STAGE 2

Mathematical Methods

CURRICULUM STATEMENT 2007

SSABSA
SENIOR SECONDARY ASSESSMENT BOARD OF SOUTH AUSTRALIA

SACE
SOUTH AUSTRALIAN CERTIFICATE OF EDUCATION

EQUITY STATEMENT

This curriculum statement has been accredited by the Senior Secondary Assessment Board of South Australia and is consistent with equal opportunity and human rights legislation.

Each curriculum statement is constructed using the principles of the SSABSA Curriculum and Assessment Policy which identify the student as the centre of the teaching, learning, and assessment processes within the SACE. Inclusivity and flexibility are guiding standards that SSABSA uses in determining curriculum and assessment practices that support students in achieving the requirements of the SACE.

SACE STUDENT QUALITIES

It is intended that a student who completes the SACE will:

1. be an active, confident participant in the learning process (*confidence*).
2. take responsibility for his or her own learning and training (*responsibility*, *self-direction*).
3. respond to challenging learning opportunities, pursue excellence, and achieve in a diverse range of learning and training situations (*challenge*, *excellence*, *achievement*).
4. work and learn individually and with others in and beyond school to achieve personal or team goals (*independence*, *collaboration*, *identity*).
5. apply logical, critical, and innovative thinking to a range of problems and ideas (*thinking*, *enterprise*, *problem-solving*, *future*).
6. use language effectively to engage with the cultural and intellectual ideas of others (*communication*, *literacy*).
7. select, integrate, and apply numerical and spatial concepts and techniques (*numeracy*).
8. be a competent, creative, and critical user of information and communication technologies (*information technology*).
9. have the skills and capabilities required for effective local and global citizenship, including a concern for others (*citizenship*, *interdependence*, *responsibility towards the environment*, *responsibility towards others*).
10. have positive attitudes towards further education and training, employment, and lifelong learning (*lifelong learning*).

ESSENTIAL LEARNINGS AND KEY COMPETENCIES

This curriculum statement offers a number of opportunities to develop essential learnings and key competencies as students engage with their learning. Curriculum support materials, with examples of how these can be developed while students are undertaking programs of study in this subject, will be available on the SSABSA website (www.ssabsa.sa.edu.au).

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CONTENTS

Rationale.....1
Advice for Students.....3
Goals.....4
Strands.....5
Learning Outcomes.....7
Structure and Organisation.....8
Scope.....9
Topic 1: Working with Statistics.....10
Topic 2: Algebraic Models from Data — Working from Observation.....18
Topic 3: Calculus — Describing Change.....22
Topic 4: Linear Models — Managing Resources.....26
Assessment.....29
Moderation.....33
Support Materials.....34

1-unit and 2-unit Subjects

A 1-unit subject consists of 50 to 60 hours of programmed time. It is normally considered to be a one-semester or half-year subject. A 2-unit subject consists of 100 to 120 hours of programmed time. It is considered to be a full-year subject.

Accreditation

This curriculum statement was accredited by the Board from 2003. This accreditation is effective until further notice.

SSABSA informs schools of changes and amendments approved during the period of accreditation. Refer to the curriculum statement on the SSABSA website (www.ssabsa.sa.edu.au) for future changes.

RATIONALE

Mathematics is an integral part of the education and training of all students. It is an important component of the senior secondary studies of many students as they complete their schooling and prepare for further study, training, or work, and for personal and community life.

Mathematics is a diverse and growing field of human endeavour. Mathematics can make a unique contribution to the understanding and functioning of our complex society. By facilitating the current and new technologies and institutional structures, mathematics plays a critical role in shaping that society.

In a time of major change, nations, states, and their citizens are having to operate successfully in an emerging global, knowledge-based economy. Major social, cultural, and environmental changes are occurring simultaneously with changing commercial relationships, new computing and communications technologies, and new sciences such as biotechnology and nanotechnology. Mathematics plays an important part in all of these.

By virtue of mathematics, computers can offer a wide and ever-changing variety of services to individuals and enterprises. It is therefore important that citizens have confidence in their mathematical abilities to understand the services offered and make informed judgments about them.

It is equally important that students have the opportunity to gain the grasp of mathematics that will allow them to be designers of the future, and leaders in various fields. They may be involved in product design, industrial design, production design, engineering design, or the design of new financial and commercial instruments.

The same considerations apply to the new sciences and the new technologies that they support. As systems for information-searching, data-handling, security, genetic design, molecular design, and smart systems in the home and at work, become more and more sophisticated, users will need to have a basic facility with mathematics and the designers of such technologies will need to have increasing understanding of mathematics.

The elements of traditional mathematics — arithmetic, geometry, algebra, calculus, and probability — are involved in the creation and understanding of technologies. It is important that students understand these elements in the context of current and emerging technologies because they transform our understanding of mathematics. Statistics and statistical arguments abound in the mass media and other sources of information, and many students will use statistics in their chosen careers. It is vital that citizens, as they make decisions that affect their lives and future, are able to interpret and question claims of advertisers and other advocates.

Computers and graphics calculators allow a whole new range of mathematical experience to be brought into the classroom through, for example, interactive geometry and drawing tools, techniques of numerical computation, and the analysis of graphs. Some concepts and calculations that were once too difficult to describe or implement can now be demonstrated relatively easily. Technology opens up new applications, allows new explanations, and provides new learning environments for the development of mathematical understanding and confidence.

All students, regardless of gender or background, should have access to mathematical opportunities that accommodate and extend their experiences, broaden their perspective on mathematics, and allow them to appreciate the variety of its past and present roles in society.

ADVICE FOR STUDENTS

This subject asks students to examine what has happened and what is happening in the world round them, and to interact with their findings. It enables students to see mathematics as a creative human response to an external environment through a study of contemporary situations or case-studies.

