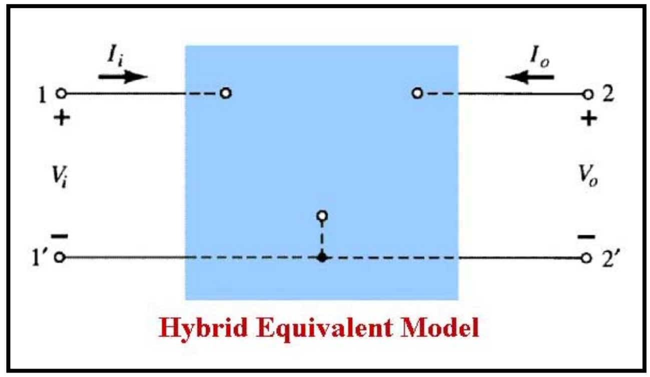
TWO PORT NETWORKS – h-PARAMETER BJT MODEL

The circuit of the basic two port network is shown on the right. Depending on the application, it may be used in a number of different ways to develop different models. We will use it to develop the h-parameter model. Other models may be covered in EE III.



Hybrid Equivalent Model

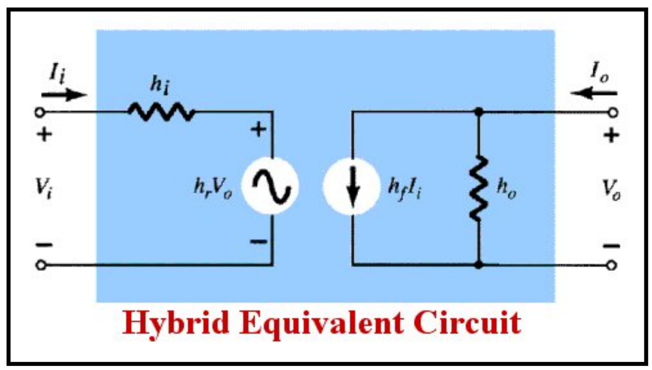
The h-parameter model is typically suited to transistor circuit modeling. It is important because:

1. its values are used on specification sheets
2. it is one model that may be used to analyze circuit behavior
3. it may be used to form the basis of a more accurate transistor model

The h parameter model has values that are complex numbers that vary as a function of:

1. Frequency
2. Ambient temperature
3. Q-Point

The revised two port network for the h-parameter model is shown on the right.



Hybrid Equivalent Circuit

At low and mid-band frequencies, the h-parameter values are real values. Other models exist because this model is not suited for circuit analysis at high frequencies.

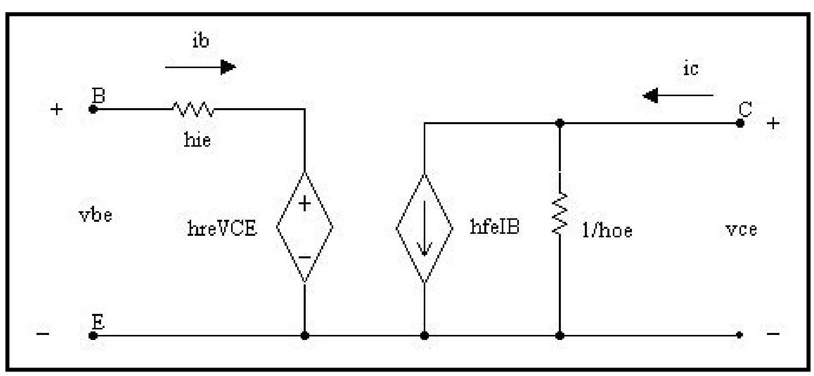
The h-parameter model is defined by:

$$\begin{vmatrix} V_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} \begin{vmatrix} I_1 \\ V_2 \end{vmatrix}$$

$V_1 = h_{11}I_1 + h_{12}V_2$ (KVL)

$I_2 = h_{21}I_1 + h_{22}V_2$ (KCL)

The h-parameter model for the common emitter circuit is on the right. On spec sheet:



$h_{11} = h_{ix}$
$h_{12} = h_{rx}$
$h_{21} = h_{fx}$
$h_{22} = h_{ox}$

h_{rx} and h_{fx} are dimensionless ratios
h_{ix} is an impedance <Ω>
h_{ox} is an admittance <S>

