Duke University Math News

February 14, 2000

Summer Opportunities

Duke math majors have many excellent prospects for graduate study, fellowships, actuarial opportunities, and math related summer research or internships. A large collection of information about such opportunities is located in the outer office of Math-Physics 121. See http://www.math.duke.edu/undergraduate/ for more information and for links to other sites of interest such as http://forum.swarthmore.edu/students/college/urp/.

The National Science Foundation sponsors many six to eight week research programs around the country with stipends from $2000 to $3000. A listing of Research Experiences for Undergraduates is available at http://www.nsf.gov/mps/dms/reulist.htm. Many past and current Duke students have attended such programs and can provide first hand experiences.

Of particular interest to Duke students is the Park City/Institute for Advanced Study summer program in Computational Complexity (http://www.admin.ias.edu/ma/SummerSession2000.htm) organized in part by Professor Robert Bryant.

Pending formal approval by the National Science Foundation, the Duke mathematics department will sponsor a summer program, Practical Research for Undergraduates with VIGRE (PRUV) from mid May until late June this year.

For more information, contact dkraines@math.duke.edu.

> *"Contrariwise," continued Tweedledee, "if it was so, it might be; and if it were so, it would be: but as it isn't, it ain't. That's logic."*
>
> —Lewis Carroll
> *Alice in Wonderland*

Competitions and Awards

High School Math Meet

On November 13, 108 high school students participated in the Duke Math Meet, including teams from North Carolina, South Carolina, Georgia, and Virginia. The A-team from Thomas Jefferson High School in Fairfax, Virginia dominated the competition, receiving the highest score in every round. They acheived a perfect score on the team round, and were the only team to successfully complete the relay. The top five teams were, in order, Thomas Jefferson A, Chapel Hill High School A, North Carolina School of Science and Mathematics A, Thomas Jefferson B, and Paideia School A from Paideia, Georgia.

Greg Price from Thomas Jefferson A took first place individually, as the only participant to receive a perfect score on the individual round. The next four winners were Vladimir Novakovski from Thomas Jefferson A, David Mermin from Chapel Hill A, Michael Carr from Paideia School A, and Josh Dezube from Thomas Jefferson B.

See the DUMU home page http://www.math.duke.edu/dumu/ for this year's problems and solutions, as well as information about the next Duke Math Meet in November 2000.

Virginia Tech Math Contest

One Saturday morning late last October, 255 undergraduates from 40 colleges and universities throughout the Southeast competed in the 21st annual Virginia Tech Math Contest. Students from Duke University continued their domination of this contest with Kevin Lacker '02 finishing first with a score of 44, an impressive 5 points above the next contestant from the US Naval Academy. The median score was about 10 out of 60. John Clyde '02 came in fourth and Melanie Wood '03 and Daniel Wong '03 tied for seventh.

Other Duke contestants ranking among the top 10% were Jeff Mermin '00, Taren Steinbrickner-Kauffman '03 and John Thacker '01. In addition, Matthew Atwood '03, James Richard Berg '03, Sooraj Bhat '03, Peter Fishman '02, Jonathan Godshall '03, Andy Goss '02, Ning Lin '01, Carl Miller '01, and Daniel Neill '01 finished in the top 40%. Lacker and Clyde each received a cash prize. The Virginia Tech Math Contest greatly expanded this year from its average of 180 participants at 32 schools.

The two and a half hour contest is considered to be a warm-up for the prestigious six-hour William Lowell Putnam Mathematical Competition, taken by about 2500 of the best undergraduates at over 400 colleges and universities in the United States and Canada. Since 1990, two Duke Putnam teams have won the competition and two have placed second. See http://www.math.duke.edu/news/awards/competitions.html#putnam for more details.

ACM Programming Team

A team of three Duke students has been invited to the international ACM Programming Contest to be held in Orlando in March. This year's team of juniors Mark Baumann and John Clyde and graduate student Patrick Reynolds, finished third in the mid-Atlantic region last fall. Mathematics major Clyde was also a member of the Duke team that ranked fifth in the International ACM Contest in the Netherlands in April, 1999, the highest among US teams that year. See http://acm.baylor.edu/acmicpc/ for more information.

