BOARD OF STUDIES
NEW SOUTH WALES

General Mathematics

Stage 6

Syllabus

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Contents

1 The Higher School Certificate Program of Study

The purpose of the Higher School Certificate program of study is to:

- provide a curriculum structure that encourages students to complete secondary education;
- foster the intellectual, social and moral development of students, in particular developing their:
  - knowledge, skills, understanding and attitudes in the fields of study they choose
  - capacity to manage their own learning
  - desire to continue learning in formal or informal settings beyond school
  - capacity to work together with others
  - respect for the cultural diversity of Australian society;
- provide a flexible structure within which students can prepare for:
  - further education and training
  - employment
  - full and active participation as citizens;
- provide formal assessment and certification of students' achievements;
- provide a context within which schools also have the opportunity to foster students' physical and spiritual development.

2 Rationale for General Mathematics in the Stage 6 Curriculum

Mathematics involves observation, representation, investigation and comparison of patterns and relationships in social and physical phenomena. It allows the creative solution of problems and is also a powerful, precise and concise means of communication. At an everyday level, it is concerned with practical applications in many branches of human activity. At a higher level, it involves abstraction and generalisation. As such, it has been integral to most of the scientific and technological advances made in Australia and elsewhere.

Effective participation in a changing society is enhanced by the development of mathematical competence in contextualised problem-solving. Experience in such problem-solving is gained by students by:

(i) gathering, interpreting and analysing mathematical information and
(ii) applying mathematics to model situations.

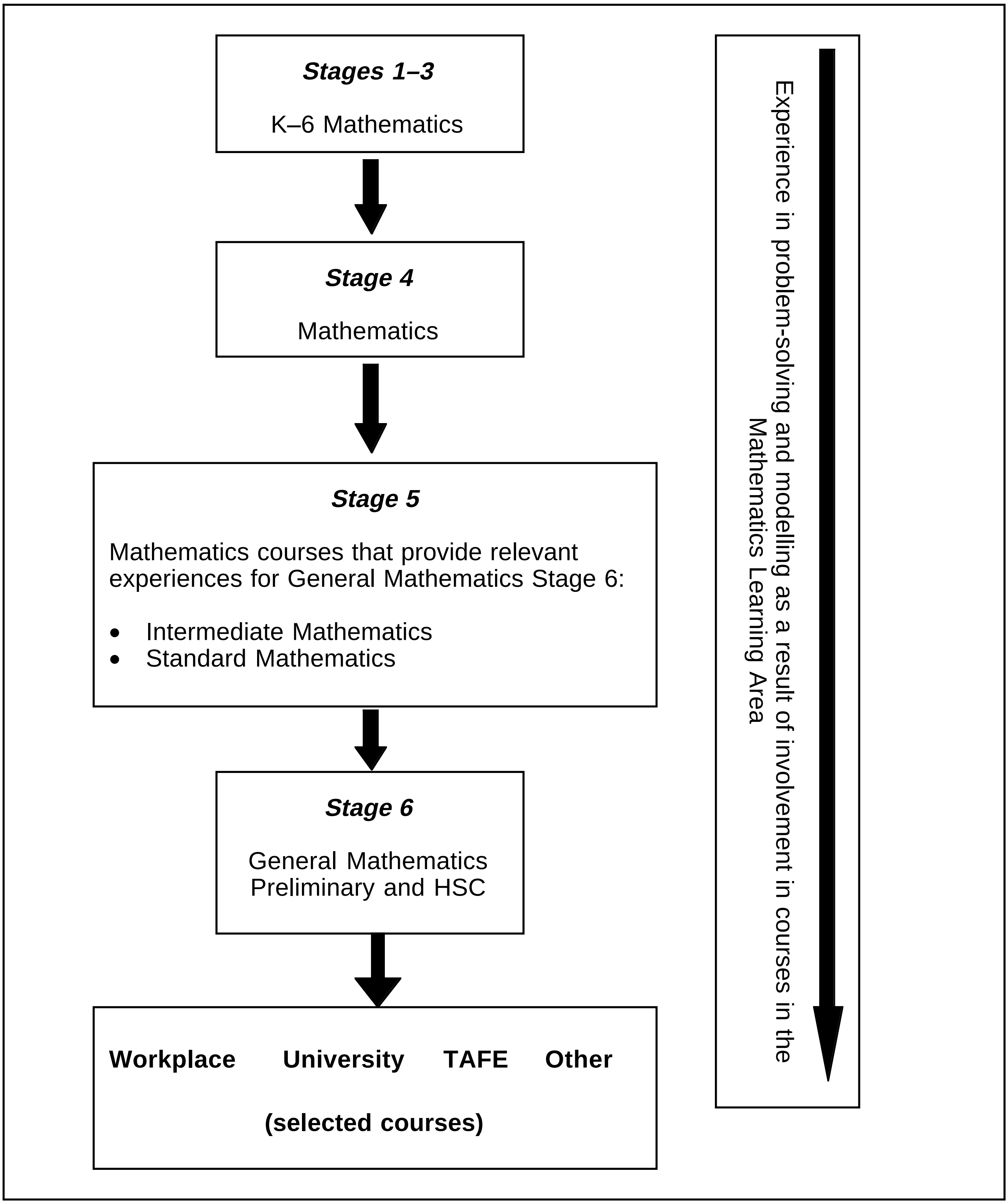
The opportunities for creative thinking, communication and contextualised problem-solving provided by this course, assist students to find positive solutions for the broad range of problems encountered in life and work beyond school.

General Mathematics supports the other Stage 6 mathematics courses in catering for the wide variation in students' mathematical competence at the conclusion of Year 10, and provides opportunities for continuing mathematical growth.

The purpose of General Mathematics is to provide an appropriate mathematical background for students who wish to enter occupations which require the use of basic mathematical and statistical techniques. The direction taken by the course, in focusing on mathematical skills and techniques that have direct application to everyday activity, contrasts with the more abstract approach taken by the other Stage 6 mathematics courses.

The study of General Mathematics provides students with valuable support in a range of concurrent Stage 6 subjects, in fostering development of mathematical skills and techniques that assist students who undertake associated research and projects. The course provides a strong foundation for vocational pathways, either in the workforce or in further vocational training studies, and for university courses in the areas of business, the humanities, nursing and paramedical sciences.

3 Continuum of Learning for Stage 6 General Mathematics Students



4 Aim

General Mathematics is designed to promote the development of skills, knowledge and understanding in areas of mathematics that have direct application to the broad range of human activity. Students will learn to use a range of techniques and tools to develop solutions to a wide variety of problems related to their present and future needs and aspirations.

5 Objectives

Students will develop:

- appreciation of the relevance of mathematics
- the ability to apply mathematical skills and techniques to interpret practical situations
- the ability to communicate mathematics in written and/or verbal form
- skills, knowledge and understanding in financial mathematics
- skills, knowledge and understanding in data analysis
- skills, knowledge and understanding in measurement
- skills, knowledge and understanding in probability
- skills, knowledge and understanding in algebraic modelling.

6 Course Structure

The following schematic view provides an overview of the arrangement and relationship between components of the Preliminary course and HSC course for General Mathematics Stage 6.