The successful completion of Mathematical Methods can provide pathways into university courses in accounting, management, computer studies, health sciences, business, commerce, and psychology.

For accurate information about courses, prerequisites, and assumed knowledge, students should consult current publications from the institutions or providers and the South Australian Tertiary Admissions Centre.

Assessment in Mathematical Methods consists of two school-based components, together weighted at 50% (skills and applications tasks, and a portfolio of directed investigations and a project), and an external examination component weighted at 50%.

Students who undertake this subject should have studied at least 2 units of Stage 1 Mathematics, including the successful completion of:

- Topic 7: Statistics, Subtopics 7.6 to 7.8
- Topic 8: Geometry and Mensuration, Subtopics 8.3 and 8.4
- Topic 9: Models of Growth, all subtopics
- Topic 10: Quadratic and Other Polynomials, all subtopics
- Topic 11: Coordinate Geometry, Subtopics 11.1 to 11.6
- Topic 12: Functions and Graphs, Subtopics 12.4 and 12.5.

Student Research

When conducting research, in both the school and the wider community, students must be aware that their actions have the potential to affect other people positively or negatively. SSABSA's policy on students as researchers sets out SSABSA's commitment to supporting students in ethical research. Students who are conducting research should follow SSABSA's Guidelines on Conducting Ethical Research for the SACE. See the SSABSA website (www.ssabsa.sa.edu.au).

SACE Classification

For the purposes of SACE completion, Mathematical Methods is classified as a Group 2 subject.

GOALS

This subject is designed to develop students':

- confidence with mathematical concepts and relationships, and use of mathematical skills and techniques in a range of contexts;
- appreciation of the power, applicability, and elegance of mathematics in analysing, investigating, modelling, and describing aspects of the world;
- facility with mathematical language in communicating ideas and reasoning;
- skills of problem-solving and abstract thinking;
- appreciation of the importance of electronic technology in mathematics;
- mathematical knowledge and skills so that they may become informed citizens capable of making sound decisions in the world of work and in their personal environments.

STRANDS

The study of Mathematical Methods is described in the following six strands:

- Exploring, Analysing, and Modelling Data
- Measurement
- Number
- Patterns and Algebraic Reasoning
- Spatial Sense and Geometric Reasoning
- Analysing and Modelling Change.

Exploring, Analysing, and Modelling Data

This strand deals with ways of displaying, organising, and processing information. In statistical and other algebraic modelling investigations, students can examine everyday situations or events that generate data, examine trends, make and test conjectures, and make projections from mathematical models. Students reach and present appropriate conclusions to mathematical questions based on data, and model a variety of observed phenomena, with the use of graphics calculators and/or computers.

Measurement

This strand deals with the theme of ‘measures’. Aspects of measurement are found in a variety of forms such as scales, rates, ratios, angles, dimensions, precision, location, dispersion, and association. Students extend their capacity to think mathematically and to organise, analyse, and communicate results by examining the interrelationships of changes in measures. They address matters of practical importance in the everyday world.

Number

This strand deals with new understandings of number in different contexts, including: a conceptual grasp of natural logarithms, their properties, and their use in modelling growth; the use of numbers as data with different levels of complexity; the use of ‘proportions’ as new extensions to earlier ‘number’ ideas. A more sophisticated understanding of precision, error, and estimation should result. Students will be able to identify the strengths and limitations of certain procedures for estimating values numerically.

Patterns and Algebraic Reasoning

This strand deals with data-fitting and graphical and symbolic modelling, working from observation and attempting to describe change mathematically. The thorough treatment of the function concept and its multiple representations is reviewed to model particular kinds of behaviour or events. Students fit real data and information with an apt mathematical description in order to conjecture and predict how various changes might affect a dynamic situation.

Spatial Sense and Geometric Reasoning

This strand deals with the exploration, conjecture, examination, and validation of geometric, algebraic, and statistical relationships in different ways and in different contexts. An emphasis on visual learning is aided by the use of graphics calculators and/or computers. Students extend their capacity to identify, understand, and develop appropriate plans to predict the future from past events.

Analysing and Modelling Change

This strand deals with the key elements of mathematical modelling by examining relationships between variables numerically, dynamically, and algebraically. Study in this strand is based on developing an understanding of the elements of ‘change’. Students assess practical situations and represent their mathematical elements graphically and algebraically, using graphics calculators and/or computers. Students describe relationships between variables to model change, and make reasoned conjectures about future events.

LEARNING OUTCOMES

At the end of the program in Mathematical Methods students should be able to:

1. plan courses of action after using mathematics to analyse data and other information elicited from the study of situations taken from social, scientific, economic, or historical contexts;
2. understand fundamental concepts, demonstrate mathematical skills, and apply routine mathematical procedures;
3. think mathematically by posing questions, making and testing conjectures, and looking for reasons that explain the results of the mathematics;
4. make informed and critical use of electronic technology to provide numerical results and graphical representations, and to refine and extend mathematical knowledge;
5. communicate mathematically, and present mathematical information in a variety of ways;
6. work both individually and cooperatively in planning, organising, and carrying out mathematical activities.

STRUCTURE AND ORGANISATION

This 2-unit subject consists of the following four topics, with the approximate allocation of programmed time shown:

- Topic 1: Working with Statistics 26 hours
- Topic 2: Algebraic Models from Data — Working from Observation 24 hours
- Topic 3: Calculus — Describing Change 24 hours
- Topic 4: Linear Models — Managing Resources 18 hours.

Approximately 8 hours of programmed time should be spent on the portfolio, which is one of the assessment components. Additional time should be allocated to consolidation and 6 to 8 hours to skills and applications tasks.

Although the topics are presented separately, their interrelationships could be made clear to students in relevant contexts involving mathematical, physical, and social phenomena.

