\mathbf{q}^{t} \mathbf{I}^{\beta}=L_{\beta} ; t=t^{\prime}+1, \ldots T$. Accordingly, $\overline{\mathbf{c}}^{t}=\overline{\mathbf{q}}\left(\mathbf{I}-\mathbf{A}_{\beta}\right) ; t=t^{\prime}, \ldots$, $T-1$; and $\overline{\mathbf{c}}^{T}=\overline{\mathbf{q}}-\mathbf{f}$, where $\mathbf{f}$ happens to equal $\overline{\mathbf{q}} \mathbf{A}_{\beta}$. Thus, two stationary states have been connected, as far as quantities are concerned.

The next step consists of the fixation of prices. The series of discounted prices given by

$$
\mathbf{A}_{\alpha} \mathbf{p}^{t-1}+w_{t} \mathbf{I}^{\alpha}=\mathbf{p}^{t} ; t=1, \ldots, t^{\prime}
$$

is proportional to long-run prices but falls at the general rate of interest $r_{\alpha}$, if we define

$$
\mathbf{p}^{0}=\mathbf{p}^{\alpha}, \mathbf{s p}^{\alpha}=1
$$

where $\mathbf{s}$ is the numéraire. The discounted prices then are

$$
\mathbf{p}^{t}=\mathbf{p}^{0} /\left(1+r_{\alpha}\right)^{t} ; t=0, \ldots, t^{\prime}
$$

Because prices are here proportional to long-run prices $\mathbf{p}_{\alpha}$, the own rates of interest are equal to the rate of profit $r_{\alpha}$, and undiscounted prices $\overline{\mathbf{p}}$ in terms of numéraire $\mathbf{s}$ are equal to $\mathbf{p}^{\alpha}$.

Prices later are determined successively. At the beginning of period $t^{\prime}+1$ (when technique $\beta$ is begun to be used) the price vector is still defined as $\mathbf{p}^{t^{\prime}}=\mathbf{p}^{\alpha} /\left(1+r_{\alpha}\right)^{t^{\prime}}$. Then we have

$$
\mathbf{A}_{\beta} \mathbf{p}^{t-1}+w_{t} \mathbf{I}^{\beta}=\mathbf{p}^{t} ; t=t^{\prime}+1, \ldots, T
$$

These prices can be shown to converge asymptotically to a scalar multiple of $\mathbf{p}^{\beta}$ for $T$ large enough, since we postulate that the own rate of interest in terms of $\mathbf{s}$ shall be equal to $1+r_{\beta}$. The price equations are thus complemented by the equation $\mathbf{s p}^{t-1} / \mathbf{s p}^{t}=1+r_{\beta} ; t=t^{\prime}+1, \ldots, T$. This equation, for any period $t$, together with the price equations, will determine $\mathbf{p}^{t}$ and $w_{t}$, given $\mathbf{p}^{t-1}$. The corresponding undiscounted prices in terms of numéraire $\mathbf{s}, \overline{\mathbf{p}}^{t}=\mathbf{p}^{t} / \mathbf{p}^{t}$

$$
\overline{\mathbf{p}}^{t}=\frac{\mathbf{\mathbf { p p } ^ { t - 1 }}}{\mathbf{s p}^{t}} \mathbf{A}_{\beta} \overline{\mathbf{p}}^{t-1}+\bar{w}_{t} \mathbf{I}^{\beta}=\left(1+r_{\beta}\right) \mathbf{A}_{\beta} \overline{\mathbf{p}}^{t-1}+\bar{w}_{t} \mathbf{I}^{\beta}
$$

converge to $\mathbf{p}^{\beta}$, and $\bar{w}_{t}$, if undiscounted, converge to $w_{\beta}$.
Technique $\alpha$ must be cost-minimizing for $t=1, \ldots, t^{\prime}$, therefore

$$
\mathbf{A}_{\alpha} \mathbf{p}^{t-1}+w_{t} \mathbf{l}^{\alpha} \leq \mathbf{A}_{\beta} \mathbf{p}^{t-1}+w_{t} \boldsymbol{\beta}^{\beta}
$$

and conversely

$$
\mathbf{A}_{\beta} \mathbf{p}^{t-1}+w_{\mathbf{l}} \mathbf{l}^{\beta} \leq \mathbf{A}_{\alpha} \mathbf{p}^{t-1}+w_{l} \boldsymbol{\alpha}^{\alpha}
$$

for $t=t^{\prime}+1, \ldots, T$. It has here been assumed that technique $\beta$ is adopted immediately, although $\mathbf{p}^{\prime}$ is proportional to $\mathbf{p}^{\alpha}$, because the reduction of the wage rate in the transition from $r_{\alpha}$ to $r_{\beta}$ renders the more labour-intensive technique ( $\mathbf{I}^{\beta} \geq \mathbf{l}^{\alpha}$ ) profitable without delay. This simplifying assumption will be justified in contrast with the transition in the case of reswitching which is inherently more complicated.
The intertemporal path of quantities and prices emerges as an intertemporal equilibrium and as a (unique) optimum if we join it with the utility function

$$
U=\sum_{i, t}\left(1-\bar{c}_{i}^{t}+c_{i}^{t}\right)^{p_{i}^{t}}
$$

where we assume that quantities have been normalized such that $\bar{c}_{i}^{t}<1$ and prices such that $p_{i}^{t}<t$; the utility function then is strictly concave for all variations of $c_{i}^{t}$ in the semi-positive orthant; a slightly different utility function is used in Schefold (1997, 2000a).

One finds that $\partial U / \partial c_{i}^{t}=p_{i}^{t}$ for $c_{i}^{t}=\bar{c}_{i}^{t}$. This means that utility is at a maximum on the path constructed where $c_{i}^{t}=\bar{c}_{i}^{t}$ for all $i, t$. The path is an optimum, given the quantity conditions. It is also an equilibrium, for it is easy to show that the budget equation of the consumer

[^7]$$
\mathbf{c}^{1} \mathbf{p}^{1}+\ldots+\mathbf{c}^{T} \mathbf{p}^{T}=\mathbf{q}^{0} \mathbf{p}^{0}-\mathbf{f} \mathbf{p}^{T}+w_{1} L_{\alpha}+\ldots+w_{T} L_{\beta}
$$
is fulfilled on our path with $c_{i}^{t}=\bar{c}_{i}^{t}$. Hence this is an equilibrium for the utilitymaximizing consumer, given the budget constraint for prices compatible with competitive production. In both maximizations, optimum and equilibrium, we now regard the utility function as given, with parameters $p_{i}^{t}$ which turn out to be the prices on the equilibrium path. Note that it also turns out that $\lambda=1$ in the solution of the problem for the household, where $\lambda$ is the Lagrange multiplier associated with the budget constraint. ${ }^{8}$

Reswitching: What changes if there is reswitching? Here we also assume an initial stationary state in $w_{\alpha}$ according to figure 2 . An immigration takes place


Figure 2. Transition from technique $\alpha$ to $\beta$ with reswitching. The capital-labour ratios are given by the tangent of $\rho_{\alpha}$ and the tangent of $\rho_{\beta}$.

[^8]at the end of period $t^{\prime}$. The only technique available to absorb the additional labour at a lower capital-labour ratio involves reswitching, hence a rise of the wage rate in response to the increased supply of labour, as shown in figure 2 in a steady-state comparison. ${ }^{9}$

Apart from the direction in the change of distribution, only one important detail changes in the formulae which we have constructed for demechanization. When the increase of labour takes place between period $t^{\prime}$ and $t^{\prime}+1$, it is not sufficient to change distribution at the end of period $t^{\prime}$ in order to ensure that technique $\beta$ is adopted by entrepreneurs according to the principle of profit maximization. We first have that technique $\alpha$ shows a cost advantage with respect to $\beta$ as long as we are in the stationary state corresponding to $r_{\alpha}$ in figure 2, with prices proportional to long-run prices $\mathbf{p}^{\alpha}$. This means explicitly

$$
\mathbf{a}_{1} \mathbf{p}^{t-1}+w_{t} l_{1}<\mathbf{a}_{0} \mathbf{p}^{t-1}+w_{t} l_{0}
$$

That technique $\alpha$ is cheaper in the stationary state where the own rates of interest are equal to $r_{\alpha}$ shows in the cost advantage in the first sector (supposing that technique $\alpha$ and technique $\beta$ employ the same methods of production in sectors $2, \ldots, n$ ). Once the transition has been made, the cost advantage in sector 1 is reversed in favour of technique $\beta$ :

$$
\mathbf{a}_{0} \mathbf{p}^{t-1}+w_{t} l_{0}<\mathbf{a}_{1} \mathbf{p}^{t-1}+w_{t} l_{1}
$$

Prices are defined such that the own rate of interest of $\mathbf{s}$ equals $r_{\beta}$, and the corresponding undiscounted prices converge to $\mathbf{p}^{\beta}$, and we have here $r_{\beta}<r_{\alpha}$, in the previous case $r_{\alpha}<r_{\beta}$.

The difficulty now is this: if reswitching is involved, the change in the cost advantage presupposes not only a change in distribution but also a considerable change in relative prices, for reswitching presupposes that we have neither $\mathbf{a}_{1}-\mathbf{a}_{0}>\mathbf{0}$ nor $\mathbf{a}_{1}-\mathbf{a}_{0}<\mathbf{0}$. The transition is made to preserve full employment, hence we still have $l_{o}>l_{1}$. The lowering of the intensity of capital during the transition means that prices must change so that we have $\left(\mathbf{a}_{1}-\mathbf{a}_{0}\right) \mathbf{p}^{t-1}<w_{t}\left(l_{0}-l_{1}\right)$ for $t \leq t^{\prime}$ and conversely $\left(\mathbf{a}_{1}-\mathbf{a}_{0}\right) \mathbf{p}^{t-1}>w_{t}\left(l_{0}-l_{1}\right)>0$ when the new technique is to be used from period $t^{\prime}+1$ (when the immigration has taken place) onwards. The latter condition implies $\mathbf{a}_{1} \mathbf{p}^{t-1}>\mathbf{a}_{0} \mathbf{p}^{t-1}$ at

[^9]$r_{\beta}$, while $\mathbf{a}_{0} \mathbf{p}^{t-1}>\mathbf{a}_{1} \mathbf{p}^{t-1}-w_{t}\left(l_{0}-l_{1}\right)$ at $r_{\alpha}$. If $w_{t}$ is small, $w_{t}\left(l_{0}-l_{1}\right)$ is small. Reswitching means that the technique which employs more labour becomes more profitable at higher wages because the sum of the prices of the capital goods employed fall. And the fall of prices must compensate for the fact that more of at least one capital good will be used in the transition to the less capital-intensive technique.

The change of distribution from $r_{\alpha}$ to $r_{\beta}$ can, at unchanged prices proportional to $\mathbf{p}^{\alpha}$, not induce a change in technique from $\alpha$ to $\beta$ for that change in distribution raises the relative wage costs of $\beta$; to that extent, $\beta$ becomes less profitable. Hence capital costs $\mathbf{a p}_{0}$ must fall strongly relative to wage costs. In order to prepare for this change in the cost relationships, distribution must change prior to the 'immigration', at the end of some period $t^{\prime \prime}<t^{\prime}$, for the drastic change of relative prices, which eventually induces the transition, necessarily lags behind the change in distribution. This means that, for equilibrium to be possible, the lagging-behind must be correctly foreseen, i.e. the change of distribution has to be anticipated by a number of periods which depends on all the parameters of the model in such a way that distribution and prices have changed sufficiently exactly when immigration takes place so that the increased amount of labour can immediately be absorbed.