Preliminary Course	HSC Course
Financial Mathematics • FM1: Earning money • FM2: Investing money • FM3: Taxation	**Financial Mathematics** • FM4: Credit and borrowing • FM5: Annuities and loan repayments • FM6: Depreciation
Data Analysis • DA1: Statistics and society • DA2: Data collection and sampling • DA3: Displaying single data sets • DA4: Summary statistics	**Data Analysis** (A) DA5: Interpreting sets of data (B) DA6: The normal distribution (C) DA7: Correlation
Measurement • M1: Units of measurement • M2: Applications of area and volume • M3: Similarity of two-dimensional figures • M4: Right-angled triangles	**Measurement** • M5: Further applications of area and volume • M6: Applications of trigonometry • M7: Spherical geometry
Probability • PB1: The language of chance • PB2: Relative frequency and probability	**Probability** P1.1 PB3: Multi-stage events P1.1 PB4: Applications of probability
Algebraic Modelling • AM1: Basic algebraic skills • AM2: Modelling linear relationships	**Algebraic Modelling** • AM3: Algebraic skills and techniques • AM4: Modelling linear and non-linear relationships

7 Objectives and Outcomes

7.1 Table of Objectives and Outcomes

Objectives	Preliminary Outcomes	HSC Outcomes
Students will develop:	**A student:**	**A student:**
• appreciation of the relevance of mathematics	P1 develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation	H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society
• the ability to apply mathematical skills and techniques to interpret practical situations • skills, knowledge and understanding in algebraic modelling	P2 applies mathematical knowledge and skills to solving problems within familiar contexts	H2 integrates mathematical knowledge and skills from different content areas in exploring new situations
	P3 develops rules to represent patterns arising from numerical and other sources	H3 develops and tests a general mathematical relationship from observed patterns
	P4 represents information in symbolic, graphical and tabular forms	H4 analyses representations of data in order to make inferences, predictions and conclusions
	P5 represents the relationships between changing quantities in algebraic and graphical form	H5 makes predictions about the behaviour of situations based on simple models
• skills, knowledge and understanding in measurement	P6 performs calculations in relation to two-dimensional and three-dimensional figures	H6 analyses two-dimensional and three-dimensional models to solve practical and mathematical problems
	P7 determines the degree of accuracy of measurements and calculations	H7 interprets the results of measurements and calculations and makes judgements about reasonableness

Objectives	Preliminary Outcomes	HSC Outcomes
Students will develop:	**A student:**	**A student:**
• skills, knowledge and understanding in financial mathematics	P8 models financial situations using appropriate tools	H8 makes informed decisions about financial situations
• skills, knowledge and understanding in data analysis	P9 determines an appropriate form of organisation and representation of collected data	H9 develops and carries out statistical processes to answer questions which she/he and others have posed
• skills, knowledge and understanding in probability	P10 performs simple calculations in relation to the likelihood of familiar events	H10 solves problems involving uncertainty using basic principles of probability
• the ability to communicate mathematics in written and/or verbal form	P11 justifies his/her response to a given problem using appropriate mathematical terminology	H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others

7.2 Key Competencies

General Mathematics provides a context within which to develop general competencies considered essential for the acquisition of effective, higher-order thinking skills necessary for further education, work and everyday life.

Key competencies are embedded in the *General Mathematics Stage 6 Syllabus* to enhance student learning. The key competencies of ***collecting, analysing and organising information*** and ***communicating ideas and information***, reflect core processes of statistical inquiry and are explicit in the objectives and outcomes of the syllabus. The other key competencies are developed through the methodologies of the syllabus and through classroom pedagogy. Students work as individuals and as members of groups to engage with applications and modelling tasks, and through this, the key competencies ***planning and organising activities*** and ***working with others and in teams*** are developed. At all levels of the course, students are developing the key competency ***using mathematical ideas and techniques***. Through the advice provided on the selection and use of appropriate technology, students can develop the key competency of ***using technology***. Finally, students' continual involvement with seeking solutions to problems, both large and small, contribute towards their development of the key competency ***solving problems***.

Presentation of Content

The course content is presented in five *areas of study*, aspects of which are studied in each of the Preliminary and HSC courses. Within each area of study, the material is divided into cohesive *units of work*, each of which contributes to the students' achievement of one or more of the course outcomes. It is intended that the prescribed skills, knowledge and understanding be developed through the study of appropriate tasks and applications that clearly demonstrate the need for such skills.

Units of work are presented in the following format:

Name of Unit

Focus statement: a brief summary of content and purpose of the unit.

Outcomes addressed

An indication of the specific course outcomes that will be informed through study of the unit.

Students learn and acquire the following skills, knowledge and understanding

An identification of the mandatory mathematical content of the unit.

Terminology introduced in this unit

An identification of words or phrases that may be new to students, and which may be used in relevant assessment tasks. Students are expected to understand these terms but will not be asked to define them.

Technology that may be used in support of this unit

The provision of advice about the nature and suggested use of technology that is appropriate to the unit.

Suggested applications and modelling tasks

The provision of examples of a range and style of applications that may be used to introduce and illustrate the mathematical content of the unit. These applications are not mandatory, and are presented here for the support of teachers. The selection provided in the syllabus is necessarily brief. Further examples and resources for modelling activities will be found in the support document.

The use of formulae in the teaching and assessment of the course

Any formulae that have been required in the *Mathematics Stage 4 (Years 7–8) Syllabus* are considered to be assumed knowledge. These include formulae for:

- calculations involving the theorem of Pythagoras
- perimeter
- the circumference of a circle
- the area of a rectangle, triangle and circle.

Formulae required in the themes of the Stage 5 Standard course are considered to be assumed knowledge. These include formulae for the:

- area of a parallelogram, trapezium and rhombus
- volume of a right prism.

Students are not required to learn other formulae that are introduced or referred to in this syllabus. A list of formulae will be provided with the HSC examination. This list is included with the sample HSC examination that accompanies this syllabus.

8 Preliminary Course Content

Hours shown are indicative only.

Financial Mathematics	**24 hours**

FM1: Earning money
FM2: Investing money
FM3: Taxation

Data Analysis	**32 hours**

DA1: Statistics and society
DA2: Data collection and sampling
DA3: Displaying single data sets
DA4: Summary statistics

Measurement	**32 hours**

M1: Units of measurement
M2: Applications of area and volume
M3: Similarity of two-dimensional figures
M4: Right-angled triangles

Probability	**16 hours**

PB1: The language of chance
PB2: Relative frequency and probability

Algebraic Modelling	**16 hours**

AM1: Basic algebraic skills
AM2: Modelling linear relationships

8.1 Financial Mathematics

FM1: Earning money

The principal focus of this unit is the range of ways in which individuals earn and manage their money.

Outcomes addressed

A student:

P1 develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation

P2 applies mathematical knowledge and skills to solving problems within familiar contexts

P7 determines the degree of accuracy of measurements and calculations

P8 models financial situations using appropriate tools

P11 justifies his/her response to a given problem using appropriate mathematical terminology.

Students learn and acquire the following skills, knowledge and understanding

- calculation of monthly, fortnightly, weekly, daily and hourly payments from salary
- calculation of wages incorporating hourly rate, penalty rates such as overtime, special allowances for, for example, wet work, confined spaces, toxic substances, heat, heights
- calculation of annual leave loading
- calculation of earnings based on commission, piecework, royalties
- calculation of income based on government allowances, such as youth allowance, pensions
- determination of deductions such as union fees, superannuation contributions, health fund instalments and tax instalments
- calculation and comparison of user costs associated with maintaining accounts with financial institutions
- calculation of net pay following deductions
- creation and management of budgets
- reading information from household bills, including those for electricity, gas, telephone, council rates and water rates.

