

# Statements of Learning for Mathematics

Statements of Learning for Mathematics ISBN-13: 978-1-86366-630-5 ISBN-10: 1 86366 630 3 SCIS order number: 1291665 Full bibliographic details are available from Curriculum Corporation.

Published by Curriculum Corporation PO Box 177 Carlton South Vic 3053 Australia Tel: (03) 9207 9600 Fax: (03) 9639 1616 Email: sales@curriculum.edu.au Website: http://www.curriculum.edu.au

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The *Statements of Learning for Mathematics* were managed by Australian Education Systems Officials Committee (AESOC) on behalf of the Ministerial Council on Education, Employment, Training and Youth Affairs (MCEETYA), and developed by Curriculum Corporation.

#### Acknowledgements

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## Foreword

At the July 2003 MCEETYA meeting, Ministers agreed to the development of Statements of Learning for Mathematics that 'define and deliver common curriculum outcomes to be used by jurisdictions to inform their own curriculum development'. The development of the Statements is a response to concerns about the lack of consistency that exists in curriculums across the nation and the impact this is having on an increasingly mobile student population.

The Statements of Learning for Mathematics have been developed collaboratively by State, Territory and Australian education authorities. They provide a description of knowledge, skills, understandings and capacities that all students in Australia should have the opportunity to learn. The development of the Statements has involved identification of what is common amongst State and Territory curriculums as well as what is essential for all students to learn.

For the many students and their families who move school within or across jurisdictions, greater consistency in learning opportunities for children at particular stages of schooling will assist in alleviating the educational and emotional impacts associated with such moves.

In line with impacts being felt across all areas of Australian society, our students are increasingly operating in a national and global society and economy. It makes sense that education jurisdictions across Australia have worked collaboratively to identify the body of knowledge, skills, understanding and capacities which are essential for that context. Jurisdictions will need to consider how they integrate these elements into their own curriculums in a manner which suits the diversity of students' needs and schools across the country.

These statements represent significant collaboration between education authorities at a State, Territory and National level, and will inform future decisions by Education Ministers on the further work to be undertaken on English, Mathematics, Science, Civics and Citizenship, and Information and Communication Technologies.

#### Ken Smith

Chair, National Consistency of Curriculum Outcomes Steering Committee Australian Education Systems Officials Committee

### Introduction

This document, *Statements of Learning for Mathematics*, is the result of collaborative work by Australian education jurisdictions to achieve greater consistency in curriculum. It sets out the knowledge, skills, understandings and capacities that students in Australia should have the opportunity to learn and develop in the Mathematics domain.

*Statements of Learning for Mathematics* is not a curriculum in itself. Instead, it contains a series of statements about essential opportunities to learn in this particular domain which education jurisdictions have agreed to implement in their own curriculum documents. As such, this document is primarily intended for curriculum developers. It is not the express intent that the document is promoted directly with teachers or the general community.

*Statements of Learning for Mathematics* is not a list of all possible opportunities to learn within the Mathematics domain. It contains only those opportunities which all education jurisdictions agree should be consistent across Australia. Jurisdictions' own individual curriculum documents will likely include additional aspects of Mathematics.

*Statements of Learning for Mathematics* contains two critical elements: the Statements themselves and their professional elaborations, which work together as a package, with the Statements also represented in expanded form in the professional elaborations. The Statements are written in a plain English form which allows them to be engaged with by a broad audience if required. As the name suggests, the professional elaborations use the professional language of the Mathematics curriculum domain.

Underpinning the Statements and professional elaborations package within the *Statements of Learning for Mathematics* is the idea of an opportunity to learn. The opportunities to learn set out in this document are those opportunities seen as reasonable, challenging and appropriate. 'Reasonable' means it is realistic to expect that most students will have actually achieved the learning within a reasonable period of their first having the opportunity to learn. Up to two years can be considered reasonable for students. 'Challenging' means that the opportunities will be a stretch and thus they represent somewhat more than a proficient student could be expected to learn initially. 'Appropriate' means that the opportunities are suitable for the majority of young Australians to experience.

The opportunities to learn in the Statements and professional elaborations sections have been developed for four year junctures – the end of years 3, 5, 7 and 9. Most of the curriculum documents of Australian education jurisdictions are organised in bands, levels or stages rather than in year junctures and so the opportunities to learn in this document will most likely be included in jurisdictions' curriculum documents in the band, level or stage where the year juncture falls.

The opportunities to learn in the Statements and professional elaborations sections are also structured around broadly defined aspects of Mathematics, known as organisers. They provide coherence and structure for this document. In implementing the opportunities to learn, jurisdictions will use whatever organisers suit their curriculum documents best.

### **School mathematics**

Mathematics educators acknowledge that there are different perspectives on the philosophy of mathematics and mathematics education and that these underpin the relationship between mathematics, the domain of *school mathematics* and the mathematics curriculum of any school system. While States and Territories have expressed a range of views on mathematics and mathematics education, with corresponding diverse interpretation and representation in curriculum design, a review of principal curriculum documents reveals significant alignment across the following broad underlying themes.

#### **Mathematics is dynamic**

Mathematical knowledge has developed across cultures throughout history and continues to develop today. Mathematics education responds to social change, developments in mathematics, new technologies and new approaches to mathematical inquiry.

#### Mathematics is an integral part of a general education

Mathematics is part of our cultural heritage. All students have a right to learn mathematics and the language of mathematics, to make sense of mathematics, to be confident in their use of mathematics and to see how it can help them make sense of their world and the worlds of others. High expectations for achievement, conceptual understanding and the opportunity to learn reasonable and challenging mathematics are fundamental to equity and social justice.

#### Mathematics contributes to individual and collective development

Mathematics and the capacity to be numerate, that is, the ability to effectively apply mathematics in everyday, recreational, work and civic life, is vital to the quality of participation in society.

#### Mathematics connects with other curriculum areas

Mathematics is a domain that supports learning and application in other curriculum areas and also draws on them for learning contexts.

#### Mathematics curriculums in Australia

The Statements of Learning and their professional elaborations draw upon the following aims which are a synthesis of those from Mathematics curriculums across Australia. They are intended to provide students with the opportunity to develop:

- knowledge and understanding of concepts and ideas, and facility with mathematical skills and processes across key areas of mathematics with
  - mental and written computation and numerical reasoning
  - function and pattern, generalisation, logical and algebraic reasoning
  - the identification and measurement of attributes or characteristics of shapes, objects, data and chance events
  - geometric reasoning and the visualisation, representation, location and transformation of shapes and objects in space

- the capacity and disposition to deploy mathematical knowledge, understanding, skills and processes in a range of situations through
  - using and building on prior knowledge, generalising to other contexts, making conjectures and incorporating new information into existing structures
  - posing and solving problems, mathematical modelling, developing proofs and conducting investigations
  - thinking creatively, generating alternatives when solving problems, and working individually and cooperatively
  - reflecting upon and discussing mathematical ideas, problems and processes, to formulate and test their own solutions, and have these tested by others
  - evaluating representations of mathematical information and challenging mathematical ideas by considering purpose and point of view
- the capacity to communicate effectively through
  - the use of informal and formal mathematical language to convey, logically and clearly, their mathematical understandings, thinking and reasoning in oral, electronic and written media
  - representation of their mathematical ideas and reasoning in different ways which reflect their conceptual understandings for various audiences and purposes
  - the selection and effective use of a range of mathematical strategies, models, information and communication technologies and related critical literacies
- enjoyment of mathematics and confidence in the use of mathematics in everyday situations through appreciation of
  - its relevance as part of their personal and working lives
  - its nature as a dynamic, diverse and complex domain with interwoven and interconnected concepts
  - the nature of mathematical thinking and its historical and cultural roles.

# Features of Statements of Learning for Mathematics and the professional elaborations

The *Statements of Learning for Mathematics* describe the knowledge, skills, understandings and capacities that all young Australians should have the opportunity to learn and develop in Mathematics.

The professional elaborations build on the Statements of Learning by providing more specific detail and by making use of the technical language related to Mathematics.

As systems over time will integrate the *Statements of Learning for Mathematics* into their curriculum documents, teachers' application of them will be through their own State or Territory curriculums.

*The Statements of Learning for Mathematics* are organised by year level and are structured around five broadly defined and inter-related aspects of Mathematics curriculums that are considered essential and common.

**Working mathematically** involves mathematical inquiry and its practical and theoretical application. This includes problem posing and solving, representation and modelling, investigating, conjecturing, reasoning and proof and estimating and checking the reasonableness of results or outcomes. Key aspects of working mathematically, individually and with others, are formulation, solution, interpretation and communication. The processes of working mathematically draw upon and make connections between the knowledge, skills and understandings acquired in Number, Algebra, function and pattern, Measurement, chance and data, and Space. It is noted that not all jurisdictions use a separate organiser such as Working mathematically in their curriculum documentation. For these jurisdictions, Working mathematically is understood to be integrated across the other curriculum organisers.

**Number** involves the study of representation and models for number, counting, magnitude, order and computation. This includes number systems, their properties, and exact or approximate calculation with number carried out mentally, by hand using written algorithms and using technology.

**Algebra, function and pattern** involves the study of general relationships between objects and their representation by the informal or formal use of variables. This includes working with functions and relations applied to everyday and mathematical objects, patterns in number and space, and general forms (rules, formulas, tables, graphs, equations and equivalences) expressed using words, symbols or diagrams.

**Measurement, chance and data** involves the study of unit, measure and error, events and likelihood, and data and inference. This includes length, area, volume, angle, mass, time, temperature, probability and statistics. A key aspect of measurement is the use of formulas or technology for indirect measurement and the calculation of rates.

**Space** involves the study of shape and location. Shape includes identification, classification and representation (by hand and using instruments or technology) of two-dimensional and three-dimensional shapes and objects, and investigation of their geometric properties. Location includes consideration of ways in which the position of an object can be described in terms of features in a given context, distance and orientation, and how shapes can be moved in space by transformations.

#### **Reading the Statements of Learning and the professional elaborations**

The *Statements of Learning for Mathematics* have been designed to describe progressions of learning that are accessible and challenging at four year junctures of years 3, 5, 7 and 9.

Each Statement of Learning and professional elaboration subsumes the knowledge, skills, understanding and capacities of the Statements and professional elaborations that precede it. It is important for curriculum writers to consider the Statements of Learning and the professional elaborations as a whole, in conjunction with the Introduction.

In a number of instances examples, including various mathematical expressions and formulas, have been incorporated to assist curriculum writers to clearly identify the intended depth and breadth. Whenever examples are included, they are for the purpose of clarification only and should not be taken as prescriptive.

As noted above, the professional elaborations expand upon and provide more specific detail that clarifies the intent of the Statements of Learning and makes use of the technical language of Mathematics.

The *Statements of Learning for Mathematics* do not attempt to address pedagogical issues. Learning experiences may include a variety of strategies to support the learners.

# **Year 3 Statements of Learning for Mathematics**

#### Year 3 Working mathematically

Students actively investigate everyday situations as they identify and explore mathematics. They experiment with different ways of changing numbers and shapes, and try to predict the effects of those changes as they search for patterns and relationships they can describe. They use simple strategies such as searching for similarity, difference and repetition and use these to make sense of the mathematics they are learning.

Students interpret situations where mathematics is involved. They choose and use concrete materials, drawings, lists, tables and some mathematical symbols to represent these situations and describe them in their own words. They interpret these different representations of mathematical situations and see the connections between them.

Students interpret and work through different mathematical situations, make and test conjectures and predictions, and solve a variety of mathematical problems. They use suitable approaches and check their reasoning, describe the solutions or findings, and attempt to convince others about their reasonableness.

Students identify different types of mathematical situations, and describe the important aspects of those situations in their own words or in other ways such as diagrams. They talk freely about their observations, ideas and approaches, why particular approaches might be used, explaining which facts, strategies and procedures they expect will assist in the solution.

#### Year 3 Number

Students recognise and match different representations of whole numbers at least to three digits. They use place value to compare and order these numbers, and can position them accurately on a number line. They make whole numbers larger or smaller by adding or subtracting 1, 10 or 100 and count collections fluently by 1s, 2s, 5s and 10s. They recognise different representations of halves and quarters, and also mixed numbers involving these fractions.

Students recall addition and subtraction facts or use efficient strategies for working them out. They can apply those strategies to calculate mentally with larger numbers. They use mental and written methods to add and subtract any two-digit numbers. They recognise simple multiplication and division situations, and work them out using suitable strategies including the use of arrays, skip counting and sharing with appropriate concrete materials. They use technology to assist in the exploration, development and refinement of these strategies and also for calculations beyond the scope of these strategies and capabilities.

Students interpret and distinguish mathematical situations and problems that involve any one of the four arithmetic operations: addition, subtraction, multiplication and division. They create problems based around a particular operation and use concrete materials, sketches and diagrams to model and solve number problems. They explain the approaches they use, compare them with other approaches and check the reasonableness of their calculations.

### Year 3 Algebra, function and pattern

Students begin to recognise and describe simple relationships, including those related to order, sequence and arrangement. They describe relationships in their own words and create and follow step-by-step guidelines and instructions about simple procedures where order, or the sequence of actions, is important.

Students identify and represent relationships, including inverse and equivalence, and describe them in their own words. They make use of concrete materials, drawings and measuring equipment such as balance scales to explore and represent equivalence. They are aware of how inverse operations enable them to work out related number facts and solve for unknowns in simple equations involving addition and subtraction, and that other inverse relationships exist.

Students recognise and describe patterns and sequences, and identify whether they involve repetition, or regular increases or decreases. They analyse patterns, determine and describe the rules that apply and continue them, work out elements by considering their place in the pattern, or create similar patterns.