Each topic consists of a number of subtopics. The subtopics are presented in the 'Scope' section in two columns, as a series of key questions and key ideas side-by-side with considerations for developing teaching and learning strategies.

A problems-based approach is integral to the development of the mathematical models and associated key ideas in each topic. Through key questions teachers can develop the key concepts and processes that relate to the mathematical models required to address the problems posed. The considerations for developing teaching and learning strategies present suitable problems for consideration, and guidelines for sequencing the development of the ideas. They also give an indication of the depth of treatment and emphases required. It is hoped that this form of presentation will help teachers to convey the concepts and processes to their students in relevant social contexts.

The key questions and key ideas cover the prescribed areas for teaching, learning, and assessment in this subject. It is important to note that the considerations for developing teaching and learning strategies are provided as a guide only. Although the material for the external examination will be based on key questions and key ideas outlined in the four topics, the applications described in the considerations for developing teaching and learning strategies may provide useful contexts for examination questions.

SCOPE

The mathematics in this subject is developed to facilitate planning. Students are asked to examine what has happened and what is happening in the world round them, and to consider their findings. This empowers them to describe their world and changes in it. As a result, students see the relevance of the mathematics inherent in planning decisive courses of action to resolve issues.

The interrelationships of the topics are indicated and used in relevant contexts involving mathematical, physical, and social phenomena.

In Topic 1: Working with Statistics, students move from asking statistically sound questions to a basic understanding of how, and why, statistical decisions are made. The topic provides students with opportunities and techniques to examine argument and conjecture from a ‘statistical’ point of view. This involves working with categorical and interval data, and discovering and using the power of the central limit theorem, and understanding its importance in statistical decision-making about means and proportions.

In Topic 2: Algebraic Models from Data — Working from Observation, students move from an analysis of data and models of best fit to methods of prediction, by interpolation and extrapolation. Linear, exponential, and power models are three useful empirical models of growth. Students can experience the range of applications of these models to different situations involving economics, business, and the sciences.

In Topic 3: Calculus — Describing Change, students gain a conceptual grasp of introductory calculus and facility in using its techniques in applications. This is achieved by working with various kinds of mathematical models in different situations, which provide a context for the examination and analysis of the mathematical function behind the mathematical model.

In Topic 4: Linear Models — Managing Resources, students represent problem situations, using a linear function and linear inequalities as constraints or a system of equations as models; solve such representations; and interpret their solution(s) in the context of the original model. Working with the linear programming model and systems of linear equations is yet another context for modelling with mathematics in practical situations.

The coherence of the subject comes from its focus on using mathematics to model practical situations. Modelling, which links the four mathematical areas studied, is made practicable by the use of electronic technology.

TOPIC 1: WORKING WITH STATISTICS

Subtopic 1.1: Normal Distributions

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
Why do normal distributions occur?	Students are reminded of the sets of data in Subtopics 7.6 to 7.8 of Stage 1 Mathematics and the bell-shaped distributions that frequently resulted. They would note that they are the value of the quantity of the combined effect of a number of random errors. In investigating why normal distributions occur, students experience the building of a spreadsheet that generates a large amount of data from the sum of a finite number of random numbers.
What are the features of normal distributions? • Bell-shaped • The position of the mean • Symmetry about the mean • The characteristic spread — the 68:95:99.7% rule • The unique position of one standard deviation from the mean.	A refined spreadsheet from this activity will allow students to see the features of normal distributions unfold. All normal distributions have approximately 68% of the data within one standard deviation of the mean.
Why are normal distributions so important? • The variation in many quantities occurs in an approximately normal manner, and can be modelled using a normal distribution • Use of the model to make predictions and answer questions.	Students are introduced to a variety of quantities whose variation is approximately normal (e.g. the volume of a can of soft drink or the lifetime of batteries), and in these contexts calculate proportions or probabilities of occurrences within plus or minus integer multiples of standard deviations of the mean. Extend these calculations further by using symmetry and complementary properties of the normal distribution to calculate other proportions or probabilities, for example, $P(Z<1)$ where Z is the number of standard deviations from the mean.
What is the standard normal distribution?	Students develop an understanding that the standard normal distribution is one distribution that models every normal distribution, using the property that every normal distribution has the same characteristic spread properties.
How is a normal distribution standardised? • $Z=\dfrac{X-\mu}{\sigma}$.	Students appreciate that any measurement scale (X) is transformed to a scale (Z) in units of standard deviations (σ) from the mean (μ), using the formula $$Z=\frac{X-\mu}{\sigma}.$$

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
Why standardise? • Formula $Z = \dfrac{X - \mu}{\sigma}$	Students use the process of standardising and determining the proportion of a population within given limits to explore meaningful situations such as the length of a warranty for car batteries. They note that proportions within certain limits can be determined simply from one rather than many distributions and vice versa.
• Reasons for standardising.	Standardised normal probability statements provide a basis for the derivation of confidence intervals in Subtopic 1.3. Examples of standardised normal probability calculations should be restricted to variables that are continuous. This avoids the need for continuity corrections, which are discussed in Subtopics 1.4 and 1.5.