Terminology introduced in this unit

annual leave	annual leave loading
deductions	double time
gross pay/gross wage	overtime
royalty	time-and-a-half.

Technology that may be used in support of this unit

- use of a prepared spreadsheet to calculate earnings
- use of a prepared spreadsheet to create a budget.

Suggested applications and modelling tasks (FM1)

- Use a spreadsheet to calculate wages for a small company including the calculation of overtime.
- Calculate earnings based on current Enterprise Agreements. These can be obtained from unions or the human resource department of an organisation.
- Review a previously prepared budget to reallocate funds for a sudden contingency.

FM2: Investing money

The principal focus of this unit is to use formulae and tables to perform calculations related to the value of investments over a period of time.

Outcomes addressed

A student:

P2 applies mathematical knowledge and skills to solving problems within familiar contexts
P3 develops rules to represent patterns arising from numerical and other sources
P7 determines the degree of accuracy of measurements and calculations
P8 models financial situations using appropriate tools
P11 justifies his/her response to a given problem using appropriate mathematical terminology.

Students learn and acquire the following skills, knowledge and understanding

- calculation of simple interest using $I = \mathrm{Pr}\,n$, where P = principal, r = percentage interest rate per period expressed as a decimal (eg if the rate is quoted as 8.2%, then r = 0.082), and n = number of periods

P1.1 for fixed values of P, using tables of values and hence drawing and describing graphs of I against n for differing values of r

Note: these are linear graphs whose gradient is determined by the value of r (see AM2, AM4)

- calculation of monthly, quarterly, six-monthly interest rates based on quoted rates per annum (pa)
- use of formulae to calculate future value, compound interest and present value with pen and paper

$A = P(1+r)^n$, where A (amount) = final balance (*future value*), P (principal) = initial quantity (*present value*), n = number of compounding periods, r = interest rate per compounding period

Note: In the financial world, the compound interest formula quoted above is generally presented as $FV = PV(1+r)^n$, where FV = future value and PV = present value

- for fixed values of P, using tables of values and hence drawing and describing graphs of A against n for differing values of r

  Note: these are examples of exponential growth (see AM3, AM4)
- calculation of dividend paid on a share holding and the dividend yield, excluding franked dividends
- extrapolating from the information shown on a prepared graph of share performance to suggest possible future movement
- calculating future and present value of an investment from prepared tables (see table below)
- calculation of the price of goods following inflation
- calculating the appreciated value of items such as stamp collections, memorabilia.

Compounded values of $1

	Interest rate per period				
Periods	1%	5%	10%	15%	20%
1	1.010	1.050	1.100	1.150	1.200
2	1.020	1.103	1.210	1.323	1.440
3	1.030	1.158	1.331	1.521	1.728
4	1.041	1.216	1.461	1.750	2.074
5	1.051	1.276	1.611	2.011	2.488
6	1.062	1.340	1.772	2.313	2.986

Eg: to calculate the future value of $3000 invested for 6 years compounded annually at 5%,

A = 3000 x 1.340 = $4020

Terminology introduced in this unit

compounding period
dividend
dividend yield
future value
present value
shares.

Technology that may be used in support of this unit

- use of a spreadsheet's chart tool to create graphs of functions based on the compound interest formula, eg compare the value of an investment of $1000 over 5 years at 10% pa compounded
  (a) annually
  (b) bi-annually
  (c) monthly.

Suggested applications and modelling tasks (FM2)

- If the interest rate is quoted as 6% pa, what amount needs to be invested in order for the investment to be worth $850 at year's end?
- Jan and Bob wish to save $10 000 for their granddaughter's university expenses. They wish to have this amount available in 8 years' time. Calculate the single sum to be invested at 5% pa compounded annually.
- Determine the single sum to be deposited if $10 000 is required in 5 years' time and terms of 3% pa (compounded quarterly) are available.
- A principal of $1000 is to be invested for three years. Determine which is the better investment option: (i) 6% pa simple interest, (ii) 5.9% pa compounded annually or (iii) 5.85% pa compounded half-yearly.
- Use a spreadsheet to create tables to determine the future and present values of $1.
- Students could access and analyse historical data on share prices. This data is available from financial journals, newspapers and the Internet.

- An investor purchases 1000 shares in a company at a price of \$3.98, with a dividend yield of 5.5%. Brokerage costs are 1% of the purchase price. One year later, the shares sold at \$4.80. Calculate the total earnings over the year, after costs.
- Use published share data to create a table of a company's share price over a period of time.
- Use the charting facilities of a spreadsheet to illustrate performance of a company's shares over a period of time.
- Students could create and track a portfolio of shares and publish their results each month. The value of the portfolio could be compared to the changes in the All Ordinaries Index in the time period examined.
- Students could participate in the 'ASX Game'. This game is administered by the Australian Stock Exchange. Players nominally invest \$50 000 in listed shares. This could occur in conjunction with other subjects, with the emphasis, in this instance, on mathematics and modelling activities. For details visit the Australian Stock Exchange website.

FM3: Taxation

The principal focus of this unit is the calculation of tax payable on income and goods and services.

Outcomes addressed

A student:

P2 applies mathematical knowledge and skills to solving problems within familiar contexts
P5 represents the relationships between changing quantities in algebraic and graphical form
P7 determines the degree of accuracy of measurements and calculations
P8 models financial situations using appropriate tools
P11 justifies his/her response to a given problem, using appropriate mathematical terminology.

Students learn and acquire the following skills, knowledge and understanding

- calculation of the amount of allowable deductions from gross income
- calculation of taxable income
- calculation of Medicare levy (basic levy only — see Tax Pack for details)
- calculation of PAYE (Pay As You Earn) tax payable or refund owing, using current tax scales
- given rates of tax from a range of countries, calculation of the Value Added Tax (VAT) payable on a range of goods and services
- calculation of the goods and services tax (GST) payable on a range of goods and services
- creating graphs to illustrate and describe different tax rates
  Note: a graph of tax paid against taxable income is a piecewise linear function (see AM2).

Terminology introduced in this unit

goods and services tax (GST)
group certificate
income tax
Medicare levy
PAYE
tax deduction
taxable income
value added tax (VAT).

Technology that may be used in support of this unit

- use of a prepared spreadsheet to calculate PAYE tax (see example below).

Suggested applications and modelling tasks (FM3)

- Calculate tax refund (or amount payable) based on a sample Group Certificate, taking into account gross income, deductions, taxable income, tax payable on taxable income, Medicare levy and tax already paid as per the Group Certificate.
- Students could complete a tax return form (as included in the Tax Pack) using a typical PAYE employee's earnings and deductions. The aim is to calculate the 'refund from' or 'amount owed to' the Australian Taxation Office (ATO).

	A	B	C	D	E	F
1	**Tax calculator Version 1.0**			Created by C. Books		3/7/2000
2						
3	This spreadsheet calculates the yearly, weekly and fortnightly tax based					
4	on yearly taxable income not including the medicare levy					
5						
6		Variables			Results	
7		**Taxable Income**	$30,000		**Yearly Tax**	$6,222.00
8					**Weekly Tax**	$119.65
9					**Fortnightly Tax**	$239.31
10		Calculation Section				
11	Bracket			Tax Payable		
12	1	$1	$5,400			
13	2	$5,401	$20,700			
14	3	$20,701	$38,000	$6,222		
15	4	$38,001	$50,000			
16	5	$50,001				

8.2 Data Analysis

DA1: Statistics and society

The focus of this unit is the importance of statistical processes and inquiry in society. The material treated is essential background for the later units in Data Analysis, and may be integrated with the later units if desired.