#### Year 3 Measurement, chance and data

Students identify, distinguish and name the attributes of shapes and objects with respect to length, area, volume and mass. They directly compare or measure these attributes using informal units. They use appropriate language when comparing and ordering several objects in relation to the same attribute.

Students identify and use a limited range of metric units (metre, centimetre, litre and kilogram) and select appropriate instruments for measuring. They make reasonable estimates using these units and use various strategies to judge whether a measure is less, about the same as, or more than a given unit.

Students read all times on digital clocks and times to the hour, half-hour and quarter-hour on analogue clocks. They interpret basic calendars, locate specific days and dates, and use effective strategies to work out the details about earlier or future dates.

Students know that some questions and issues, including statements and questions they have created themselves, can best be answered by collecting data. They work out suitable ways to do this, including the use of technology to access existing data, and explore the best ways of organising it. They present the data in ways that assist its interpretation. They make simple statements, including predictions about likelihood, what is possible and what is not, whether variation exists within the set of data or with existing data, and decide whether additional data should be collected.

#### **Year 3 Space**

Students recognise common two-dimensional shapes and three-dimensional objects, describing them using both everyday language and geometric names. They sort and group them using common characteristics, draw sketches and construct reasonable models using a range of materials, drawing tools and other technology. They recognise angles both as parts of shapes and objects, and in turns.

Students distinguish when shapes and designs are symmetrical or not and use strategies such as folding or using mirrors to confirm this. They explore how flips, slides and turns change common shapes and use them to complete simple visual puzzles, to make patterns, and in exploring the characteristics of those shapes.

Students interpret simple maps and plans and identify the most obvious features that have been marked. They make reasonable sketches of familiar local environments such as the school grounds or a particular room. They interpret the language of turns (half, full, quarter, three-quarter) as they follow and give directions for moving around these environments or for locating specific features.

# Year 5 Statements of Learning for Mathematics

### Year 5 Working mathematically

Students actively engage in mathematical inquiry as they explore new mathematics and begin to link this with their existing knowledge. They look for pattern and repetition and try to generalise about various situations. They restate problems or investigations in their own words to ensure they understand what is required or break a task into simpler steps. They select and use strategies and approaches that suit each new situation.

Students identify and interpret some of the symbols and conventions used to represent mathematical situations. They choose and use concrete materials, sketches, diagrams, physical models and a range of mathematical symbols when interpreting and representing these situations, including some simple inequalities. They see the links between different representations of the same situation and use those which make most sense to them.

Students make and test straightforward statements, propositions and conjectures as they explore and attempt to explain patterns and relationships. They reflect on their approaches and conclusions, and describe and generalise about them using specific instances they have observed.

Students identify and describe the mathematical nature of various problems and investigations, and specify the significant aspects of those situations. They communicate about their ideas, findings and approaches, including how they may have used technology. They make judgements about whether they were successful, and compare them with the ways other students dealt with the same situations.

### Year 5 Number

Students recognise and represent whole numbers at least to thousands and decimal fractions at least to hundredths, and use them in familiar contexts such as in measurements. They apply their knowledge of place value to compare and order numbers and place them on number lines. They use concrete materials and technology to represent and explore numbers, and describe the place value changes as numbers (including decimal fractions) are multiplied and divided by 10 and 100.

Students represent and describe simple common fractions and mixed numbers involving denominators to tenths. They use collections of objects, lines and areas of shapes to provide examples of these fractions and to solve practical problems involving fractions. They use reference points such as  $0, \frac{1}{2}$  and 1 to decide where to locate fractions on a number line. They use a variety of concrete models to compare and order fractions, to recognise when they are equivalent, and to assist mental calculations for fractions with the same or easily related denominators.

Students recall addition and subtraction facts, and recall or use suitable strategies to work out multiplication and related division facts. They add and subtract whole numbers to thousands and decimal fractions to hundredths in familiar contexts, and multiply and divide whole numbers by whole numbers to 10. They explore whole numbers by listing all of their factors and identify prime numbers because they have two distinct factors. They apply number properties to modify computations so that they can more easily be carried out, and use inverse operations to solve relevant problems. They choose when to use mental or written methods or technology, and they form quick estimates to check any calculations.

#### Year 5 Algebra, function and pattern

Students recognise and describe relationships and represent them using concrete and pictorial materials, lists, tables or graphs. They analyse simple relationships such as that between the length, width and perimeter of a rectangle, and make predictions based on the information they have. They create their own relationships in relevant situations, and make up their own rules or criteria for sorting, ordering and arranging data and objects.

Students make generalisations associated with the four operations that are built upon properties (commutative, associative and distributive) and inverse operations. They use these to illustrate links between related number facts, to extend their range of mental computations, and to simplify some written computations.

Students interpret problems and other relevant mathematical situations based around a single operation and write equations (number sentences) or make models that represent them. They use various strategies to solve them including the use of measuring equipment to illustrate the concept of equivalence or balance. They use technology where appropriate to assist their reasoning and apply this reasoning to more complicated situations.

Students represent and interpret patterns in number and space. They use materials to model and continue spatial patterns such as those based around triangles, squares and letter patterns. They analyse these and other patterns, describe the rules that describe the pattern, and work out further elements. They are aware of the role that position, such as the 20th term, plays in patterns and attempt to work out ways of predicting terms in given positions.

#### Year 5 Measurement, chance and data

Students measure, compare and order lengths, areas, volumes, angles and masses by selecting and using suitable informal or formal units (millimetres, centimetres, metres, square centimetres, square metres, millilitres, litres, degrees, grams, kilograms). They select appropriate instruments and measure to the nearest whole unit. They arrange a set of measurements of the same attribute in order of magnitude. They make reasonable estimates by applying strategies that suit the situations and the objects concerned.

Students recognise and link the different ways of recording the same measurement such as using metres, centimetres or millimetres. They interpret and read the graduated scales on a range of measuring instruments with respect to the units involved. They understand and use the relationship between the lengths of the sides and the perimeters of irregular and regular polygons.

Students tell the time of the day to the nearest minute using a range of analogue and digital timepieces, and recognise and use a.m. and p.m. to provide greater detail. They are aware that durations can be calculated if starting and finishing times or dates are available, and they use efficient methods when estimating or calculating them. They use their knowledge about times, calendars and timetables (including electronic and digital formats), and timelines to seek specific information or to schedule and sequence events.

Students identify and describe all of the possible outcomes for familiar events involving chance. They make judgments about their likelihood, predict whether some are more likely than others using suitable language including most unlikely, never, probably. They may collect data from experiments or observations to justify or adjust these predictions. They distinguish situations that involve equally-likely events from those that do not.

Students know that data they collect can be used to answer questions or respond to issues that have been raised. They use a range of ways of collecting data including surveys, observations and experiments, depending on the circumstances. They choose suitable tables or graphs including technology-generated graphs to present the information. They use these to support statements or predictions they have made, or to convince them that additional data is required. They look for and describe expected or unexpected variation within the sets of data they use.

### Year 5 Space

Students recognise and name a range of two-dimensional shapes and three-dimensional shapes and objects, and sort them into broad groups according to their main features. They identify and give more specific names to some of the shapes and objects within those groups, such as 'isosceles triangle', because of their particular features. They use appropriate spatial language including parallel, perpendicular, vertex, edge, base, acute, right, obtuse and reflex angles when describing these features.

Students make reasonably accurate representations of known two-dimensional shapes and threedimensional shapes and objects. They are accurate in terms of the essential features such as the number of lines and edges, sizes of angles and whether lines are parallel or not. They sketch representations of objects from different viewpoints and know that the same two-dimensional shapes can be drawn in different orientations. They construct skeletal models of three-dimensional objects using straws or sticks, and solid shapes using clay, play-dough or similar materials. They recognise and construct nets of common three-dimensional objects.

Students identify shapes and designs that are symmetrical (or not), explore basic transformations (flips, slides and turns) of shapes and describe the changes that occur. They experiment with multiple copies of shapes to create patterns and designs, and to identify whether they tessellate. They identify and create two-dimensional shapes that have one or more lines of symmetry.

Students recognise and interpret the symbols and conventions used on different maps, plans and grids to locate key features and landmarks. They use the North symbol, the symbols within the legend and alpha-numeric grids to plan movement around those environments. They understand the relationship between the four major compass points and the amount of turn (quarter, half, three-quarter and full turns) and how these can be used when giving directions. They use simple scales to estimate distances on maps and plans.

# Year 7 Statements of Learning for Mathematics

### Year 7 Working mathematically

Students extend their use of mathematical inquiry and employ a range of investigative, modelling and problem solving strategies and processes, including the use of technology. They develop models, investigate and test propositions, hypotheses and conjectures, and identify key assumptions and conditions that apply to working mathematically in different contexts.

Students pose questions and formulate statements amenable to straightforward mathematical analysis. They choose and use words, mathematical symbols and conventions, diagrams, tables and graphs to develop suitable representations of concepts and relationships and to apply skills and processes in mathematical inquiry. They interpret and evaluate symbols used to represent variables in simple algebraic expressions and formulas. They are aware that representations in mathematics have evolved over time and are familiar with common variations in their use.

Students apply a range of mathematical skills, processes and strategies to make judgments about whether statements are true or false, for particular cases, or in general. They systematically check reasoning in context, follow simple deductions, and use technologies as appropriate to assist them to explore the possible truth of statements. They make generalisations in cases where there appear to be no counter-examples and develop informal arguments to justify generalisations.

Students communicate about their own or collaborative work, informally and formally in verbal or written forms. They present problems, describe the background, ideas and approaches, and report on progress, outcomes or results. They use technology as appropriate to assist mathematical inquiry and in presentation of their work.

#### Year 7 Number

Students use their knowledge of the base 10 number system and its number properties to compare and order sets of positive and negative numbers, and decimal fractions. They use mental, written and technology-assisted methods for addition, subtraction, multiplication including small whole number powers, and division using one- and two-digit whole number divisors. They apply their understanding of the meaning and order of operations when carrying out more complicated calculations, in particular when using technology to assist with computation. They interpret and solve practical problems, using an appropriate sequence of operations and suitable methods when dealing with integers, decimals and percentages.

Students represent and order common fractions and identify families of equivalent fractions, including those expressed in simplest form and as decimals and percentages. They use mental, written and technology-assisted methods to carry out related computations involving addition and subtraction where a common denominator is readily identifiable, and computations involving multiplication and simple division.

Students read and interpret problems that involve simple percentages, proportions, ratios and rates in practical situations including money, time and other measurements. They solve them by choosing and

using a range of strategies and approaches, including the use of technology and their knowledge of the relationships between whole numbers, decimal fractions, percentages and common fractions.

Students form estimates for calculations involving whole numbers, decimal fractions and common fractions using their knowledge of number systems and single digit mental calculation. They make estimates about the magnitude of answers based on powers of ten, and rounding when carrying out numerical approximations in a range of practical situations.

#### Year 7 Algebra, function and pattern

Students use words, diagrams, materials and symbols to represent variables and to write expressions and relationships including formulas and equations. They read and interpret representations of practical and other situations such as simple formulas, describe them with the assistance of suitable materials and diagrams, and evaluate expressions for whole number and simple fraction values.

Students use a variety of approaches, including words, materials, diagrams and symbols, to represent, manipulate and re-arrange simple algebraic expressions that involve the operations of addition, subtraction and multiplication. They explore general number properties and apply these to computation, and identify and establish equivalences between linear expressions.

Students determine when linear and other simple algebraic equations involving the operations of addition, subtraction and multiplication are satisfied or not for a given set of values. They use a variety of approaches to solve these equations including the use of materials, flow charts, tables, graphs, inverse operations and algebra, and explain their reasoning.

Students construct tables of values for functions defined by simple rules, using whole number values as inputs, and plot the corresponding set of ordered pairs, including the use of technology. They interpret simple functions, the set of input values used and the set of output values obtained, in context.

#### Year 7 Measurement, chance and data

Students choose and use metric units and are familiar with International System (SI) units and the relationships between units. They select appropriate instruments and other technology when measuring, including those involving scales where not all of the graduations are numbered. They use these to measure and compare the magnitudes of lengths, areas, volumes, masses, angles, times, including those on 24-hour clocks, schedules, timelines and time elapsed, and temperatures as applicable to various objects and events.

Students understand that all measurement involves error, and describe a reasonable range of values for a given measurement. They make estimates of quantities with respect to common everyday measures within a given range.

Students develop and use simple formulas to calculate perimeter, area, surface area and volume of common regular shapes from the relevant measured linear dimensions, and apply these to practical problems. They make judgments about the reasonableness of results obtained using formulas. They carry out calculations that involve 12-hour and 24-hour time cycles, duration of events and schedules in practical situations, and take into account time zones.

Students comprehend that many events, in familiar situations, have different likelihoods of occurrence, and make and interpret empirical estimates of probabilities related to these events. They compare experimental data for simple chance events with theoretical probability obtained from proportions expressed as percentages, fractions or decimals between 0 and 1, based on counting or area. They distinguish events that are equally likely from those that are not.

Students identify data as discrete or continuous, and use a variety of representations including twoway tables to summarise sample data obtained from a given population. They use frequency, relative frequency and choose suitable measures of location (mean, median, mode) as summary statistics to describe the distribution of sample data from a given context.

Students analyse and comment on data related to a particular situation, issue or topic of interest. They identify and interpret variation in the available data, calculate and compare the measures of location, and make informal inferences, noting possible causes of bias.

#### Year 7 Space

Students identify, describe and classify common two-dimensional and three-dimensional objects and geometric shapes with respect to properties involving line, length, angle and surface using everyday language and geometric conventions. They identify and describe the properties of part and composite shapes.

Students sketch by hand representations of common two-dimensional shapes and three-dimensional shapes and objects with attention to their geometric properties. They use drawing instruments and software to construct accurate representations of two-dimensional shapes according to specification. They construct three-dimensional objects from plans, nets and isometric diagrams.

