COMPOSITE WALLPAPER PATTERNS AMONG THE INTUITIVE ETHNOMATHEMATICAL DISCOVERIES OF THE ANCIENT CRETE AND URARTU

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Abstract: *Characteristic composite patterns were found in the Romanesque Cathedral Art in Hungary on the doorways from the 11^{th}-13^{th} centuries AD. Looking for the Eurasian art heritage of the period of the 2. and 1. millennium we study artistic patterns from Crete and Urartu which also contain such composite structures. Their main characteristic is that they can be divided into two subsystems that are simple wallpaper patterns. The separated wallpaper subsystems have their own symmetry group. Superposition of the two single, separated subsystems results in – according to the Curie principle – that the symmetry group of the lower order subsystem will determine the symmetry group of the whole pattern. Finally, subgroup structure of the composite patterns is also discussed.*

INTRODUCTION

Arts formulate knowledge of human discoveries about the surrounding world. One kind of formulation is the scientific one, but this is the product of the last centuries. The most general formulation in the world cultures has been the artistic formulation, the depiction in artworks. Among the wide range of themes of these artworks, the patterns form an important group. Patterns with natural or technological origin represent recognitions about natural or technological phenomena of the age of the artist. Studying the art of great number of cultural communities, we can see many examples that ornamental structures contain intuitive mathematical discoveries. These cultural communities belonged to various ethnic groups of the history. The cultural heritage of these groups represent unexhaustable sources both for historical knowledge and for catching the moment when the concept was first represented, even if in an intuitive form. That is why we may call ornamental art the source of ethnomathematics (Bérczi 1986, 1989, 2000; Crowe 1996; Gerdes 1989).

In one of our earlier works we have found that many ornamental structures were constructed by doubling ornamental threads (Bérczi 1986, 1989). Doubling is a kind of technology where we duplicate a unit. This unit may be other then thread and we may look for other types of multiplications, which were used in arts, but which have origin in a technology. The archaeological material of Eurasia gives a rich store of artistic sources and we select those patterns from it, where a kind of multiplication or composition principle can be recognized. We study the wallpaper patterns that were composed of two subsystems of plane symmetry structures.

COMPOSITE PATTERNS, SEPARABLE SUBSYSTEMS IN THE ROMANESQUE ARCHITECTURAL ART IN HUNGARY: GYULAFEHÉRVÁR, SZÉKESFEHÉRVÁR

If a pattern with a symmetry group can be separated to two such subsystems of patterns both of which alone form a pattern, then we call the initial pattern *composite pattern*. The two separated patterns are called subsystem patterns. The most frequently occurring composite patterns are those where one subsystem pattern is formed from a net. First we show some patterns and we use in their description the subgroup structure showed in details about the colored symmetry (Coxeter 1986).

In the Romanesque architectural art many traditions of the pattern formation survived. In the centuries of 11th-13th AD many cathedrals and village churches were built in the Kingdom of Hungary. Some of these architectural heritage survived the centuries, others could be found on excavations. We show one from a recently standing cathedral of Gyulafehérvár (Alsó Fehér County, Transylvania, Alba Iulia, Romania) and one from an excavation from the medieval crowning town of Hungary, Székesfehérvár which was destroyed in the Turkish wars in the 16th-17th centuries AD.

The first example shows the southern doorway of the Cathedral of Gyulafehérvár (Alba Julia, Transylvania, Romania, Figure 1). It consists of 2 separable subsystems. One is a net of tendrill with *pg* symmetry group of this subsystem pattern. The second is a fill of the cells with palmette type leaves which form a *cm* type symmetry pattern. Unification (superposition) of the two subsystems results in violation of the mirror symmetries of *cm* subsystem pattern, therefore the final symmetry of the pattern is the symmetry of the lower order symmetry subsystem: *pg*. The symmtery structure of the whole southern doorway pattern is *cm/pg* subgroup structure.