Outcomes addressed

A student:

P1 develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation

P9 determines an appropriate form of organisation and representation of collected data

P11 justifies his/her response to a given problem using appropriate mathematical terminology.

Students learn about and acquire the following skills, knowledge and understanding

- the importance of analysing data in planning and decision-making by governments and businesses
- the process of statistical inquiry, including the following steps:
  - posing questions
  - collecting data
  - organising data
  - summarising and displaying data
  - analysing data and drawing conclusions
  - writing a report
- the role of statistical methods in quality control in manufacturing industries
- issues of privacy and ethics in data collection and analysis
- organisations that collect and/or use statistics, including the Australian Bureau of Statistics (ABS), the United Nations (UN), the World Health Organisation (WHO).

Terminology introduced in this unit

data
information
quality control
statistics
statistical inquiry.

Suggested applications and modelling tasks (DA1)

- Explore some of the different forms of information collection and recording that have been and are being used — the tally stick, church records, census, sport and weather information, the Guinness Book of Records, computer networks, data banks, credit ratings …
- Investigate the role of statistics in shaping and describing aspects of life in our society (eg forecasting economic trends, forecasting weather, consumer opinion polls, public perception surveys, TV/radio surveys).
- Investigate the contribution made to the field of statistics by people such as:
  - John Graunt
  - Carl Friedrich Gauss
  - Florence Nightingale
  - Sir Ronald Fisher
  - W. Edwards Deming.

DA2: Data collection and sampling

The principal focus of this unit is the planning and management of data collection. In some cases, complete data sets may be available, and in other cases a survey may be needed that involves the use of a sample. Although the emphasis is on quantitative data, students should be aware of processes related to categorical data.

Outcomes addressed

A student:

P1 develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation

P9 determines an appropriate form of organisation and representation of collected data

P11 justifies his/her response to a given problem using appropriate mathematical terminology.

Students learn and acquire the following skills, knowledge and understanding

- identification of the target population to be investigated
- determining whether data for the whole population is available (eg the results of a round of a sporting competition), or whether sampling is necessary
- recognising that the purpose of a sample is to provide an estimate for a particular population characteristic when the entire population cannot be accessed
- classification of data as:
  - **Quantitative**, either discrete or continuous
  - **Categorical**

    eg gender (male, female) is *categorical*; height (measured in cm) is *quantitative*, *continuous*; quality (poor, average, good, excellent) is *categorical*; school population (measured in individuals) is *quantitative, discrete*
- distinguishing between the following sample types:
  - random
  - stratified
  - systematic
- determination of which of the above sample types is appropriate for a given situation
- relating sample selection to population characteristics, eg if 20% of the Australian population is aged under 20, your sample should include 20% of under 20s
- generating random numbers with a table or a calculator to assist in establishing random samples
- describing and using the ‘capture-recapture’ technique for estimating the size of populations, eg the number of fish in a lake
- recognising the effect of sample size in estimating the nature of the population, eg using the number of boys and girls in a particular Year 11 class to estimate the gender ratio in Year 11 across NSW

- use of the principles for effective questionnaire design, such as
  - simple language
  - unambiguous questions
  - respect for privacy
  - freedom from bias
  - consideration of number of choices, if given, eg an even number of choices may force an opinion in particular circumstances, but on occasions it may be appropriate to allow a neutral choice.

Terminology introduced in this unit

bias	categorical data	census	continuous
database	discrete	poll	population
questionnaire	random sample	sample size	strata
stratified sample	systematic sample.		

Suggested applications and modelling tasks (DA2)

- Prepare and present oral and written reports to describe collected data, including the source of the data and the type of data. If a sample is used, describe how the sample was chosen and how the measurements were made.
- Prepare questionnaires and discuss consistency of presentation and possible different interpretations of the questions.
- Discuss when it is appropriate to use a sample rather than a census.
- Discuss and present conclusions drawn from published data.
- Discuss and decide on methods of random number generation.
- Determine the best method of sampling in a range of situations.
- Discuss the reasonableness of drawing conclusions about populations from particular samples (eg whether it is appropriate to draw conclusions about the whole school from data based on a single class).
- Discuss and make judgements about arguments in the media for which statistical information is presented.
- Discuss the difference between a census and a sample, and illustrate with examples of where samples and censuses are used.
- Consider different ways of presenting questions, eg open questions, yes/no questions, tick boxes, response scales versus continuum scales etc.
- Discuss bias, the representativeness of the sample chosen and other issues that may affect the interpretation of the results.
- Select a range of samples from a fixed population and record the characteristics of each sample.
- The ‘capture-recapture’ technique can be simulated by using a large (but uncounted) number of toothpicks in a bowl to represent the population of fish in a particular waterway. Discuss why it may not be practical to catch all of the fish in order to count them. Take a small number (10 or 20) of the toothpicks and mark them with a coloured pen. Return them to the bowl and shake them around. This simulates the capture, tagging and release of a known number of fish. Now take a handful of the toothpicks (fish) from the bowl and establish the percentage of tagged fish in the handful. Since the total number of tagged fish is known, this percentage may be used to estimate the size of the total population.

DA3: Displaying single data sets

In this unit, students prepare a variety of data displays and consider the appropriateness of each for the stated purpose. The power of statistical displays both to inform and misinform should be emphasised.

Outcomes addressed

A student:

P1 develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation

P4 represents information in symbolic, graphical and tabular forms

P7 determines the degree of accuracy of measurements and calculations

P9 determines an appropriate form of organisation and representation of collected data

P11 justifies his/her response to a given problem using appropriate mathematical terminology.

Students learn and acquire the following skills, knowledge and understanding

- creation of tally charts and frequency tables to organise ungrouped and grouped data
- creation of dot plots, sector graphs (pie charts), bar graphs, histograms and line graphs, with attention being paid to the scale on each axis
- selection of a suitable scale for each axis of a graph
- noting the capacity of statistical displays for misrepresentation, particularly in the selection of the scale used on the axes
- creation of a stem-and-leaf plot to illustrate a small data set
- drawing a radar chart to display data such as sales figures, temperature or rainfall readings (see example at end of unit)
- division of data into deciles and quartiles
- determination of the range and interquartile range as measures of the spread of a data set
- creation of frequency graphs and cumulative frequency graphs (ogives)
- determining the median and upper and lower quartiles of a data set from a cumulative frequency polygon
- establishment of a five number summary for a data set (lower extreme, lower quartile, median, upper quartile, upper extreme)
- development of a box-and-whisker plot from a five number summary
- linking types of data with appropriate displays, eg continuous quantitative data is best represented by a histogram; categorical data is best represented with a bar graph or sector graph (pie chart)
- describing the strengths and/or weaknesses of sector graphs, bar graphs, histograms, frequency polygons and radar charts, including suitability for data represented.