Students recognise congruence of shapes where one shape can be superimposed on another through a sequence of transformations (reflections, rotations and translations), and similarity of shapes, where one shape is an enlargement or reduction of another. They identify points, lines and planes of symmetry in shapes and objects, and relate these to transformations and tessellations of suitable shapes in the plane.

Students use grids and simple coordinate systems, major and intermediate compass points, and the corresponding degrees of turn, straightforward scales, distance, and annotations such as arrows, to interpret and construct simple maps and plans to specify location. They provide and follow instructions for moving from one location to another based on maps or plans, and use the scale to estimate or calculate distances between locations.

### Year 9 Working mathematically

Students develop the breadth and depth of their mathematical inquiry in familiar and unfamiliar situations, and choose and use a broad range of strategies and processes, including technology. They identify and describe key features of a context or situation for investigation, plan and carry out inquiries, stating key assumptions and conditions. They compare different models for a given context, make predictions, solve problems and reflect on solution methods, carry out mathematical investigations, and interpret their work in the original context.

Students pose questions and formulate propositions, conjecture and hypotheses amenable to mathematical analysis. They choose and use appropriate mathematical symbols and notations, diagrams, tables, graphs, variables, relations, and equations, to represent concepts and relationships, to apply skills and processes, and to clarify, modify and refine statements. They understand that mathematics has been refined over its historical development across cultures and explore different approaches to problems.

Students apply a broad range of mathematical and logical skills, processes and strategies as they make deductions, and verify and generalise their reasoning. They seek counter-examples or explore proofs to verify the truth, or otherwise, of various mathematical propositions, conjectures and hypotheses. They use technology to explore pattern and structure and hence develop generalisations for further consideration.

Students communicate about their own and collaborative work, informally and formally in verbal and written form. They attend to the nature, purpose and scope of the communication, and describe background, ideas and approaches used as they report on progress, outcomes or results. They use technology as appropriate to assist mathematical inquiry and in presentation and discussion of their work.

#### Year 9 Number

Students work with fractions, decimal numbers and percentages. They are familiar with different representations of numbers, including the identification of prime factors and the use of scientific notation for very large or very small numbers in practical situations. They readily recognise and use the most suitable equivalent form when comparing and ordering rational numbers.

Students work with irrational numbers related to lengths arising in space and measurement problems. They locate integers, rational numbers and decimal approximations to some irrational numbers, on the real number line.

Students apply a range of number facts, properties and strategies to carry out computations involving integers and rational numbers for the four arithmetic operations. They use mental and written methods when dealing with simple powers and square roots, and use technology as appropriate. They apply the relevant operations, with attention to the meaning and order of the operations involved, in practical and theoretical situations.

Students are familiar with rational numbers in different forms and use these to formulate and solve ratio, proportion, percentage and rate problems, using mental, written and technology-assisted methods. They interpret irrational numbers arising in space and measurement contexts geometrically, and calculate with these numbers using decimal approximations and technology.

Students use a range of strategies to form estimates for computations involving rational and some irrational numbers. They form upper and lower bounds for estimates, and round values correct to a suitable level of accuracy with respect to the context for computation.

#### Year 9 Algebra, function and pattern

Students use words and symbols to represent variables and constants when writing expressions for algebraic relations and functions, including linear functions, involving arithmetic and other mathematical operations. They use technology as appropriate to evaluate these expressions which include simple inequalities, using integer, decimal and fraction values of variables. They model and interpret the expressions and relationships in context, and use known relationships such as formulas to develop new relationships.

Students construct tables of values for linear and some simple non-linear functions using integer, decimal and fraction values of variables. They draw graphs of these functions in all four quadrants using technology as required. They interpret and apply variables and functions in context and make related predictions.

Students select and apply the identity, inverse, associative, commutative and distributive properties to manipulate and re-arrange algebraic expressions that involve the four arithmetic operations, reciprocals, whole number powers and square roots. They use a variety of approaches such as concrete materials, technology, algebra, diagrams, flow charts and backtracking to identify and establish equivalences between linear expressions and between simple non-linear expressions.

Students determine when equations and inequalities are satisfied or not for a given set of values. They construct and solve linear equations including simple simultaneous linear equations, and some non-linear equations using tables, graphs, algebra and technology.

Students draw graphs for families of linear and some simple non-linear functions generated by changing constants used to define the rule of the function. They use technology to explore and describe the effects of varying these constants and interpret the shape of a graph and its key features in context.

#### Year 9 Measurement, chance and data

Students work routinely with International System (SI) and other units with respect to both everyday and technical measurement contexts, including derived measures, and choose units appropriate to the order of magnitude involved. They use instruments, technologies, strategies and formulas to estimate or calculate (as appropriate) various measures including mass, duration, temperature, angle, and simple derived measures such as rates. They recognise equivalent forms of the same measure, making conversions as required, and apply Pythagoras' theorem, scale and rates in appropriate situations to work out measures. Students record a measurement as a value that lies within a given interval of measurement error and make judgments about acceptable or reasonable error in a measurement context. They estimate values that lie between marked graduations on scales of measuring instruments. They understand that error can be compounded by repetition and calculation.

Students use a variety of sources, including samples and surveys, published data, data-bases, experiments and simulations to estimate probabilities associated with events. They assign, or make estimates of, probabilities based on personal experiences.

Students specify sample (event) spaces for single and straightforward compound events using a variety of suitable representations. They determine corresponding probabilities using counting, measure and symmetry. They are familiar with the notion of equally likely events, and the use of random, or nearly-random, event generators, including technology.

Students choose and use a variety of suitable representations and descriptive statistics to summarise (with the assistance of technology for larger data sets) and interpret discrete and continuous data obtained by random sample from a population. They explore how bias can arise, and determine the effect of outliers on the measures of location.

Students use proportions, simple measures of spread and centre (location), and informal consideration of the distribution of data to make informal inferences in response to their own and others' questions and hypotheses. They critically analyse articles in the media that make use of statistics in an attempt to support a case or argument.

#### **Year 9 Space**

Students identify, describe and classify a broad range of two-dimensional shapes and three-dimensional shapes and objects and composite shapes, including those with curved surfaces, with respect to properties involving line, length, angle and surface using everyday language and geometric conventions.

Students draw by hand representations of common two-dimensional shapes and three-dimensional shapes and objects (and their cross-sections) with attention to their geometric properties and scale. They use drawing instruments and software to construct accurate representations of two-dimensional shapes and three-dimensional shapes and objects according to specification. They construct three-dimensional objects from plans, cross-sections, nets, isometric and perspective diagrams.

Students use congruence, similarity and sequences of transformations to analyse the geometric properties of shapes and patterns. They make deductions about the geometric properties of shapes and objects, and follow simple geometric proofs including those related to angle properties associated with parallel, perpendicular and transversal lines, and polygons. They relate symmetry to transformations, and tessellate suitable regular shapes and composite shapes, in the plane and on surfaces.

Students interpret, construct and use maps, diagrams and plans to specify location, represent relationships spatially, and to move from one location to another. They use grids, coordinate systems, bearings, scale, distance, angle and various keys, references and annotations as applicable to the context, to interpret and construct these maps, diagrams and plans.

# Year 3 Professional Elaborations – Opportunities to Learn for Mathematics

#### Year 3 Working mathematically

Students engage in simple mathematical inquiry and see the mathematics in everyday situations. They conduct experiments and seek additional information to assist their inquiries. They use simple strategies to identify relationships, change and pattern in a variety of situations where mathematics is involved.

Students have the opportunity to:

- recognise and describe simple relationships and change (eg properties for classifying shapes, a sequence of shapes being rotated through a quarter turn, a set of numbers that has been doubled)
- experiment with ways of changing numbers or shapes (eg *adding or subtracting* 10 *from whole numbers, using materials to make one angle in a triangle larger and larger*), recognise the effect of the change, predict further changes and make suggestions that lead to further experimentation
- use simple strategies such as recognition of similarity, difference and repetition, to identify and describe regularity and pattern (eg *explore the symmetry of triangles using paper folding or computer software, and notice that only those with two or three equal sides are symmetrical*).

Students interpret and use some of the different representations associated with mathematics, including concrete materials, words, with drawings and diagrams, and a basic collection of mathematical symbols. They pose and respond to questions related to these representations.

Students have the opportunity to:

- interpret problem situations and express them mathematically by drawing simple diagrams, writing number sentences, making sets of things, lists, tables, geometric constructions and by using concrete materials that assist with the solution (eg *How many teams of eight can we make from the children in our class? Which pet is most common? How can we arrange the desks to create a bigger mat area?*)
- interpret various ways of representing problem situations and investigations (eg *interpret a representation such as* ◊ + Δ = 20 *by asking 'how many ways can you make* 20 *using addition?'*) and pose related questions (eg *what if the representation becomes a subtraction situation such as* ◊ Δ = 20?)
- interpret number sentences and describe them using their own words (eg *describe number sentences like* □ ÷ 2 = 12 by saying 'what number do I halve to get 12?' Write a word problem that can be represented by 45 17 = □).

Students attempt to explain and justify their reasoning to others as they work on problems using a range of approaches. They reflect on their findings or solutions, and describe some solutions more generally. They check their reasoning and the results of their work with respect to the original situation.

Students have the opportunity to:

• analyse simple mathematical statements, and choose methods of testing them including the use of technology (eg *use a calculator or spreadsheet to support their reasoning about the pattern of adding 9 to any two-digit number*)

- check their reasoning and work by ensuring the reasonableness of the results with respect to the original problem (eg check mental or written calculations by doing them again, by making estimates or by using an inverse operation)
- make simple predictions or conjectures, think through and choose ways to test them, and attempt to convince themselves and others about whether they are true (eg *the length of a person's shadow changes most quickly later in the day*).

Students discuss the mathematical nature of problems by locating relevant information and expressing this mathematically. They talk about their work, describe what they are doing and how they are approaching it.

Students have the opportunity to:

- describe the relevant mathematics in problems and express it in ways that assist with solution (eg *describe and demonstrate with materials, a general process or rule that works out any odd number such as the 50th odd number*)
- explain why a particular approach has been taken (eg *make an organised list to find all of the pairs of whole numbers that add to a given number, using technology where appropriate*) when responding to mathematical questions
- describe their interpretations of problem situations and investigations in their own words and identify strategies (eg *guess and check, draw a diagram, solve a simpler problem*) that could be used to solve those situations.

#### **Year 3 Number**

Students recognise, represent, count and order whole numbers at least to three digits and recognise different representations of some simple common fractions in everyday life.

Students have the opportunity to:

- represent numbers in various ways (eg using symbols, concrete materials, calculators and number expanders)
- use place value to explain why one number is larger or smaller than another, and arrange up to four numbers in order of size (eg *use materials to show that any four-digit whole number must be larger than any three-digit whole number*)
- count collections by 1s, 2s, 5s and 10s and make given numbers larger or smaller by 1, by 10 or by 100
- recognise and link symbolic representations of the same number (eg 317 *is three hundred and seventeen or* 300 + 10 + 7; *nine hundred and nine is* 909 *or* 9 *hundreds*, 0 *tens*, 9 *ones*)
- place numbers, including 0, on a number line and explain their reasoning (eg say why 70 is placed on the number line between 50 and 100 but closer to 50)
- recognise and represent halves and quarters using collections (eg counters or toys)
- identify when partitions of linear and area models show halves and quarters and name the common fractions or mixed numbers shown (eg *identify half- and quarter-hour segments on clock faces*).

Students recall or use strategies to work out and extend the addition and subtraction facts. They represent and solve simple multiplication and division situations and problems.

Students have the opportunity to:

- recall single digit addition facts and the related subtraction facts, or use efficient strategies such as doubles, think of addition or count back to work them out (eg *work out the subtraction facts related to a given addition fact* 6 + 2 = 8, 8 2 = 6, 8 6 = 2)
- use known strategies and results to assist mental computation involving the addition and subtraction of one- and two-digit numbers (eg solve 60 23 *mentally by first taking away 20 to get 40 then taking away 3 to get 37, knowing that 26 + 9 = 35 work out related examples such as 26 + 19, 26 + 29*)
- choose between mental and written methods, including the students' own written recordings, when adding and subtracting one- and two-digit numbers and make estimates to check these calculations
- use arrays and other grouping strategies to represent and solve multiplication situations, involving single digit numbers, and show how these also relate to division
- skip count by 2s, 5s and 10s, use technology and materials such as hundreds boards to support skip counting by other numbers and show how skip counting relates to multiplication
- model and solve both sharing and grouping division situations and problems involving single digit divisors using concrete materials
- use technology to assist in the exploration and development of mental computation strategies and for single operations involving two-digit or three-digit numbers.

Students identify and distinguish situations and problems that require addition, subtraction, multiplication and division.

Students have the opportunity to:

- interpret problems based around a single operation, decide which operation is required and represent it using concrete materials, sketches, technology or a combination of these (eg *recognise that a problem involves aspects such as difference, equal groups or sharing, link it to the relevant operation and use appropriate materials to represent it*)
- create problems based around a selected operation and identify similar problems based on the same operation
- explain their methods for solving problems, and compare and discuss other methods for solving the same problems.

### Year 3 Algebra, function and pattern

Students recognise, describe and use simple relationships.

Students have the opportunity to:

- establish simple correspondences between sets (eg students in the class and their favourite colour)
- order sets and lists of things and explain reasoning used (eg arrange a list of names of students in a class in alphabetical order, use place value to order a set of three-digit numbers)
- create and follow sequences of actions and instructions (eg follow a set of instructions and use the constant addition function on a calculator or other technology, follow a tourist guide around a precinct, create 'think of a number' problems or the rules for a simple dice game).

Students identify and describe relationships such as inverse and equivalence in a variety of ways.