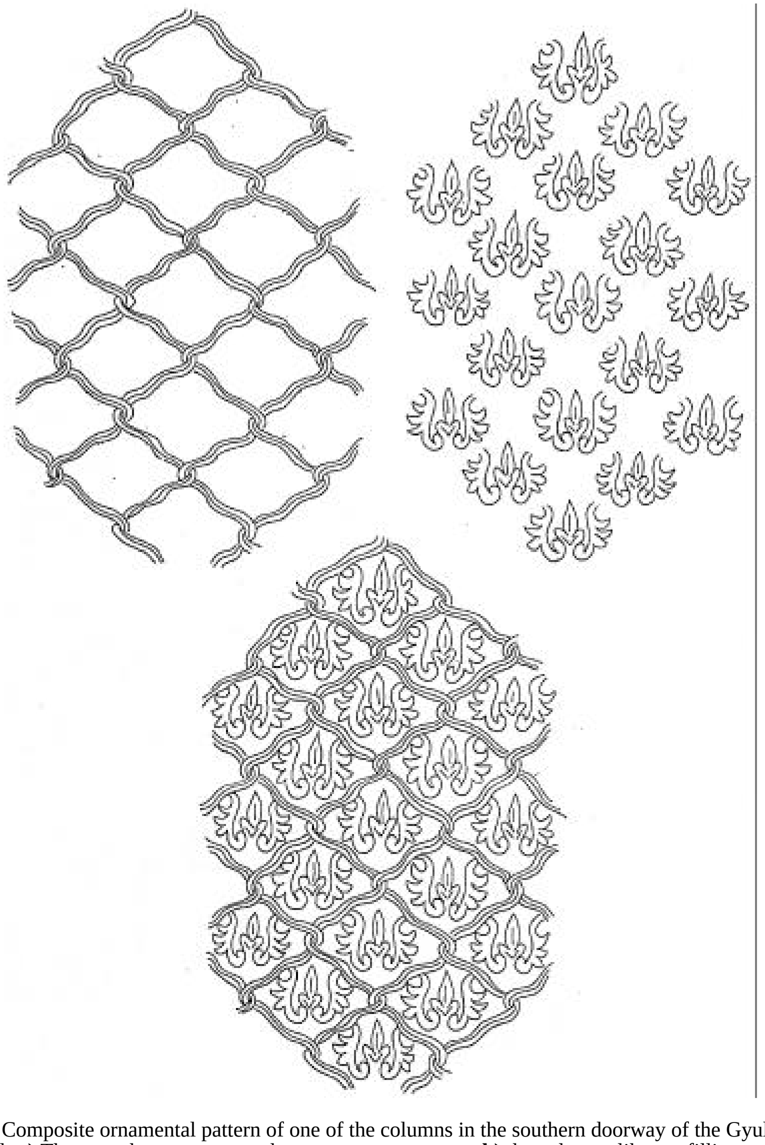

Figure 1: Composite ornamental pattern of one of the columns in the southern doorway of the Gyulafehérvár Cathedral: **a)** The net subsystem pattern has *pg* symmetry group, **b)** the palmette like net filling pattern has *cm* symmetry group, and **c)** after unification the composite pattern has the symmetry group of the lower degree, however, all the system may be marked with *cm/pg* pattern type.