The time-path of quantities can thus be constructed as in the example of demechanization, with gross outputs being kept constant, but consumption will now not increase for all commodities any more at the end of $t^{\prime}$ but fall for at least one, since we do not have $\mathbf{a}_{0}<\mathbf{a}_{1}$ any more. Consumption per head will again fall in long-term prices since $w_{\alpha}(0)>w_{\beta}(0)$. Prices are proportional to $\mathbf{p}^{\alpha}$ up to period $t^{\prime \prime}$. Then, distribution is changed by introducing the condition $\mathbf{s p}^{t-1} / \mathbf{s p}^{t}=r_{\beta}$. For some periods, technique $\alpha$ continues to be used nevertheless, till at the end of $t^{\prime}$, relative prices have changed sufficiently for method $\mathbf{a}_{0}$ to come more profitable in period $t^{\prime}+1$, and, thanks to $l_{o}>l_{1}$, the additional labour will be absorbed.

Numerical examples can be constructed for any $n \geq 2$, and one is given in Schefold (2000a) for $n=2$.

## 3. DISCUSSION

Mandler agrees that the assumption of perfect foresight is difficult to accept in the case of lagging-behind. If we take the assumption of a one-consumer-economy literally, this consumer supplies the additional labour in a future period and is expected to foresee this. But who takes the action necessary to induce the change of distribution prior to the immigration? Who generates this market signal? Equilibrium exists in our case because the
consumer exhibits a change of his intertemporal preferences from $t^{\prime \prime}$ onwards. But, in so doing, he does not react to a signal. And even in this model there is not only the consumer but there are also producers, and they must interpret a future change in relative amounts of consumption as indicating a change in the intertemporal rates of substitution of the consumer to the effect that the own rate of interest of $\mathbf{s}$ falls to $r_{\beta}$ and wages are changed accordingly, several periods prior to the changes in the quantities of labour demanded and labour supplied. The discovery of the lagging-behind effect therefore represents a novel critique of intertemporal equilibrium. The equilibrium path exists, but the required signals to 'find' the correct timing and deviation of the distributional change emerge only with the equilibrium itself.
The lagging-behind effect has caused two different kinds of reactions to earlier presentations. It was said that it was not new (a) and that it contradicted the assumption of perfect foresight (b). ${ }^{10}$ I start with the latter.

Perfect foresight does not mean that the agents foresee the result of market processes, i.e. the equilibrium prices and quantities, but that they know what they want to offer or demand at alternative future prices (possibly contingent upon states of nature). They thus know that the 'immigration' (inelastically) will take place, and both producers and the consumer know how they will react to consequent price changes, but the consumer does not know which technique will be adopted by producers to adapt to the situation, and producers naturally will expect falling wage rates. The traditional way to analyse such a tendency of actual markets in the theory is to examine stability, as we shall do below. That lagging-behind is surprising for the agents is confirmed by the tendency to instability which we shall find, but it is compatible with perfect foresight in that at equilibrium prices (including the distributional change preceding 'immigration' in a 'paradoxical' deviation), equilibrium quantities are demanded and supplied. If perfect foresight included the knowledge of equilibrium prices, stability analysis (and indeed market processes) would not be necessary.

Lagging-behind seems not to be new in that it resembles ordinary market anticipations: too much rain in July damages the crops and grain prices rise immediately, although the harvest is still some weeks away. This is an analogue, but with a difference: the rise in grain prices serves to preserve stocks, and labour is not stocked; instead, distribution changes to induce a change of technique. A better analogy is the rule of Hotelling in intertemporal

[^10]equilibrium: prices must rise early in order to speed up the use of a finite resource and so as to induce the transition to a more expensive backstop technology. But there is still the difference: at least in the ordinary cases of applications of Hotelling's rule, prices begin to rise as soon as the future scarcity is known, and the price path rises gently according to the simple rule that the resource owners must be at the margin of indifference between enjoying the capital gain of resource left in the ground and profiting from selling the resource above ground. There is no rule (except for a planner who is able to calculate the equilibrium) which tells the agents when to change distribution and in which direction. For they do not know the optimal technique which allows to absorb the 'immigration', before they know the prices, and the prices they expect (falling wages) do not lead to a substitution generating full employment.

It should be admitted, however, that the difficulties regarding the assumption of perfect foresight are only a matter of degree in the comparison of demechanization and reswitching. Lagging-behind may occur in the case of demechanization, too, where some change of relative prices may also be needed to induce a change of technique, although this is primarily triggered by the change of distribution, as has been stated.

Mandler is not convinced that the intertemporal equilibrium involving reswitching is paradoxical, and he adds that there might be problems of stability only as an afterthought in footnotes:

> Schefold's model . . . does not present a real paradox, since along with the increase in labour supply the model also shifts the agent's per period utility function... Since, according to any neoclassical price theory, the direction of prices changes can be arbitrary when both supply and demand shift, it is not obvious that Schefold's equilibrium is in any way implausible. (Mandler, 2002, p. 216)

I should like to state that, contrary to Mandler, I regard the transition within the intertemporal equilibrium which has been constructed and which involves reswitching as 'obviously' paradoxical. It is, of course, true that the price can go either way if both demand and supply shift according to 'any' version of neoclassical theory. But if we know what causes the shifts and their direction, we can separate 'normal' from 'paradoxical' reactions. Consider the following analogy: suppose that the supply of oil is increased at no additional cost and that this oil can be used fully only if its price is raised. Usually, more applications for the use of raw material are found if its price is lowered-this is the 'technical' aspect corresponding to the 'reswitching' of methods - and more consumers are ready to buy the consequent productthe demand aspect in the analogy-if the price is lower. Why should more
oil be used if the price is higher? Because of a snob effect (Veblen effect)? But oil is a mean of production, not an object of fashion. So is labour. Why should one have to raise the wage to sell more labour?

Cause and effect can be distinguished if alternative outcomes can be identified as resulting from alternative initial givens, and paradoxical constellations are recognized by comparing them with what is regarded as normal. The case of reswitching is compared with that of demechanization where the increased supply of labour is absorbed by lowering the wage. It should be noted that no direct influence of the amount of labour supplied on the utility of the consumer has been postulated. The increased wage therefore is not to be justified by a rising disutility of work. The rise of the wage in the transition with reswitching is a reasonable reaction only for an observer who understands the peculiar interdependence of distribution, changes in relative prices and the intensity of capital, therefore for an observer who can calculate the changes in relative prices consequent upon distributional changes, therefore for an ideal planner, not for an economic agent who sees only what happens in his particular market.

Mandler tries to formulate what he would regard as paradoxical:

> To establish genuine paradox, one must show that an increase in labour supply alone will increase the wage above what it would have been absent the increase . . one might show that an economy . . . would have higher wages . . . with a larger labour supply. (Mandler, 2002, p. 217)

This is in fact impossible with an invariant per-period utility function, and Mandler's doubt is justified. As far as the conditions of accumulation are concerned, we do have a simultaneous rise of the labour supply and of the wage rate in our model. But this is achieved, hence 'wanted' only if the consumer exhibits different rates of time preference before and after the transition. For the wage rate and the own rate of interest of numéraire $\mathbf{s}$ in period $t$, using technique $\sigma$, are given by $\mathbf{A}_{\sigma} \mathbf{p}^{t-1}+w_{l} \mathbf{I}^{\sigma}=\mathbf{p}^{t}$ and $r=\mathbf{s p}^{t-1} / \mathbf{s p}^{t}-1$, hence by $1+r=\mathbf{s p}^{t-1} /\left(\mathbf{s} \mathbf{A}_{\sigma} \mathbf{p}^{t-1}+w_{t} \mathbf{s}^{\sigma}\right)$, where $\mathbf{p}^{t-1}$ are the output prices determined in the previous period. The wage rate thus is inversely related to $r$, and $r$ is regulated in the present model by the rates of intertemporal substitution which are implicit in the utility function. With the utility function shown above, the rate of time preference along the equilibrium path for any commodity changes as its price changes over time, and the price changes are given by the fixed parameters of the utility function.

Mandler's postulate could perhaps be fulfilled by using another theory of preferences where the level of accumulation and consumption is linked in a non-trivial way to a variable rate of time preference. I do not have an explicit

[^11]example to meet Mandler's postulate, but the absence of reswitching and reverse capital deepening plays a crucial role in models where the rate of time preference varies. If utility functions are not homothetic, the rates of time preference may rise or fall with the level of steady-state consumption. This is a possibility to which we shall come back below. Another possibility is the use of recursive utility functions which provide a richer framework to incorporate such an idea (Schefold, 1993, 1997, ch. 18.1). It has been shown that models of accumulation in an intertemporal equilibrium with perfect foresight based on recursive preferences have a remarkable turnpike property and converge under rather general conditions to an ultimate stationary state with a uniform rate of profit which is equal to the rates of time preference adopted by each individual consumer in that terminal stationary state. One requirement for the result to hold is (on the consumption side) that the rates of time preference rise with the accumulation of individual wealth, for if the rates of time preference fall, the rich will continue to accumulate and the concentration of wealth will continue to increase. ${ }^{11}$ Another crucial hypothesis for this turnpike result to hold, however, is that reswitching and paradoxes of capital theory are excluded. It was precisely this observation that led me in 1990 to examine the stability properties of intertemporal equilibria which we are discussing here. I then concluded:

> It would thus appear that the assumption of a 'neoclassical technology', i.e. one which excludes reswitching and perverse Wicksell effects, is necessary not for the existence of an intertemporal equilibrium but for the possibility of interpreting it as an explanation of distribution in a long-period equilibrium by affording the possibility of a transition towards it. (Schefold, 1990, in Schefold, 1997, ch. 15, p. 382)

The observation that reswitching may contribute to the destabilization of intertemporal equilibria thus first emerged in the context of the discussion of the stability of intertemporal paths approaching a terminal state.

To exclude reswitching in order to obtain convergence to terminal steady states has been a crucial assumption of the turnpike theorems for intertemporal equilibria since the 1980 s. Mandler himself suggests to use the turnpike theorems for comparisons between classical and neoclassical approaches at the end of his article. We agree on this as on a possible basis for the comparison, and although he does not enter a discussion regarding the conceptual differences between classical long-period positions and neoclassical

[^12]terminal states and only states the formal analogy which exists in the form of a tendency towards the formation of a uniform rate of profit.

Note first that terminal states are steady states, but the comparison of states with a uniform rate of profit need not be one of steady states. Neoclassical theory distinguishes between the time-path of intertemporal equilibrium and the terminal state towards which it gravitates if the assumptions are sufficient to ensure the turnpike property to hold. The terminal state is characterized by an equality of all own rates of interest with a general rate of profit and with the rates of time preference of the consumers. Compared with this final state of rest, the path towards it is only a transient state of adaptation of stocks which are initially given in arbitrary amounts as inherited from the past, and which must adapt in their relative amounts to future requirements. However, even in neoclassical theory, the uniform rate of profit does not presuppose a stationary or even steady state (with equal rates or growth in all sectors). If there are constant returns to scale and there is no change of techniques of production due to special scarcities of unproduced factors, the proportions in which consumption goods are produced may vary with the growth of incomes according to Engel elasticities, and if these shifts are foreseen, all own rates of interest may be equal. It is easy to construct intertemporal equilibria with consumption changing over time and with uniform rates of profit, using the model of this paper. ${ }^{12}$

But let us be modest and confine the discussion here to steady states. Since Mandler sees a correspondence between terminal states and classical longperiod positions, the differences discussed in Schefold (1997, ch. 18.1) should also be emphasized: as stressed in the beginning, the classical models are based on different theories of distribution and consumption and, at least in my view, also on different visions of accumulation. The method adopted by the classical economists was to assume that market prices already had gravitated to long-run prices (i.e. the arguments usually were exposed on the basis of the assumption that prices were equal to long-run or normal prices) because this process of gravitation of market prices was thought to proceed at a quicker pace than accumulation itself. Although market prices were

[^13]always disturbed by new events so that actual prices could hardly ever be expected to be identical with normal prices, this gravitation was thought to be completed when slower movements of accumulation and of changes in distribution were contemplated. That the movement of market prices might influence the course of accumulation, e.g. through changes in expectations and speculative stocks, could be left out of consideration as long as the causes for the deviation of market prices from normal prices (e.g. harvest fluctuations) were different from the determinants of normal prices (e.g. technology and necessary wages). (It is more controversial whether path dependence could come in on a second stage and be compatible with the notion of longperiod analysis.)