Students have the opportunity to:

- express simple relationships in their own words (eg 'whatever number you say, I'm going to double it')
- record data in tables and on graphs, notice simple relationships and make appropriate comments (eg plot the hourly temperature during the day on a simple scale and observe that it rises between 8 in the morning and 12 midday, then stays the same until school finishes for the day)
- use visual images to identify and describe some attributes of equivalence in measurement and spatial situations (eg compare objects and decide whether they have the same mass or length regardless of shape, select shapes from a collection that have the same number of straight sides and name the families)
- use materials such as arrays or balances to identify and describe equivalence in number situations (eg show why 14 + 8 can be changed to 12 + 10 without affecting the equivalence, use coloured materials to show the equivalence of number expressions such as 5 + 6, 9 + 2 and 3 + 4 + 4)
- explore situations where inverse operations can be applied and describe how inverse relationships apply to other situations and problems (eg *interpret*  $13 = \Delta + 8$  *as a subtraction situation, use a* 4 *by* 3 *array to work out related multiplication and division facts, observe that more small squares are needed to cover the same area as* 12 *large squares*).

Students identify and describe patterns and sequences that show increase, decrease and repetition, and create and continue patterns and sequences.

Students have the opportunity to:

- analyse spatial arrangements and patterns, and describe the repeating elements or changes between elements (eg *study a growing matchstick pattern and identify the change from one element to the next; state the rule for a pattern that is two squares after every three hexagons*)
- analyse, describe and create simple patterns and make general statements and predictions about them (eg {*red, green, green, red, green, red ...*} *and say that if there are* 10 *green, there will be at least* 5 *reds, and that the* 12th *element will be green*)
- analyse number sequences, generalise about changes between elements and continue them (eg *constant increase* {2, 5, 8, 11 ...}, *constant decrease* {91, 86, 81, 76 ...}, *changing differences* {5, 6, 8, 11, 15 ...}).

#### Year 3 Measurement, chance and data

Students identify and distinguish the attributes of shapes and objects with respect to length, area, volume and mass. They directly compare, measure using informal units, and order a set of objects according to a specified attribute.

- use appropriate language to describe length, width and height attributes of objects, and the distance between two points, and measure this attribute using informal units such as paces, string and straws
- explore area as coverage of surfaces and use a range of informal units to measure the area of various surfaces (eg *measure the top of a desk with A4 sheets or cover a basketball with cupped hands*)
- use suitable strategies to measure how much a container holds including the use of informal units (eg compare and order up to three containers according to their volume)
- use appropriate language (eg *light, heavy*) to describe mass and use hefting and a range of informal units on simple balances when comparing the mass of different objects
- make direct comparisons between objects for a given attribute (eg *arrange a group of people from smallest to largest by visually comparing their heights*).

Students choose and use the metric units of metre, centimetre, litre and kilogram. They estimate measurements, compare measurements, and use appropriate instruments to measure to the nearest unit.

Students have the opportunity to:

- choose and use the appropriate unit and instrument to measure different lengths (eg measure to the nearest centimetre using a ruler marked and numbered in centimetres and select a trundle wheel to measure length in metres)
- choose appropriate instruments to measure and compare the mass of objects (eg *use balance scales to compare a range of objects with a 2-kilogram bag of rice*)
- decide whether containers hold less, about the same or more than a litre (eg *use a one-litre measuring jug to fill, calibrate and compare the volume of other containers*)
- make reasonable estimates of length, volume and mass using appropriate strategies (eg *three of my normal steps make 2 metres*).

Students read and say times and dates and apply these to events in their lives.

Students have the opportunity to:

- read o'clock, half and quarter hour times on a range of analogue clocks and read the time on digital clocks (eg *know that they will be picked up after school at about half past three or* 3:30)
- use calendars to identify specific information about days and dates (eg *identify the dates of every Tuesday in a month; identify the date that is a week later or earlier than a given date*).

Students collect and analyse data and present the data in different ways. They make a range of statements based on their experiences, observations and sets of data including events that are likely or unlikely, and decide whether additional data should be collected.

Students have the opportunity to:

- pose their own questions and raise issues of interest, and know that some questions are best answered and some issues explored by collecting appropriate data
- identify ways of collecting data and analyse, organise and present data using a suitable format for the context under consideration (eg *lists, tallies, tables and simple graphs such as pictographs*)
- use technology to access data and to assist them to record and present their data (eg *access CensusAtSchool data, bar graphs and spreadsheets*)
- make qualitative judgements about data obtained from observations or experiments, and explain whether it supports or disagrees with a particular view using the appropriate language of chance (eg *Do heavy black clouds on the horizon mean that it is likely to rain here today? Say which totals are more likely to occur when two dice are tossed and the numbers added*)
- identify and describe variation between and within sets of data especially as seen in graphs (eg boys' and girls' views about sport, two packets of jelly beans having different numbers of black beans).

#### **Year 3 Space**

Students recognise and describe familiar two-dimensional shapes and three-dimensional shapes and objects. They identify them within nature and the built environment, and represent them in various ways including with the assistance of technology.

Students have the opportunity to:

- identify and describe families of three-dimensional shapes and objects including prisms, pyramids, cones, cylinders and spheres, and make models and sketches of objects they can see or handle, illustrating their key features
- identify common two-dimensional shapes including squares, rectangles, triangles and circles, draw them using technology when appropriate, and distinguish the families by their characteristics
- identify and describe the differences between two-dimensional shapes and three-dimensional objects and also how they are linked (eg *two-dimensional shapes have two dimensions but no depth; three-dimensional objects have length, width and depth, and their surfaces are two-dimensional shapes which may be flat or curved*)
- recognise angle in shapes and objects and in turns (eg *a slice of pizza, corners of a box, opening and closing a book or a door*).

Students recognise line (reflection) symmetry in the environment and also where there is none. They use simple transformations to manipulate shapes.

Students have the opportunity to:

- use mirrors, folding and other techniques to explore and identify line symmetry in a variety of shapes
- explore and describe the effect of a single flip, slide or turn on a range of shapes
- use symmetry and/or transformations to create or continue patterns including tessellations.

Students use simple maps, grids and plans of familiar environments to identify pathways and specific locations.

- identify the key features of simple maps, grids and plans (eg *describe the way roads, parks, buildings, doors and other features are marked or shown*), follow directions for moving around and find specific locations
- make sketches of and interpret maps of generally familiar environments, and give directions for moving from one point to another
- use and interpret the language of turns (eg *half, full, quarter, three-quarter, left, right, clockwise, anticlockwise*) when giving or following directions (eg *walk out the door and make a quarter turn to the right*).

# Year 5 Professional Elaborations – Opportunities to Learn for Mathematics

#### Year 5 Working mathematically

Students engage in mathematical inquiry, and build new knowledge through exploring problems and investigations in familiar and some unfamiliar situations. They identify and use suitable strategies that assist their solution.

Students have the opportunity to:

- use different strategies such as looking for consistent change to identify and generalise about regularity and pattern (eg *use diagrams to explore whether* 0.27 *is less than* 0.5; *use the constant function on a calculator to multiply and divide whole numbers and numbers with decimal fractions by ten, and see whether a pattern results*)
- choose and apply strategies suited to the mathematical structure of various problems and investigations (eg *use a hundreds board to identify all whole numbers to* 100 *that have a remainder of* 1 *when divided by* 4,{1, 5, 9, 13, ..., 97}), and use this knowledge to pose related problems
- recognise connections between mathematical ideas and use this knowledge to describe situations more simply, to break tasks down into more manageable steps, or to restate problems in their own words.

Students recognise and interpret some mathematical symbols and conventions, use these to represent a variety of situations, and pose and respond to questions arising from them. They use concrete materials, sketches, diagrams, pictures and physical models to represent appropriate mathematical concepts or to illustrate particular processes, including those related to combinations of operations and simple inequalities. They make links between different representations of the same situation and choose those which make the most sense to them and are best suited to current circumstances.

Students have the opportunity to:

- represent problem situations and investigations with appropriate materials and use them to justify their findings (eg *use grid paper to work out different rectangles that have the same perimeter and record the findings in a table*)
- interpret and explain a range of mathematical symbols associated with equality (eg 23 × 10 = 230 also means that 230 ÷ 10 = 23), with unknowns (eg 200 [] = 151) and with simple inequalities (eg *identify whole numbers that fit* 12 × 5 < [] or 15 + 14 > [] × 4 and investigate whether all solutions have been found)
- create mathematical situations or pose questions that match given number sentences (eg *interpret*  $\Box \div 7 = 14$  by 'how many days would be equal to 14 weeks?'), write number sentences to match given problems (eg *even if I save* \$12 *each week for the next year I won't have enough money to buy the bike I want or* \$12 × 52 < *cost of bike*) and pose similar questions of their own.

Students make and test a range of straightforward statements, propositions and conjectures as they engage with mathematical situations, including where they investigate and identify relationships, change and pattern. They attempt to explain and justify their observations and ideas, and make generalisations where these are appropriate.

Students have the opportunity to:

- make and test statements about relationships, using technology as appropriate, and explain why they are true or false or whether more examples need to be investigated to make a decision (eg *test the statement that any number can be divided by* 5 *by doubling it and dividing by* 10)
- make and test simple conjectures (eg *that every rectangle made using the same number of identical squares has the same perimeter*), and explain the approach taken and the conclusions reached (eg *say that the perimeter becomes smaller as the length and width become closer to each other in size*)
- generalise about mathematical situations with respect to key attributes and explain how the generalisation was reached (eg *classify shapes as prisms in terms of the defining features, solve logic problems based on combinations of attributes, identify whole numbers that satisfy certain properties such as primes or perfect numbers*)

Students describe the mathematical structure and requirements of problems and investigations. They experiment with different approaches including the use of technology, and compare their approaches with those of other students. They communicate and justify their findings in appropriate ways.

Students have the opportunity to:

- describe the structure of a problem, nominate other problems that have the same structure and explain why they are the same (eg see the connection between a problem about a collection of 25 coins, all 5- and 10-cent pieces worth \$2, a problem about a farmyard of sheep and chickens where there are 20 animals and a total of 56 legs, and a problem about using a 3 L and a 5 L container to make up various volumes such as 22 litres)
- describe approaches used successfully within mathematical situations and compare these with other effective approaches (eg *investigate ways of reducing the complexity of computations such as* 505 198 *including by making both numbers larger by* 2 *to give* 507 200, *discuss other possibilities such as reducing both numbers by* 5 *and use an 'add on' strategy*)
- explain reasoning and the sequence of steps used to interpret and solve problems and how they have checked results (eg there are 22 students in my class who play tennis and 19 who play basketball. This is more than the total of 30 students in the class, meaning some must play both sports, but some might play neither sport. I need more data and I'll experiment with using a diagram to present the information; explain a strategy for winning a game for two players which starts at 50, players elect whether to subtract {1, 2, 3 or 4} from the previous number in the sequence, the player who finishes on 1 wins the game)
- communicate their ideas, suggestions and findings to others using informal reports, concrete materials, technology, diagrams and graphs.

#### Year 5 Number

Students recognise, represent, count and order whole numbers at least to thousands and decimal fractions at least to hundredths in familiar contexts.

- use a variety of manipulatives and other materials to model and compare different representations of whole numbers and decimal fractions
- use place value to compare and order numbers and locate them, relative to zero, on a number line
- multiply and divide numbers by 10 and 100 mentally and using technology, and describe the changes using models such as a place value chart (eg *use technology to multiply* 1.5 *repeatedly by* 10, *record each change on a place value chart and describe the pattern of the changes*)

- recognise different representations of numbers involving decimal fractions (eg *recognise* 2.12 as  $2 + \frac{12}{100}$ ,  $2 + \frac{1}{10} + \frac{2}{100}$ , 2 + 0.1 + 0.02, *two and twelve-hundredths*) and explore related contexts involving money and measures (eg \$2.12 and 2.12 m)
- illustrate and explain the connection between whole numbers and decimal fractions using relevant contexts (eg *use the scale on a tape measure to assist counting forwards by* 0.05 *m* (5 *cm*) *from a given length* {0.90 *m*, 0.95 *m*, 1.00 *m*, 1.05 *m*, 1.10 *m*...}) and use technology or other appropriate means to support reasoning.

Students represent and describe simple common fractions to tenths using a range of models and link fractions to practical situations.

Students have the opportunity to:

- identify equal partitions within models of fractions and name the fractions shown
- use number lines to identify suitable reference points including those for consecutive whole numbers and midpoints to locate numbers involving common fractions with reasonable accuracy (eg  $\frac{3}{5}, \frac{4}{8}, \frac{7}{3}, 3\frac{1}{4}$ )
- interpret symbolic representations and use concrete representations to compare and order common fractions (eg *illustrate with diagrams and number lines why given fractions such as*  $\frac{2}{5}$ ,  $\frac{1}{3}$  *and*  $\frac{5}{8}$  *are more or less than*  $\frac{1}{2}$ ) including when fractions are equivalent
- use area, set and linear models such as fraction walls, arrays and number lines as well as simple equivalences to perform mental calculations with common fractions (eg *work out*  $\frac{1}{4} + \frac{1}{2}$ ;  $\frac{5}{8} \frac{3}{8}$ )
- recognise and use fractions in everyday and practical situations (eg *explain when the third quarter in a game of netball or football will occur, work out that if the third quarter goes for 20 minutes, the whole game should go for 80 minutes*).

Students recall addition and subtraction facts, work out multiplication and related division facts, and apply number properties and mental computation strategies to larger numbers. They add and subtract whole numbers to thousands and decimal fractions to hundredths in familiar contexts, and multiply and divide whole numbers by numbers up to 10.