Math Contest in Modeling

The 2000 MCM, sponsored by the Consortium for Mathematics and its Applications (COMAP), began at 12:01 am on Friday, February 4, when the team of Sam Malone '02, Jeff Mermin '00 and Daniel Neill '01 picked up the contest problems. They were asked to design a mathematical model to improve air safety and reduce workloads on air traffic controllers. Students are encouraged to write computer programs, search through research journals and textbooks, surf the internet and make use of other inanimate sources as they work on these open ended projects. At 5 pm on Monday February 7, the team finished their 35 page paper, supported by extensive computer simulations, that studied several possible methods for reducing risks at airports.

Last year, 478 teams from nine countries submitted solution papers in this annual contest. Those that rank in the top 15% are designated as Meritorious and the best two or three papers in each category are named Outstanding. Since 1993, Duke has fielded five Meritorious and two Outstanding teams. Mermin was a member of the 1998 Outstanding team and Malone was a member of the 1999 Outstanding team. Their solution papers have been published in the UMAP Journal.

See http://www.comap.com/undergraduate/contests/mcm/ and http://www.math.duke.edu/news/awards/competitions.html#modeling for more information.

Alice T. Schafer Award

Sarah E. Dean '00 has been chosen as runner up (second place) in the tenth annual Alice T. Schafer Award given to undergraduate women in the United States in recognition of excellence in mathematics. Dean was presented with a certificate and cash prize at the annual joint meetings of the American Mathematical Society, the Mathematical Association of America, and the Association of Women in Mathematics. These meetings were held from January 19–22, 2000 in Washington DC. Dean, a Goldwater Scholar and a Faculty Scholar, expects to attend graduate school in mathematical physics next year.

This award (http://www.awm-math.org/schaferprize.html) was established in 1990 in recognition of the work of Alice Schafer, mathematics professor and chairman emeritus at Welleley College. Duke students who have been honored with this prestigious award have been Jeanne Nielsen Clelland '91 (first place) and Jennifer Slimowitz '93 (honorable mention).

> *The essence of mathematics is its freedom.*
> —Cantor

New Courses

Several new courses will be introduced next year:

Cryptography and Society
Proposed number: 065S
Prerequisites: none
Curriculum 2000: STS, QID-0
Introduction to basic ideas of modern cryptography with emphasis on implementation, applications in daily life, and implications for the individual and society. Topics covered include: the history of cryptography and cryptanalysis, public and private key cryptography, digital signatures, limitations of modern cryptography, applications to electronic communications and electronic commerce, privacy, computer security, and law enforcement. Related ethical questions will be considered including the debate over personal privacy versus public security.

Perspectives on Science
Proposed number: 061 and 062
Credits: 0.5
Curriculum 2000: STS
Weekly seminars showcase research directions that use quantitative methods. Students conduct interviews and library research leading to a web-based report and oral presentation. Projects must include a focused quantitative example and an analysis of the broader impact or development of the field including historical developments and impact on society.

MTH 061 emphasizes biological and medical sciences. MTH 062 emphasizes engineering, physical, and social sciences.

This course will be taught by professor Andrea Bertozzi in the fall and spring of 2000-01 as part of the NSF sponsored project ADVANCE, designed to build a resilient cohort of women in the quantitative sciences. Students attend weekly seminars given by prominent scientists, many of whom are women, on quantitative methods (mathematics, statistics, computation) in the physical, biological, environmental, engineering, and social sciences.

Topics in Mathematical Physics
Proposed number: 207
Prerequisites: MTH 103, MTH 104
Curriculum 2000: QID-1
Topics selected from: gravitational lensing, general relativity, classical mechanics, quantum mechanics, critical phenomena and statistical mechanics, or other areas of mathematical physics.