In contrast, intertemporal equilibrium paths and their terminal states (if they exist) are determined by the same data: the preferences, the technology and the endowments. From a classical point of view, a neoclassical intertemporal equilibrium which converges towards a terminal state looks like a model of market prices converging towards a long-period equilibrium. But the model is special in that markets always clear, speculative elements are absent, and the terminal state is special in that distribution (the rate of profit) remains determined by supply and demand for labour and 'capital'. If recursive preferences are used, reswitching must be excluded to demonstrate convergence, as was stated above.

In the present setting, reswitching introduces an element of instability for the intertemporal equilibrium even within a short time horizon-in the limit, in the consideration of only one period as shown in the Appendix. The instability arises in the course of a change of technique. It is a more general phenomenon than a paradox of capital as an obstacle to the long-run convergence towards a terminal state. For the obstacle to convergence also is caused by a change of technique taking place at a particular time. Hence the 'short-run' problem to be considered here seems to be the relevant case which must be analysed.

## 4. STABILITY: PRELIMINARY CONSIDERATIONS

We first show intuitively why reswitching leads to local instability in a simple case. Consider the point of reswitching between $r_{\beta}$ and $r_{\alpha}$ in figure 2; we denote the rate of profit associated with this intersection of the wage curves by $\bar{r}$. Let $\varepsilon>0$ be small and $\mathbf{p}^{\beta}$ and $w_{\beta}$ the normal prices and the wage rate associated with $\bar{r}-\varepsilon$ (where technique $\beta$ is chosen) and $\mathbf{p}^{\alpha}$ and $w_{\alpha}$ the normal prices and the wage rate associated with $r+\varepsilon$ (where technique $\alpha$ is chosen). There is then an instability at $\bar{r}$ in the following sense: suppose we are in a
steady state at $\bar{r}-\varepsilon$, at full employment. If somehow a transition is made to $\bar{r}+\varepsilon$, the same gross output can be produced with less labour since $\mathbf{l}^{\beta} \geq \mathbf{l}^{\alpha}$. Unemployment results, and this would be observed also by an auctioneer who would test the possibility of such a transition. The unemployment would justify the lowering of the wage from $w_{\beta}(\bar{r}-\varepsilon)$ to $w_{\alpha}(\bar{r}+\varepsilon)$.

The reader is kindly invited to check that the same argument holds if the move is made in the opposite direction from a position of full employment at $w_{\alpha}$ to a position of overemployment at $w_{\beta}$ : then the overemployment justifies the raising of the wage. Hence there appears to be an instability which needs closer analysis. Before we attempt that, the reader is also invited to confirm that the argument does not hold in a transition across the 'ordinary' switchpoint between $r=0$ and $r_{\beta}$ or the 'neoclassical' switchpoint of figure 1. In these cases, the change in employment and the change of the wage rate are in opposite directions so that the equilibria seem stable.

The instability of the equilibrium at $\bar{r}$ does not easily get corrected, for if one moves on to $\bar{r}+2 \varepsilon, \bar{r}+3 \varepsilon$ and so on, the unemployment seems to persist and one might conclude that an auctioneer could continue to lower the wage all the way down to $w_{\alpha}=0$ and $r=R_{\alpha}$, and that this would constitute a second equilibrium of the system.

But here the intuitive argument, based on simple steady-state comparisons, fails, at least for one-consumer economies which have unique equilibria.

In a first attempt to refine the argument, we assume that we have an intertemporal equilibrium lasting one period only (with consumption $\mathbf{c}^{0}$ and $\mathbf{c}^{1}$ at the beginning and the end of the period) and no terminal stocks at $\bar{r}-\varepsilon$, with one consumer maximizing his utility, and the utility function and the endowments happen to be such that this is a steady state, $\mathbf{p}^{\beta}$ and $w_{\beta}$ being the undiscounted prices and the wage rate respectively. An auctioneer tests the stability of this position by crying prices $\mathbf{p}^{\alpha}$ and $w_{\alpha}$ pertaining to $\bar{r}+\varepsilon$. The producers then adopt technique $\alpha$. The consumer will now change his demand and the consequent change in employment which might be necessary in order to meet the demand might lead to less unemployment or even overemployment in spite of the reduced requirement of labour per unit of output because $\mathbf{l}^{\alpha} \leq \mathbf{l}^{\beta}$. However, we need not calculate this change of demand as long as $\varepsilon$ is small. For the demand functions $\mathbf{c}^{0}$ and $\mathbf{c}^{1}$ are continuous functions of the prices and of distribution, and normal prices change continuously as $r$ is raised from $\bar{r}-\varepsilon$ to $\bar{r}+\varepsilon$, since the normal prices of systems $\beta$ and $\alpha$ are the same in the switchpoint at $\bar{r}$. If $\varepsilon$ is small, the demand change is negligible.

The instability caused by reswitching is therefore at least a local phenomenon. A thorough analysis, taking finite demand changes into account, is

[^14]complicated. In this paper, we shall focus on comparisons between 'neoclassical' switches and reswitching, making various assumptions about the procedures ascribed to the auctioneer, the utility functions of the consumer and the length of the intertemporal equilibrium (number of periods). It will be seen that the character of the instability depends much on these assumptions, and this finding has convinced me that it is more useful first to explore this variety of possibilities before attempting a generalized mathematical approach which would be closer to the methods employed by neoclassical economists.

To examine stability at all is like opening Pandora's box. A myriad of evil spirits such as different kinds of expectations and speculative behaviour fly out and should in principle be incorporated in a complete model of disequilibrium. Since no complete model can be built, sage model builders will try to open the lid of the box only a little, taking into account only a small number of causes for disequilibrium. And only few models of disequilibrium can be constructed which do not introduce some kind of path dependence. Walrasian tâtonnement has a dual use: as a conceptional instrument to search the equilibrium solution, and as a model of how prices in special kinds of markets might tend to equilibrium without changing the initial data (Negishi, 1987).

We are here only interested in tâtonnement as a concept to examine the stability of equilibrium, and it can here be only of limited use, since continuous excess demand functions do not exist for producers in the case of constant returns and perfect competition, unless special assumptions are made as above, in the intuitive argument.

Some readers might be tempted, like Mandler, to abandon constant returns and to admit 'slightly' diminishing returns to scale and a given number of firms in order to work with continuous excess demand functions of producers. But we wish to retain free entry. To abandon the assumption of linear activities not only means to rule out reswitching in favour of a strictly convex technology. It also means to forget one of the fundamental insights of the 1920s which was gained in the debates on returns to scale in the Economic Journal (Sraffa, 1926). If the factors of production have been enumerated completely, there must be constant or increasing returns to scale. For if the returns are diminishing, there must be a reason why a process operated at a given level of production cannot be operated with the same costs per unit of output at twice that level. Such a hidden reason may exist, e.g. in the form of entrepreneurship, but this is a factor of production which must be taken into account-if necessary, by abandoning perfect competition.

We do not examine stability in general, but only perturbations of an existing equilibrium. Prices of commodities are equal to the cost of production,
given distribution, since we assume that the producers (entrepreneurs) in our economy are perfectly competitive. Deviations from equilibrium (and movements back to it) must therefore primarily be based on changes in distribution. We express this idea using the fiction of the auctioneer. We therefore assume that the auctioneer announces wage rates and prices of endowments, using a certain numéraire. The producers report the prices which then obtain for future periods, after the choice of the cost-minimizing technique. The auctioneer announces these prices to the consumer. The consumer thus gets a new budget constraint. He maximizes his utility, assuming that he obtains the income deriving from full employment and from the full use of the endowments at the prices and the wage rate announced by the auctioneer and the producers, and he reports his demand for consumption goods in all periods within the time horizon considered. This consumption demand is announced to the producers. We avoid the problem that producers might wish to supply infinite amounts (with constant returns and perfect competition), if demand prices are above cost of production (or nothing in the opposite case), by the assumption already made that the cost of production prices are reported by the producers and by assuming further (which means to grant much to the theory) that the plans for production made by producers are aggregated by the auctioneer to obtain total levels of investment. Either the producers use the initial endowments fully. What is not consumed at the beginning of the first period will then be invested during that period. What is not consumed at the end of the first period will be invested in the second period and so on. Thus, activity levels will be determined, going forward in time. These activity levels determine employment in each period. Alternatively, activity levels are determined recursively, beginning in the last period, as the gross production which would be necessary in each period to satisfy consumption demand and gross investment. Then a discrepancy will show in the beginning as a difference between endowments offered and quantities demanded for consumption and investment needed in the first period. The markets for all quantities of goods and commodity prices will thus be cleared by heroic assumptions, except for the last (if quantities demanded are calculated forward) or the first (if quantities are calculated recursively). Moreover, the labour markets may be in disequilibrium. If equilibrium does not obtain, the procedure must be repeated at changed prices which we shall define, using either the 'forward' or the 'recursive' approach.

The meaning of disequilibrium in the labour market requires special attention. The wage rate in this model is thought to represent a 'surplus' or 'luxury' wage. The necessary wage goods are contained in the means of production. If there is full employment, the necessary wage will be complemented by a surplus wage-the worker will get a few per cent more wage, say. He lacks

[^15]the leverage to get a surplus wage if there is unemployment. As explained in my earlier contributions, the expression 'unemployment' is, like the expression 'immigration', only a façon de parler in order to facilitate comparison between the neoclassical and classical and Keynesian theories. 'Unemployment', if it obtains in equilibrium, is 'voluntary' according to the standard interpretation, of course. It may be called 'unvoluntary' in disequilibrium.

It is clear what the auctioneer must do if a mistake was made regarding distribution. If, e.g. a zero wage was announced and the amount of employment resulting from the calculated activity levels exceeds the labour supply available, the auctioneer will have to start again and announce a positive wage. By changing the wage rates, he may grope towards equilibrium. In a model with constant returns he cannot adapt all prices in the tâtonnement process, however, since prices of commodities are determined by cost of production, except for the prices at the beginning of the first period. These are the prices at which the initial endowments are sold either as consumption goods or as investment goods. A quantity in this market will balance in any case (assuming positive levels of production and consumption), if activity levels are calculated moving forward, since what is not consumed at the beginning of the first period will be invested during that period. A mismatch will then show at the end of the last period. Conversely, it shows in the beginning, if activity levels are calculated recursively. Different assumptions may be made as to how the auctioneer will change the initial prices if equilibrium is not attained through the change of wage rates alone. We shall see that the stability of the economy depends crucially on the assumptions which can be made in this regard. But it is clear already at this stage that the primary tool of the auctioneer to grope for equilibrium in this model is to change wage rates and prices of endowments, since all other prices result from cost of production.