- recall or calculate mentally addition and multiplication for any pair of whole numbers to 10, and decide when to apply the corresponding inverse operation
- identify factors and multiples of some two- and three-digit numbers (eg say that the factors of 80 are {1, 2, 4, 5, 8, 10, 16, 20, 40, 80}, say that the first six multiples of 25 are 25, 50, 75, 100, 125 and 150)
- investigate prime numbers, determine that they have exactly two factors and identify the prime numbers at least to 20
- apply the commutative, associative and distributive properties to assist calculations and choose whether to use mental, written or technology-assisted methods or a combination of these (eg 47 + 95 + 13 = 47 + 13 + 95 = 60 + 95 = 155, calculate 4 lengths of 3.25 metres as 3.25 m × 4 by writing 3 m × 4 makes 12 m and 0.25 m × 4 makes 1 m more, total 13 m) and discuss the method used
- read and interpret practical problems, identify appropriate operations to use, express them mathematically and solve them (eg *recognise that a problem involves repeated subtraction, link it to division and use a suitable method to solve it*)
- form reasonable mental estimates to computations involving a single operation (eg 45.3 + 6.8 + 190 *is about* 50 + 200 or 250, 85 ÷ 8 *is about* 10).

### Year 5 Algebra, function and pattern

Students recognise, represent, describe and use relationships in a variety of ways.

Students have the opportunity to:

- use lists, tables or graphs to represent, analyse and predict change (eg *analyse a table showing the price of petrol at a particular petrol station every morning this month and predict whether some days next month might have lower prices than other days*)
- explore, describe and contrast simple relationships using concrete and pictorial materials where appropriate (eg *describe changes in the perimeter of a square when the side length increases (or decreases) and contrast these with changes in the area)*
- order sets of things according to a given condition (eg order of arrival of participants at an event)
- identify or create criteria for arranging and sorting information, and use these to make decisions in practical contexts (eg *sorting data, refining a web search*).

Students make generalisations about arithmetic operations and expressions based on the inverse, commutative, associative and distributive properties.

Students have the opportunity to:

- use inverse operations to make links between multiplication and division facts and to check computations (eg *use* 23 × 7 *to check* 161 ÷ 7)
- use known facts to work out extended calculations and check ideas, where appropriate, using technology (eg 14 8 = 6 to work out and explain 44 8 and 104 8)
- use a range of strategies including the number properties to simplify, manipulate and calculate expressions (eg 7 × 4 × 5 *is the same as* 4 × 5 × 7 *and* 72 ÷ 3 *is the same as* 60 ÷ 3 *plus* 12 ÷ 3 *or is the same as* 12 × (6 ÷ 3))
- generalise about the operations and make changes to computations that maintain equivalence but reduce the complexity to enable mental computation (eg *change* 24 ÷ 5 to 48 ÷ 10, *change* 3000 – 1564 to 2999 – 1563, *change* 35 × 4 to 7 × 5 × 4 to give 7 × 20, develop and use rules for adding or subtracting numbers such as 98 or 99 mentally).

Students write and solve simple equations (number sentences) arising from problem situations and from models, pictures and other materials.

- interpret arrays and write equations that match them (eg for 20 counters in a  $4 \times 5$  array, write  $4 \times 5 = 20$  and other interpretations such as two lots of  $2 \times 5$  make 20 and  $(3 \times 5) + (1 \times 5) = 20$ )
- use measuring equipment such as balance scales or other materials to demonstrate and explore equivalence (eg *use tape measures or arrays of counters to show that*  $3 \times 12 = 6 \times 6 = 4 \times 9 = 36$ )
- discuss strategies used to solve simple equations, including the use of technology to assist reasoning, and apply them to more complex situations (eg *reason that*  $4 + \square = 7$  *can be solved using subtraction, and use the same reasoning with*  $26 + \square = 70$ )
- develop various strategies such as writing simple equations that can be used when solving problems and finding unknowns (eg use division, known number facts or guess and check with multiplication to solve x 4 = 120, think of a number, add 7, multiply by 4 and finish with 60, what number did we begin with?).

Students represent, interpret and analyse numerical and spatial patterns.

Students have the opportunity to:

- use materials to represent number and spatial patterns (eg use blocks to model and identify triangular and square numbers and use matchsticks to develop growth patterns such as those based around letters)
- specify elements in a sequence or pattern in terms of their position (eg for the sequence  $\{3, 6, 9, 12, 15 \dots\}$  the 20th term will be 60 as  $20 \times 3 = 60$ )
- use simple strategies to analyse and/or continue patterns including the relationship with previous terms, alternating terms and repetition (eg {0, 1, 1, 2, 3, 5, 8 ...}, {1, 3, 9, 27, 81 ...}).

#### Year 5 Measurement, chance and data

Students estimate, measure, compare and order lengths, areas, volumes, angles and masses selecting and using suitable informal units and formal units (millimetres, centimetres, metres, square centimetres, square metres, millilitres, litres, degrees, grams, kilograms) and appropriate measuring instruments and scales.

Students have the opportunity to:

- choose the appropriate attribute when comparing objects or solving practical problems (eg *clarify what is meant by 'biggest' when asked to identify the biggest container*)
- measure and compare different lengths and identify the precision required for the context (eg choose centimetres in preference to metres for measuring the height of a door frame and know that millimetres could be used if greater accuracy is necessary; read 64 mm on a ruler where the scale is marked in millimetres but numbered in centimetres)
- measure and compare areas of surfaces using a range of units such as grids of square units, A4 sheets of paper, square centimetres and square metres (eg making sure there is enough material to cover a library book, identifying which classroom needs the most carpet tiles)
- measure and compare volumes of liquids using appropriate instruments and units (eg measure doses of medication to the nearest 10 millilitres using a dropper or medicine glass with an appropriately graduated scale)
- measure and compare the masses of different objects using appropriate instruments and units (eg use a pan balance to measure different amounts of flour in grams to make several different recipes; read the mass shown on kitchen scales to the nearest 100 grams)
- arrange recorded measurements in increasing or decreasing order of magnitude (eg *know that* 1.9 *kilograms is more than* 1700 *grams and less than* 2000 *grams*)
- estimate, measure and compare angles to the nearest 10 degrees
- use known measures to make reasonable estimates of length, area, mass and volume (eg *own height, pace and hand span, area covered by their hand, mass of a margarine container, volume of drink cans*).

Students recognise different ways of reading and recording metric measures, and understand and use the relationships between perimeters of polygons and the lengths of their sides.

- identify and link different forms of recording metric measures
  - (eg 2.5  $L = 2500 \ mL$ , 1.5  $m = 150 \ cm = 1500 \ mm$ ,  $3\frac{1}{4} \ kg = 3.25 \ kg = 3250 \ g$ )
- interpret and read measures from a variety of scales associated with measuring instruments (eg *identifies the measures shown by the marked graduations on a litre measuring jug*)

• investigate perimeter as a measure of boundary of a closed shape, determine the perimeters of polygons and identify special cases where shortcuts can be applied (eg *measure the length of one side of an equilateral triangle and multiply it by 3 to find the perimeter*).

Students identify the time of day to the nearest minute, and use efficient methods to calculate the duration of specific events. They use clocks, calendars, timetables and timelines to sequence events.

Students have the opportunity to:

- use a range of analogue and digital timepieces to tell the time to the nearest minute and recognise equivalent forms of saying and recording the time (eg 9:56 *a.m. is nine fifty-six or four minutes to ten in the morning*)
- interpret and use a range of calendars, timetables, including electronic and digital formats, and timelines to record and locate specific information (eg *dates occurring in eight weeks, locating information about past events, organising a schedule of forthcoming events, illustrating sequences of events over time*)
- use efficient strategies to identify and calculate durations of specific events including those lasting minutes, hours, days, weeks, months and years.

Students identify the set of all possible outcomes for familiar events involving chance, predict their comparative likelihood, and use experimental data to review predictions.

Students have the opportunity to:

- identify situations involving equally likely outcomes and distinguish them from situations where outcomes are not equally likely (eg *rolling a die compared to tossing a matchbox*)
- use descriptive chance language (eg *never, sometimes, most likely, highly unlikely, probably*) when making predictions and ordering events in terms of their likelihood
- design chance experiments to collect data and make predictions based on that data (eg *after* 100 *drawings, make predictions about the next* 20 *colours drawn from a jar of coloured counters*).

Students collect data in response to questions and issues, conduct trials, present data in appropriate forms, including with the use of technology, analyse data and make simple statements or predictions.

- compare different data collection methods (eg *simple surveys, observations, experiments and simulations*) and select one suited to a given context
- decide whether collection and recording procedures need to be changed and whether new data should be collected as a consequence
- use a range of tabular and graphical displays with suitable scales including technology-generated graphs (eg *bar graphs, pie graphs, pictographs*) to represent the same data set and compare their effectiveness
- analyse data and make statements and predictions that respond to questions or issues, and use tabular and graphical displays to support those views (eg *predict*, *within a reasonable range*, *the shoe size of someone joining this class next week*)
- identify and describe variation within sets of data (eg *the difference in children's views of a popular television show*) and between sets of data (eg *children's views compared to adults' views on the same show*) and comment on whether the variation is consistent with their expectations.

### Year 5 Space

Students identify families of two-dimensional shapes and three-dimensional shapes and objects, and describe their features using appropriate language and terms.

Students have the opportunity to:

- generalise about the features of families of shapes and objects (eg *all of the side (lateral) faces of pyramids are triangles*)
- identify the unique features of some shapes within those families (eg *a parallelogram is a quadrilateral with two pairs of parallel sides*)
- use appropriate spatial language including parallel, perpendicular, vertex, edge, base, acute, right, obtuse and reflex angles, when describing the features of shapes (eg ▲ *this shape is an irregular quadrilateral it has three acute angles and a reflex angle*).

Students represent shapes and objects in various ways that illustrate the essential features.

Students have the opportunity to:

- make drawings of two-dimensional shapes and drawings or models of three-dimensional shapes and objects that accurately reflect the size and significant features of those shapes (eg *use dotted lines to represent edges of prisms that are not in direct view, and show where lines are parallel or perpendicular*)
- represent and describe two-dimensional shapes in different orientations and three-dimensional shapes and objects from different perspectives, highlighting relevant features, using technology as appropriate
- use given features to identify or predict two-dimensional shapes, three-dimensional shapes and objects, or the family to which they belong
- visualise and construct nets of prisms and pyramids, and match given nets to the corresponding three-dimensional shapes and objects.

Students identify shapes and designs that are symmetrical as well as those that are not (asymmetrical). They use transformations to create patterns and arrangements including tessellations.

Students have the opportunity to:

- describe the result of combinations of transformations of a shape (eg *a slide then a rotation*) and create patterns and designs
- identify symmetrical shapes and designs, create some using transformations (eg *flips*) and explain why others are not symmetrical
- use multiple copies of different shapes to observe that some families of shapes will always tessellate (eg *triangles and quadrilaterals will tessellate but pentagons will not*).

Students understand the importance of symbols and conventions when making or reading maps, grids and plans.

- interpret and use symbols (eg the North symbol,  $\uparrow$ , the key or legend) and conventions (eg alpha-numeric grids) when planning directions or placement of key features on maps
- make links between the four major compass points and quarter, half, three-quarter and full turns when following or giving directions
- estimate lengths and distances on maps, grids and plans with respect to a straightforward scale (eg *use a scale of* 1 *cm* = 10 *m to estimate the length of a street*).

# Year 7 Professional Elaborations – Opportunities to Learn for Mathematics

### Year 7 Working mathematically

Students extend their use of mathematical inquiry and choose and use a range of strategies and processes, including technology. They carry out investigations, develop and compare models for given situations, and solve problems in contexts amenable to straightforward mathematical analysis.

Students have the opportunity to:

- make and test simple propositions related to a given context (eg given any three whole numbers it is always possible to construct a triangle with sides corresponding to these lengths)
- investigate propositions, hypotheses and conjectures (eg *converting to solar water heating is a practical proposition in a given capital city, no natural number has a square with the last digit 2, every natural number is the sum of two prime numbers*)
- develop a practical model, describing assumptions and conditions, and reflect on its usefulness (eg a sport tipping strategy based on data from previous years and adjusted from time to time in terms of current season data).

Students use a combination of everyday and mathematical terms, symbols, diagrams, materials and conventions to pose questions, represent and apply concepts, skills and processes, and are familiar with common variations in use.

Students have the opportunity to:

- choose and use sets of things, lists, tables, diagrams and graphs to represent, interpret and analyse data, relations and functions (eg *tree and other diagrams, statistical and coordinate graphs*)
- apply operation and relation symbols and brackets to write and evaluate numerical expressions, and use common alternative expressions (eg *relate a geometric diagram with suitable labelling to the area of a rectangle with length L and width W to the product of L and W and the expressions* '*Area of a rectangle equals Length times Width*', *Area = Length × Width*,  $A = L \times W$  or A = LW)
- choose, use and interpret symbols to represent variables in simple algebraic expressions and word problems, and formulas
- explore how different forms of representation are used in a given context and discuss their strengths and limitations (eg *different bases involved in counting and measuring such as with time*).

Students make generalisations, judge whether reasoning is convincing or not, and explore mathematical demonstrations.

- systematically test and check the validity of propositions (eg the proposition that it is likely that two students in a large class will have the same birthday, the apparent contradictions in dissection and 'missing area' problems)
- follow deductions and reasoning used to establish the truth of a proposition or otherwise (eg a visual demonstration that the angles of a triangle fit exactly around a point on a straight line, and see why a plane triangle cannot have two right angles; and show that the sum of two even numbers or the sum of two odd numbers is always an even number)

- make generalisations based on systematic exploration of particular cases, recognise regularity, and attempt to develop general case arguments to support their truth or otherwise (eg *establish the conditions for a simple connected diagram such as*  $\boxtimes$  *to have a path where every edge is travelled only once, and explain why this is the case*)
- explore the possible truth of propositions using technology where appropriate (eg *use geometry software to observe that the angle bisectors of any triangle appear to coincide, investigate when the sum of two numbers is less than their product*).