Computation in Algebra and Geometry
Proposed number: 250
Coerequisites: MTH 251 or consent of instructor
Application of computing to problems in areas of algebra and geometry, such as Linear Algebra, Algebraic Geometry, Differential Geometry, Representation Theory, and Number Theory; use of general purpose symbolic computation packages such as Maple or Mathematica; use of special purpose packages such as Macaulay, PARI-GP, and LiE; programming in C/C++. Previous experience with programming or the various mathematical topics not required.

Employment

The mathematics department wants to hire an office assistant. The hours are flexible and the pay is negotiable. For more information, contact Ms. C. B. Wilkerson in room 121 Math-Physics Building.

Leonard Carlitz (1907-1999)

Leonard Carlitz, the first James B. Duke Professor of Mathematics, died on September 17, 1999 in Pittsburgh. Carlitz had a long and distinguished career as a member of the Duke Mathematics Department, joining the faculty in 1932 and continuing until his retirement in 1977. His research career spanned 60 years (1930-1990); during that time he wrote an amazing number of research papers, 770 in all, mainly in the areas of finite fields, number theory, and combinatorics. It is projected that his collected works, now in progress, will be over 6800 pages long. As late as 1990, a paper entitled *Tensor powers of the Carlitz module*, extending work that Carlitz had published in 1935, appeared in the *Annals of Mathematics*. In addition to his research and teaching duties at Duke, Carlitz was on the editorial board of the *Duke Mathematics Journal* from 1938 to 1973. He was named a James B. Duke Professor in 1964.

Beginning in 1940, and continuing to 1976, Carlitz supervised the Ph.D. theses of 44 students. Carlitz was a kind and gentle man who cared deeply about his students and was admired by students and colleagues alike. Indeed, Carlitz was a mentor not only to his students but to a number of younger mathematicians as well. He and Richard Scoville, a recently retired member of the department, collaborated on a number of papers in combinatorics.

Carlitz was born on December 26, 1907 in Philadelphia. His obvious mathematical talent surfaced early. He was given a scholarship to the University of Pennsylvania and there he completed his A.B. degree in 1927 and his Ph.D. degree in 1930. Carlitz spent the academic year 1930-31 at Cal Tech studying with E.T. Bell and the academic year 1931-32 at Cambridge studying with G.H. Hardy.

In the summer of 1931, before leaving for Cambridge, Carlitz married Clara Skaler, also of Philadelphia. Together they raised two children, Michael (1939) and Robert (1945); they remained together for 59 years until Clara died in 1990. He is survived by two sons and two granddaughters.

Leonard and Clara Carlitz often entertained faculty and graduate students in their beautifully landscaped home (Clara's work) near the Duke campus. Carlitz's favorite form of exercise was walking; twice daily on most weekdays during his 45 years on the faculty he would walk the mile-long trip between his house and his office.

Carlitz was a legend even as a graduate student at Penn. On his oral examination for the Ph.D. he was asked to sketch Gauss' proof of the law of quadratic reciprocity. Carlitz replied: "During his lifetime Gauss gave seven different proofs of this result. Which one would you like to see?"

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—*Richard Hodel*

Problem Corner

Solutions from Last Issue

Problem 1: *Fibonacci Sum*

Let $F_n = (1, 1, 2, 3, 5, ...)$ be the Fibonacci sequence (defined by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$). Compute $\sum_{k=1}^{\infty} \frac{F_n}{2^n}$.

Solution to Problem 1:

Let $s = \sum_{k=1}^{\infty} \frac{F_n}{2^n}$.

$$\begin{aligned} s &= \frac{1}{1} + \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{5}{16} + \frac{8}{32} + \ldots \\ \frac{s}{2} &= \phantom{\frac{1}{1} +{}} \frac{1}{2} + \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{5}{32} + \frac{8}{64} + \ldots \\ s - \frac{s}{2} &= \frac{1}{1} + \phantom{\frac{1}{2} +{}} \frac{1}{4} + \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \frac{5}{64} + \ldots \\ &= 1 + \frac{s}{4} \end{aligned}$$

Thus $\frac{s}{2} = 1 + \frac{s}{4}$, which implies $s = 4$.

Problem 2: *Massive Matrix*

Let A be a 2000×1998 matrix with real entries. Show that AA^T contains at least one nonnegative entry which is not on the main diagonal.