Subtopic 1.2: Central Limit Theorem

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
What results from plotting the mean of each sample, when sampling repeatedly from any population?	Students are reminded of the process of taking a random sample from a population in Subtopics 7.6 and 7.8 of Stage 1 Mathematics. Students start by taking random samples generated by, for example, rolling dice and progress to using graphics calculators and/or computers to simulate and streamline the process of sampling. Once students understand the ideas behind the simulation, they can take many samples from a population, calculate their means, and analyse the distribution of the means. The sizes of samples taken are varied so that the characteristics of sampling distributions can be induced.
The result of the central limit theorem	Repeated sampling from numerous populations of various forms (e.g. normal, skewed, U-shaped, and uniform) results in many sample means so that students can induce relevant aspects of the central limit theorem. Students become aware that, as the sample size n becomes larger, the distribution of the sample mean approaches normality, with a mean equal to μ, the population mean, and with a standard deviation equal to $\sigma / \sqrt{n}$, where σ is the standard deviation of the population. This result holds, regardless of the form of the distribution from which the samples are drawn.
Simple applications of the central limit theorem	For example, calculating the likelihood of buying an individual can of soft drink containing less than 375 millilitres compared with the likelihood of buying a six-pack of the same soft drink with an average contents of less than 375 millilitres.
What is the importance of the central limit theorem for statistics?	In statistical inference of means or proportions, it can be assumed that sample means and sample proportions have approximately normal distribution, regardless of the original distribution from which observed values were sampled.

Subtopic 1.3: Confidence Intervals for a Population Mean (μ) for Interval Data

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
How can the likely range of plausible values for a population mean be described by taking one random sample and calculating its sample mean? • Calculation of an approximate 95% confidence interval for μ.	By taking one random sample from a population and determining its sample mean, it is possible to determine an interval within which, at some level of confidence, the true population mean will lie. The algebraic derivation comes directly from a standard normal probability statement about
What does a confidence interval for μ mean?	$Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}$, where $\overline{X}$ is the sample mean from a sample of n observations drawn from a population with mean μ and standard deviation σ. A graphical explanation is explored; for example, students consider probable positions of an individual sample mean lying on the corresponding x-axis of the sampling distribution. This gives rise to the development of the appropriate interval for the population mean. To illustrate this the example of cans of soft drink can be further developed. An exploration of levels of confidence other than 95% would add to students' understanding.
Assessing a claim with a confidence interval	A confidence interval can be used as a guide to the validity of a claim about a population mean. It also gives an idea of how inaccurate the claim may be. It is an interval within which, at some level of confidence, it is certain that the true population mean will lie.
Other simple applications	
• The effect of the sample size on the width of the confidence interval.	Students are aware that the greater the sample size is, the narrower the confidence interval will be.
• Calculation of the approximate sample size required to return a confidence interval of a desired width.	Students consider the relationship between confidence intervals and sampling.

Subtopic 1.4: Review of Continuous and Discrete Interval and Categorical Data

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
What is the difference between continuous and discrete distributions?	Review ideas and representations of continuous and discrete data to reinforce ideas of continuous and discrete distributions. The idea of a continuity correction comes naturally and could be considered at this point.

Subtopic 1.5: Binomial Distributions

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
How would a binomial probability be calculated? • Tree diagram • Binomial tables.	It is envisaged that students would use either a spreadsheet (or other software package) or a graphics calculator to determine the value of probabilities that they are asked to calculate. Students are required to understand the theory of binomial probability. Students start by using tree diagrams to explore success / failure situations. They are led to see that tree diagrams become more cumbersome as the size of the sample increases. The 'Pascal' patterns evident in the trees could be seen as a link to a simpler representation. Counting combinations are then addressed in the context in which the students are working. It is not envisaged that these ideas would be derived from an in-depth exploration of 'counting'. It is now a small step to the binomial formula $C_r^n p^r (1-p)^{(n-r)}$. Students could see this formula as a 'powerful tree'. Students use electronic technology to explore these ideas. Binary trials have only two possible outcomes, for convenience called 'success' and 'failure', with a chance of success = p on each independent trial.
What are binomial distributions?	Counting binomial outcomes (counts or proportions) by Pascal's triangle, binomial coefficients, and the binomial expansion, students understand that the number of successes in n independent binary trials, all with the same success probability p, has a binomial distribution with parameters n and p.
How are probabilities calculated from binomial distributions? • $P(X=k) = C_k^n . p^k .(1-p)^{n-k}$ • Use of calculator in place of formula.	Formula for the binomial distribution.
What if the sample size is larger than the calculator can compute? • Normal approximation for the distribution of a binomial variable, provided that $np \geq 10$ and $n(1-p) \geq 10$.	Look at binomial distributions for various large n. Approximate by the normal. What is the normal approximation to the binomial? If p is the probability of success in any one trial, and X is the number of successes in n trials, then intuition can be used to appreciate that the mean of X is equal to $n.p$ and simulation (also deduced in Stage 1 simulation work) can be used to appreciate that the variance of X is equal to $n.p(1-p)$.

Key Questions and Key Ideas

- Normal approximation for a sample proportion, provided that $np \geq 10$ and $n(1 - p) \geq 10$.

Considerations for Developing Teaching and Learning Strategies

Then a binomial distribution with parameters n and p is approximately normal, with mean $n.p$ and variance $(X) = n.p(1 - p)$.

If $\hat{p}$ is the proportion of some characteristic occurring X times in a sample size of n from some population where the proportion of that characteristic is p, then $\hat{p} = X / n$ has an approximately normal distribution with mean equal to p and the variance of $\frac{p(1 - p)}{n}$ usually approximated by $\frac{\hat{p}(1 - \hat{p})}{n}$.

Subtopic 1.6: Confidence Intervals for a Population Proportion (*p*) for Binomial Data

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
How can the likely range of plausible values for a population proportion be described by taking one random sample and calculating its sample proportion? • Calculation of an approximate 95% confidence interval for *p*.	By taking one random sample from a population and determining its sample proportion, it is possible to determine an interval within which, at some level of confidence, the true population proportion will lie. The algebraic derivation comes directly from the normal approximation to the binomial distribution. A graphical explanation is explored to make this derivation clear; for example, students consider probable positions of an individual sample proportion lying on the corresponding *p*-axis of the sampling distribution. This gives rise to the development of the appropriate interval for the population proportion.
What does a confidence interval for *p* mean?	An exploration of levels of confidence other than 95% would add to students' understanding. Students develop the understanding that a confidence interval is an interval within which, at some level of confidence, it is certain that the true population proportion will lie.
Assessing a claim with a confidence interval	A confidence interval can be used as a guide to the validity of a claim about a population proportion. It also gives an idea of how inaccurate the claim may be. Apply it to various student-generated questions of interest.
Other simple applications	
• The effect of the sample size on the width of the confidence interval.	Students are aware that the greater the sample size is, the narrower the confidence interval will be.
• Calculation of the approximate sample size required to return a confidence interval of a desired width.	Students consider the relationship between confidence intervals and sampling.