Our analysis of stability should ideally be based as directly as possible on what our critique aims at, i.e. neoclassical theory. But there is no 'canon' for the analysis of stability in the case of constant returns. There is therefore no 'absolute' criterion for stability. Processes which 'prove' stability are easy to invent if one is prepared to endow the auctioneer with abilities that transcend those of the market which he was initially introduced to represent. If the mathematical space of suitably normalized prices is subdivided into cubes of the length of $\delta$, an ideal auctioneer can try one price vector within each of those cubes, take the best resulting approximation to equilibrium and will thus end up within an $\varepsilon$-environment of equilibrium, and $\delta$ will be positive for a pre-assigned small $\varepsilon$ at least in the case of continuity. If the auctioneer is assumed to be sufficiently stupid, he will not know how to get out of an initial state of disequilibrium in any case. The point then is to ascribe
abilities to the auctioneer which are compatible with neoclassical assumptions and which represent the potential and the limitations of actual market processes. And there are many ways of doing this. For the usual assumptions that we have markets for every present and future commodity in the system and that the auctioneer gropes for equilibrium prices and quantities in all markets do not define how he combines the information obtained by his tâtonnement in each market. Our primary aim is not to examine complete stabilization processes but to compare conditions which obviously are such that deviations lead back to equilibrium with others which obviously, at least at first and for some iterations, lead away from equilibrium. We hope to convince the reader that our results are of theoretical significance for the critique of neoclassical theory despite some special assumptions, in other words that they are relevant for the essential version of the neoclassical theory of equilibrium and its stability.

## 5. INSTABILITY: A COMPARATIVE APPROACH

We examine the stability of the state which the economy has reached immediately after the immigration, when the less capital-intensive technique (demechanization or reswitching) has been adopted. We now consider the periods after the immigration as a separate equilibrium, with the endowments consisting of the stocks left over. We therefore have a given utility function for these periods, as Mandler requests (the Appendix examines the case he prefers of a given and uniform rate of time preference). We go beyond the local analysis of instability presented at the beginning of the last section. The deviation from equilibrium, which we assume to test stability, now consists of the assumption that the market by accident, or the auctioneer by erroneous design, set the wage rates for the coming periods equal to zero. The motivation for this assumption is simple: the market may have failed to react immediately to the fact that technology has been changed at just the right moment to absorb the immigration fully. Falling wages are the obvious first (but, with reswitching, false) reaction to 'immigration'. We consider only two periods (only one in the Appendix), and eventually we shall set $n=2$. The periods to be considered are periods $t^{\prime}+1$ and $t^{\prime}+2$, according to the numbering adopted earlier. But it is simpler if we call these periods now 1 and 2; hence $t^{\prime}=0$.

The technique used after immigration was called $\beta$ in figures 1 and 2. The equilibrium path which serves as reference path for the stability analysis is, with gross outputs $\mathbf{q}$ still being kept constant, as follows for both demechanization and reswitching:

[^16]$$
\mathbf{q}=\overline{\mathbf{c}}^{0}+\mathbf{q} \mathbf{A}_{\beta}, \mathbf{q}=\overline{\mathbf{c}}^{1}+\mathbf{q} \mathbf{A}_{\beta}, \mathbf{q}=\overline{\mathbf{c}}^{2}+\mathbf{f}
$$
therefore
$$
\overline{\mathbf{c}}^{0}=\overline{\mathbf{c}}^{1}=\overline{\mathbf{c}}^{2}=\mathbf{q}\left(\mathbf{I}-\mathbf{A}_{\beta}\right)
$$
and we add
$$
\mathbf{f}=\mathbf{q} \mathbf{A}_{\beta}
$$

Given initial prices of the first period $\mathbf{p}^{0}, \beta$ must show a cost advantage in both periods ( $\mathbf{p}^{0}$ is proportional to $\mathbf{p}^{\alpha}$ if there was no lagging-behind at $t^{\prime}$ ):

$$
\begin{aligned}
& \mathbf{A}_{\beta} \mathbf{p}^{0}+w_{1} \mathbf{1}^{\beta}=\mathbf{p}^{1} \leq \mathbf{A}_{\alpha} \mathbf{p}^{0}+w_{1} \mathbf{l}^{\alpha} \\
& \mathbf{A}_{\beta} \mathbf{p}^{1}+w_{2} \mathbf{l}^{\beta}=\mathbf{p}^{2} \leq \mathbf{A}_{\alpha} \mathbf{p}^{1}+w_{2} \mathbf{l}^{\alpha},
\end{aligned}
$$

where $w_{1}$ and $w_{2}$ are the discounted wage rates which result from the assumption that the own rate of interest $r_{\beta}$ is given in terms of numéraire $\mathbf{s}$.

Those wage rates are set equal to zero in our stabilization experiments. This assumption may seem extreme. It can easily be mitigated, however, by extending the argument of section 4 . The following calculations can, at the cost of rendering the formulas more complicated, be adapted to the assumption that only small changes of the wage rate are considered. In the neoclassical case, the wage rate must be lowered so as to jump over a switchpoint such as in figure 1 or such as the upper switchpoint in figure 2 . These moves reveal stability. In the case of reswitching, it suffices to lower the wage from $w_{\beta}$ in figure 2 to a point immediately below the second switchpoint (where the reswitching takes place). It will be found in the latter case that the auctioneer may encounter unemployment, and he will then have to set the wage rate equal to zero for subsequent iterations of the tâtonnement process. ${ }^{13}$ We here assume that this is done directly. $\mathbf{p}^{0}$ and $\mathbf{q}$ remain, to begin with, as given. We first derive prices, then consumption demand in the tâtonnement process, and the magnitudes which result from the first round of the examination of stability will be denoted by an asterisk.

Case 1: In the case of demechanization, starting from technique $\beta$, we first verify that $\mathbf{A}_{\beta} \mathbf{p}^{0} \leq \mathbf{A}_{\alpha} \mathbf{p}^{0}$, whatever $\mathbf{p}^{0}$, since $\mathbf{A}_{\alpha} \geq \mathbf{A}_{\beta}$. Setting $w_{1}^{*}=w_{2}^{*}=0$

[^17]here does not change the fact that $\beta$ is the dominating technique in the first period (reswitching will be different in this). Prices are thus determined as follows:
\[

$$
\begin{aligned}
& \mathbf{A}_{\beta} \mathbf{p}^{0}=\mathbf{p}^{1 *}=\mathbf{p}^{1}-w_{1} \mathbf{l}^{\beta}<\mathbf{p}^{1} \\
& \mathbf{A}_{\beta} \mathbf{p}^{1 *}=\mathbf{p}^{2 *}=\mathbf{A}_{\beta}\left(\mathbf{p}^{1}-w_{1} \mathbf{l}^{\beta}\right)=\mathbf{p}^{2}-w_{2} \mathbf{l}^{\beta}-w_{1} \mathbf{A}_{\beta} \mathbf{l}^{\beta}<\mathbf{p}^{2}
\end{aligned}
$$
\]

(Note that these equations would also hold if only one technique had been given, with an arbitrary $\mathbf{p}^{0} \geq \mathbf{o}$.) Profits (interest) are not explicit since we reckon in discounted prices. The disequilibrium prices fall faster than prices in the reference case. On the one hand, the income of the consumer is reduced because wages are equal to zero. On the other, income is increased because the value of terminal stocks is diminished and because the fall of prices increases purchasing power. These effects cancel in such a way that the consumer could buy the consumer goods of the reference case at these changed prices. This follows from a short calculation. ${ }^{14}$ Utility in disequilibrium therefore must be higher than in equilibrium. Since prices on the disequilibrium path fall faster than on the reference path, we may expect consumption later in time to be higher, therefore in particular $\mathbf{c}^{2 *}>\overline{\mathbf{c}}^{2}$, also $\mathbf{c}^{1 *}>\overline{\mathbf{c}}^{1}$. In contrast, $\mathbf{c}^{0 *}<\overline{\mathbf{c}}^{0}$, for although initial prices have not changed $\left(\mathbf{p}^{0 *}=\mathbf{p}^{0}\right)$, the price of initial consumption rises relatively to that of late consumption. ${ }^{15}$ The Appendix examines a special case in greater detail to show this.

Since consumption falls $\left(\mathbf{c}^{0 *}<\overline{\mathbf{c}}^{0}\right)$ initially, the activity levels rise in the first period $\left(\mathbf{q}^{0}=\mathbf{c}^{0 *}+\mathbf{q}^{1 *} \mathbf{A}_{\beta}\right.$, if we calculate activity levels forward). But, at full employment, this is not possible, the excess demand in the labour market (which may, e.g. be reported by the producers) will make it clear to the auctioneer that the wage rate must rise. The effect of deferred consumption

[^18]therefore acts here as an immediate stabilizer. The Appendix makes clear that in this case, which does not involve a change of technique, utility functions can be found such that they generate the same reference path, but with an arbitrarily small effect of deferred consumption.

Case 2: The stability problem in the case of reswitching will be seen to arise in the context of a change of technique. A change of technique had not to be considered so far in the case of demechanization, since we started from the situation after the immigration, hence from a situation with full employment at a low intensity of capital and at a low wage so that no change of technique intervened when the wage was lowered to zero. For comparison, we therefore also consider the stability of the stationary state prior to immigration in the case of demechanization. Surprisingly, the result is here even more pronounced. The economy is in a stationary state corresponding to $w_{\alpha}$ and $r_{\alpha}$ in figure 1. If the wage rate is here set equal to zero, an immediate transition to technique $\beta$ takes place, since $\mathbf{A}_{\beta} \leq \mathbf{A}_{\alpha}$ implies $\mathbf{A}_{\beta} \mathbf{p}^{0} \leq \mathbf{A}_{\alpha} \mathbf{p}^{0}$, but $\mathbf{l}^{\beta} \geq \mathbf{l}^{\alpha}$. Even if activity levels were kept constant, the demand for labour increases to above the level of full employment prior to immigration, i.e. $\mathbf{q l}^{\beta} \geq \mathbf{q} \mathbf{l}^{\alpha}$. We therefore have overemployment and the auctioneer will have to raise the wage rate above zero. Here, what we shall call the technology effect is added to the effect of deferred consumption which is also present but which we do not even need to consider in detail (it is clear that prices of later dates will fall), since we have overemployment anyway. In fact, prices fall faster than in case 1 (i.e. by more than wage costs):

$$
\mathbf{p}^{1 *}=\mathbf{A}_{\beta} \mathbf{p}^{0} \leq \mathbf{A}_{\alpha} \mathbf{p}^{0}<\mathbf{A}_{\alpha} \mathbf{p}^{0}+w_{1} \mathbf{1}^{\alpha}=\mathbf{p}^{1}
$$

therefore

$$
\mathbf{p}^{1 *} \leq \mathbf{p}^{1}-w_{1} \mathbf{1}^{\alpha}
$$

Case 3: We finally turn to the case of reswitching, also involving a change of technique. In period one, the economy has just switched to technique $\beta$ after the immigration. There is full employment at the high level corresponding to immigration, but the auctioneer now tries $w_{1}^{*}=w_{2}^{*}=0$.

There is a general argument for stability, based on the uniqueness of equilibrium in one-consumer economies which seems to work in this case, too. Instability means that a deviation is not corrected; the auctioneer will not move away from a position encountered in the tâtonnement process if, having moved away from one equilibrium, he has reached another. Instabilities
therefore must exist if there are several equilibria, but this is not the case here. Do we therefore necessarily have stability? To set wage rates equal to zero and to fulfil all other equilibrium conditions is equivalent to looking for an optimum (maximizing the utility of the one and only consumer with the conditions for reproduction fulfilled), but without a labour constraint. Such an optimum $\left(\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}, \tilde{\mathbf{c}}^{2}\right)$ will exist, clearly $U\left(\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}, \tilde{\mathbf{c}}^{2}\right)>U\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}, \overline{\mathbf{c}}^{2}\right)$ because a binding constraint has been relaxed. Hence, employment will be larger than $L_{\beta}-$ otherwise, this optimum would have been chosen from the start. The auctioneer therefore observes excess demand in the labour markets at ( $\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}, \tilde{\mathbf{c}}^{2}$ ) and must raise wages. But this argument does not ensure stability. It is questionable whether the auctioneer, in groping for a solution, will reach ( $\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}, \tilde{\mathbf{c}}^{2}$ ) and will hence have to raise wages. For we shall see that we can get complications such that he may be trapped at zero wages, in a situation from which he does not move away, because he is systematically mistaken.