Students communicate about their work informally and formally with the assistance of technology as appropriate.

Students have the opportunity to:

- present and discuss a problem, its formulation, possible solution strategies, and processes and results obtained (eg *an oral presentation on why square numbers have an odd number of factors*) with the assistance of technology as applicable (eg *to display a list of factors of whole numbers*)
- develop a report describing background, ideas and approaches, and report on progress and results (eg a poster presentation or a written report on a model for seeding of athletes in a competition, investigation into the history of prime numbers including the search for large primes).

#### Year 7 Number

Students identify and represent integers and decimal fractions, and compare and order them using a variety of methods and models. They calculate with the four operations, powers of 10 and small whole number powers of other numbers, by mental, written and technology-assisted methods.

- order rational numbers on a suitably scaled part of the real number line
- identify and use factors of numbers including prime factors to assist mental computation, and to recognise number properties (eg 27 × 3 = 9 × 3 × 3 = 9 × 9 = 81, and 72 = 6 × 12 = 2 × 3 × 2 × 6 = 2 × 3 × 2 × 2 × 3 = 2<sup>3</sup> × 3<sup>2</sup>)
- mentally calculate simple problems given in terms of two numbers and a single operation (eg 89 + 27 = 90 + 26, 152 - 86 = 156 - 90, 1.1 × 10<sup>3</sup> = 1.1 × 1000 = 1100, 20<sup>2</sup> = 2 × 10 × 2 × 10 = 4 × 100 = 400 and 240 ÷ 6 = (24 ÷ 6) × 10 = 40) and discuss the efficiency of the method used (eg 40 ÷ 0.8 is the same as 400 ÷ 8 which equals 50)
- use a number line or materials to solve practical problems involving the interpretation of sums and differences of integers in context (eg show that an overnight change in temperature from 5°C to -7°C is a drop of 12°C, find the difference, which could be positive, zero or negative, between the initial and final balance of a student's bank account after several deposit and withdrawal transactions)
- apply effective written methods to carry out computations with decimals to at least thousandths (eg 2.852 × 12.3, 16.8347 ÷ 0.2)
- use technology such as calculators or spreadsheets to carry out more complex or repetitive computations with attention to order of operation (eg *include GST and fixed packaging and postage costs for a list of items from a mail order catalogue to calculate total cost for several orders, calculate the area to be covered when painting a cube with side length* 9.7 cm as 6 × 9.7<sup>2</sup> square cm)
- interpret problem situations to select and use an appropriate sequence of operations, and apply suitable methods of computation (eg *realise that calculating a 20% discount involves multiplication and subtraction; know that multiplying a given number by a decimal number between 0 and 1 will produce a smaller number, such as 60 × 0.3 = 18, while dividing a given number by a decimal number between 0 and 1 will produce a larger number, such as 60 ÷ 0.3 = 200, and apply this in practical situations such as finding the cost of 0.3 kg of meat at \$8.45 per kilogram).*

Students represent and describe common fractions, including simplest form, and find their equivalent representations as decimals and percentages. They order them, and use mental and written methods with addition and subtraction (where a common denominator is readily identifiable), multiplication and simple division, and use technology for more complicated computations.

Students have the opportunity to:

- recognise fractions in both mixed number and numerator-denominator (improper fraction) form and their equivalence (eg  $4\frac{2}{3} = \frac{14}{3}$ )
- know decimal equivalents for  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{8}$ ,  $\frac{1}{10}$  and use these to find and interpret decimal and percentage equivalents for fractions (eg  $\frac{3}{8} = 0.375 = 37.5\%$ ,  $\frac{5}{3} = 1.666 \dots \approx 167\%$ )
- use concrete materials, and mental or written methods, for addition and subtraction of same denominator fractions and fractions with readily identifiable common denominators, and explain their reasoning (eg  $\frac{3}{4} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8} = \frac{11}{8}, \frac{8}{9} \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$ )
- use concrete materials, and mental or written methods, for multiplication and division of a whole number by common fractions (eg  $\frac{2}{5} \times 40 = 16$ , *the number of one-eighth slices in three pizzas is twenty-four*)
- use technology to carry out and check calculations involving rational numbers and represent answers in numerator-denominator, mixed number or decimal form as required (eg  $4\frac{3}{4} 2\frac{1}{3} = 2\frac{5}{12} = 2.41666 \dots$ ) and discuss the suitability of each form in a given context for computation.

Students apply a range of strategies and approaches to calculate simple proportion, percentages and simple rates in practical situations involving money, time and measurements.

Students have the opportunity to:

- represent a set to set (part to part) relationship as a ratio (eg *the number of boys compared to the number of girls in a school*) or a subset to set (part to whole) relationship as a ratio (eg *the number of girls in a school*) to the total number of students in a school), and express these in simplest form
- calculate proportions of a given ratio using multiplication and division by whole numbers (eg convert a recipe for 4 people to a recipe for more or fewer people such as 1, 2, 6, 8 or 12 using the given quantity of ingredients for the original recipe; interpret a scale of 1 cm to 100 m to know that a length of 3 cm on a map corresponds to a distance of 300 m, and a length of 5.5 cm on the map corresponds to a distance of 550 m)
- calculate with fractions and percentages based on multiples of 10% and 25% of a given unit or quantity (eg <sup>3</sup>/<sub>5</sub> or 60% of a bag of cherries, if a breakfast drink contains 10% mango juice, 20% pineapple juice, 30% orange juice and the rest is water, how much of each juice is there in a 2-litre container of the breakfast drink?) using informal and formal methods
- calculate proportions and simple rates involving money and whole number quantities, using technology as appropriate (eg compare two similar products such as the cost of a packet of breakfast cereal compared to the mass of cereal it contains to determine best value).

Students use a variety of methods to form estimates and make approximations.

- form estimates and make approximations arising from practical situations involving calculations with whole numbers, decimals and fractions, and interpret and justify their reasoning in context (eg total of a shopping bill, the mean of a small set of numbers {10.2, 6.8, 4.5, 8.3, 7.1}, and simple proportions of a quantity such as <sup>3</sup>/<sub>5</sub> of a 44-litre cooler)
- choose and use the number line, rounding and powers of 10 to form estimates (eg  $43.27 \times 531.8$  *is between* 20 000 *and* 30 000; 29 643 ÷ 87 *will be approximately* 30 000 ÷ 100 *or* 27 000 ÷ 90, *that is* around 300;  $4\frac{3}{4} + 2\frac{1}{3}$  *is between* 6 *and* 8).

# Year 7 Algebra, function and pattern

Students use words, diagrams, materials and symbols to represent variables and construct and interpret simple algebraic expressions and relationships. They evaluate these expressions for whole number and simple fraction values.

Students have the opportunity to:

- define variables, explore and interpret mathematical expressions (eg the average of two numbers a and b is half their sum or  $\frac{a+b}{2}$ ; if m is the unknown but fixed number of matches in a collection of identical matchboxes, then the number of matches in two matchboxes and three extra matches is  $2 \times m + 3$ )
- read and interpret symbolic representations of practical situations (eg the perimeter P of a square is four times its side length S, read  $P = 4 \times S$  or P = 4S) and explain their meaning using suitable diagrams and materials
- use variables and constants to represent formulas and simple algebraic relationships and interpret these in context (eg represent 'the area of a triangle is half base times vertical height', or represent the relationship 'Mary is three years older than John' as M = J + 3 where M represents Mary's age and J represents John's age)
- develop simple algebraic relationships for practical problems using suitable diagrams and materials, and evaluate them for various values of the variables involved (eg *relate the worded expression* 'perimeter of a rectangle is equal to twice its length plus twice its width' to the corresponding diagram and the formulas perimeter = 2 × length + 2 × width or p = 2 × l + 2 × w or p = 2l + 2w, where l represents the length of the rectangle and w represents the width of the rectangle; find the perimeter of a rectangle with length 8 cm and width 5<sup>1</sup>/<sub>2</sub> cm).

Students use words, materials, diagrams and symbols to assist in the representation and manipulation of simple algebraic expressions involving addition, subtraction and multiplication, and to establish equivalences between them.

Students have the opportunity to:

- apply commutative and associative properties to expressions to explore general mathematical properties of number (eg show how 6 lots of 15 000 could be calculated as 3 lots of 30 000, and generalise this principle to a family of related calculations; explain why, in general, for two numbers m and n,  $m + (2 \times n) = (2 \times n) + m$  but  $m + (2 \times n)$  is not equal to  $n + (2 \times m)$  and identify exceptions)
- apply the distributive property to demonstrate equivalence (eg use materials, diagrams and number examples to explain why (2 × 6) + (3 × 6) = 5 × 6 generalises to (2 × n) + (3 × n) = 5 × n; or why 2 lots of (Δ + 5) is equivalent to 2 × Δ + 10 by interpreting the symbol as an unknown but fixed quantity in a given context).

Students determine when simple equations involving the operations of addition, subtraction and multiplication are satisfied, and solve such equations using materials, inverse operations and algebra or flow charts and backtracking, tables and graphs.

Students have the opportunity to:

• identify when numbers satisfy a given equation or not (eg the numbers four and six satisfy the equation 'the sum of two numbers is equal to ten' and also the equation 'two numbers multiplied together equal twenty-four' but not the equation 'the second number is twice the first number')

• use a variety of methods and approaches to solve simple equations and explain reasoning (eg 'multiply a number by itself, then add 2. The result is 38, what is the number?' 'Find the value of the number represented by "a" for which  $2 \times a - 3 = 11$  is true').

Students construct tables of values for functions given by simple rules and input–output function machines and graph the corresponding set of ordered pairs, in the first quadrant of the cartesian plane. They interpret simple functions informally by considering the set of input values and the corresponding set of output values in context.

Students have the opportunity to:

- explore and interpret functions described graphically, and make predictions from these graphs (eg describe the \$AUD exchange rate with respect to the \$US for a given month, observe its range of values and discuss what is likely to happen to this value for a short trip overseas at the beginning of the following month)
- identify and continue number patterns, describing the patterns in words (eg {4, 6, 8, 10, 12 ...}, {3, 6, 12, 24, 48 ...}, {2, 5, 10, 17 ...}, {2, 5, 7, 12 ...})
- use whole number values to construct tables for functions (eg the sum of two numbers m and n is twenty with rule expressed in the form m = 20 - n; a function machine where the output value is onehalf of the input value; the area of a square A expressed in terms of its side length L as  $A = L \times L$ ) and draw corresponding graphs by plotting points and using technology
- specify rules of linear functions using words and symbols from tables of values, and use these to make predictions (eg note the set of odd numbers can be specified by the rule 'double the number then subtract one' or  $\Delta = 0 \times 2 1$ , from the table:

| $\diamond$ | 1 | 2 | 3 | 4 | 5 |
|------------|---|---|---|---|---|
| Δ          | 1 | 3 | 5 | 7 | 9 |

and then use this function to predict the value of the 20th odd number).

#### Year 7 Measurement, chance and data

Students are familiar with SI units, and use instruments and technology to measure and compare lengths, areas, surface areas, volumes, angles, masses, times and temperatures as applicable to various objects and events.

Students have the opportunity to:

- use SI units for measurement, and an appropriate instrument or other technology to measure a required attribute or characteristic
- choose and use other units (eg kilometre, millimetre, tonne, gram, degree, hour, minute, kilolitre, megalitre and millilitre) as more suitable in various contexts, and identify relationships between units (eg the duration of a video is usually expressed in minutes rather than seconds, a small garden water tank has a capacity of 2.75 kL = 2750 L)
- specify area and volume in terms of unit<sup>2</sup> and unit<sup>3</sup>
- measure time elapsed to the nearest appropriate unit, angle in terms of fraction of a whole turn or in degrees using a protractor, and temperature to the nearest degree Celsius
- read and record measurements from calibrated scales in which intermediate graduations are not numbered (eg *a medicine measuring glass, a speedometer*)

Students understand that measurement involves error, estimate quantities with respect to common everyday measures, and describe a reasonable range for a given measurement.

- record measurements and explain why a particular range of values is likely (eg *if several students* use a stop-watch to time a 100 m race, the winner's time is only likely to be accurate to the nearest one-tenth of a second because of different reaction times)
- estimate quantities to within a given range (eg *the amount of drink left in a partially full drink bottle to the nearest* 50 *mL, the weight of a school bag or back-pack to the nearest* 1 *kg*).

Students investigate situations from which they develop simple measurement formulas using words and symbols, and apply these to practical problems.

Students have the opportunity to:

- develop and apply formulas for the perimeter and area of triangles and parallelograms given the relevant linear dimensions
- investigate and use the relationship between the radius and diameter of a circle and its circumference and area
- calculate the surface area of shapes and objects, such as cubes and rectangular and triangular prisms, and make a judgment about the reasonableness of the result
- calculate the volume of cubes, rectangular and triangular prisms (eg *using base area* × *height*) and make a judgment about the reasonableness of the result
- calculate time elapsed (duration) in terms of hours, minutes and seconds from analogue and digital timepieces, for both 12- and 24-hour cycles (eg *time of travel during a flight across different time zones*).

Students make and interpret empirical estimates of probabilities in familiar situations. They compare experimental data with theoretical probability obtained from proportions based on counting or area.

Students have the opportunity to:

- identify events as more or less likely or equally likely (eg although each card is equally likely to be drawn at random from a 52-card pack, an ace is less likely to be drawn than a red card)
- determine empirical estimates of probability (eg based on a previous season's performance a player may have a one-in-two or 50% chance of scoring on a single free shot during a basketball game)
- compare experimental data from simple trials involving coins, various-sided dice, spinners, and other devices with theoretical probability
- describe probabilities in terms of a number between 0 (impossible) and 1 (certain).