Technology that may be used in support of this unit

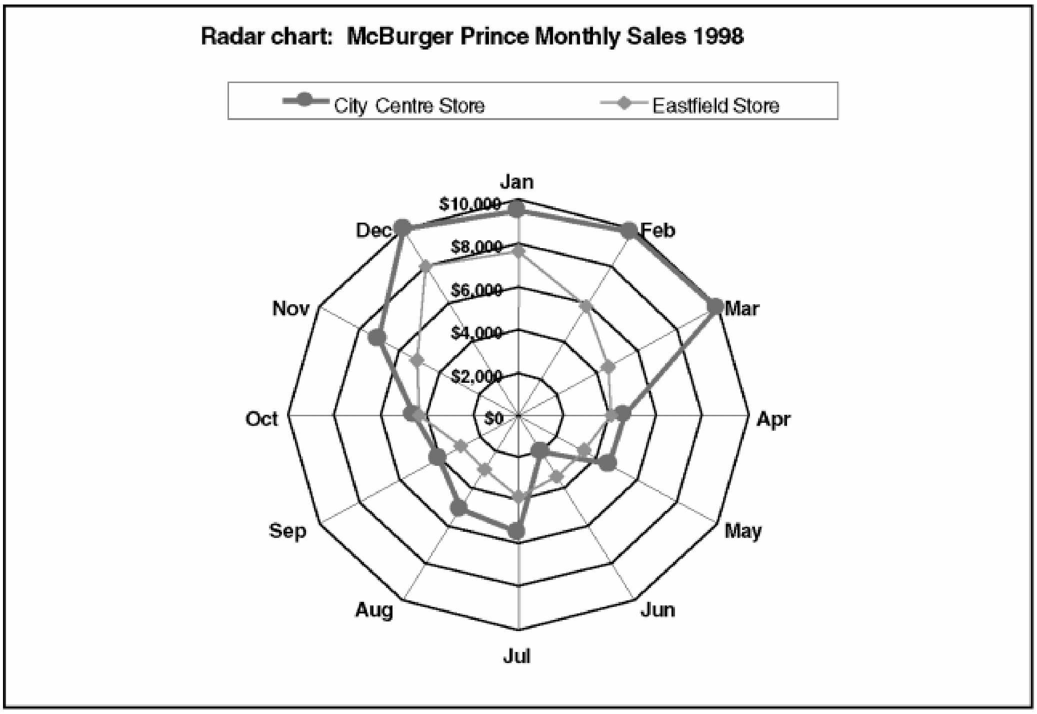
- use of a spreadsheet or a graphing calculator to create frequency tables and statistical graphs

Terminology introduced in this unit

bar graph
box-and-whisker plot
column graph
cumulative frequency
decile
dot plot
five number summary
frequency table
grouped data
histogram
interquartile range
line graph
ogive
polygon
quartile
radar chart
range
relative frequency
sector graph
stem-and-leaf plot
upper/lower extreme
upper/lower quartile.

Suggested applications and modelling tasks (DA3)

- Prepare rough dot plots for initial exploration of data.
- Suggest possible reasons for the occurrence of any unusual scores in a data set.
- Having gathered data, use pen and paper, a scientific calculator, graphing calculator and/or spreadsheets, other appropriate software or databases to organise the data and draw graphs.
- Decide on the most appropriate type(s) of display for a particular data set, choosing from a histogram, stem-and-leaf plot or other type of graph.
- Change the type of display on the computer to see if a different impression is given of the shape of the data display.
- Perform experiments and record and analyse data to determine whether or not performance of a simple task can be improved with practice.
- Compare and contrast the suitability of different types of statistical displays, eg a radar chart can be used to illustrate seasonal change or hourly change of a quantity over one day; a line graph is useful to show trends in data over equal time intervals.
- Students could collect examples of misleading statistical displays and prepare accurate versions of each. They could describe the inaccuracies, and their corrected version, in appropriate mathematical language.



DA4: Summary statistics

The principal focus of this unit is the calculation of summary statistics for single data sets and their use in interpretation.

Outcomes addressed

A student:

P2 applies mathematical knowledge and skills to solving problems within familiar contexts
P4 represents information in symbolic, graphical and tabular forms
P7 determines the degree of accuracy of measurements and calculations
P11 justifies his/her response to a given problem using appropriate mathematical terminology.

Students learn and acquire the following skills, knowledge and understanding

- calculation of the mean of small data sets, using the formulae $\bar{x} = \frac{\sum x}{n}$, $\bar{x} = \frac{\sum fx}{\sum f}$ where $\bar{x}$, represents the mean of the sample
- determination of the mean for larger data sets of either grouped and ungrouped data using the statistical functions of a calculator
- calculation of the means of a range of samples from a population
- informal description of standard deviation as a measure of the spread of data in relation to the mean
- determination of the population standard deviation using the σ_n button of a calculator and the sample standard deviation as an estimate of the population measure, using the σ_{n-1} button
- determination of the median and mode(s) of a data set, either from a list or from a frequency table
- selection and use of the appropriate statistic (mean, median or mode) to describe features of a data set, eg median house prices, modal shirt size
- comparison of the summary statistics of various samples from the same population.

Terminology introduced in this unit

mean
median
mode
standard deviation
summary statistic.

Technology that may be used in support of this unit

- calculate summary statistics from inbuilt statistical functions of a graphing calculator
- use a spreadsheet to create frequency tables and calculate summary statistics

Suggested applications and modelling tasks (DA4)

- Investigate the use of the word 'average' in a range of social contexts and the use of different 'averages' to serve different purposes, and sometimes, to mislead.
- Interpret and evaluate data from students' own surveys and draw conclusions that can be justified.
- Report orally and in writing on investigations, discussing what was investigated, how the investigation was planned and the data collected, the display and the analysis, together with the conclusion that could be drawn.
- Write a letter to a newspaper summarising the results of an investigation, suggesting the implications, and explaining and justifying conclusions.

8.3 Measurement

M1: Units of measurement

In this unit, the principal focus is on metric units of measurement, and rates and ratios. The students learn about making judgements about measurement errors.

Outcomes addressed

A student:

P2 applies mathematical knowledge and skills to solving problems within familiar contexts
P5 represents the relationships between changing quantities in algebraic and graphical form
P7 determines the degree of accuracy of measurements and calculations.

Students learn and acquire the following skills, knowledge and understanding

- determination of appropriate units to use when measuring physical attributes
- conversion between commonly used units of measurement using standard prefixes
- recognition that accuracy of physical measurement is limited to $\pm\frac{1}{2}$ of the smallest unit of which the measuring instrument is capable
- calculation of the percentage error in a measurement, eg if the measured height was 155 cm ± 0.5 cm (ie to the nearest cm), the percentage error for this measurement is $\pm\left(\frac{0.5}{155}\right)\times 100\%$
- determination of possible sources of error in measuring
- repeating and averaging measurements to reduce likelihood of error
- determination of the significant figures to be used in recording measurements, in relation to the accuracy of the measuring instrument being used
- use of positive and negative powers of ten in expressing numbers in scientific notation
- calculation of rates eg pay rates, speeds, rates of flow
- conversion between units for rates, eg km/h to m/s
- calculation of concentrations expressed as weight/weight, weight/volume or volume/volume
- Note: these calculations have particular applications to nursing and agriculture
- determination of overall change in a quantity following repeated percentage changes, eg an increase of 20% followed by a decrease of 20%
- finding the ratio of two quantities in familiar contexts
- division of quantities in a given ratio
- use of unitary method to solve problems.

Terminology introduced in this unit

concentration
percentage error
prefix
ratio
significant figures
unitary method.