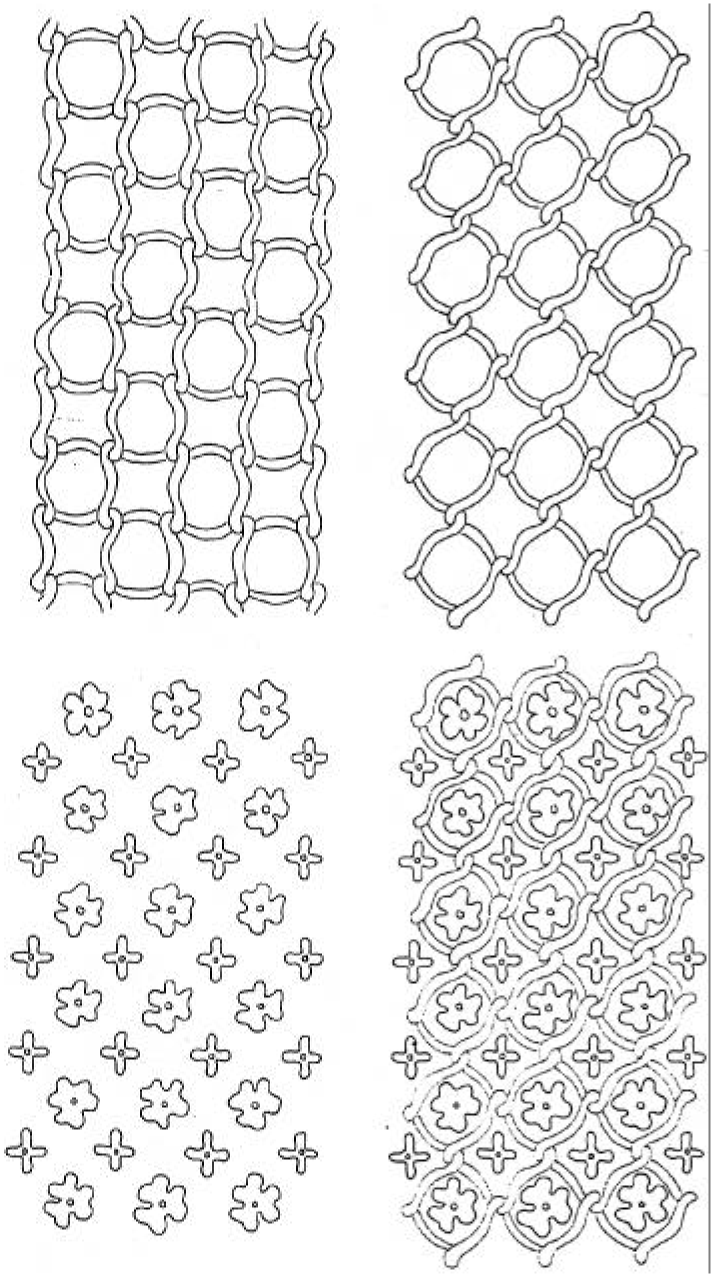

Figure 2: Composite ornamental pattern of the other columns in the southern doorway column fragment of the Gyulafehérvár Cathedral: **a)** The net subsystem pattern has *p4m* symmetry group, if it is considered in a rolled down to sheet surface, **b)** when it is rolled up, it is reduced to *p2*, **c)** the rosette like cell filling pattern (except the 3-fold flowers) has *p4m* symmetry group, and **d)** after unification the composite pattern has the symmetry group of the lower degree of *p2*, however, all the system may be marked with *p4m/p4g/p2* complex pattern type.

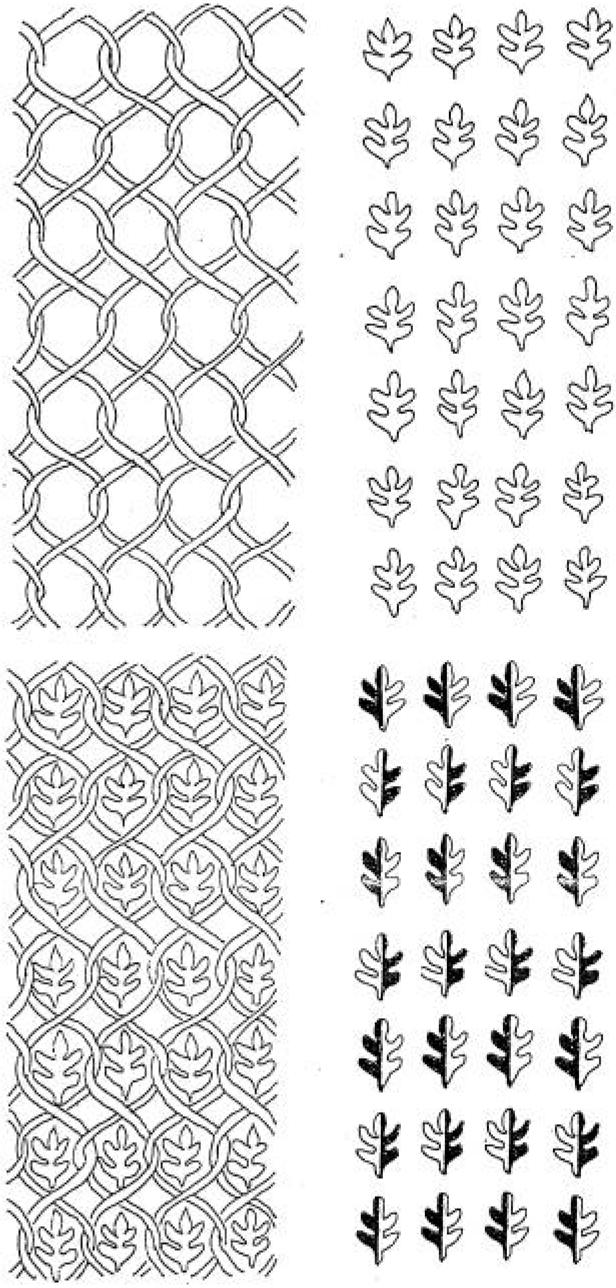

Figure 3: Composite ornamental pattern of a column from the Székesfehérvár Cathedral: **a)** The net subsystem pattern has *pg* symmetry group, **b)** the leaf like cell filling pattern has pm symmetry group, and **c)** after unification the composite pattern has the symmetry group of the lower degree pg, however, all the system may be marked with *pm/pg* pattern type.

Our second example shows a pattern of another doorway column from the Gyulafehérvár Cathedral (Henszlmann 1896). Its pattern consists of 2 separable subsystems. One is a tendrill type net with *p4g* symmetry if it is considered on a plane. However, it is placed on the column in a tilt form which position violates its mirror reflections and reduces its symmetry to a lower *p2* group of this subsystem pattern. If we disregard the shape of the *D3* like flowers, then the second pattern is a fill of the cells with *D4* flowers, which form a *p4m* type symmetry pattern. The 45 degrees tilt of the net allowed only *p2* symmetry and this net violates all mirror reflections in the flower pattern. Unification (superposition) of the two subsystems results in that the final symmtery structure of the whole column pattern is *p4m/p2* subgroup structure. However, if we should like to note the earlier states of the pattern, then it is better to read: *p4m/(p4g)/p2*. Really, if the whole pattern was rolled down and placed onto a sheet, the pattern allows using the mirror reflection and a *p4g* structure can be recognized.