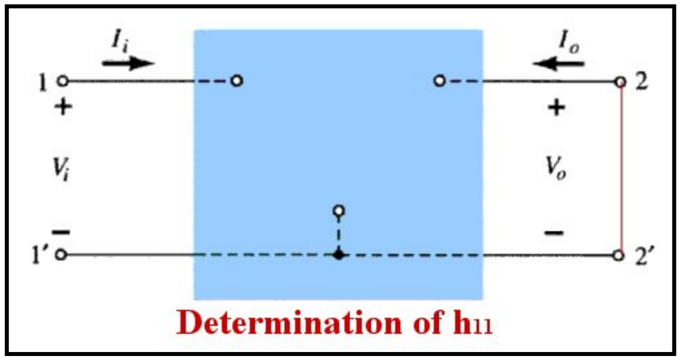
where x = lead based on circuit configuration
e = emitter for common emitter
c = collector for common collector
b = base for common base

Short Circuit Input Impedance

$h_{11} = Z_{IN}$ with output shorted <Ω>

$$h_{11} = \left.\frac{V_i}{I_i}\right|_{V_O = 0}$$

1. Short terminals 2 2’
2. Apply test source V_i to terminal 1 1’
3. Measure I_i
4. Calculate h_{11}



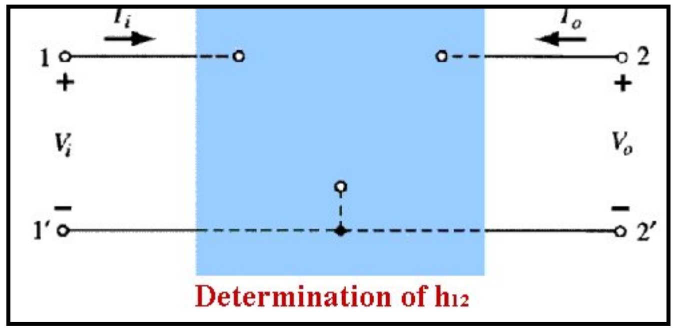
Determination of h_{11}

Open Circuit Reverse Transfer Ratio

h_{12} <dimensionless>

$$h_{12} = \left.\frac{V_i}{V_o}\right|_{I_i = 0}$$

1. Open terminals 1 1’
2. Apply test source V_2 to terminal 2 2’
3. Measure V_i
4. Measure V_o
5. Calculate h_{12}



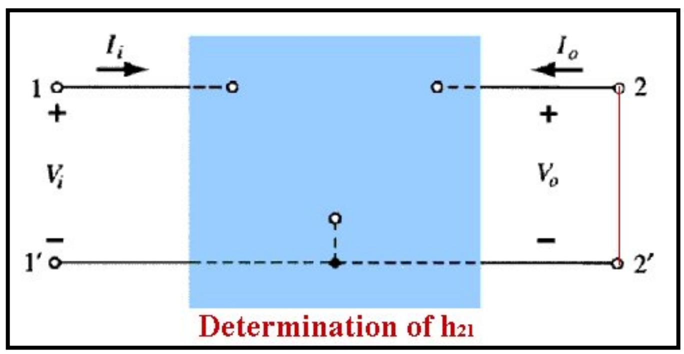
Determination of h_{12}

Short Circuit Forward Transfer Ratio

h_{21} = <dimensionless>

$$h_{21} = \left.\frac{I_o}{I_i}\right|_{V_O = 0}$$

1. Short terminals 2 2’
2. Apply test source V_i to terminal 1 1’
3. Measure I_i
4. Measure I_o
5. Calculate h_{21}



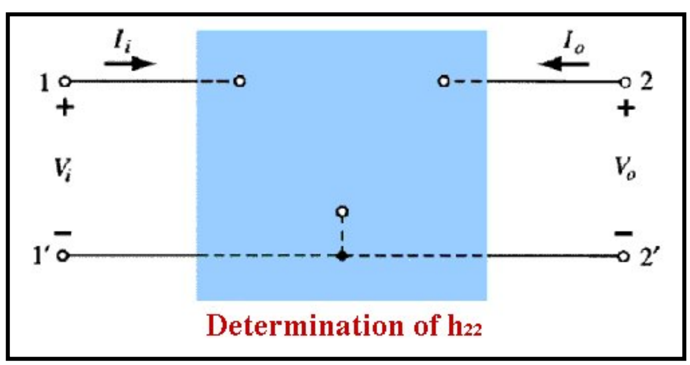
Determination of h_{21}

Open Circuit Output Admittance

h_{22} <Siemans>

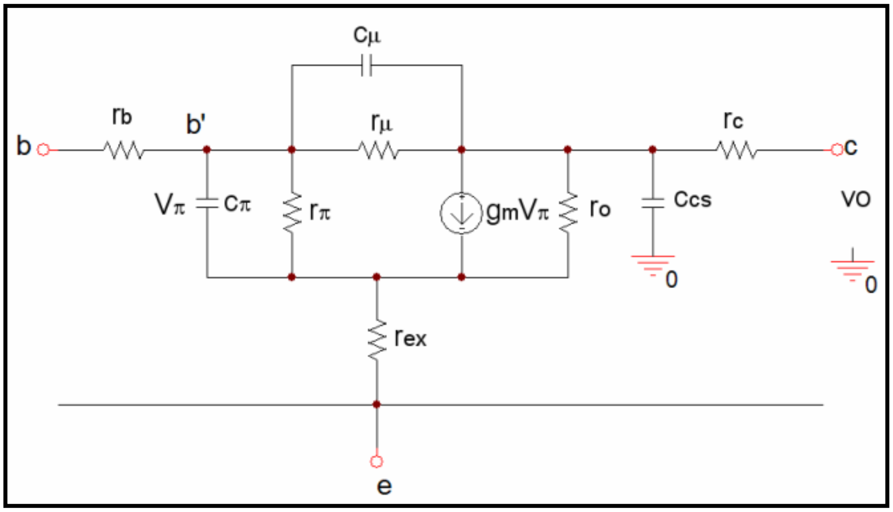
$$h_{22} = \left.\frac{I_o}{V_o}\right|_{I_i = 0}$$

1. Open terminals 1 1’
2. Apply test source V_2 to terminal 2 2’
3. Measure I_o
4. Measure V_o
5. Calculate h_2



Determination of h_{22}

BJT Hybrid Model π



Hybrid Model π
All frequencies
Better model than h parameter model since it contains frequency sensitive components.
These are ac small signal parameters which are determined at the Q point

Parasitic Resistances

$r_b = r_{b'b}$ = ohmic resistance – voltage drop in base region caused by transverse flow of majority carriers, $50 \le r_b \le 500$