Suggested applications and modelling tasks (M1)

- Students measure their heights and calculate the percentage error in their measurements.
- Modify given recipes by varying quantities to provide for various numbers of people.
- Applications from nursing may be considered, eg a patient fed intravenously by means of drops of fluids. The patient needs 3 L of fluid per day. One mL of fluid contains 15 drops. Find the rate at which the intravenous drip must run, expressing the answer in the form of number of drops fed to the patient each minute.
- Calculate the quantity of each component for a fertilising operation, given the ratio of each component in the mixture.
- Calculate and compare freight costs for a variety of modes of transport.
- Calculate the rates of application of chemicals used in agriculture, such as those for pesticides and feed additives.
- Your vet has recommended a vitamin supplement, for your sick guinea pigs, at the rate of 10 g per litre in their drinking water. From the chemist you can obtain the vitamin supplement in solution and the label states ‘300 g/300 mL’. How many mL of the solution should you add to the guinea pigs’ 500 mL water bottle?

M2: Applications of area and volume

The principal focus of this unit is the calculation and application of area and volume in the solution of problems.

Outcomes addressed

A student:

P2 applies mathematical knowledge and skills to solving problems within familiar contexts

P6 performs calculations in relation to two-dimensional and three-dimensional figures

P7 determines the degree of accuracy of measurements and calculations.

Students learn and acquire the following skills, knowledge and understanding

- calculation of the area of triangles and quadrilaterals (review only)
- using a field diagram to calculate the area of irregularly shaped blocks of land
- classifying polyhedra into prisms (named with respect to their constant cross-section), pyramids or other
- construction of nets of solids and matching nets to solids
- sketching 3D solids using isometric paper and vanishing points
- using appropriate formulae in calculating surface area of right prisms, square and rectangular pyramids
- using appropriate formulae in calculating volume of right prisms, cylinders, pyramids, cones, spheres
- application of the relationship between units of capacity and units of volume.

Technology that may be used in support of this unit

- use of drawing tools to create 2D and 3D drawings.

Suggested applications and modelling tasks (M2)

- Use appropriate software applications for drawing accurate figures and investigating geometrical properties.
- Survey an irregular area using offsets and use this information to calculate its approximate area.
- Discuss and report on possible sources of error, eg experimental, instrumental and constant error in the above activity.
- Estimate the painted surface area of a classroom. Plan and prepare a budget for the redecoration of a favourite room at home.
- Investigate the dimensions that maximise the area for a given shape and perimeter, such as in the design of playpens and stock paddocks.

M3: Similarity of two-dimensional figures

The principal focus of this unit is to apply similarity properties to problems in everyday life.

Outcomes addressed

A student:

P2 applies mathematical knowledge and skills to solving problems within familiar contexts
P6 performs calculations in relation to two-dimensional and three-dimensional figures
P7 determines the degree of accuracy of measurements and calculations.

Students learn and acquire the following skills, knowledge and understanding

- establishment of properties of similar figures
- recognition of similarity in everyday life
- finding scale factors of similar figures
- recognising that similar figures related by a scale factor of 1 are said to be congruent
- use of the relevant enlargement or reduction factor to calculate actual dimensions
- development of scale drawings of objects and images
- use of scale factor to solve problems involving similar figures
- transferring measurements between floor plans and elevations
- obtaining measurements from plans of buildings and rooms
- calculation of lengths and areas from a floor plan
- interpretation of commonly used symbols on house plans.

Terminology introduced in this unit

congruent
elevation
enlargement
reduction
scale factor
similar.

Suggested applications and modelling tasks (M3)

- Investigate the scale factor of enlargements obtained using an overhead projector. Is there a relationship between the distance of the projector from the screen and the scale factor of the resulting projection?
- By measuring the shadow thrown by a metre rule, students use similarity and shadow lengths to find the height of tall objects, eg tree, flag pole etc.
- Students accurately construct a scaled floor plan of the classroom.
- Students collect floor plans of new houses (from newspapers, new home exhibition centres…) and discuss features of the houses that appeal to them or features that they would like to change. They could compare the cost of the houses with their floor areas and determine cost per square metre. From this they could determine the most expensive/economical house to build. Students could complete this in groups and present their findings to the class.
- Find ceiling heights from plans.
- Use house plans to cost carpeting, tiling, painting rooms etc.
- Use a grid over a free-form diagram to draw an enlargement or reduction.
- Undertake an investigation relating to scale drawings and area, eg design a space-efficient car park to hold a certain number of cars. Students could construct scale models of cars and the car park to ensure that there is enough space for vehicles to reverse park, move safely about the car park etc.
- Use the properties of its diagonals to determine whether or not a room is rectangular.
- Students build similar rectangular prisms and cubes, finding the volume of each. They could then tabulate results and establish patterns.
- Students could investigate what happens to costs associated with area, surface area and volume if lengths are doubled, tripled…?
- For investigation: If all the dimensions on earth doubled overnight, how would you know when you got up the next morning?

M4: Right-angled triangles

In this unit, students learn to solve practical mathematical problems involving right-angled triangles.

Outcomes addressed

A student:

P2 applies mathematical knowledge and skills to solving problems within familiar contexts
P3 develops rules to represent patterns arising from numerical and other sources
P6 performs calculations in relation to two-dimensional and three-dimensional figures
P7 determines the degree of accuracy of measurements and calculations
P11 justifies his/her response to a given problem using appropriate mathematical terminology.

Students learn and acquire the following skills, knowledge and understanding

- use of Pythagoras' theorem to find an unknown side in a right-angled triangle
- application of Pythagoras' theorem to:
  - determine whether or not a triangle is right-angled
  - solve problems based on single right-angled triangles
  - calculate perimeters of irregularly shaped blocks of land
- defining sine, cosine and tangent ratios
- use of trigonometric ratios to find the length of an unknown side in a right-angled triangle
- use of trigonometric ratios to find the size of an unknown angle in a right-angled triangle using a calculator to approximate the angle to the nearest minute
- solution of problems involving angles of elevation and depression, given the appropriate diagram
- determining whether an answer seems reasonable by using a diagram drawn roughly in proportion.

Terminology introduced in this unit

adjacent
angle of elevation
hypotenuse
sine
trigonometry.

angle of depression
cosine
opposite
tangent

Suggested applications and modelling tasks (M4)

- Find possible lengths of two sides of a right-angled triangle, given the length of the hypotenuse.
- Investigate the trigonometric ratios for angles of, say, 30°, 45°, 60° in a number of similar right-angled triangles.
- In groups, students use a clinometer to find the heights of school buildings etc. Different groups compare and discuss their answers. They could use angles of elevation or depression or use the fact that $\tan 45° = 1$.
- Construct and use a 'height measurer' to measure the heights of trees.
- Using a metre rule leaning against a wall along with a blackboard protractor, students measure, as a decimal fraction of a metre:

  (a) the height the ruler reaches up the wall;

  (b) the distance of the foot of the ruler from the wall, for a range of different angles.
- Plan and carry out an orienteering event at the school.

8.4 Probability

PB1: The language of chance

In this unit, students learn to use the language of probability, count outcomes and describe the sample space of an event.

Outcomes addressed

A student:

P1 develops a positive attitude to mathematics and appreciates its capacity to provide enjoyment and recreation

P3 develops rules to represent patterns arising from numerical and other sources

P10 performs simple calculations in relation to the likelihood of familiar events

P11 justifies his/her response to a given problem using appropriate mathematical terminology.

Students learn and acquire the following skills, knowledge and understanding

- ordering everyday events from the very unlikely to the almost certain
- using a list or table to identify the sample space (set of all possible outcomes) of a simple experiment or game
- performing experiments and determining whether or not the outcomes are equally likely
- determining the number of outcomes for a multi-stage event by multiplying the number of choices at each stage, eg the total number of ways to place 3 different letters in 3 envelopes is 3 x 2 x 1
- using systematic lists to verify total number of outcomes for simple multi-stage events.