Our third example shows a fragment of a doorway column from the excavations in the ruins of the Cathedral of Székesfehérvár. Its pattern consists of 2 separable subsystems. One is a tendrill type net with *pg* symmetry, the other is a fill of leaves with pm symmetry. Superposition of the two subsystems results in violation of the mirror symmetries of the pm subsystem pattern, therefore the final symmetry of the pattern is *pg*. The symmetry of the whole system is: *pm/pg*. We show how the net of tendrills can be replaced by coloring the leaves. This balck-and-white colored pattern has also the *pm/pg* type symmetry structure. This shows an example to the using of constraints with equivalent role.

In the following chapters first we study why can we use the Curie Principle in the description of the composite patterns. Then we look for earlier occurrences of such structures in the Eurasian communal arts. Because the superposed subsystem patterns can be considered as constraints to each other, we summarize our pattern studies with pointing the role of the Curie principle in pattern studies.

THE COMPOSITE PATTERNS AND THE CURIE PRINCIPLE

If a pattern is a composite pattern with at least two subsystem patterns then we can consider the subsystem patterns (components) are interacting. Their interaction is their superposition (unification). In this interaction the Curie principle can be used. This principle states that after superposition (addition) of the two subsystems, the symmetry group of the lower order subsystem determines the symmetry group of the whole pattern (Shubnikov, Koptsik, 1977).

When the two subsystems are superposed on each other, they form constraints for each other. As a result of the superposition the final symmetry of the original composite pattern is determined. This can be formulated in the relation of the groups of the two component subsystem patterns. The product of this superposition is in strong relation with the results of the coloring of a pattern with a given symmetry group (Coxeter 1986). In order to use this formalism we give the correspondence between the two types of structures. There the symmetry of a 2-colored pattern was written in a form, where *G* noted the group of the uncolored pattern and *G1* was the group of the colored pattern. *G/G1* noted the type of the pattern, and this meant the order of the coloring, too. (The possibilities were given in Table I of Coxeter 1986). Similarly, in our case, the composite pattern contains a pattern with higher order of symmetry (with symmetry group *G*) and its symmetry is constrained (restricted) by a pattern (superposed on it) with a lower (*G1*) symmetry group. The quotient group of *G/G1* gives the symmetry group of the composite pattern according to the normal subgroup relations of the 17 plane crystallographic groups. The colouring of a composite plane symmetry pattern is possible according to these subgroup relations (Coxeter 1986, Macgillavry 1976). For our studies, however, the subgroup relations are enough, and therefore the composite wallpaper patterns are also „halfway" between plane symmetry groups and coloured plane symmetry groups – similarly as double threads were somewhere halfway between friezes and wallpaper patterns, in Bérczi (1989).

COMPOSITE PATTERNS FROM CRETE (ORCHOMENOS, KNOSSOS, HERAKLEION)

We looked for some early occurrences of such type of composite patterns. Visiting Museums in Crete we can find such patterns from Orchomenos, Knossos and Herakleion. Our first example from the Cretean art is the *p2*-*p4*-like pattern from the palace of Orchomenos, Crete (Figure 4).

The composite pattern of *Orchomenos* consists of tendrill of *S*-edges which form a network and of leaves placed in the free spaces between the tendrill cells. The two types of motifs suggest separation of the pattern to two subsystems. In the tendrill net at each vertex 4 *S*-walls meet and the *S*-edges form a subsystem pattern with *p4* symmetry. The space remaining open in the *p4* net were filled by leaves. The leaves are mirror symmetric „molecules" and they form a subsystem pattern with *p2* symmetry. So the original pattern is composed of (added from) a *p4* and a *p2* subsystem pattern.