On the reference path, the labour-intensive technique $\beta$ is being used with. $\mathbf{l}^{\beta}>\mathbf{l}^{\alpha}$. This technique can be profitable with

$$
\mathbf{A}_{\beta} \mathbf{p}^{0}+w_{1} \mathbf{l}^{\beta} \leq \mathbf{A}_{\alpha} \mathbf{p}^{0}+w_{1} \mathbf{l}^{\alpha}
$$

only if in the first process, the only one in which the two techniques differ, technique $\beta$ employs a vector of capital goods such that $\mathbf{a}_{0} \mathbf{p}^{0}+w_{1} \mathbf{l}^{\beta}<\mathbf{a}_{1} \mathbf{p}^{1}+$ $w_{1} \mathbf{l}^{\alpha}$, where $\mathbf{a}_{1}$ continues to denote the vector of capital goods employed by $\alpha$. As we saw above, this implies that, when the wage rate falls to zero, $\mathbf{a}_{0} \mathbf{p}^{0}<\mathbf{a}_{1} \mathbf{p}^{0}$ and $\mathbf{A}_{\beta} \mathbf{p}^{0} \leq \mathbf{A}_{a} \mathbf{p}^{0}$, because $l_{0}>l_{1}$, hence $\beta$ still is used, although $w_{1}^{*}=0$. Technique $\alpha$ must prevail in the long run if the wage rate is equal to zero and if the comparison is made either in terms of the normal prices pertaining to the maximum rate of profit of $\alpha, \mathbf{p}^{\alpha}$, or in terms of a series of prices which converge to $\mathbf{p}^{\alpha}$, since $R_{\alpha}>R_{\beta}$, but there is a lagging-behind effect in the short run also in this disequilibrium transition.

To simplify matters, we assume that the lagging-behind takes one period only. We therefore assume that we have in the first period

$$
\mathbf{A}_{\beta} \mathbf{p}^{0}=\mathbf{p}^{1 *}=\mathbf{p}^{1}-w_{1} \mathbf{l}^{\beta} \leq \mathbf{A}_{\alpha} \mathbf{p}^{0}
$$

and in the second

$$
\mathbf{A}_{\alpha} \mathbf{p}^{1 *} \leq \mathbf{A}_{\beta} \mathbf{p}^{1 *}
$$

so that

$$
\mathbf{A}_{\alpha} \mathbf{p}^{1 *}=\mathbf{p}^{2 *}=\mathbf{A}_{\alpha} \mathbf{p}^{1}-w_{1} \mathbf{A}_{\alpha} \mathbf{l}^{\beta}
$$

[^19]It is clear that $\mathbf{p}^{1 *}<\mathbf{p}^{1}$ and

$$
\mathbf{p}^{2}-\mathbf{p}^{2 *}=\left(\mathbf{A}_{\beta}-\mathbf{A}_{\alpha}\right) \mathbf{p}^{1}+w_{2} \mathbf{I}^{\beta}+w_{1} \mathbf{A}_{\alpha} \mathbf{l}^{\beta}
$$

which is positive since all components of $\mathbf{A}_{\beta}-\mathbf{A}_{\alpha}$ vanish, except those on the first row, and $\left(\mathbf{a}_{0}-\mathbf{a}_{1}\right) \mathbf{p}^{1}>\mathbf{0}$ because technique $a$ is dominant in the second period. Hence we again have the effect of deferred consumption so that activity levels rise in both periods and employment rises. But, in the second period, technique $\alpha$ is used with $\mathbf{l}^{\alpha} \leq \mathbf{l}^{\beta}$; to this extent, there is a loss of employment in the second period. The magnitude (but not the direction) of the deferred consumption effect depends on the shape of the utility function. Our construction so far only requires that it be concave and have marginal utilities equal to prices on the reference path. By making the curvature of the function more pronounced without altering its properties on the reference path, as indicated in the Appendix, we get a utility function which results in a smaller effect of deferred consumption. We may therefore assume that the employment loss due to the change of technique exceeds the employment gain due to the effect of deferred consumption at least during the first iterations. Hence there is unemployment in the second period and the assumption of the auctioneer to set the second wage rate equal to zero has been confirmed.

It should now be clear that it will depend on the precise policy of the auctioneer whether his process of groping for equilibrium will actually arrive at it or whether the auctioneer will be trapped by trying a low wage rate in period 1 and a zero wage rate in period 2 again and again. The technology effect will disappear only if the wage rate for the first period is raised sufficiently to render technique $\beta$ dominant not only in the first but also in the second process.

And it seems that the auctioneer can get trapped in a disequilibrium process with zero wages indefinitely. For suppose that wages for both periods are set equal to zero and that the auctioneer announces a $\mathbf{p}^{0 *}$ such that $\mathbf{A}_{\alpha}$ is dominant from the start, in both periods under consideration. As the Appendix shows, it is possible to set the price for one of the commodities, say $i$ at $t=0$, so low that it attracts most of the demand and activity levels in the first periods are low and such that technique $a$ dominates. Hence, little is produced and consumption and activity must be low in the second period as well. Suppose, moreover, that a utility function is chosen with the same first derivatives in $\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}, \overline{\mathbf{c}}^{2}\right)$ as so far, but such that the distance to $\mathbf{c}^{0 *}, \mathbf{c}^{1 *}, \mathbf{c}^{2 *}$ is small so that any remaining effect of deferred consumption (which increases employment) is smaller than the technology effect (which creates unemployment). The zero wage rates will then be confirmed. The process may
now be iterated. New initial prices $\mathbf{p}^{0 *}$ have to be defined which will reflect the relative discrepancies between the demand and supply of consumption goods, as expressed in the activity levels of the first or the last period, depending on whether they are calculated forward (which is not always possible) or recursively. If this process leads to repeated iterations, with the effect of deferred consumption permanently being dominated by the technology effect, the auctioneer never raises the wage rates to their equilibrium value.

In order to render this idea more precise, we distinguish between an ideal auctioneer, similar to an ideal planner, and an uninformed auctioneer whose actions reflect the problem of anticipating today what will happen in the markets tomorrow. The auctioneer, having announced $w_{1}^{*}=w_{2}^{*}=0$ and $\mathbf{p}^{0 *}$ (such that $\mathbf{A}_{\beta}$ is dominant), gets $\mathbf{p}^{1 *}$ and $\mathbf{p}^{2 *}$ from the entrepreneurs and $\mathbf{c}^{0 *}$, $\mathbf{c}^{1 *}, \mathbf{c}^{2 *}$ from the consumer with budget $\mathbf{q p}^{0 *}-\mathbf{f p}^{2 *}$. This will not yield the equilibrium solution ( $\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}, \tilde{\mathbf{c}}^{2}$ ), pertaining to the optimization without the labour constraint, since $\mathbf{p}^{0 *}$ are (except by accident) not equilibrium prices. The ideal planner is able to calculate backwards (recursively) in order to obtain the activity levels at the beginning of the first period which can then be compared with the endowments. He first gets $\mathbf{q}^{2 *}=\mathbf{c}^{2 *}+\mathbf{f}$, then $\mathbf{q}^{1 *}=\mathbf{c}^{1 *}$ $+\mathbf{q}^{2 *} \mathbf{A}_{\alpha}$, then $\mathbf{q}^{0 *}=\mathbf{c}^{0 *}+\mathbf{q}^{1 *} \mathbf{A}_{\alpha}$, hence $\mathbf{q}^{0 *}=\mathbf{c}^{0 *}+\mathbf{c}^{1 *} \mathbf{A}_{\alpha}+\left(\mathbf{c}^{2 *}+\mathbf{f}\right)\left(\mathbf{A}_{\alpha}\right)^{2}$. He thus knows today what is needed for tomorrow, not by groping but, as far as quantities are concerned, directly. The next step is obvious: if $\mathbf{q}_{i}^{0 *}>\mathbf{q}_{i}\left(\mathbf{q}_{i}^{0 *}<\right.$ $\left.\mathbf{q}_{i}, \mathbf{q}_{i}^{0 *}=\mathbf{q}_{i}\right)$, some $\mathbf{p}_{i}^{0 * *}>\mathbf{p}_{i}^{0 *}\left(\mathbf{p}_{i}^{0 * *}<\mathbf{p}_{i}^{0 *}, \mathbf{p}_{i}^{0 * *}=\mathbf{p}_{i}^{0 *}\right)$ will be announced for the next and similarly for later iterations. ${ }^{16}$ If labour is disregarded, and if the iterations converge, they converge to $\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}, \tilde{\mathbf{c}}^{2}$, but if wage rates are raised in cases of overemployment and lowered in cases of unemployment, the iterations converge to $\left(\overline{\mathbf{c}}_{0}, \overline{\mathbf{c}}_{1}, \overline{\mathbf{c}}_{2}\right.$ ). In the example mentioned above ( $\mathbf{p}_{i}{ }^{*}$ low, but $\mathbf{A}_{\alpha} \mathbf{p}^{0 *} \leq \mathbf{A}_{\beta} \mathbf{p}^{0 *}$ ), employment in period 1 and 2 can become arbitrarily small (apart from what is needed to produce terminal stocks $\mathbf{f}$ ), but $\mathbf{c}_{i}^{0 *}$ will be large. This may be termed a relative price effect which works (in this case) against the deferred consumption effect. Since $\mathbf{c}_{t}^{0 *}$ is large, the auctioneer will set $\mathbf{p}_{i}^{0 * *}>\mathbf{p}_{i}^{0 *}$ and consumption demand sooner or later shifts to the future. The technology effect, helped by the relative price effect, may dominate the deferred consumption effect for a while, but nor for ever, for otherwise we should get a convergence towards ( $\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}, \tilde{\mathbf{c}}^{2}$ ), and this is impossible since the

[^20]equilibrium is unique and a unique optimum. A convergence to ( $\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}, \tilde{\mathbf{c}}^{2}$ ) would mean that a second optimum existed. A lower terminal employment than $L_{\beta}$ is impossible, hence wage rates eventually must rise.

The uninformed auctioneer is an incarnation of Say's law: he begins by announcing $\mathbf{q}-\mathbf{c}^{0 *}$ to the producers, and if $\mathbf{q}-\mathbf{c}^{0 *}>\mathbf{o}$ and if there is $\mathbf{q}^{1 *} \mathbf{A}_{\alpha}$ such that $\mathbf{q}^{\mathbf{1}}>\mathbf{0}$, they can report $\mathbf{q}^{1 *}$ to the auctioneer as activity levels for the first period, but if negative quantities or activity levels come in, the auctioneer varies $\mathbf{p}^{0 *}$. Once $\mathbf{q}^{1 *}>\mathbf{0}$, the auctioneer can grope for a positive $\mathbf{q}^{2 *}$ and finally verify whether $\mathbf{q}^{2 *}-\mathbf{c}^{2 *}=\mathbf{f}$. But while it is clear how to vary $\mathbf{p}^{0 *}$ in order to obtain $\mathbf{c}^{0 *}<\mathbf{q}$, the rules are increasingly more difficult to design for obtaining $\mathbf{q}^{1 *}>\mathbf{0}, \mathbf{q}^{2 *}>\mathbf{0}$ and finally the match $\mathbf{q}^{2 *}=\mathbf{c}^{2 *}+\mathbf{f}$. The helpless auctioneer therefore may get stuck in an unending process of mistaken adaptations (e.g. a cycle) which would not have started without the technology effect.