  Note: factorial notation is not required.

Terminology introduced in this unit

equally likely
event
frequency
multi-stage event
outcomes
sample space.

Suggested applications and modelling tasks (PB1)

- Practical experiments could involve coin tossing, dice rolling, picking cards from a pack.
- Comment critically on statements involving probability, such as: ‘Since it either rains or is fine, the probability of a fine day is 50-50’.
- What is a ‘one in 300-year flood’? Investigate expressions used in other disciplines and in everyday life to describe likely or unlikely events, eg ‘once in a blue moon’.
- Statements involving the language of probability could be collected from various media (newspapers, magazines, radio, television, the World Wide Web) and discussed.
- Investigate the number of different meals that can be chosen from a menu.
- Determine the number of combinations of raised dots that are possible in the Braille system for reading and writing. Investigate whether or not they are all used. Repeat for Morse Code.
- Investigate whether we will ever run out of postcodes, car numberplates or phone numbers.

PB2: Relative frequency and probability

The main focus of this unit is to compare relative frequency and calculated probability.

Outcomes addressed

A student:

P2 applies mathematical knowledge and skills to solving problems within familiar contexts
P4 represents information in symbolic, graphical and tabular forms
P10 performs simple calculations in relation to the likelihood of familiar events
P11 justifies his/her response to a given problem using appropriate mathematical terminology.

Students learn and acquire the following skills, knowledge and understanding

- estimating the relative frequencies of events from recorded data
- performing simple experiments to obtain relative frequencies from recorded results
- using relative frequencies to obtain approximate probabilities
- using the following definition of the probability of an event where outcomes are equally likely:

  $$P(event) = \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$$
- calculating probabilities in terms of the fractional, decimal, or percentage chance
- demonstrating the range of possible probabilities, $O \leq P(E) \leq 1$, through examination of a variety of results
- comparing calculated probabilities with experimental results
- illustrating the results of experiments through statistical graphs and displays (see DA3)
- defining and using the relationship between complementary events

  $$P(an\ event\ does\ not\ occur) = 1 - P(the\ event\ does\ occur)$$

Terminology introduced in this unit

complementary events
percentage chance
probability
relative frequency.

Suggested applications and modelling tasks (PB2)

- Data could be generated from simple experiments, and also be obtained from other sources where experiments are not possible, eg weather and sporting statistics from the newspaper, other data from Australian Bureau of Statistics (ABS) Yearbooks, websites.
- Statements involving the language of probability could be collected from various types of media (newspapers, magazines, radio, television, Internet) and discussed.
- If it is appropriate for the school population, plan a school raffle for fundraising.
- Experiments could be carried out in which the probability is not intuitively obvious, eg P(drawing pin landing point up).
- Examine the birth notices on a particular day in a daily paper. Record the number of boys and the number of girls. On this basis, estimate the probability that a child born is (a) male or (b) female. Compare these results with those published by ABS.

8.5 Algebraic Modelling

AM1: Basic algebraic skills

The main focus of this unit is to provide a foundation in basic algebraic skills required for this area of study and to use these skills to solve real and abstract problems.

Outcomes addressed

A student:

P2 applies mathematical knowledge and skills to solving problems within familiar contexts
P3 develops rules to represent patterns arising from numerical and other sources
P7 determines the degree of accuracy of measurements and calculations.

Students learn and acquire the following skills, knowledge and understanding

- identification and generalisation of simple linear number patterns
- adding and subtracting like terms
- evaluation of the subject of a formula through substitution of numerical values, using a wide variety of formulae such as

  $C=\frac{5}{9}(F-32)$, $v=u+at$, $S=\frac{D}{T}$, $V=\frac{4}{3}\pi r^3$, $A=P(1+r)^n$, $B=\frac{m}{h^2}$, $D=\frac{mA}{150}$, $D=\frac{yA}{(y+12)}$, $D=\frac{kA}{70}$
- solution of linear equations involving up to 3 steps (fractions with numerical denominators only)
- expansion and simplification of expressions such as

  $3p(p-2)$, $2x^2(7-x)+x(x-1)$
- multiplication of algebraic terms such as $3M^2\times 5M$, $\frac{9y}{4}\times 5y$
- division of single terms (linear, quadratic and cubic) such as $\frac{9L^3}{3L^2}$
- solving equations following substitution and evaluation,

  eg find a, given that v = 10, u = 5, s = 8 and $v^2=u^2+2as$.

Terminology introduced in this unit

cubic
linear
power
quadratic.

Suggested applications and modelling tasks (AM1)

- 'Practical' formulae for evaluation, eg $C = \frac{5}{9}(F - 32)$, $v = u + at$, $S = \frac{D}{T}$, $v^2 = u^2 + 2as$, $V = \frac{4}{3}\pi r^3$, $h^2 = a^2 + b^2$.
- Teachers may decide to introduce some of the formulae from the Financial Mathematics section here to emphasise how new variable names may be used, for example, $FV = PV(1+r)^n$. Note that FV is a single number, the Future Value of an investment, and PV is a single number, the Present Value.
- Algebra can be used to describe numerical patterns in calendars: take any 2 x 2 square, the sum of the diagonals is the same. To describe this, call the top left hand number x, then the number below it is $x + 7$, the number to its right is $x + 1$, the last number in the square is $(x+1) + 7$. The result for the diagonals can be described as $x + (x+1) + 7 = (x+1) + (x+7)$.
  Other patterns can be found in larger squares and represented algebraically.
- Linear number patterns can be found in matchstick activities such as 'line of squares'.
- Trade examples: A formula for calculating the bend allowance in sheet metal is $B = 2\pi\left(R + \frac{T}{1}\right) \times \frac{A}{360}$, where B is the bend allowance or arc length, T is the thickness in mm, A is the number of degrees in the angle of the bend, and R is the radius of curvature in mm.
- Fried's rule gives the medicine dosage for an infant in comparison with the adult dosage: $D = \frac{mA}{150}$ where D = infant dosage, m = age of infant in months and A = adult dosage. Two similar formulae are Young's rule: $D = \frac{yA}{(y+12)}$ where y = age of child in years, and Clark's rule: $D = \frac{kA}{70}$ where k = mass of child in kilograms. Compare the results of each rule for an 18-month-old child of mass 8.5 kg, if the adult dosage of the particular medication is 10 mL.
- For adults, the Body-Mass Index $B = \frac{m}{h^2}$ where m = mass in kilograms and h = height in metres. The medically accepted healthy range for B is $21 \le B \le 25$. Calculate the value of B for Carlo, who is 1.8 m tall and weighs 90 kg. If Carlo were to consult you as his doctor, what positive suggestions might you make to him about lifestyle?

AM2: Modelling linear relationships

Outcomes addressed

A student:

P3 develops rules to represent patterns arising from numerical and other sources
P4 represents information in symbolic, graphical and tabular forms
P5 represents the relationships between changing quantities in algebraic and graphical form.