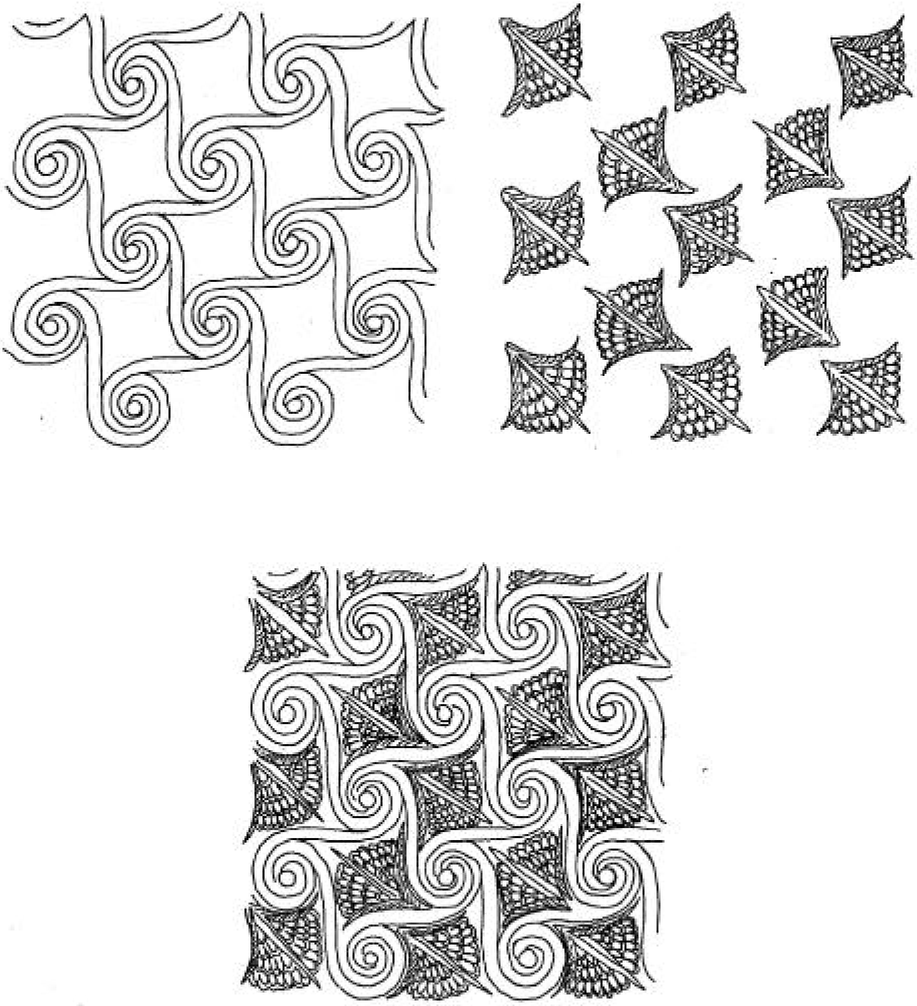

Figure 4: Composite ornamental pattern from Orchomenos Crete: **a)** The net subsystem pattern has *p4* symmetry group, **b)** the papirus like cell filling pattern has *p2* symmetry group, and **c)** after unification the composite pattern has the symmetry group of the lower degree of *p2*, however, all the system may be marked with *p4/p2* pattern type.

Unification (superposition) of the two separated subsystems results in violation of the 4-fold rotation of the *p4* net subsystem pattern. Therefore the final symmetry of the pattern is the symmetry of the lower order symmtry subsystem: *p2*. However, we may mark in the symbol of this pattern that it is a composite pattern. Therefore the *p4/p2* subgroup sign is used here for the Orchomenos pattern (15^{th} century BC).