TOPIC 2: ALGEBRAIC MODELS FROM DATA — WORKING FROM OBSERVATION

Subtopic 2.1: Algebraic Generation of Linear Models

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
Obtaining, assessing, and using linear models	The use of small sets of data for models developed using electronic technology will reduce the time spent on repetitive tasks. After a scatter plot of suitable data has been produced, a line of best fit will lead to estimation of slope and intercept, allowing a linear model to be proposed. An introductory set of data from an initial-value, constant-adder example such as a taxi fare will remind students of the underlying properties of a linear model.
When is a linear model appropriate? The use of: • best fit by eye • residual analysis • r^2 values.	Students examine the appropriateness of a line of best fit by eye, paying attention to the scale used on the axes. The residuals are used as a tool to critically evaluate the suitability of the model. The use of sets of data that are strongly correlated but have a residual pattern suggesting another model (i.e. not linear) demonstrates the importance of the use of the residuals in decision-making. Access to r^2 and r values allows the degree of linear association between the variables to be assessed; the residuals produced in the calculation of these statistics can be examined to determine the existence of any pattern (or lack of it).
How are linear models used? • Interpolation • Extrapolation.	Once the model for the relationship between the variables has been determined, values of the explanatory variable can be used to predict values for the response variable (and vice versa), either between the known data limits (interpolation) or outside the known data limits (extrapolation). The reliability of such predictions would be discussed with reference to the correlation statistic and the residual analysis and, in the case of extrapolation, to the validity that the conjectured trend will continue past the data limits.

Subtopic 2.2: Algebraic Generation of Exponential Models

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
Exponential growth, compound interest, and natural logarithms	The use of small sets of data for models developed using electronic technology will reduce the time spent on repetitive tasks. Just as the taxi-fare example provides a convenient introduction to linear models, compound interest provides a ready-made, initial-value, constant-multiplier example to initiate discussion of exponential growth. In this context, the value of an investment can be looked at over a number of adjustments, and over an equivalent period, using adjustment periods of increasing smaller duration. This process can be taken to its ultimate conclusion and, with continuous compounding, the compound interest calculation $$P\left(1+\frac{r}{100N}\right)^{Nt}$$ becomes, for $P = 1$, $t = 1$, and $r = 100$, $$\lim_{N\to\infty}\left(1+\frac{1}{N}\right)^{N} = 2.7818\ .\ .\ .$$ This introduces Euler's number e as the limit to which 1 dollar will grow in 1 year when compounded as frequently as possible at 100% per annum. Continuous compounding, $P_t = P_0\, e^{kt}$, can be proposed as a standard for the comparison of different examples involving compound interest, $P_t = P_0\left(1+\frac{r}{100N}\right)^{Nt}$, and natural logarithms can be introduced as a method of calculating the equivalent continuous rate $$k = N\ln\left(1+\frac{r}{100N}\right).$$
Laws of logarithms	The laws of logarithms and their relevance to the algebraic methods used here are studied in the context of natural logarithms.
Exponential regression models • The use of natural logarithms to transform the linear relationship $\ln y = mx + c$ to the exponential relationship $y = Ab^x$, $A = e^c$, $b = e^m$.	With e and natural logarithms as tools, exponential regression can be tackled by using the natural logarithms to progress to a linearisation of the data. At this stage, with small sets of data, the linearisation is done in a table and the transformed data plotted to assess the effect of the transformation. Given a suitably linear plot, regression statistics on the transformed data are produced using the formulae for linear models, with a suitable inverse transformation applied to the linear equation to produce the exponential model.

Key Questions and Key Ideas

Assessing and using exponential models

- Residual analysis
- r^2 values
- Interpolation
- Extrapolation.

Considerations for Developing Teaching and Learning Strategies

Exponential models can be assessed using the residuals from the transformed data and r^2 values. With a model assessed as a good fit, it is possible to make a fairly confident prediction about values within the range of data extremes (interpolation); however, any prediction outside the range of data extremes (extrapolation) is based on the additional assumption that the trend displayed by the data extends into the region for which the prediction is made.

Subtopic 2.3: Algebraic Generation of Power Models

Key Questions and Key Ideas

Obtaining power models

- The use of the laws of natural logarithms to transform the linear relationship $\ln y = m \ln x + c$ to the power relationship $y = Ax^m$, $A = e^c$.

Assessing and using power models

- Residual analysis
- r^2 values
- Interpolation
- Extrapolation.

Considerations for Developing Teaching and Learning Strategies

The use of small sets of data for models developed using electronic technology will reduce the time spent on repetitive tasks. Some sets of data from physical applications may be useful at this stage. Experiments involving distance travelled by an object falling from rest, the time for a dropped ball to stop bouncing, and the time for a pendulum to swing, should all yield sets of data that lead to models of this type.

Natural logarithms are used to transform the linear relationship $\ln y = m \ln x + c$ to the power relationship $y = Ax^m$, $A = e^c$.

Another context for the study of this model would be a previously encountered linear model with r^2 value close to one, and a residual plot indicating that a non-linear model would be more appropriate. A more appropriate result may be obtained by fitting a power model.

As for the previous models, ‘goodness’ of fit can be assessed from residual plots and r^2 values. The model can be used for interpolation and extrapolation, with the qualifications given on page 20.