This may suffice to show, without going into further details, how difficult it is to design a suitable process of adaptation for models with linear production, which reflects the possibilities of an auctioneer who in turn reflects the possibilities of the market. ${ }^{17}$ Convergence cannot be expected to be as smooth as the possibilities of the ideal auctioneer suggest, and the technology effect will delay it in the case of reswitching, as it accelerates it in the case of demechanization.

We thus find that reswitching adds an important element of instability, the importance of which depends on the process of adaptation, but also on the utility function. Mandler was right to point out that the assumption of only one well-behaved consumer helps to stabilize the intertemporal equilibrium. To use his analogy with demand and supply curves: the Marshallian cross of demand and supply curves is stable if both curves have the required shape, i.e. if the demand curve falls and the supply curve rises. If the supply falls because of increasing returns due to external effects and the demand curve cuts it from above, there is stability according to Marshall and instability according to Walras. The instability is unambiguous only if both curves have reversed slopes. We have here, so to speak, analysed only one half of the problem in order to isolate the contribution of one cause of instability. A

[^21]deeper analysis will require to modify the assumption on the side of the consumer, e.g. by introducing recursive preferences, but also the tâtonnement process.

The introduction of recursive preferences is beyond the scope of this paper, but variable rates of time preference may help to demonstrate that the deferred consumption effect need not always obtain. Consider the case of reswitching according to figure 2 , with a one-period equilibrium for technique $\beta$ and with prices proportional to normal prices pertaining to $r_{\beta}$, $\mathbf{A}_{\beta} \mathbf{p}^{0}+w \mathbf{l}^{\beta}=\mathbf{p}^{1}$, therefore $\mathbf{p}^{1}=\mathbf{p}^{0} /\left(1+r_{s}\right)$. Suppose that the auctioneer sets $w^{*}=0$ and $\mathbf{p}^{0 *}$ equal to the normal prices of technique $\alpha$ at $R_{\alpha}$ so that $\mathbf{A}_{\alpha} \mathbf{p}^{0 *}=\mathbf{p}^{1 *}=\mathbf{p}^{0 *} /\left(1+R_{\alpha}\right) \leq \mathbf{A}_{\beta} \mathbf{p}^{0 *}$. The endowment $\mathbf{q}^{0}$ is used as the numéraire, $\mathbf{q}^{0} \mathbf{p}^{0}=\mathbf{q}^{0} \mathbf{p}^{0 *} .{ }^{18}$ The equilibrium is now represented in the plane for commodity $i(i=1,2)$ at time 0 and 1 according to figure 3 , supposing that the budget of the consumer happens to be divided equally between both commodities. The figure then illustrates that the deferred consumption effect can be reversed by what might be called an intertemporal Giffen or income effect in the comparison of the original steady state at $\overline{\mathbf{c}}_{i}$ (point $Q$ ) and the consumption demand $\mathbf{c}_{i}^{*}\left(\right.$ point $\left.Q^{*}\right)$ resulting from tâtonnement. ${ }^{19}$

It is shown in the Appendix (case 3) that $\overline{\mathbf{c}}_{i}$ is within the budget set resulting from tâtonnement so that the budget lines must cross below the $45^{\circ}$ line in figure 3.

We thus get the remarkable result that the tendency to instability of equilibrium prices due to reswitching is increased from the side of utility precisely in the constellation opposite to that where the convergence of intertemporal equilibria to a terminal state with a stable distribution of wealth is in danger. We saw above (section 3): if the rate of time preference falls with the level of accumulation (absolute value of the slope of the indifference curves where they cut the $45^{\circ}$ line), there is, to any level of interest, always a level of accumulation where people still prefer more future than present consumption; hence a steady state cannot be reached. Hayek (1941, p. 221) remarked that the rates of time preference of different consumers had to increase with accumulation if there was to be a terminal state where they could be equalized. Here we see that the stabilizing deferred consumption effect may be absent

[^22]

Figure 3. Rates of time preferences and rates of substitution with an intertemporal income effect. $\operatorname{tg} \gamma=1+R_{\alpha}, \operatorname{tg} \delta=1+r_{\beta}$.
if this condition holds; the technology effect of reswitching then is not counteracted. We conclude that stability problems of different kinds, one concerning equilibrium prices, the other the distribution of wealth, may arise, whatever assumption about time preference is made.
The neoclassical theory of general equilibrium must exclude reswitching in order to avoid instability. To say that reswitching does not affect intertemporal equilibrium because equilibria involving reswitching exist therefore is only a half-truth. This much may be asserted. And what a half-truth is, explains the following story:

The thief went to confess and told the priest, furtively stretching his arm round the confessional: "I am stealing". "That is very bad", admonished him the priest, "say at least: 'I repent, I used to steal.'" "I repent, I used to steal", replied the thief obediently, pocketing the watch which he had just taken from the unsuspecting priest.

[^23]The priest asked: "What did you steal, my son?"-"A watch." "That is very bad, you must give it back", said the priest. "Father, may I give it to you?", asked the thief politely. "No, you must give it to the owner", answered the priest. "The owner said 'no', when I offered the watch to him", explained the thief. "Then you may keep it", concluded the priest.

## APPENDIX

The aim of the Appendix is to confront the deferred consumption effect and the technology effect more rigorously in a simplified model. Moreover, we want to show, following Mandler's suggestion, that the stability analysis remains the same, if a utility function is not constructed as above in section 2 but, more conventionally, regarded as given, with intertemporal utility represented as a discounted sum of an unvarying per-period utility function (Mandler, 2002, p. 216). We consider only one period, hence $T=1$, with $n=2$. The techniques $\alpha$ and $\beta$ still differ in that $a$ uses inputs $\mathbf{a}_{1}$ and labour $l_{1}$ to produce the first commodity, and $\beta$ uses $\mathbf{a}_{0}$ and $l_{0}>l_{1}$. There are no terminal stocks: $\mathbf{f}=\mathbf{0}$. The utility function for consumption vectors $\mathbf{c}^{0}, \mathbf{c}^{1}$, given the rate of discount $\rho$, is

$$
U=\sum_{\substack{i=1,2 \\ t=0,1}} \rho^{-t} \sqrt{c_{i}^{t}}
$$

The following calculations can also be performed, using the model of the main text, with essentially the same result. In the main text, a special utility function was chosen in order to represent a specified path as an intertemporal equilibrium. Here, with a given utility function, a first simple example will be constructed by assuming that initial endowments are available in suitable proportions.

Case 1: Only one technique, $\alpha$, is used. We construct a reference path which shall be optimal and analyse its stability, i.e. we analyse the deferred consumption effect which results from setting $w=0$ in the tâtonnement process. Since the utility function here is given, the reference path (a steady state) will be shown to be optimal and an equilibrium by assuming that the endowments happen to be available in the proportions needed for the path $\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}$. We take $\mathbf{p}^{0}=\mathbf{p}^{\alpha}$, where $\mathbf{p}^{\alpha}$ are the normal prices associated with rate of profit $r_{\alpha}=\rho-1, \mathbf{s p}^{\alpha}=1, \mathbf{A}_{\alpha} \mathbf{p}^{0}+w \mathbf{l}^{\alpha}=\mathbf{p}^{1}, \mathbf{s p}^{0} / \mathbf{s p}^{1}=1+r_{\alpha}$, hence $\mathbf{p}^{1}=\mathbf{p}^{\alpha} /\left(1+r_{\alpha}\right)$, $w=w_{\alpha}$. The vectors $\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}$ are chosen so that marginal utilities on the path $\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}$ shall be equal to prices:

[^24]$$
\frac{\partial U}{\partial c_{i}^{t}}=\frac{\rho^{-t}}{2} \frac{1}{\sqrt{\bar{c}_{i}^{t}}}=\lambda p_{i}^{t}
$$
hence
$$
\bar{c}_{i}^{t}=\left(2 \rho^{t} \lambda p_{i}^{t}\right)^{-2}
$$

Since $\mathbf{f}=0$, activity levels $\mathbf{q}^{1}$ during the period equal $\overline{\mathbf{c}}^{1}$. Hence, if initial endowments $\mathbf{q}^{0}$ happen by assumption to be equal to $\overline{\mathbf{c}}^{0}+\mathbf{q}^{1} \mathbf{A}_{\alpha}=\overline{\mathbf{c}}^{0}+\overline{\mathbf{c}}^{1} \mathbf{A}_{\alpha}$, we have an optimum which is also an equilibrium for the household, for $\overline{\mathbf{c}}^{0}$ satisfy the budget equation $\mathbf{c}^{0} \mathbf{p}^{0}+\mathbf{c}^{1} \mathbf{p}^{1}=\mathbf{q}^{0} \mathbf{p}^{0}+w \mathbf{q}^{1} \mathbf{l}^{\alpha}$, provided there is full employment (proof: by adding the price equations, multiplied by $\mathbf{q}$, and the equality of the endowments and of consumption and investment, multiplied by $\mathbf{p}^{0}$ ). Since $\mathbf{p}^{1}=\mathbf{p}^{0} /\left(1+r_{\alpha}\right), 1+r_{\alpha}=\rho$, we have $\overline{\mathbf{c}}^{0}=\overline{\mathbf{c}}^{1}$, hence the activity levels (which determine employment) $q_{i}^{1}=\bar{c}_{i}^{1}=\left(2 \lambda p_{i}^{0}\right)^{-2}$. If an amount of labour $L_{\alpha}=\mathbf{q}^{1} \mathbf{1}^{\alpha}=\left[l_{1} /\left(p_{1}^{\alpha}\right)^{2}+l_{2} /\left(p_{2}^{\alpha}\right)^{2}\right] / 4 \lambda^{2}$ is supplied, all the conditions for an optimum and an equilibrium are fulfilled. This optimization is called $\bar{P}$.

The Lagrange multiplier so far turns out to be indeterminate, since the budget equation is homogeneous in $\lambda$. Not only the $\bar{c}_{i}^{t}$, but also $\mathbf{q}^{0}$ and $\mathbf{q}^{1}$ are proportional to $\lambda^{-2}$. We insert $\bar{c}_{i}^{t}$ in the budget equation:

$$
\begin{aligned}
\sum_{i, t} \bar{c}_{i}^{t} p_{i}^{t} & =\sum_{i, t} \frac{1}{4 \lambda^{2} \rho^{2 t} p_{i}^{t}}=\mathbf{q}^{0} \mathbf{p}^{0}+w \mathbf{q}^{1} \mathbf{l}^{\alpha} \\
& =\left(\overline{\mathbf{c}}^{0}+\overline{\mathbf{c}}^{1} \mathbf{A}_{\alpha}\right) \mathbf{p}^{0}+w \overline{\mathbf{c}}^{1} \mathbf{l}^{\alpha}
\end{aligned}
$$

where each $\bar{c}_{i}^{t}$ on the right-hand side is proportional to $\lambda^{-2}$. The choice of $\lambda$ regulates the 'size' of the economy constructed in the example. A larger $\lambda$ means a smaller $\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}$ and $\mathbf{q}^{0}$. It is well known that $\lambda$ measures the marginal gain in utility engendered by a marginal increase of the household budget. This gain, given the increase, is smaller in a larger economy because of diminishing marginal utility. Alternatively, one might choose a level for $\mathbf{q}^{0}$, and $\lambda$ would be determined. Without loss of generality, we assume $\lambda=1$ in the formulae above and for what follows.