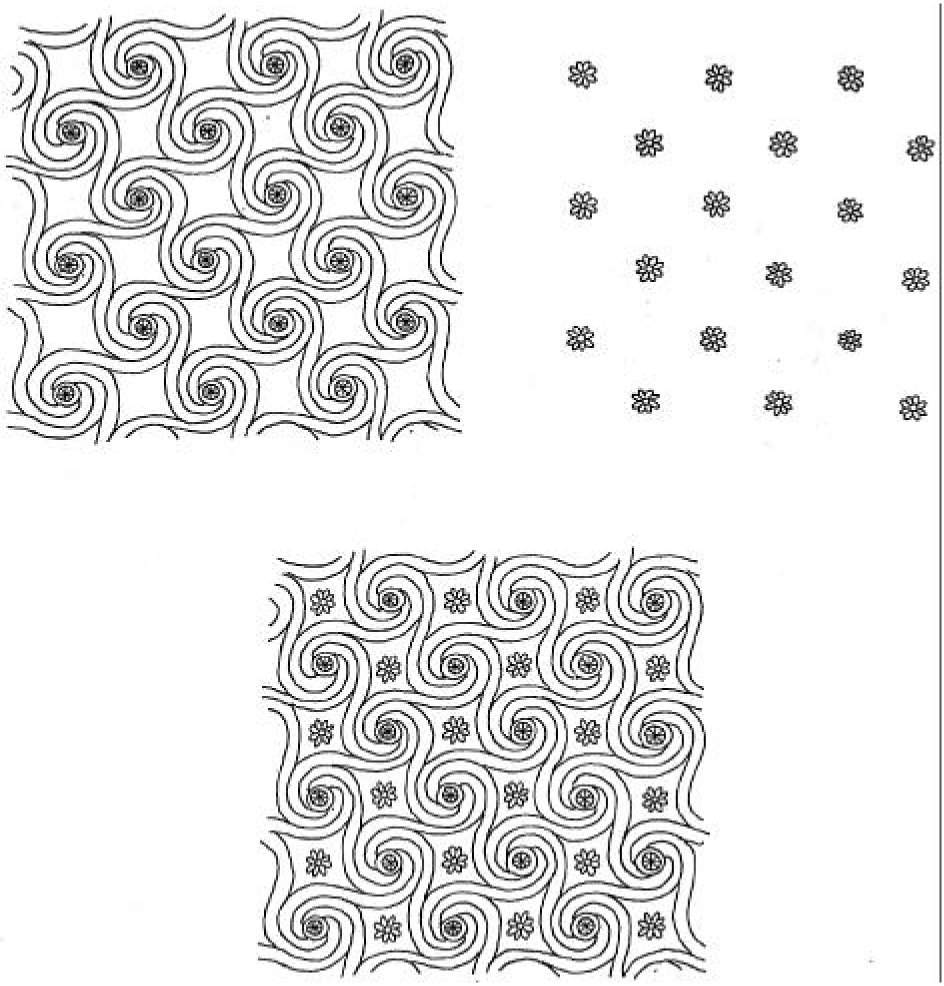

Figure 5: Composite ornamental pattern from the Knossos palace, Crete: **a)** The net subsystem pattern has p4 symmetry group, **b)** the rosette like cell filling pattern has *p4m* symmetry group, and **c)** after unification the composite pattern has the symmetry group of the lower degree of *p4,* however, all the system may be marked with *p4m/p4* pattern type.

There is a pattern from the *Knossos Palace* which is also a composite pattern (Figure 5). It also consists of tendrill of *S*-edges which form a network and small rosette flowers sitting in the free spaces between the tendrill cells (Hood 1978). When we separate the pattern, we get the following subsystems. The tendrill net form a subsystem pattern with *p4* symmetry. The flowers are *D4* symmetric rosettes and they form a subsystem pattern with *p4m* symmetry. So the Knossos pattern is composed of a *p4* and a *p4m* subsystem pattern.