TOPIC 3: CALCULUS — DESCRIBING CHANGE

Subtopic 3.1: Rate of Change

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
What is an average rate of change? • Average rate of change as a ratio of changes in connected quantities.	This idea can be covered in the context of, for example, finding average speeds, cost per kilogram, litres of water used per day, or watts of power used per day. To emphasise the connection between the quantities involved, the averages could be explored by, for example, considering average rates of consumption of water, power, and so on, for one billing period, for different billing periods, for several billing periods, and for a year. After this numerical work, a graphical approach could be used, with an average rate of change represented as a chord across a graph (possibly modelled from data in one of the contexts used in the numerical work).
What is an instantaneous rate of change?	Moving from average rates of change to an instantaneous rate of change can be motivated by considering average rates over relatively small time intervals (e.g. a day in the context of a year, a minute in the context of a day, a second in the context of an hour).
Gradient as a rate of change	This could best be done: • algebraically, using a model derived in a previously studied context; • graphically (and geometrically), using a chord across a curve, approximating a tangent as the interval diminishes, and by zooming into a relatively small interval and observing ‘local linearity’. The idea of a tangent to a curve can be developed as an extension of the concept as it applies to circles, and the connection between a line’s gradient and the associated rate of change can be made.
How is the gradient of the tangent found?	An immediate association regarding the description of such tangents could be posed: the gradient of a tangent to a circle can be found if the centre and the point of contact are known. Is it possible to do something similar for tangents to other curves?

Subtopic 3.2: Derivatives

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
The notion of a derivative for: • x^n	A working familiarity with the rule for differentiation of powers will emerge through investigations of tangents to simple polynomial curves, using graphical software or a graphics calculator, and/or through algebraic/numerical work with simple polynomials in a spreadsheet. Inherent in this will be the notion of the derivative: the rule for finding the gradient of the tangent involves the *x*-coordinate of the point of contact, and systematic changes to the curve's equation.
• e^{kx} and ln x.	A working knowledge of the derivatives of exponential and logarithmic functions will emerge through similar investigations, using the equations of exponential and logarithmic models derived in Topic 2: Algebraic Models from Data — Working from Observation.

Subtopic 3.3: Differentiation

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
What is the algebraic structure of function notation?	Functions can be classed as sums, compositions, products, or quotients through an analysis of the use of grouping symbols (brackets) and a working knowledge of the order of operations. This is needed for correct differentiation of the function.
What are the differentiation rules for: • sums? • compositions? • products?	The investigative work done with tangents to simple polynomials adequately demonstrates the principle behind differentiating sums. The chain rule can be verified for a number of simple examples by using graphing software or a graphics calculator to establish numerical results for the function's derivative, and evaluating the product of the two factors proposed by the chain rule to establish the same results algebraically. An algebraic approach would be to apply the chain rule and simplify, then simplify and differentiate term by term. Either or both methods used to introduce the chain rule would work well here, with suitable examples. The quotient rule could be developed through a directed investigation, and included in school assessment pieces. It will not be examined. The quotient can be obtained from the product rule: rewrite the quotient $Q(x)=\frac{f(x)}{g(x)}$ as the product $f(x)=Q(x).g(x),$ then use the product rule to find $f'(x),$ then put $Q(x)=\frac{f(x)}{g(x)}$ into the right-hand side, and rearrange to make $Q'(x)$ the subject.

Subtopic 3.4: Properties of Regression Models

Key Questions and Key Ideas

Non-asymptotic growth models

- Linear
- Exponential
- Logarithmic $y = b + a\ln t$
- Power.

Polynomial models

- Quadratic
- Cubic.

Asymptotic growth models

- Logistic $y = \dfrac{C}{1 + Ae^{-bt}}$
- Terminal velocity $y = A(1 - e^{-bt})$
- Surge $y = Ate^{-bt}$.

Considerations for Developing Teaching and Learning Strategies

The emphasis now turns to the:

- use of technology in efficiently generating and assessing algebraic models for large sets of data;
- use of technology in efficiently generating and assessing different models for the same set of data;
- use of technology in investigating new models for sets of data with features that are not consistent with linear, exponential, or power models;
- algebraic and graphical properties of the models themselves, and also to the propensity of models such as these to convey information about the changes that occur within the known data limits, and that will occur if the trends in the data consistently extend past the known data limits.

Logarithmic models are useful in exponential situations where the roles of the explanatory and response variables are interchanged.

Logistic models are useful in limited-growth situations that exhibit first an exponential phase, then a (quasi-) linear phase, and finally an asymptotic approach to the limiting population size.

Rational and terminal velocity models can be used in contexts where the response variable asymptotically approaches a limiting value at a decreasing rate.

- Rational $y = k + \dfrac{a}{bx + c}$
- The rational model could be used as an example for a directed investigation.

The surge model, used extensively in the study of medicinal doses, models an initial rapid increase in the response variable, which reaches a maximum and then decays to zero.

TOPIC 4: LINEAR MODELS — MANAGING RESOURCES

Subtopic 4.1: Linear Programming

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
What is linear programming? Description of the general linear programming problem	Linear programming (or linear optimisation) is a method of solving certain types of problems that arise in many practical situations. The typical form of questions is: What is the best way to • allocate resources? • plan production? • provide services? • allot working hours? • compose blends of ingredients? given that certain limitations (or constraints) are imposed by, for example, availability, capacity, costs, minimum quality requirements, safety standards.
Optimising a linear function with linear inequalities as constraints	There will usually be many ways in which the constraints can be satisfied. The 'best' solution will be defined in a specific way. For example, it might be the solution that maximises profit, or minimises cost or time. Linear programming is a method that leads systematically to the best solution of problems such as those listed above.
In what contexts does linear programming arise? A variety of applications should be presented	Examples can be found in business, manufacturing, marketing, and planning contexts, and might include: • allocating manufacturing resources; • planning an investment portfolio; • blending ingredients; • designing balanced diets; • formulating marketing plans. Students recognise the similarity in the mathematical models formulated from these diverse examples in which there are linear inequalities as constraints and linear functions to be optimised.
How is mathematics used in these contexts? • Assignment of variables • Formulation of constraints • Recognition of assumptions • Identification of the feasible region • Interpretation of the solution.	An important aspect of this work is learning to represent given information correctly in mathematical terms. The following questions are dealt with: • What are the unknowns? • How are the unknowns to be denoted? • What are the constraints? • Are there unstated assumptions (such as negative values that are meaningless)?