Solution to Problem 2:

Suppose A is a 2000×1998 matrix which contains no nonnegative values except on the main diagonal. Represent by $\mathbf{a}_i = (a_{i1}, a_{i2}, \ldots, a_{i1998})$ the i^{th} row vector of A. Choose an orthogonal matrix O such that $\mathbf{v}_1 O = (0, 0, \ldots, 0, |\mathbf{v}_1|)$; then AO has $(0, 0, \ldots, 0, |\mathbf{v}_1|)$ as its first row vector. Let $B = AO$. Then $BB^T =$

$$\begin{pmatrix} |\mathbf{v}_1|^2 & |\mathbf{v}_1| b_{2,1998} & \cdots & |\mathbf{v}_1| b_{2000,1998} \\ |\mathbf{v}_1| b_{2,1998} & \mathbf{b}_2 \cdot \mathbf{b}_2 & \cdots & \mathbf{b}_2 \cdot \mathbf{b}_{2000} \\ |\mathbf{v}_1| b_{3,1998} & \mathbf{b}_3 \cdot \mathbf{b}_2 & \cdots & \mathbf{b}_3 \cdot \mathbf{b}_{2000} \\ \vdots & \vdots & & \vdots \\ |\mathbf{v}_1| b_{2000,1998} & \mathbf{b}_{2000} \cdot \mathbf{b}_2 & \cdots & \mathbf{b}_{2000} \cdot \mathbf{b}_{2000} \end{pmatrix}$$

And $BB^T = (AO)(AO)^T = AOO^TA^T = AA^T$, which by assumption contains only negative values off of the main diagonal. Therefore all the values b_{i1998}, $2 \le i \le 2000$ are negative, and each dot product $\mathbf{b}_i \cdot \mathbf{b}_j$ is negative for $i \neq j$.

For each i, $2 \le i \le 2000$, let $\mathbf{b}'_i$ be the vector $\mathbf{b}_i$ with the last coordinate deleted. Then $\mathbf{b}_i \cdot \mathbf{b}_j = \mathbf{b}'_i \cdot \mathbf{b}'_j + b_{i1998} b_{j1998}$, and $b_{i1998} b_{j1998}$ is positive, thus $\mathbf{b}'_i \cdot \mathbf{b}'_j < \mathbf{b}_i \cdot \mathbf{b}_j < 0$ if $i \neq j$.

So let B' be the 1999×1997 matrix formed from B by deleting the first row and last column. The row vectors of B' are $\mathbf{b}'_2, \mathbf{b}'_3, \ldots, \mathbf{b}'_{1999}$, and $B'B'^T = (\mathbf{b}'_i \cdot \mathbf{b}'_j)_{i,j}$. Thus B' has the same property as the original matrix A: every entry not on the main diagonal is negative.

Proceeding similarly we may construct a 1998×1996 matrix with this property, a 1997×1995 matrix with this property, and so forth, and we reduce finally to a 3×1 matrix $A = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ such that AA^T contains only negative values off of the main diagonal. But this is impossible: some two of the values a, b, and c must have the same sign, or one of them must be zero, so the pairwise products ab, bc, and ca (which are entries of AA^T) cannot all be negative. This is a contradiction.

New Problems

Problem 1: *Partition Puzzle*

A *partition* of a positive integer n is a non-increasing sequence of positive integers which sums to n. (For example, the partitions of 4 are (4), $(3,1)$, $(2,2)$, $(2,1,1)$, and $(1,1,1,1)$.) Let $A(n)$ be the total number of partitions of n, and let $B(n)$ be the number of partitions of n in which the first and second terms are equal. Prove that $A(n) = 1 + \sum_{k=1}^{n} B(k)$.

Problem 2: *Bijection Brainteaser*

Prove or disprove: For any bijection $f : \mathbb{Z} \to \mathbb{Z}$ of the integers, there exists a bijection $g : \mathbb{Z} \to \mathbb{Z}$ and a constant c such that $|g(f(n)) - g(n)| \le c$ for all n.

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