Students represent and summarise sample data drawn from a given population.

- distinguish between a sample drawn from a population and the population itself (eg *the ages of students in a Year 7 class compared to the ages of all Year 7 students in a state or territory*)
- identify data as discrete (eg *eye colour, movie rating such as*  $\mathcal{A}\mathcal{A}\mathcal{A}$ ) or continuous (eg *height, time elapsed*) and display the data by hand for small data sets and with the assistance of technology for larger data sets
- use two-way tables to represent categorical data (eg *the proportions of boys and girls with blue / not blue eyes*)
- calculate measures of location (mode, median and mean) and discuss their usefulness in context (eg mode for shoe size, mean for a set of measures, median for a set of house prices)
- describe variation in data in terms of relative frequency (eg over 80% of students in the basketball team are taller than 1.7 m compared with less than 20% of students in a soccer team).

Students use data to make informal inferences in response to questions and hypotheses.

Students have the opportunity to:

- use measures of location to make general statements about data sets (eg *the amount of maths homework done in different classes at Year* 7)
- interpret variation between data sets (eg the age of members in different teams, years of experience of team members in a given sport) and discuss possible effects of variation (eg how the range in years of experience of a team might affect team performance)
- make informal inferences about a population from a given sample, noting possible causes of bias (eg *location, age, gender or interest with respect to issues canvassed in an opinion poll*) and respond to questions and hypotheses (eg *taller teams do better in netball*).

#### Year 7 Space

Students identify, describe and classify common two-dimensional shapes and three-dimensional shapes and objects, and geometric shapes with respect to their properties including line, length, angle and surface.

Students have the opportunity to:

- describe and classify triangles and quadrilaterals in terms of side and angle (eg *isosceles triangles have two equal sides and two equal base angles, demonstrate that the sum of internal angles of a quadrilateral is* 360°)
- identify properties of squares, rectangles, parallelograms, trapezia, pentagons, hexagons, octagons and circles and describe part (eg *a semicircle, quarter circle, and other simple fractional sectors of a circle*) and composite shapes (eg *star shapes such as* ◆,★,★) in terms of these properties
- identify prisms, pyramids, spheres and cylinders, and describe part and composite shapes and objects (eg *a six-person tent, a truncated pyramid*) in terms of their properties.

Students use sketches, diagrams, drawing tools and geometry software to represent and construct common two-dimensional and three-dimensional objects.

Students have the opportunity to:

- draw plane shapes according to specification of geometric properties (eg use a compass and straight edge or geometry software to make a net for a kite in terms of its diagonals and their lengths, a tangram from a square)
- use plans or nets to construct prisms, pyramids and cylinders, and represent composite shapes made from identical cubes using isometric grid drawings (eg *pentominoes, a model of a group of adjacent buildings in a city using multi-link cubes*)
- explore what is necessary and sufficient information to construct uniquely a given two-dimensional shape with the assistance of suitable technology (eg *use geometry software to construct a parallelogram of given side lengths*).

Students recognise symmetry and congruence, and relate these to transformations and patterns involving shapes in the plane.

- identify points and lines of symmetry for two-dimensional shapes, and specify these in terms of
  mirror lines for reflection or a point of rotation and angle of rotation (eg *the web-shape (mage has a
  central point of rotation and symmetry by rotation through an angle of 72° about this point*) and test
  whether given shapes are symmetrical
- identify lines and planes of symmetry for common three-dimensional shapes and objects (eg *the plane containing the equator on a globe of the earth, the base-vertex perpendicular of a pyramid*), and test whether given shapes or objects are symmetrical
- apply symmetry to construct two-dimensional shapes or three-dimensional shapes and objects (eg use paper folding, compass and ruler or technology to create shapes such as X, ★, ♥ or origami designs such as a paper cup)
- use and modify congruent two-dimensional shapes to produce tessellations of the plane (eg *brick or mosaic patterns*).

Students interpret and use maps and plans to specify location and to move from one location to another.

- draw maps or plans that include scale in familiar contexts (eg place a bus stop closer to a school than a nearby shop on a map with relative distances represented accurately using a straightforward scale, such as 1 cm represents 100 m; given that a 20 m wall of a house measures 5 cm on a plan, calculate the scale as 1 cm represents 4 m)
- use distance, compass points, including NE, NW, SE, SW, fractions of a turn  $(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$  and multiples of these), angles in degrees and grids and coordinates to read and follow simple maps
- provide and follow instructions for moving from one location to another based on plans or maps with reference to distance, left and right, and angles in degrees (eg *travelling from home to a school sports event*).

# Year 9 Professional Elaborations – Opportunities to Learn for Mathematics

## Year 9 Working mathematically

Students develop the breadth and depth of their mathematical inquiry, and choose and use a broad range of mathematical strategies and processes, including technology. They carry out investigations, develop, compare and refine models, and solve problems in familiar and unfamiliar contexts. They identify and describe key features of a context and state assumptions or conditions related to investigation, modelling and problem solving, and reflect on the effectiveness of their work.

Students have the opportunity to:

- plan and conduct inquiries that require them to pose questions and formulate propositions, hypotheses or conjectures related to a given context, and use a range of strategies (eg *consider the safety of surf conditions at a given location on a particular day and time in terms of wind and current strength and direction, and tide height*)
- discriminate between important and incidental features of a context or situation, and state related assumptions and/or conditions (eg when analysing the repeated bounce of a ball from a given drop height, consider whether air resistance is significant or not when a table-tennis ball, super-ball or ball bearing is dropped from an initial height of 2 metres compared with an initial height of 1 metre)
- vary assumptions or conditions in a context (eg decide to calculate with decimal values correct to six decimal places rather than round to two decimal places when considering currency conversions for the exchange of large sums of money) and respond to new results and data
- choose and use aspects of mathematics to carry out investigations, model situations and solve problems (eg *analyse data from a data logger or video to model the distance travelled by a falling rock in a given time*) and discuss any limitations
- use relationships to model key features of a context, and apply related mathematical techniques to analyse and explore the context, and make related predictions (eg *approximate the area of a wetland in a national park and describe the distribution of a rare species of bird known to exist in the wetland, compare car hire rates for an extended trip).*

Students routinely use a combination of words, symbols, materials, diagrams and conventions when they think and reason mathematically, and understand that mathematics has been refined over its historical development across cultures. They pose questions that are amenable to mathematical analysis and involve the use of a range of different representations, as applicable, to formulate and describe concepts and relationships.

- represent, interpret and analyse measurements, shapes, data, relationships and functions, sets of things, tables, diagrams and graphs (eg *tell a story about several cars moving at constant speed along a section of road in both directions, including instances of overtaking, using tables, diagrams, graphs and functions*)
- extend the range of formal symbols used to include constants such as  $\pi$ , variables and commonly used mathematical notations such as square roots, cube roots, reciprocals and whole number powers, and employ new notations as required in context

- choose and use notations, conventions, symbols and words to pose questions, formulate mathematical propositions, express relationships, and modify and refine them (eg *the formula for the mean of a discrete set of data which may be grouped or ungrouped, describe and explain a rule for generating Pythagorean triples using numbers, diagrams and symbolic expressions*)
- explore developments in the representation of key ideas or approaches over time (eg the evolution of different approaches to efficient arithmetic computation in terms of purpose, social context and culture).

Students make generalisations, verify reasoning, and explore mathematical proofs.

Students have the opportunity to:

- test and check the validity of propositions by identifying examples which satisfy the proposition, as well as seeking possible counter-examples (eg *the claim that all dice within a given set are unbiased, the proposition that all numbers of a particular form, such as* 4 n + 1*, where n is a whole number, are prime numbers*)
- make deductions and prove statements (eg prove that if two numbers differing by 2 are multiplied and one added, then the result is always a perfect square, formulate the conjecture that 'the product of two numbers is always even unless both of them are odd' and use diagrams to show that this is the case)
- make generalisations based on propositions that do not appear to have readily identifiable counterexamples, and explore supporting general case arguments to establish their truth or otherwise (eg *a possible rule for the number of diagonals in a polygon in relation to the number of sides*)
- use technology to explore pattern and structure, and general representations of these (eg cycles in calendars, self-similarity in art and design such as in computer-generated geometric patterns, Escher-style prints or snowflakes).

Students discuss their own work and that of others with attention to its nature, purpose and scope. They use technology as appropriate to assist in presentation and discussion of their work.

Students have the opportunity to:

- provide a clear account of the mathematical reasoning behind a particular result or application (eg *how a certain type of code works*)
- give an oral presentation of a mathematical result (eg how a linear model could be used to relate the weight and height of a given cohort of people)
- make a poster or develop a structured report on a group investigation (eg how a seating plan for passengers might be devised to take account of the number of passengers with a given type of ticket and comfortable seat dimensions)
- use technology to demonstrate working related to investigation of an aspect of mathematics (eg *the infinite nature of the set of prime numbers and the validity of some possible 'rules' for generating prime numbers*).

#### **Year 9 Number**

Students extend their familiarity with and use of different representations of rational numbers using fractions, decimals, percentages and scientific notation.

- use prime numbers and factor trees to express any natural number as a product of powers of primes (eg the factor tree for  $36\ 000 = 2^5 \times 3^2 \times 5^3$ , investigation of factors of very large numbers and the existence of large prime numbers)
- use scientific notation to interpret very large or very small numbers in practical situations, including results arising from the use of technology for computation (eg where a total national debt of \$234 billion = 234 000 000 000 = 2.34 × 10<sup>11</sup> is represented as 2.34 E11, and the diameter of superfine 11 micron wool is 11 ×10<sup>-6</sup> metre is represented as 1.1 E-5)
- use equivalent fractional, decimal and percentage forms (eg  $\frac{3}{8} = 0.375 = 37.5\%$ , 12.3657  $= 12 + \frac{3}{10} + \frac{6}{100} + \frac{5}{1000} + \frac{7}{10000} = 12\frac{3657}{10000}$  and  $2\frac{4}{9} = 2.\overline{4} = 244\%$  to the nearest whole percent).

Students locate rational numbers and decimal approximations to some irrational numbers on the real number line, and use them in practical situations.

Students have the opportunity to:

- locate integers, decimals, fractions and decimal approximations to some irrational numbers on the real number line (eg {3.5,  $\frac{-2}{5}$ , 0, 1,  $\pi$ , 5.3,  $\sqrt{90}$ })
- determine a decimal approximation to the side length of a square with a given area (eg *a square with area* 90 *square metres has a side length of* 9.49 *m, correct to the nearest cm*)
- determine a decimal approximation to the circumference and area of a circle with a given radius (eg a circle of radius 5.2 metres has a circumference of 10.4 × π ≈ 32.67 metres and an area of 27.04 × π ≈ 84.95 square metres).

Students use mental, written and technology-assisted methods to carry out computations and solve practical problems with attention to the type of numbers and operations involved, and order of operation.

Students have the opportunity to:

- apply number facts and properties to carry out mental calculations (eg *the GST on* \$156 *is* \$15.60 *which gives a total of* \$171.60;  $29 \times 7 = (20 \times 7) + (9 \times 7) = 203$ ;  $(30 \times 7) 7 = 210 7 = 203$ )
- calculate problems involving two integers and a single operation using effective written methods (eg *the amount left to be paid off at a given time on a house loan based on a recent statement from the bank;* 546 × -389; 20 *billion divided by* 350 000)
- calculate simple powers and square roots mentally (eg  $3^4 = 3 \times 3 \times 3 \times 3 = 9 \times 9 = 81$ ,  $(\frac{1}{2})^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ ,  $0.6^2 = (6 \times 6) \div 100 = 0.36$ ,  $\sqrt{\frac{16}{9}} = \frac{4}{3}$ ) and use technology for more difficult cases (eg 26.531<sup>3</sup> = 18 675.010 679 291, 11<sup>5</sup> = 161 051,  $\sqrt{4509} = 67.15$  rounded correct to two decimal places)
- use technology to carry out efficient computation giving answers to a reasonable level of accuracy (eg  $(67.43 + 104.512 89.99) \div 241$ ,  $\sqrt{27.4^2 18.6^2}$ ,  $\$4800 \times (1.05)^{10}$  as a compound interest calculation).

Students solve ratio, proportion, percentage and rate problems using mental, written and technologybased approaches. They interpret square roots and  $\pi$  geometrically, and know that practical computations involving these numbers require the use of suitable decimal approximations.

- convert between different representations of rational numbers, and use these to solve practical problems (eg *if a computer file repair process is successful on average 5 out of 7 times, then it has a success rate of*  $\frac{5}{7} = 0.\overline{714285} \approx 71.43\%$ )
- determine corresponding proportions, percentages or rates given a ratio of two or more decimal numbers, and use these to solve a variety of problems (eg mixture by proportion of weight or volume of ingredients to form a given weight or volume of concrete; if the exchange rate for one Australian dollar on a given day is 0.7304 US dollars, find how many US dollars \$AUS550 obtains, and how many Australian dollars \$US200 obtains)
- carry out, with technology, computations involving decimal approximations to irrational numbers in measurement contexts (eg the diagonal of a rectangle with length 10 m and breadth 5 m is  $\sqrt{125} \approx 11.2 \text{ m}$ ; a circle with a circumference of 100 m has a diameter of  $\frac{100}{\pi} \approx 31.8 \text{ m}$ ).

Students form estimates for computations involving rational numbers, make approximations of some irrational numbers, and round results correct to a specified number of decimal places depending on the context.

Students have the opportunity to:

- find upper and lower estimates for calculations, form closer estimates within this interval for computation in a given context (eg *splitting a restaurant bill between several people*)
- form estimates for square roots (eg  $\sqrt{500}$  *is between* 20 *and* 30) and for computations involving the constant  $\pi$  (eg 2 ×  $\pi$  × 83.49 ≈ 480)
- choose a suitable level of accuracy for computation (eg calculating the cost for a quantity of soil and mulch for a garden bed at a given cost per cubic metre and deliverable in half or full cubic metre quantities).