The auctioneer now announces $w^{*}=0$ and $\mathbf{p}^{0 *}=\mathbf{p}^{0}$. Reduced prices $\mathbf{p}^{1 *}=$ $\mathbf{A}_{\alpha} \mathbf{p}^{0 *}=\mathbf{p}^{1}-w \mathbf{l}^{\alpha}$ for the end of the period result, and the household announces consumption vectors $\mathbf{c}^{0 *}, \mathbf{c}^{1 *}$, obtained from maximizing utility subject to $\mathbf{c}^{0} \mathbf{p}^{0}+\mathbf{c}^{1} \mathbf{p}^{1 *}=\mathbf{q}^{0} \mathbf{p}^{0}$. This maximization $P^{*}$ yields the conditions

$$
\frac{1}{2 \rho^{t} \sqrt{c_{i}^{\iota^{*}}}}=\lambda^{*} p_{i}^{t *}
$$

[^25]Here, a new Lagrange multiplier $\lambda^{*}$ remains to be determined, since $\mathbf{q}^{0}$ now is fixed with $\lambda=1$. Clearly, $U\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)<U\left(\mathbf{c}^{0 *}, \mathbf{c}^{1 *}\right)<U\left(\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}\right)$, where $\left(\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}\right)$ is the solution to the optimization $\tilde{P}$, which is the same as $\bar{P}$, but without the labour restriction. $U\left(\mathbf{c}^{0 *}, \mathbf{c}^{1 *}\right)<U\left(\tilde{\mathbf{c}}^{0}, \tilde{\mathbf{c}}^{1}\right)$, since $\left(\mathbf{c}^{0 *}, \mathbf{c}^{1 *}\right)$ inherits part of the labour restriction, as it were, as long as $\mathbf{p}^{0 *}=\mathbf{p}^{0}$. On the other hand, $\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)$ satisfy the budget equation of $P^{*}$ since the budget equation of $\bar{P}$ holds

$$
\overline{\mathbf{c}}^{0} \mathbf{p}^{0}+\overline{\mathbf{c}}^{1} \mathbf{p}^{1 *}=\overline{\mathbf{c}}^{0} \mathbf{p}^{0}+\overline{\mathbf{c}}^{1} \mathbf{p}^{1}-w \overline{\mathbf{c}}^{1} 1^{\alpha}=\mathbf{q}^{0} \mathbf{p}^{0}
$$

This gives rise to the following geometric representation. Let $\bar{H}$ be the three-dimensional set of non-negative consumption vectors fulfilling the budget equation of $\bar{P}$ and $H^{*}$ the corresponding set for $P^{*}$. Let $E$ be the two-dimensional plane of linear combinations of ( $\left.\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)$ and $\left(\mathbf{c}^{0 *}, \mathbf{c}^{1 *}\right)$ and $\bar{h}=\bar{H} \cap E$ and $h^{*}=H^{*} \cap E ; h$ and $h^{*}$ are straight lines. We obtain figure 4, representing ( $\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}$ ) and $\left(\mathbf{c}^{0 *}, \mathbf{c}^{1 *}\right)$ in $E$.
It is geometrically obvious that, by increasing the curvature of $U$ between $\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)$ and $\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)$, leaving the tangential plane to the indifference surface in $\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)$ unchanged, one can shift ( $\mathbf{c}^{0 *}, \mathbf{c}^{1 *}$ ) towards $\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)$ and thus reduce the deferred consumption effect to an arbitrarily small positive magnitude.
The difference between ( $\mathbf{c}^{0 *}, \mathbf{c}^{\mathbf{1} *}$ ) and ( $\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}$ ) consists primarily of a deferred consumption effect. For $\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)$ is stationary. If the figure is redrawn in the planes for $c_{i}^{0}$ and $c_{i}^{1} ; i=1$ or 2 ; with $c_{j}^{t}=\bar{c}_{j}^{t} ; j \neq i$; and with budget lines $c_{i}^{0} p_{i}^{0}+c_{i}^{1} p_{i}^{1}=\mathbf{q}^{0} \mathbf{p}^{0}+w \mathbf{q}^{1}{ }^{1}{ }^{\alpha}-\left(\bar{c}_{j}^{0} \bar{p}_{j}^{0}+\bar{c}_{j}^{1} \bar{p}_{j}^{1}\right)$ and similarly for $\left(\mathbf{c}^{0 *}, \mathbf{c}^{1 *}\right)$, as in figure 3 , but with the budget lines crossing in $\overline{\mathbf{c}}_{i}$, the derivative of the indifference curves along the $45^{\circ}$ line equals $-\rho$ (Fisher diagram). The transition to $c_{i}^{\prime *}$ involves higher rates of intertemporal substitution. In a steady state at $w=0$, they are equal to $1+R_{\alpha}>\rho$. Hence $\mathbf{c}^{o}$ falls and $\mathbf{c}^{1}, \mathbf{q}^{1}$ and $\mathbf{q}^{1 \alpha^{\alpha}}$ (employment demand) rise in the transition from ( $\left.\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)$ to $\left(\mathbf{c}^{0 *}, \mathbf{c}^{\mathbf{1} *}\right)$.

Algebraically, we get

$$
c_{i}^{t *}=\left(2 \rho^{t} \lambda^{*} p_{i}^{t *}\right)^{-2}
$$

From the budget equation

$$
\mathbf{q}^{0} \mathbf{p}^{0}=\sum_{i, t} \frac{1}{4\left(\lambda^{*}\right)^{2} \rho^{2 t} p_{i}^{t *}}
$$

therefore

[^26]

Figure 4. Equilibrium consumption demand, consumption demand $\left(\bar{c}^{0}, \bar{c}^{1}\right)$ and consumption demand in first iteration $\left(\mathbf{c}^{0 *}, \mathbf{c}^{1 *}\right)$, with indifference curves.

$$
\frac{1}{\left(\lambda^{*}\right)^{2}}=\frac{\mathbf{q}^{0} \mathbf{p}^{0}}{\frac{1}{4}\left(\frac{1}{p_{1}^{0}}+\frac{1}{p_{2}^{0}}+\frac{1}{\rho^{2} p_{1}^{1 *}}+\frac{1}{\rho^{2} p_{1}^{2 *}}\right)}
$$

The same calculation can be executed to determine the Lagrange multiplier for $\bar{P} ; \lambda$ then equals one by definition. Comparing the formulas, we have here a denominator, reduced by the wage income $w \mathbf{q}^{1} \mathrm{I}^{\alpha}$ and a nominator, which is higher insofar as $\mathbf{p}_{1}^{*}<\mathbf{p}^{1}$. Hence $1 /\left(\lambda^{*}\right)<1$ and $\mathbf{c}^{0 *}<\overline{\mathbf{c}}^{*}$, while one may expect $\mathbf{c}^{1 *}>\overline{\mathbf{c}}^{1}$ : the fall of $p_{i}^{1}$ to $p_{i}^{1 *}$ may be expected to predominate over the change in the Lagrange multiplier, because the quantities are proportional to the inverse of the square of the prices.

Case 2: It is obvious that the technology effect will increase the deferred consumption effect (to be calculated in a similar way) in the case of demechanization.

Case 3: Reswitching presents an important difference compared with Case 1, when we confront the problems $\bar{P}$ and $P^{*}$. We assume a fullemployed position at $\beta$ as in figure $2, r_{\beta}<r_{\alpha}$, which is disturbed because the auctioneer announces $w^{*}=0$. As we saw in the main text, there will be lagging-behind effect, however. Since

$$
\mathbf{A}_{\beta} \mathbf{p}^{0}+w \mathbf{I}^{\beta}=\mathbf{p}^{1} \leq \mathbf{A}_{\alpha} \mathbf{p}^{0}+w \mathbf{l}^{\alpha}
$$

with $l_{0}<l_{1}$ and $\mathbf{a}_{0} \mathbf{p}^{0}<\mathbf{a}_{1} \mathbf{p}^{0}$, the auctioneer also has to announce some $\mathbf{p}^{0 *} \neq$ $\mathbf{p}^{0}$, if the transition to $\mathbf{A}_{\alpha}$ is to be made immediately, and this we shall assume. Therefore, a $\mathbf{p}^{0 *}$ is announced such that

$$
\mathbf{p}^{1 *}=\mathbf{A}_{\alpha} \mathbf{p}^{0 *} \leq \mathbf{A}_{\beta} \mathbf{p}^{0 *}
$$

The cost of $\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)$ will now be less than the income of the consumer in situation $P^{*}$ :

$$
\begin{aligned}
\overline{\mathbf{c}}^{0} \mathbf{p}^{0 *}+\overline{\mathbf{c}}^{1} \mathbf{p}^{1 *} & =\left(\mathbf{q}^{0}-\overline{\mathbf{c}}^{1} \mathbf{A}_{\beta}\right) \mathbf{p}^{0 *}+\overline{\mathbf{c}}^{1} \mathbf{A}_{\alpha} \mathbf{p}^{0} * \\
& =\mathbf{q}^{0} \mathbf{p}^{0 *}+\overline{\mathbf{c}}_{1}^{1}\left(\mathbf{a}_{1}-\mathbf{a}_{0}\right) \mathbf{p}^{0 *}
\end{aligned}
$$

The last term on the right is negative by virtue of the transition to $\alpha$ at $w^{*}=0$ and prices $\mathbf{p}^{0 *}$. Figure 4 therefore has to be modified; using the same notation, we see in figure 5 that the budget lines $h^{*}$ and $\bar{h}$ do not cross any more in ( $\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}$ ). Therefore, one can not expect to be able to shift $\left(\mathbf{c}^{0 *}, \mathbf{c}^{1 *}\right)$ to any desired extent to $\left(\overline{\mathbf{c}}^{0}, \overline{\mathbf{c}}^{1}\right)$ by transforming the utility function; the deferred consumption effect cannot be reduced ad libitum, as long as we have constant and not varying rates of time preference as in figure 3.

But we now invoke the relative price effect: $\mathbf{p}^{0 *}$ may be chosen so as to bring employment in $P^{*}$ close to zero. Suppose $a_{01}>a_{11}, a_{02}<a_{12}$, as in the numerical example for reswitching in Schefold (2000, p. 388), with $l_{0}>l_{1}$. Suppose $p_{1}^{0 *}=1-\varepsilon, p_{2}^{0 *}=\varepsilon$ so that $\mathbf{a}_{1} \mathbf{p}^{0 *}<\mathbf{a}_{0} \mathbf{p}^{0 *}$ and $\mathbf{A}_{\alpha} \mathbf{p}^{0 *}=\mathbf{p}^{1} \leq \mathbf{A}_{\beta} \mathbf{p}^{0 *}$; $\mathbf{p}^{1 *}=(1-\varepsilon) \mathbf{a}^{1}+\varepsilon \mathbf{a}^{2}$, where $\mathbf{a}^{1}, \mathbf{a}^{2}$ are the columns of $\mathbf{A}_{\alpha}$. Hence $c_{2}^{0 *} / c_{1}^{0 *}$, $c_{2}^{0 *} / c_{2}^{1 *}, c_{2}^{0 *} / c_{1}^{1 *}$ rise beyond all limits as $\varepsilon$ tends to zero, and $c^{1 *}$ and employment become arbitrarily small. The auctioneer will redress the situation as excess demand $c_{2}^{0 *}-q_{2}^{0}$ increases by raising the price of $c_{2}^{0}$ in later iterations,

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Figure 5. Analogous to figure 3, but with a technology change (reswitching).
but the high labour productivity of technique $\alpha$ (technology effect) will even for the ideal auctioneer retard the reaching of full employment in a situation in which employment previously had been maintained on the basis of using technique $\beta$.