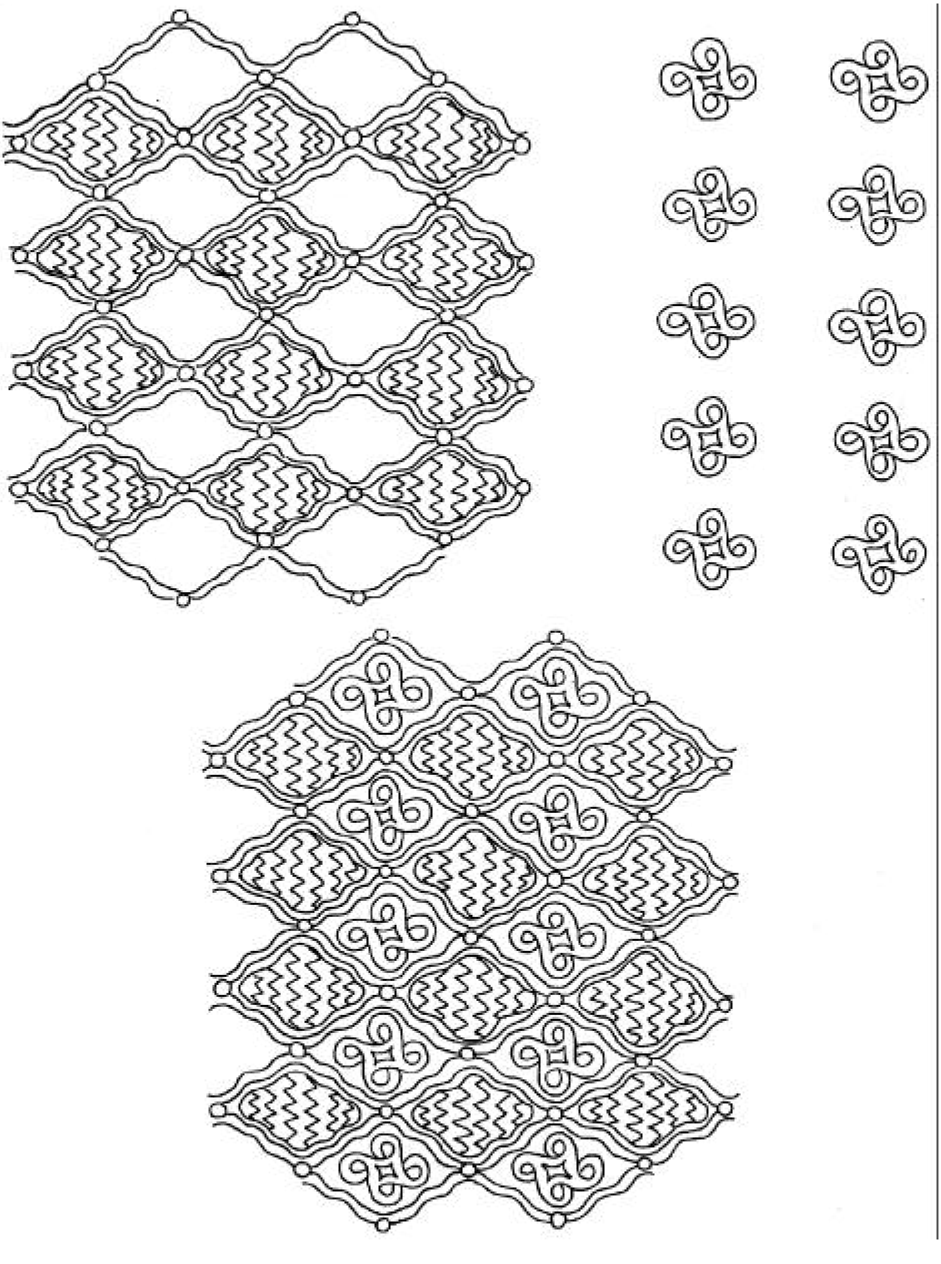

Figure 6: Composite ornamental pattern from Herakleion Museum, Crete: **a)** The tendrill net subsystem pattern has *pmm* symmetry group, **b)** the teeth-like motif filling pattern has also *pmm* symmetry group, **c)** the rounded swastica filling has *p2* symmetry group, because it is fitted to the rhombi of the net, and **d)** after unification the composite pattern has the symmetry group of the lower degree of *p2*, however, all the system may be marked with *pmm/pmm/p2* pattern type.

Unification (superposition) of the two separated subsystems results in violation of the 4-fold mirror symmetry of the *p4m* flower rosette subsystem pattern. Therefore the final symmetry of the pattern is the symmetry of the lower order symmtry subsystem: *p4*. However, now the rosettes have higher symmetry and that was violated by the *p4* (4-fold rotation only) lower order symmetry of the net. Therefore the *p4m/p4* subgroup sign is used here for the Knossos Palace pattern (15th century BC).

There is a pattern from the *Herakleion Museum* which is also a composite pattern (Figure 6.). However, it consists of 3 separate subsystems. One is a net of gentle weaving rope, forming a tendrill of ropes with pmm symmetry group of this subsystem pattern. The second is a fill of every second cells with a violated *p4* rotational form, reduced to 2-fold rotation: the symmetry of this subsystem pattern is *p2*. Finally the third subsystem is composed of a teth-like motif. This subsystem also has a pmm symmetry group.

Unification (superposition) of the three separated subsystems results in violation of the two mirror symmetries of the *pmm* subsystems patterns. Therefore the final symmetry of the pattern is the symmetry of the lower order symmetry subsystem: *p2*. Therefore the *pmm/p2* subgroup sign is used here for the Herakleion Museum pattern (15th century BC).

COMPOSITE PATTERNS FROM URARTU AND RELATED SCYTHIAN FINDS (NIMRUD, ZIWIYEH, PAZYRYK)

In the excavations at *Nimrud* a detail of a pavement tiling was found of which a mosaic pattern can be reconstructed (Ghirshman 1964). The pattern is composite and exhibits a *p4* type net subsystem pattern and a *p4m* type rosette subsystem pattern. Superposition of the two subsystems results in violation of the mirror symmetries of the *p4m* rosette subsystem pattern, therefore the final symmetry of the pattern is the symmetry of the lower order symmetry subsystem: *p4*. The mark of the whole system is: *p4m/p4* (Figure 7.)

In the excavations at *Ziwiyeh* a gold plate was found with composite pattern (see figure in Bérczi 2001, this volume). It consists of 2 separable subsystems. One is a net of gentle palmette stems forming a tendrill with *cm* (almost *cmm*) symmetry group of this subsystem pattern. The second is a fill of the cells with animals, goats and deers, alternately changing in their rows. The animals have a simple *p1* pattern, so after superpostion of the two subsystems all higher symmetry of the net pattern subsystem is violated, reduced to a *p1* symmetry of the whole *cm/p1* pattern of Ziwiyeh from the Scythian Iron Age (7th century BC). (The *cm* type net was rather frequently occurring in the steppe belt of Eurasia, and this pattern probably goes back to Urartu.)