Key Questions and Key Ideas	**Considerations for Developing Teaching and Learning Strategies**
	• How can the possible solutions be represented graphically? • What is the function to be optimised? • How is the solution to be interpreted in the original context of a problem? The use of graphics calculators and/or computers to graph feasible regions will provide opportunities for students to readily explore the effect of changes on full constraints.
How is the optimal solution found in two-variable cases?	In two-variable cases, the half-plane representing each constraint can be graphed and the feasible region found by determining their intersections. Drawing parallel lines representing possible values of the objective function across this region will lead students to the realisation that optimum values occur at vertices.
The linear programming method	There are four steps in the linear programming method (for two variables): • Graph the region. • Calculate the vertices of the region. • Evaluate the objective function at each vertex. • Compare these values to find the maximum or minimum value. What if there are more than two variables? Clearly diagrams become too difficult or even impossible to draw. However, the same ideas are valid in higher dimensions, and the first two steps can still be carried out using electronic technology. (Students would not be expected to solve examples in more than three variables, but should know that these methods are used in business, communications, and resources management for problems with hundreds, or even hundreds of thousands, of variables.) This activity would provide a rich opportunity for a directed investigation or project.

Subtopic 4.2: Matrices

Key Questions and Key Ideas	Considerations for Developing Teaching and Learning Strategies
What are matrices, and why are they used?	Matrices (including vectors) should be introduced as a means of organising and representing data or other information.
Matrices (including vectors) and their uses	Some contexts in which matrices are useful: • Inventories (quantities of types of stock at different locations). • Population (size or proportion in age or sex category in different cities). • Graphs associated with networks (existence of edges between vertices, or numbers associated with edges, such as length of roads linking towns). • Transition or Markov matrices (containing probabilities of change from one category to another). • Leslie matrices (similar to the transition and Markov matrices, but describing the growth of a population divided into age classes). • Matrix codes.
Matrices in action — multiplication	Matrix multiplication could be introduced with the multiplication of a matrix by a vector, in some of the contexts listed above: • Inventory matrix and cost vector. • Total population in different categories or cities (multiplication by a vector of ones is equivalent to totalling rows or columns). • Effect on a state vector of change modelled by a transition, Markov, or Leslie matrix. • Encoding a message expressed as a sequence of vectors. • The notion of a vector as input and output, matrix as function. This is extended to the multiplication of matrices in appropriate contexts: • The long-term effects of repeated transitions (using electronic technology). • Decoding a message, given the inverse matrix (this will involve discussion of inverse and identity matrices).

ASSESSMENT

Assessment is subject to the requirements, policies, and procedures of the Board.

One of the purposes of assessment is to measure the extent to which students have achieved the learning outcomes of a program based on this curriculum statement. The assessment tasks used to determine the SSABSA Subject Achievement Score are summative. Formative tasks are important in the learning process, but do not contribute to final grades.

Assessment in Mathematical Methods consists of the following components, weighted as shown:

Assessment Component 1: Skills and Applications Tasks (35%)
Assessment Component 2: Portfolio (15%)
Assessment Component 3: Examination (50%).

Assessment Component 1: Skills and Applications Tasks

This assessment component is designed to assess primarily Learning Outcomes 1 to 5. It is weighted at 35%.

Skills and applications tasks require students to solve mathematical problems that may:

- be routine, analytical, and/or interpretative;
- be posed in familiar and unfamiliar contexts;
- require a discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide the students with information in written format or in the form of numerical data, diagrams, tables, or graphs. The tasks should require the student to demonstrate an understanding of relevant mathematical ideas, facts, and relationships. Students should be able to select appropriate algorithms or techniques and relevant mathematical information to successfully solve routine, analytical, and/or interpretative problems. Some of these problems should be set in a personal, global, or historical context. Students should be required to provide explanations and arguments, and use notation, terminology, and representation correctly throughout the task. They may be required to use electronic technology appropriately to aid and enhance the solution of some problems.

Skills and applications tasks are to be undertaken under the direct supervision of an invigilator. The total time spent by students on assessment tasks in this assessment component should be between 6 and 8 hours.

Criteria for Judging Performance

The student's performance in the skills and applications tasks will be judged by the extent to which he or she demonstrates:

- mathematical skills and understandings (without electronic technology);
- mathematical skills and understandings (with electronic technology);
- analysis and interpretation of results and information;
- the communication of mathematical information.

Assessment Component 2: Portfolio

This assessment component is designed to assess all the learning outcomes. It is weighted at 15%.

Students are required to keep a portfolio representing approximately 8 hours of work. This may consist of a mixture of up to three directed investigations and projects *or* one substantial project.

Directed Investigation

A directed investigation is an assessment task that requires students to investigate a mathematical relationship, concept, or problem, which may be set in an applied context. The subject of the directed investigation is usually derived from one or more subtopic(s), although it can also relate to a whole topic or across topic(s).