#### Year 9 Algebra, function and pattern

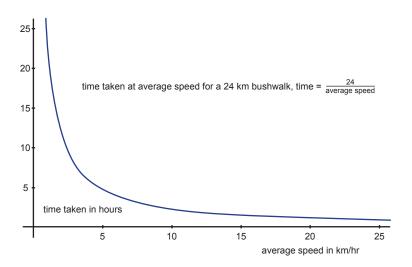
Students use words and symbols to represent variables and constants, and interpret algebraic expressions for relationships developed in context. They construct expressions that involve the four arithmetic operations, simple reciprocals, whole number powers, and substitute into and evaluate these expressions with the assistance of technology as required.

Students have the opportunity to:

- model, apply and interpret relationships, including simple inequalities, involving variables related to a given context (eg calculate a human body mass index (BMI) as weight w kilograms divided by height h metres squared, or  $\frac{w}{h^2}$  and compare with the healthy BMI range of 20 to 25; the area of a rectangle with breadth B cm and length L cm is less than 100 cm<sup>2</sup>)
- formulate linear functions to describe a situation involving constant rate of change given various data (eg *initial value and a constant rate of change, two values from the graph of a straight line*)
- use known formulas to develop, apply and interpret new relationships (eg *a cube of side length b cm* has a surface area of  $6 \times b^2$  square centimetres; the volume of a cylinder height h metres and radius r metres as circular base area  $\times$  height =  $\pi r^2 \times h m^3$ ).

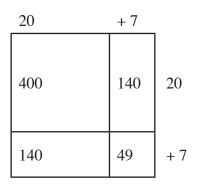
Students construct tables of values and draw corresponding graphs for functions using these values or technology as necessary. They interpret these graphs with respect to the variables involved and the relationship between them, and solve related problems.

- plot points for linear functions (eg *the temperature conversion between Celsius and Fahrenheit* scales, F = 1.8C + 32, C = <sup>5</sup>/<sub>9</sub> (F 32)) and simple non-linear functions of a discrete or continuous variable (eg *the area of a circle in terms of its radius, number patterns generated by constant multiplication from a given starting value* {1, 15, 225, 3375 ...}, {80, 40, 20, 10 ...}) and use by-hand sketches, graphing aids and technology to generate graphs of these functions
- interpret the variables involved in using linear functions to model situations and make related predictions (eg *use a linear model developed from Year 9 student data to predict the height of a Year 9 student from their forearm length*)
- interpret the variables involved in using some simple non-linear functions to model situations and make related predictions (eg *use a constant ratio to generate a pattern for compound interest such as* {\$10,000, \$10,600, \$11,236...}, *the time taken to complete a 24 km bushwalk for various average speeds*).



Students use identity, inverse, associative, commutative and distributive properties, and a variety of approaches such as concrete materials, technology, algebra, diagrams, flow charts and backtracking to explore and manipulate algebraic expressions and establish equivalences.

- re-arrange linear and some simple non-linear algebraic expressions (eg p = 3q 2 to obtain  $q = \frac{p+2}{3}$ ; given  $A = \pi r^2$  and a specific value of A, find the corresponding value of r, and obtain the general case  $r = \sqrt{\frac{A}{\pi}}$  noting that r must be positive in the related practical context)
- explore and establish linear equivalences (eg 2(4x + 8) = 4(2x + 4) = 8(x + 2)), and explore and establish simple non-linear equivalences (eg *generalise number patterns such as*  $27^2 = (20 + 7)^2 = 20^2 + 2 \times 20 \times 7 + 7^2 = 400 + 280 + 49 = 729$ :



use a diagram to show  $(a + 2) \times (a + 3) = a^2 + 5a + 6$  and  $(2x)^2 = 4x^2$ ; generalise from number examples such as  $4^3 \times 4^2 = 4^5$  and  $4^5 \div 4^2 = 4^3$  to show that  $x^3 x^2 = x^5$  and  $\frac{x^5}{x^2} = x^3$ ).

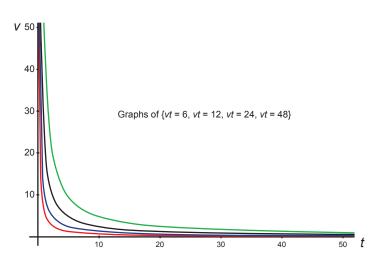
Students determine when an equation or inequality is satisfied for a given combination of values. They set up linear equations and solve them using graphs, tables and algebra. They solve some nonlinear equations using tables, graphs or algebra.

Students have the opportunity to:

- determine by substitution, with the assistance of technology for more complicated examples, whether a set of values satisfies an equation or inequality (eg '*when a* = 3 *and b* = 2 *does* 3a + 2b = 12?' *or* '*does a square of side length* 3.5 *metres have an area less than* 10 *square metres*?')
- use a variety of methods including algebra to solve linear equations of the form ax + b = c (eg find the taxi charge in dollars per kilometre if a flag-fall is \$2.80 and a 60 km trip costs \$100, or solve 3x + 7 = 22) and graphs, tables or algebra to solve simple simultaneous linear equations involving expressions of the form y = ax + b (eg solve y = 4x 3 and y = 2x + 5 simultaneously, or compare two different linear models for internet provider costs in terms of time taken to download a file, and find when the costs are the same)
- use a variety of methods to solve simple non-linear equations (eg find the time taken to travel 300 km at an average speed of 85 km/h, or find the value of x for which  $2x^2 + 3 = 53$ , or determine the approximate dimensions of a rectangle with area 100 cm<sup>2</sup>, width w and length 3 cm greater than its width).

Students draw graphs of functions, and identify the effect of changing constants used to specify the rule of a function on its graph.

- sketch graphs and use technology to explore the effect of varying the values of *a* and *b* in the rule of a linear function *f*(*x*) = *ax* + *b* on the corresponding graph, and describe these effects with reference to gradient and the *y* axis intercept (eg *y* = *x* → {*y* = 2*x*, *y* = 3*x*, *y* = -2*x*, *y* = -3*x*}, *y* = *x* → {*y* = *x* + 1, *y* = *x* + 2, *y* = *x* − 1, *y* = *x* + 1, *y* = *x* − 2}, *y* = *x* → {*y* = 3*x*, *y* = -3*x*, *y* = -3*x* + 12}, *investigate rules for a set of four linear functions that create a shape between the points of intersection of their graphs which is a parallelogram, rectangle or rhombus, such as {<i>y* = *x*, *y* = *x*, *y* = *x* − 4})
- draw graphs of some simple non-linear functions interpreted in a practical context (eg graphs of  $\{vt = 6, vt = 12, vt = 24, vt = 48\}$  where v is interpreted as average speed in km/h and t is interpreted as time of travel at a given average speed in hours) and describe the effect of changing constants used to specify the rule of the function on the corresponding graphs.



## Year 9 Measurement, chance and data

Students choose suitable informal, metric or SI units for the order of magnitude involved with measurement in a given context. They choose and use various instruments and technologies, strategies and formulas to estimate and calculate measures of shapes, objects and events including length, area, volume, mass, time, temperature, angle and rate.

Students have the opportunity to:

- use appropriate combinations of units and formulas to measure and calculate length, area and volume in a given context (eg *design and cost an automatic watering system for a garden*)
- convert between different units of measure for the same attribute (eg *express* 4.5 *ha in square metres, convert* 35.67 *tonnes to kilograms, calculate the number of seconds in* 3 *hours and* 25 *minutes, find the metric equivalent in mm of a*  $\frac{1}{8}$  *inch drill bit*)
- use a variety of instruments for indirect measures of quantities (eg *stop-watch, theodolite or data-logger*)
- apply scale and similarity, Pythagoras' theorem or trigonometry as applicable to find lengths and angles in situations where they cannot be measured directly (eg *orienteering*)
- estimate length, area, volume, mass, time of day and duration, angle and temperature by comparison with experience, and with respect to known references (eg *estimate the time of day by referring to the position of sun*)
- calculate and apply rates in familiar contexts (eg cordial mixtures mL per L to obtain desired sweetness, average speed in km/h to determine time needed to complete a journey).

Students state a reasonable range for a measurement in a practical context. They understand that measurement error can be compounded by repetition and calculation.

Students have the opportunity to:

- identify the interval within which a measurement occurs (eg the speedometer of a car typically provides a value which is accurate to  $\pm 3$  km per hour)
- read scales and make reasonable estimates where measures fall between marked graduations
- make judgments about acceptable variation in estimation of quantities based on experience (eg want 250 grams of olives from a deli, but will accept a quantity within the range of 240 grams to 260 grams)
- observe how error can be accumulated when several related measurements are made (eg three lengths of cloth measured and cut to an accuracy of 3 mm from a single piece of cloth may have a total error of up to 9 mm).

Students use a variety of sources, including samples and surveys, published data, data-bases, experiments and simulations to estimate empirical probabilities associated with events. They assign and compare probabilities based on experience.

- estimate probabilities for a range of events using materials and technology (eg gender sequences in families or school performance in a sports competition)
- explore the reasonableness of estimates of relative likelihood based on personal experience (eg *explain why a student might claim that they are twice as likely to finish a 1500 metre race ahead of their training partner than vice-versa*).

Students use a variety of approaches to determine and represent sample (event) spaces and calculate corresponding probabilities. They are familiar with the notion of random and equally-likely events.

Students have the opportunity to:

- use different representations to specify the sample (event) space for straightforward compound events (eg *a grid for the scores on a regular six-sided die and an octahedral die; a tree diagram for the combination of results obtained by spinning a six-coloured spinner and tossing a coin*)
- determine probabilities for straightforward compound events (eg the probability of having an alternating gender sequence in a family of 3 children)
- analyse situations involving random events and chance (eg playing a game of snakes and ladders, decide whether it is more likely that a person lands on a snake, a ladder or misses both of these; simulation of the Monty Hall Game Show problem).

Students use a range of representations for discrete and continuous data.

Students have the opportunity to:

- present and describe data collected from tables and data-bases using stem-and-leaf plots and histograms, as suits the nature of the data
- explore how bias can arise if random sampling is not used, and use simple random sampling in surveys
- interpret data and consider the impact of outliers on mean, median and mode, and the usefulness of range as a measure of spread (eg *house prices in a given location*)

Students use summary statistics and the distribution of data to respond to claims and questions.

Students have the opportunity to:

- make statements about proportions related to a population based on estimates formed from a random sample (eg *the proportion of people who think the leader of a political party is doing a poor, satisfactory, or good job in relation to age group*)
- use averages and features of a set of data, including graphical representation, such as clusters, the middle 50% and outliers to discuss the distribution of data in a sample, and analyse related claims and questions (eg *claims in the media about unemployment rates, questions about the fitness of a particular portion of the population*).

#### **Year 9 Space**

Students identify, describe and classify a broad range of two-dimensional and three-dimensional objects and geometric shapes, and simple composite shapes.

- describe and classify quadrilaterals in terms of side, diagonal and angle (eg *the diagonals of a rhombus bisect each other at right angles*)
- identify polygons, circles and ellipses, and simple part and composite shapes made from these
- identify and classify different representations of three-dimensional shapes and objects including cylinders, cones, the platonic solids, packages and containers with references to faces and surfaces (eg a tetrahedron-shaped package has a shape which consists of four equilateral triangles, any three of which are adjacent).

Students use drawing tools, including geometry software, models and materials to represent and construct common two-dimensional shapes and three-dimensional shapes and objects, including composite shapes and objects.

Students have the opportunity to:

- show front, side and top (plan) views and cross-sections of three-dimensional shapes and objects including simple polyhedra, cylinders, spheres and cones, and composite shapes formed from these (eg *a drink bottle*)
- draw two-dimensional shapes to specification in terms of boundary, angle and scale (eg *a symbol such as* **3** *is a regular five-pointed star inscribed within a circle of a given diameter*)
- use geometric shapes to construct accurate two-dimensional representations of three-dimensional objects (eg an isometric drawing, front-side-top view or a single point perspective drawing of an hourglass a draw various cross-sections of a tooth-paste tube, draw a suitable net for constructing a cone of a given slant edge length with a lid from a sheet of paper) and discuss which properties are preserved by the representation and which are not (eg angle, length and area)
- construct three-dimensional objects from nets and make models of three-dimensional objects from isometric diagrams (eg *a soccer-ball from the net of its stitching-pattern involving a tessellation of pentagons and hexagons*) and discuss their properties (eg *what is the difference between two tetrahedrons joined at their bases and an octahedron, and which one of these is a space filling shape?*).

Students apply symmetry, congruence and similarity of shapes and relate these to informal proofs, transformations and tessellations in the plane and on surfaces.

Students have the opportunity to:

- know and apply the angle properties related to parallel, perpendicular and transversal lines
- use congruent and similar triangles to solve geometric problems involving patterns and design
- make deductions related to geometric properties of shapes (eg when two straight lines intersect, opposite angles are equal; the sum of the interior angles of a polygon with n sides)
- explore demonstrations and informal proofs of general propositions (eg *the sum of angles in a plane triangle is always* 180°; *if corresponding angles are equal then alternate angles are equal; Pythagoras' theorem*).

Students interpret and construct various maps, diagrams and plans, and use these to specify location and to move from one location to another.

- use grids and coordinates, scale, and true bearing to read, interpret and follow maps (eg *a bush walk*, *orienteering*)
- provide directions from one location to another on a variety of maps and plans with reference to key features, distance and orientation (eg *a family holiday through central Australia*)
- draw and use diagrams to represent and analyse relationships (eg *the shortest path for a tour around a zoo, a draw for a knockout competition*).