r_c = r_{ce} = collector emitter resistance – change in I_c due to change in V_c, $20 \le r_c \le 500$

r_{ex} = emitter lead resistance – important if I_C very large, $1 \le r_{ex} \le 3$

Parasitic Capacitances

C_{je0} = Base-emitter junction (depletion layer) capacitance, $0.1pF \le C_{je0} \le 1pF$

$C_{\mu 0}$ = Base-collector junction capacitance, $0.2pF \le C_{\mu 0} \le 1pF$

C_{cs0} = Collector-substrate capacitance, $1pF \le C_{cs0} \le 3pF$

C_{je} = $2C_{je0}$ (typical)

ψ_0 = .55V (typical)

τ_F = Forward transit time of minority carriers, average of lifetime of holes and electrons, $0ps \le \tau_F \le 530ps$

C_b =

$$C_\mu = \frac{C_{\mu 0}}{\sqrt{1 + \frac{V_{cb}}{\psi_0}}}$$

$$C_{cs} = \frac{C_{cs0}}{\sqrt{1 + \frac{V_{cs}}{\psi_0}}}$$

$C_b = \tau_F \, g_m$

Hybrid Model Pi Parameters

r_π = $r_{b'e}$ = dynamic emitter resistance – magnitude varies to give correct low frequency value of $V_{b'e}$ for I_b

r_μ = $r_{b'c}$ = collector base resistance – accounts for change in recombination component of I_b due to change in Vc which causes a change in base storage

c_π = $C_{b'e}$ = dynamic emitter capacitance – due to $V_{b'e}$ stored charge

c_μ = $C_{b'c}$ = collector base transistion capacitance (C_{TC}) plus Diffusion capacitance (C_d) due to base width modulation

$g_m V_\pi = g_m V_{b'e} = I_c$ – equivalent current generator

BJT Hybrid Model π

$$g_m = \frac{I_C}{V_T}$$

$$V_T = \frac{k\,T}{q} = 26mV \ @\ 300°K$$

$$g_m = \frac{I_C}{26mV}$$

$$r_\pi = \frac{(26mV)\,(\beta)}{I_C} = \frac{26mV}{I_B}$$

$$\beta = g_m\, r_\pi$$

$$i_c = \frac{\beta\, v_\pi}{r_\pi} = g_m\, v_\pi$$

$$r_o = \frac{V_A}{I_C} \qquad \text{where } 50 \leq V_A \leq 100$$

f_T = Gain Bandwidth Product (spec sheet is 300MHz)