Typically the teacher provides students with a clear, detailed, and sequential set of instructions for part of the task or to initiate the task. Students are encouraged to demonstrate their knowledge, skills, and understanding in the investigation. The generation of data and the exploration of patterns and structures, or changing parameters, may often provide an important focus. From these, students may recognise patterns or structures and make a conjecture. Notation, terminology, and forms of representation of information gathered or produced, calculations, and results will be important considerations. Students should interpret and justify results, summarise, and draw conclusions. Students should be required to provide appropriate explanations and arguments in a report. A directed investigation may require the use of graphics calculators and/or computer technology. Students can undertake a directed investigation either individually or as part of a group. When the directed investigation is undertaken by a group, each student must make an identifiable contribution to the final product. During a directed investigation, the student's progress is guided and supported by the teacher.

A completed directed investigation should include:

- an introduction that demonstrates an understanding of the features of the problem or situation investigated;
- evidence that the student has followed instructions;
- appropriate representation of information gathered or produced, calculations, and results;
- a summary of results or findings and conclusions drawn.

The mode of presentation of a directed investigation may include:

- a written report or an oral report;
- other multimedia formats.

Project

A project is an assessment task that requires students to explore mathematical relationships, concepts, or a problem, which may be set within an applied context in a more open-ended manner than in a directed investigation. The subject of the project may be derived from one or more subtopic(s), although it commonly applies to a whole topic or across topic(s).

Being an open-ended task, the project may be initiated by the student, a group of students, or the teacher. When the teacher initiates the task, he or she should provide broad guidelines only, allowing the student or group of students sufficient scope to develop themes or aspects of their own choice. Teachers should be prepared to provide some guidance about the appropriateness of the student's choice. Although the teacher would guide and support students' progress on project work, it is expected that students would work with less guidance on a project than on a directed investigation.

Within the context of the chosen mathematical model, students are encouraged to demonstrate their knowledge, skills, and understanding. The generation and collection of data and/or information and the associated process will be an integral part of the project. Notation, terminology, and forms of representation of information, calculations, and results will also be important considerations. A project may require the use of a graphics calculator and/or computer.

Students should summarise, analyse, and interpret results, and draw conclusions. They are encouraged to recognise the limitations of the original problem, as well as to refine and/or extend it. Students should be required to provide appropriate explanations and arguments in a report.

A project provides an ideal opportunity for students to work cooperatively to achieve learning outcomes. However, when project work is undertaken by a group, each student must make an identifiable contribution to the final product, be able to discuss aspects of the whole project, and prepare an individual report or presentation.

A completed project should include:

- an introduction that outlines the problem to be explored, including its significance, its features, and the context;
- the method of solution in terms of the mathematical model or strategy to be used;
- the appropriate application of the mathematical model or strategy, including:
  - the generation or collection of relevant data and/or information, with details of the process of collection;
  - mathematical calculations and results, and appropriate representations;
  - the analysis and interpretation of results;
  - reference to the limitations of the original problem as well as appropriate refinements and/or extensions;
- a statement of the solution and outcome in the context of the original problem;
- appendixes and bibliography as appropriate.

The mode of presentation of a project may include:

- a written report or an oral report;
- other multimedia formats.

Criteria for Judging Performance

The student's performance in the portfolio will be judged by the extent to which he or she demonstrates:

- mathematical skills and understandings (without electronic technology);
- mathematical skills and understandings (with electronic technology);
- analysis and interpretation of results and information;
- the communication of mathematical information;
- the organisation and presentation of material;
- the ability to prove conjectures;
- the ability to work independently;
- the ability to work cooperatively.

Assessment Component 3: Examination

This assessment component is designed to assess primarily Learning Outcomes 1 to 5. It is weighted at 50%.

The 3-hour external examination will be based on the subtopics and key questions and key ideas outlined in the four topics. The considerations for developing teaching and learning strategies are provided as a guide only, although applications described under this heading may provide useful contexts for examination questions.

It is expected that the examination will consist of a range of questions, some focusing on knowledge and routine skills and applications, and others focusing on analysis and interpretation. Some questions may require students to interrelate their knowledge, skill, and understanding of some topics. It is also expected that the skills and understanding developed through directed investigation and project work will be assessed in the examination.

SSABSA will assume that students will have access to graphics calculators and/or computers during the external examination. It is expected that questions in the examination will be a mixture of the following types:

- Graphics calculator and/or computer *inactive* questions. There is no advantage in using a graphics calculator or computer to answer these questions.
- Graphics calculator and/or computer *neutral* questions. These questions can be solved without a graphics calculator or computer, although the electronic technology may be used.
- Graphics calculator and/or computer *active* questions. These questions require the use of a graphics calculator or computer for their solution.

Students will need to be discerning in their use of electronic technology to solve questions in examinations.

Criteria for Judging Performance

The student's performance in the examination will be judged by the extent to which he or she demonstrates:

- mathematical skills and understandings (without electronic technology);
- mathematical skills and understandings (with electronic technology);
- analysis and interpretation of results and information;
- the communication of mathematical information.

MODERATION

Moderation is subject to the requirements, policies, and procedures of the Board. SSABSA publishes the specific moderation requirements annually.

Moderation is a process designed to place different teachers' assessments of their students' performance in the same subject on the same scale so that valid comparisons between performances can be made. The purpose of moderation is to help to ensure fairness to students and to provide the wider community with reliable information about student performance. Moderation is undertaken to ensure that the school-assessed scores or SACE designations given to students who take the subject are comparable from school to school.

Assessment Component 1: Skills and Applications Tasks and Assessment Component 2: Portfolio will be statistically moderated against Assessment Component 3: Examination.

Assessment Component 3: Examination will be externally marked.

SUPPORT MATERIALS

Useful support materials are available on the SSABSA website (www.ssabsa.sa.edu.au), for example:

- annotated work samples
- assessment exemplars
- assessment plans
- illustrative programs
- performance standards
- resources
- teaching and learning strategies.