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Fachbereich Wirtschaftswissenschaften
Johann Wolfgang Goethe-Universität
Postfach 111932
D-60054 Frankfurt
Germany
E-mail: schefold@wiwi.uni-frankfurt.de

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[^1]:    ${ }^{1}$ Classical economists did not always confine their attention to capitalist systems; their central concern was to understand changes in social and historical influences on distribution. Even if the system is capitalist and competitive, different rules of distribution may emerge; Sraffa distinguished between a constellation in which (in a closed economy with capitalists and workers) all the surplus except the necessary wages remains with the capitalists and another where the rate of profit is regulated by the monetary rates of interest so that real wages rise to include a surplus element. Other theories of distribution have been proposed which apply in other circumstances (Schefold, 2000b).

    The neoclassical approach also exists in different varieties, but they do not correspond to different historical conditions but to different levels of abstraction in the attempt to represent the same reality: different models represent capital in an aggregate and in a disaggregated form, with perfect foresight or with other assumptions regarding the formation of expectations. Modifications of the theory according to special historical conditions are considered, if at all, only at later stages - usually, neoclassical economists believe that there is only one economic theory.
    ${ }^{2}$ 'The Sraffa critique, which is effective for certain versions of neoclassical theory, has no impact on the Arrow-Debreu model.'

[^2]:    ${ }^{3}$ Reswitching is only the most drastic, but not the empirically most important form of the anomalies. For simplicity we here concentrate on reswitching. A recent PhD thesis by Zonghie Han, written in Frankfurt under my supervision, finds one example of reswitching and many of reverse capital deepening in an analysis based on input-output tables.
    ${ }^{4}$ The 'scenario' 'accumulation with a constant labour force', although possibly most characteristic for the neoclassical approach, is here left out; a tentative discussion of its stability is in Schefold (1997, pp. 488-9, 490-1, 496, 500-1).
    ${ }^{5}$ More extensive descriptions of the scenarios are in Schefold (1997, pp. 484-9, 2000a); proofs of the underlying theorems in Schefold (1997, pp. 464-7, 476).

[^3]:    ${ }^{6}$ More capital because raw materials, e.g. the cotton in the textile industry which Ricardo had in mind, were still necessary to produce cloth while, in addition, more machines were needed (Schefold, 1997, ch. 11). Ricardo, however, was sceptical whether the labour set free by laboursaving technical progress would automatically find new employment. In a Keynesian perspective, growth with full employment requires a sufficiently high rate of investment, and rising demand must absorb the rising supply of consumption goods. The role of wages is ambiguous, for high wages raise costs and may thus hinder employment, but they are also necessary as a source of purchasing power. The Keynesians spoke in rather abstract terms of the 'animal spirits' of entrepreneurs which could drive a process of accumulation at full employment.

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[^5]:    ${ }^{7}$ Actually, two wage curves of two techniques which have a switchpoint in common can both be strictly linear in the following fluke case: let $\alpha$ be a technique for which the labour theory of value holds, hence $\left(1+R_{\alpha}\right) \mathbf{A}_{\alpha}{ }^{\alpha}{ }^{\alpha}=1^{\alpha}$ (Schefold, 1997, p. 52). If the wage curve of technique $\beta$ has one switchpoint in common with $\alpha$, one method is different in one industry, and $l^{\beta}$ cannot be an eigenvector of $\mathbf{A}_{\beta}$, but $w_{\beta}$ can be made linear by taking the standard commodity of $\beta$ as the numéraire for the wage curves of both techniques.

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[^8]:    ${ }^{8}$ For the necessary equilibrium condition is $\partial U / \partial c_{i}^{t}=\lambda p_{i}^{t}$, where $p_{i}^{t}$ are the prices and $\lambda$ is the Lagrange multiplier. But the derivatives of $U$ on the path are $\partial U / \partial c_{i}^{t}=p_{i}^{t}$, where $p_{i}^{t}$ are the parameters which serve as exponents in $U$. Hence $\lambda p_{i}^{t}=p_{i}^{t}$, and $\lambda=1$.

[^9]:    ${ }^{9}$ The wage curve $w_{\alpha}$ cannot be strictly linear unless the standard commodity of technique $\alpha$ is chosen as the numéraire. For if there is reswitching, the price vectors at the two switchpoints, common to both techniques, must be different (technique $\beta$ is regular).

[^10]:    ${ }^{10}$ a: Christian Bidard, b: Fabio Petri, in written communications for which I am grateful.

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[^12]:    ${ }^{11}$ As long as the rich have higher rates of time preference than the poor, they will exchange some of their wealth for present consumption (Neumann, 1990, p. 46).

[^13]:    ${ }^{12}$ An example may be constructed as follows: if there is only one technique, represented by input-output matrix $\mathbf{A}$ and labour vector $\mathbf{l}$, and if in periods $1, \ldots, T$ consumption vectors $\mathbf{c}^{1}, \ldots, \mathbf{c}^{T}$ shall be available, the required activity levels are $\mathbf{q}^{T}=\mathbf{c}^{T}, \mathbf{q}^{T-1}=\mathbf{c}^{T-1}+\mathbf{q}^{T} \mathbf{A}, \mathbf{q}^{T-2}=\mathbf{c}^{T-2}$ $+\mathbf{q}^{T-1} \mathbf{A}, \ldots, \mathbf{q}^{1}=\mathbf{c}^{1}+\mathbf{q}^{2} \mathbf{A}$. If the initial endowments $\mathbf{q}$ happen to be equal to $\mathbf{q}^{1}$, we have $\mathbf{q}=\mathbf{c}^{1}$ $+\mathbf{c}^{2} \mathbf{A}+\ldots+\mathbf{c}^{T} \mathbf{A}^{T-1}$, and if these quantitative data are foreseen, the quantity equations will be compatible with prices proportional to long-run prices and an intertemporal equilibrium can be constructed using the utility function proposed above. The stability problem to be considered below focuses on finding the vectors of future consumption so that $\mathbf{q}^{1}=\mathbf{q}$.

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[^17]:    ${ }^{13}$ This idea is also pursued in Schefold (2003).

[^18]:    ${ }^{14}$ Expenses for consumption on the reference path at disequilibrium prices are $\overline{\mathbf{c}}^{0} \mathbf{p}^{0}+\overline{\mathbf{c}}^{1} \mathbf{p}^{1 *}+$ $\overline{\mathbf{c}}^{2} \mathbf{p}^{2 *}$ and are equal to disequilibrium income $\mathbf{q}^{0} \mathbf{p}^{0}-\mathbf{f} \mathbf{p}^{2 *}$, since the former equals $\overline{\mathbf{c}}^{0} \mathbf{p}^{0}+\overline{\mathbf{c}}^{1} \mathbf{p}^{1}-$ $w_{1} \overline{\mathbf{c}}^{1} \boldsymbol{p}^{\beta}+\overline{\mathbf{c}}^{2} \mathbf{p}^{2}-w_{2} \overline{\mathbf{c}}^{2} \boldsymbol{1}^{\beta}-w_{1} \overline{\mathbf{c}}^{2} \mathbf{A}_{\beta^{\beta}}{ }^{\beta}$ and $\mathbf{q}^{0} \mathbf{p}^{0}-\mathbf{f p}^{2}+w_{2} \mathbf{f}^{\beta}+w_{1} \mathbf{f} \mathbf{A}_{\beta} \boldsymbol{\beta}^{\beta}$ the latter; hence both are equal if the budget equation on the reference path holds $\overline{\mathbf{c}}^{0} \mathbf{p}^{0}+\overline{\mathbf{c}}^{1} \mathbf{p}^{1}+\overline{\mathbf{c}}^{2} \mathbf{p}^{2}=\mathbf{q}^{0} \mathbf{p}^{0}-\mathbf{f p}^{2}+w_{1} \mathbf{q}^{\beta}+w_{2} \mathbf{q} \mathbf{l}^{\beta}$, inserting the formulas for the disequilibrium prices, $\mathbf{q}=\overline{\mathbf{c}}^{2}+\mathbf{f}$ and $\mathbf{q}=\overline{\mathbf{c}}^{1}+\mathbf{q} \mathbf{A}_{\beta}$.
    ${ }^{15}$ The marginal utility $\partial U / \partial c_{i}^{0}=\left(\partial / \partial c_{i}^{0}\right) \sum_{i, t}\left(1-\bar{c}_{i}^{t}+c_{i}^{t}\right)^{p_{i}^{t}}$ is equal to $p_{i}^{o}$ on the path $c_{i}^{o}=\bar{c}_{i}^{o}$, $c_{i}^{1}=c_{i}^{1 *}>\bar{c}_{i}^{1}, c_{i}^{2}=c_{i}^{2 *}>\bar{c}_{i}^{2}$, but the Lagrange multiplier in disequilibrium can be expected to be such that $\mathbf{c}^{0 *}<\overline{\mathbf{c}}^{0}$; compare the rigorous proof for deferred consumption in the Appendix. (Prices are normalized such that the Lagrange multiplier in equilibrium equals one; see Schefold, 1997, pp. 466-7 and footnote 1 above).

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[^20]:    ${ }^{16}$ The excess demands for endowments at $w^{*}=0$, with $T=1, f=0$ and $n=2$, have opposite signs so that the auctioneer necessarily has to change the relative price of endowments. This follows from Walras' Law: from $\mathbf{q}^{0} \mathbf{p}^{0 *}+w^{*} L=\mathbf{c}^{0 *} \mathbf{p}^{0 *}+\mathbf{c}^{1 *} \mathbf{p}^{1 *}=\mathbf{c}^{0 *} \mathbf{p}^{0 *}+\mathbf{c}^{\mathbf{1} *} \mathbf{A} \mathbf{p}^{0 *}+w^{*} \mathbf{c}^{\mathbf{1} *} \mathbf{l}$, we get Walras' Law: $\left(\mathbf{c}^{0 *}+\mathbf{c}^{1 *} \mathbf{A}-\mathbf{q}^{0}\right) \mathbf{p}^{0 *}+\left(\mathbf{c}^{\mathbf{1}} \mathbf{I}-L\right) w^{*}=0$, hence $\left(\mathbf{q}^{*}-\mathbf{q}^{0}\right) \mathbf{p}^{0 *}=0$, where $\mathbf{q}^{*}=\mathbf{c}^{0 *}+\mathbf{c}^{1 *} \mathbf{A}$ and $\mathbf{q}^{*}-\mathbf{q}^{0}$ is the excess demand for endowments.

[^21]:    ${ }^{17}$ Hence the temptation to introduce diminishing returns. For is it not clear that prices must vary in each period and that, e.g. $\mathbf{p}^{2 *}$ must be raised if $\mathbf{q}<\mathbf{c}^{2 *}+\mathbf{f}$, and that the supply response to an increase in $\mathbf{p}^{2 *}$ cannot be infinite? But it may be more realistic not to assume full employment (so that production is not limited by rising factor prices), to retain constant or increasing returns and to admit some degree of imperfect competition: the increase in profits consequent upon the rise of $\mathbf{p}^{2 *}$ may then in part be retained or invested in new technology.

[^22]:    ${ }^{18}$ The budget constraint of the consumer thus is doubly affected by tâtonnement: by setting the wage equal to zero and by changing the prices of the endowments.
    ${ }^{19}$ The budget of the consumer in terms of $c_{i}^{o}$ falls if relative prices of technique $\alpha$ do not change much so that $p_{i}^{0}$ and $p_{i}^{0 *}$ are not very different and the loss of the wage predominates. The purchasing power of the consumer is not diminished in the tâtonnement with the assumptions of case 1 in the Appendix and increased in case 3 , taking the cost of equilibrium consumption as the basis for the comparison.

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