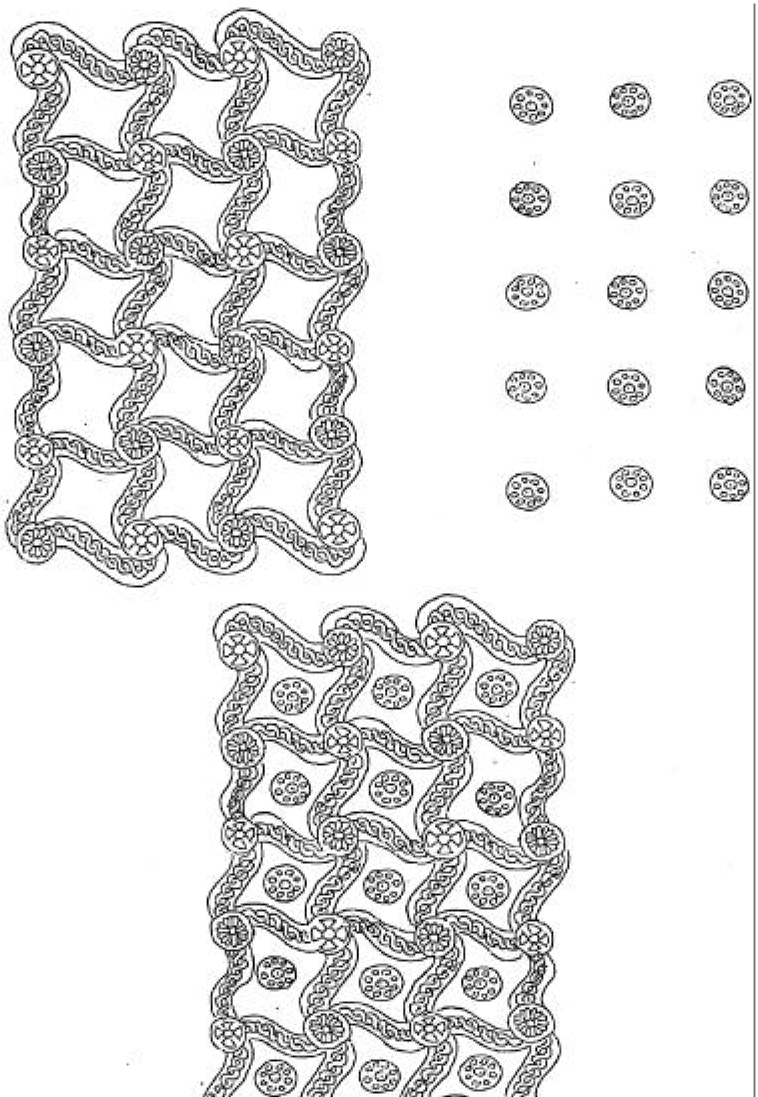

Figure 7: Composite ornamental pattern from Nimrud palace, Urartu: **a)** The net subsystem pattern has *p4* symmetry group, **b)** the rosette like cell filling pattern has *p4m* symmetry group, and **c)** after unification the composite pattern has the symmetry group of the lower degree of *p4*, however, all the system may be marked with *p4m/p4* pattern type.

In the kurgan excavations at *Pazyryk* a shabrack with applicated pattern was found

(Rudenko 1953, Bérczi 2000). It was composite pattern with two separable subsystem patterns. One of them was a chesstable like pattern with *p4m* symmetry group (if the squares and circular ornaments were considered). The second subsystem pattern consisted of horsehead figures and could be seen as dark squares in the chess-table. The horseheads pattern had *cm* type symmetry group. Superposition of the two subsystem patterns results in reduction of all the mirror reflections of the p4m, except that of which is in coincidence with the mirror reflections of cm. So the final structure of the composite Pazyryk shabrack pattern is *p4m/cm* (5^{th} century BC) (see figure in Bérczi 2001, this volume).

Such type of composite structures survived in artistic applications in cultures of the Eurasian steppe. Later use is a composite pattern on the murals of Afrasiab where a *cmm* type net subsystem pattern is superposed with a *pg* type animal (senmurv) filling of the net. Their united structure can be written in *cmm/pg* form, and the mirror reflections are reduced to *pg* by the glide reflection of the pattern with lower symmetry group. The frequent use of the *cm* type net in the steppe belt of Eurasia probably goes back to Urartu. (Bérczi 2001, this volume)

SUMMARY

Composite structure on the romanesque cathedral doorways from Hungary was studied. The main characteristic of such composite structures is that they can be divided into two subsystems which are simple wallpaper patterns with their own symmetry group. Unifying them results in violation of the symmetry of that subsystem that has the higher order symmetry group, according to the Curie principle. This superposition of the two single, separated subsystems can be considered as some ethnomathematical discovery of ancient communities therefore we looked for the Eurasian art heritage of the period of the 2. and 1. millenium.

We found that patterns from Crete, Urartu and some Scythian archaeological finds also contain such composite structure. In Crete mainly the *p4m* and *p4* patterns occurred and were reduced by *p4* and *p2* type patterns. In the Urartu region both the Cretean *p4m/p4* type pattern accurred and other new types were found (Bérczi 2001). The Urartuan *cmm*, *pmm*, *cm* and *pg* patterns with reduction to a final cm-type pattern were found later in Eurasian Steppe belt. The Cretean and the Steppe belt composite patterns meant the initial steps in mapping the symmetry patterns of the ethnographical mathematical past in Eurasia.

We can conclude only that it seems, rotational type *p4*, *p2* related wallpaper patterns were most favourite in the Mediterranian region, while the *cm* and pmm related were more frequently used in the Eurasian steppe belt region. However, both regions used the composite type patterns which seem more cemplex than the simple patterns only. So the complex wallpaper patterns witness that both regions had higher level of intuitive mathematical mental world which were preserved in their cultural heritage.

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