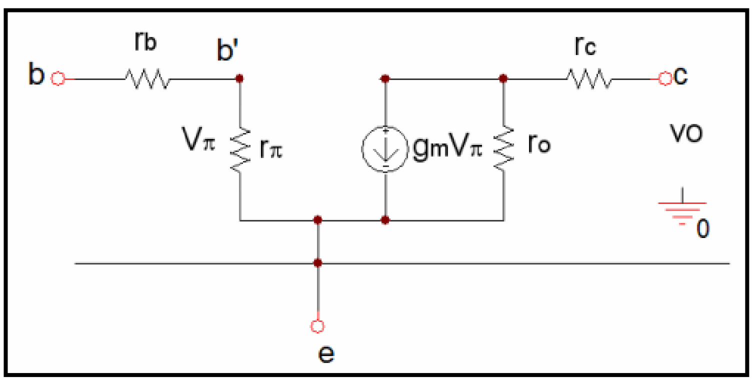
C_μ corresponds approximately to C_{OBO} (on spec sheet is 8pF for 2N2222A)

τ_{CB} = collector base time constant (spec sheet is 150ps for 2N2222A)

$$r_b = \frac{\tau_{CB}}{C_\mu}$$

$$C_\pi = \frac{\beta}{2\,\pi\, r_\pi\, f_T} - C\mu$$

Low and Midband Frequency Hybrid Model π



At low frequencies, all Xc for hybrid model π are very large. Since they are in parallel with a much lower than their associated resistances, the X_C component may be ignored (replace with an open). At frequencies over a few 100kHz, they must be considered. At high frequencies, they shunt (short) the parallel resistances.

r_μ is very large and may be ignored (replace with an open)

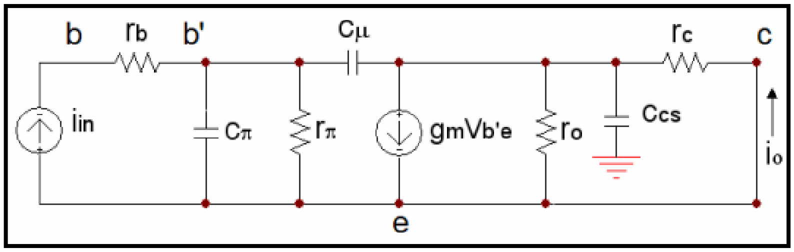
r_b is much smaller than r_π. Since they are in series, r_b is often ignored (replaced with a short or lumped with r_π) since the current, I_B the current through both, is also very small.

r_{ex} is very small and is often ignored (replace with a short) unless I_E is very large.

r_o is very large and may be ignored at low frequencies

$i_c = i_o \cong g_m\, v_\pi$ (where $v_{b'e} = v_\pi$)

BJT Hybrid Model π



High Frequency Model

f_t transition frequency due to parasitic capacitances

Apply i_{in}, and short the output

Ignore the following parameters:

r_b = small

r_{ex} = very small

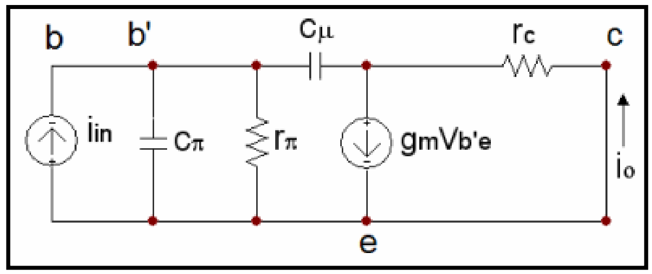
r_μ = very large

$r_c << r_o$ Since they are in parallel, ignore r_o

X_{CCS}

In the text and the following, s and jω are used interchangeably

$\omega = 2\pi f$



The equivalent model with a shorted output and current source applied to the input is shown on the right.

Note: in the following equations, we manipulate the equations until they are in the form s, 1 + s, etc. Remember that s = jω

$$V_{b'e} = i_{in}\left(\frac{r_\pi}{1 + r_\pi(c_\pi + c_\mu)\, s}\right)$$

$$i_o \cong g_m\, V_{b'e}$$

$$i_o \cong g_m\, i_{in}\left(\frac{r_\pi}{1 + r_\pi(c_\pi + c_\mu)\, s}\right)$$

$$i_o \cong i_{in}\left(\frac{g_m\, r_\pi}{1 + r_\pi(c_\pi + c_\mu)\, s}\right)$$

But $\beta_0 = g_m\, r_\pi$

$$i_o \cong i_{in}\left(\frac{\beta_0}{1 + \beta_0\left(\frac{c_\pi + c_\mu}{g_m}\right) s}\right)$$

$$\frac{i_o}{i_{in}}(j\omega) = \beta(j\omega) \cong \left(\frac{\beta_0}{1 + \beta_0\left(\frac{c_\pi + c_\mu}{g_m}\right) j\omega}\right)$$

At high frequencies, the imaginary component of the denominator dominates. Therefore:

BJT Hybrid Model π

$$\beta(j\omega) \cong \frac{\beta_0}{\beta_0\left(\frac{c_\pi + c_\mu}{g_m}\right) j\omega}$$

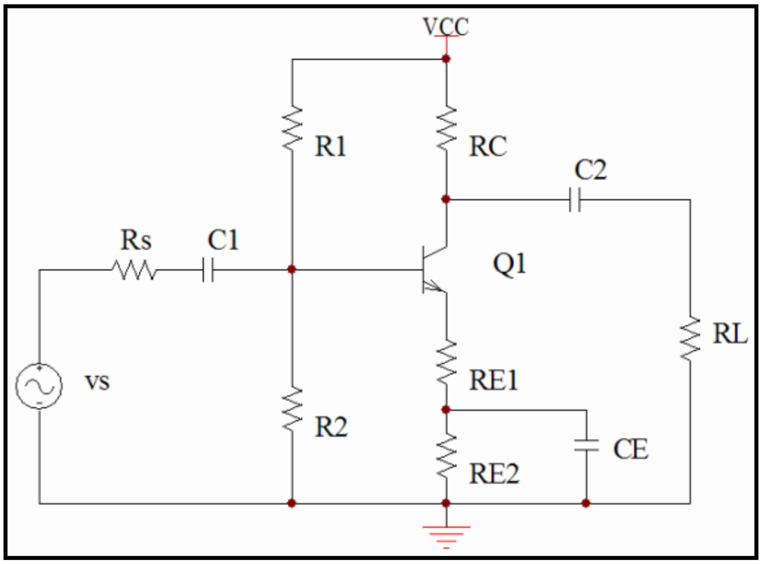
$$\beta(j\omega) \cong \frac{g_m}{(c_\pi + c_\mu)\, j\omega}$$

$\beta(j\omega) = 1$ when $\omega = \omega_T$

$$\omega_T = \frac{g_m}{c_\pi + c_\mu}$$

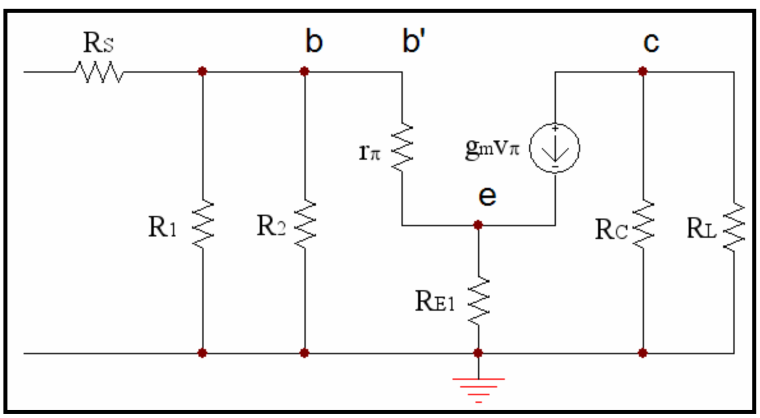
$$f_T = \frac{g_m}{2\pi\,(c_\pi + c_\mu)}$$

Common Emitter Amplifier at Low and Mid-band Frequencies



The circuit on the right is a common emitter amplifier using voltage divider bias with a partially bypassed emitter resistor. There is a load R_L, and a source resistance R_S. Note that R_S is the internal resistance of v_S.

After we solve for the quiescent operating point, we are ready to perform calculations for voltage gain and perhaps frequency response. To do so requires us to use a model for the transistor. One model that is typically used is the hybrid model pi.



In the circuit on the right, we have the equivalent circuit for the common emitter amplifier shown above. At mid band frequencies, X_{C1}. X_{C2}, and X_{CE} are shorts, but $X_{C\mu}$ and $X_{C\pi}$ are still very large and treated as open. Since X_{CE} is in parallel with R_{E2}, R_{E2} and X_{CE} are both replaced with a short at mid band frequencies.
As discussed earlier, we will assume that the effects of r_b, r_{ex}, and r_o have little effect on the circuit operation due to their value and are removed.