Students learn and acquire the following skills, knowledge and understanding

- sketching graphical representations of quantities that vary over a period of time or in relation to each other

  Note: Students should develop an understanding of a function as

  input $\rightarrow$ *processing* $\rightarrow$ *output*. It is not intended that students learn a formal definition of a function.
- identifying independent and dependent variables in practical contexts
- graphing of linear functions derived from everyday situations (eg cost of an excursion = fixed cost + cost per student x number of students) by plotting ordered pairs from tables of values
- calculating the gradients of such graphs with ruler and pencil
- establishing a meaning for the gradient in the given context
- establishing a meaning for the intercept on the vertical axis in the given context
- sketching graphs of linear functions expressed in the form $y = mx + b$
- development of a linear graph of the form $y = ax$ from a description of a situation in which one quantity varies in a direct linear fashion with another, given one ordered pair
- using the above graph to establish the value of a (the gradient) and to solve problems related to the given variation context
- interpreting linear functions as models of physical phenomena
- using stepwise and piecewise linear functions to model situations encountered in daily life, eg parking charges, taxi fares, tax payments, mobile phone bills
- recognising the limitations of such models, eg a person’s height as a function of age may be approximated by a straight line for a limited number of years, but not over a complete lifetime
- using graphs to make conversions from one measurement to another eg $AUD to Euros
- interpreting the graphical solution of simultaneous linear equations drawn from practical situations
- drawing a line of best fit on a graphed set of ordered pairs with a ruler and pencil.

Terminology introduced in this unit

decreasing function
direct linear variation
function
increasing function
interpolate
proportional to
relation
y-intercept.
dependent variable
extrapolate
gradient
independent variable
piecewise linear function*
rate of change
step function*

* in practical contexts only

Technology that may be used in support of this unit

use of a graphing facility to:

- graph linear functions
- determine graphically the intercepts using ‘zooming’ techniques and built-in commands
- find points of intersection graphically
- inspect ordered pairs on a graph by ‘tracing’.

Suggested applications and modelling tasks (AM2)

- Functions and associated mathematical concepts to be introduced via everyday examples, such as:
  - distance travelled as a function of time elapsed
  - cost of postage as a function of mass (a step function)
  - amount of fine for speeding as a function of amount by which speed limit is exceeded (a step function)
  - a person’s height as a function of age (not strictly linear but can be approximated as a linear function)
  - cost of a mobile phone plan as a function of time and/or as a function of the number of calls
  - income tax rate as a function of taxable income (a piecewise function).
- Practical examples of linear functions include:
  - printing costs which involve an initial setup cost and a dollar rate per item (or hundreds of items…) printed
  - cost of taxi fare which may be approximated by hiring charge + (dollar rate per kilometre) x (number of kilometres travelled). Hiring charge is the vertical intercept and the gradient is the dollar rate per kilometre
  - catering costs which may involve a base amount for a set number of people plus a rate for extra guests
  - drawing conversion graphs (usually lines through the origin).
- Compare different mobile phone plans (graphically) to determine the most economical plan for a person’s likely pattern of use.

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9 HSC Course Content

Hours shown are indicative only.

Financial Mathematics	**30 hours**

FM4: Credit and borrowing
FM5: Annuities and loan repayments
FM6: Depreciation

Data Analysis	**26 hours**

DA5: Interpreting sets of data
DA6: The normal distribution
DA7: Correlation

Measurement	**24 hours**

M5: Further applications of area and volume
M6: Applications of trigonometry
M7: Spherical geometry

Probability	**16 hours**

PB3: Multi-stage events
PB4: Applications of probability

Algebraic Modelling	**24 hours**

AM3: Algebraic skills and techniques
AM4: Modelling linear and non-linear relationships

9.1 Financial Mathematics

<u>FM4: Credit and borrowing</u>

This unit of work focuses on the mathematics involved in borrowing money, the different types of loans available and credit cards.

Outcomes addressed

A student:

H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H5 makes predictions about the behaviour of situations based on simple models

H8 makes informed decisions about financial situations

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- calculation of principal, interest and repayments for flat-rate loans
- calculation of values in a table of home loan repayments (see below)

	Home Loan Table			
	Amount =	**$50 000**	*This table assumes the same number of days in each month,* ie Interest = Rate/12 x Principal	
	Annual Interest Rate =	**10%**		
	Monthly Repayment **(R) =**	**$600**		
N	**Principal (P)**	**Interest (I)**	**P + I**	**P + I - R**
1	$50 000	$416.67	$50 416.67	$49 816.67
2	$49 816.67	$415.14	$50 231.81	$49 631.81
3	$49 631.81	$413.60	$50 045.40	$49 445.40
4	$49 445.40	$412.05	$49 857.45	$49 257.45
5	$49 257.45	$410.48	$49 667.93	$49 067.93

- comparison of different options for borrowing money in relation to total repayments, fees, interest rates and flexibility
- calculation of credit-card payments, incorporating fees, charges, rates and interest-free periods
- use of published tables from financial institutions to determine monthly repayments on a reducing balance loan.

Terminology introduced in this unit

flat rate loan
reducing balance loan
term of loan.

Technology that may be used in support of this unit

- use of a prepared spreadsheet to simulate loans
- naming of cell ranges to make the construction of formulae easier to understand and implement
- use a graphing calculator to calculate, graph and analyse loan repayments.

Suggested applications and modelling tasks (FM4)

- Construct and use a simple loan spreadsheet initially by using paper, pen and calculator and then by using spreadsheet software. Consider car loans, travel loans, loans for capital items, home loans and others.
- Use a spreadsheet to create a loan-repayment table for amount borrowed versus interest rate.
- Calculate the effective interest rate (E) charged on a loan in relation to the stated rate (r), using $E = \frac{(1+r)^n - 1}{n}$ where E = effective interest rate per period, expressed as a decimal.
- Students could use a home loan spreadsheet to vary the amount borrowed, the interest rate and the repayment amount. Determine the answers to suitable ‘what-if’ questions, eg ‘How much can the interest rate rise before the amount of the repayment has to be increased?’ ‘What would the effect of this be on the term of the loan?’
- Use a prepared graph of ‘amount outstanding’ against ‘repayment periods’ to determine when a particular loan will be half-paid.

FM5: Annuities and loan repayments

The principal focus of this unit is the nature and mathematics of annuities; the processes by which they accrue and the ways of maximising their value as an investment. Emphasis should be placed on using formulae and tables.

Outcomes addressed

A student:

H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H5 makes predictions about the behaviour of situations based on simple models

H8 makes informed decisions about financial situations

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- recognition that an annuity is a form of investment involving periodical, equal contributions to an account, with interest compounding at the conclusion of each period
- calculation of the *future value* (A) of an annuity (or the contribution per period), using

  $$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

  where M = contribution per period, paid at the end of the period.

  Note: the future value of an annuity is the total value of the investment at the conclusion of the last period for payment.

  For example, I am planning to take the trip of a lifetime in ten years' time and estimate that the amount of money I will need at that time is \$30 000. I am advised to contribute \$2500 each year into an account that pays 4% pa, compounded annually. Will I have enough money in ten years time to make my dream come true? By how much will I fall short of or overshoot my goal?

- calculation of the present value (N) of an annuity (or the contribution per period), using

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\} or\ N = \frac{A}{(1+r)^n}$$

  Note: the present value of an annuity is the single sum of money which, if invested today at the rate of compound interest which applies to the annuity, would produce the same financial result over the same period of time.

  For example, which would give the better financial result at the end of 20 years — a lump sum of $100 000 invested today at 12% pa compounded annually, or a monthly payment of $1 000, commencing today, with interest of 12% pa compounded monthly?
- using tables to solve problems involving annuities
- use the present value formula for annuities to calculate loan instalments, and hence the total amount paid over the term of a loan
- investigate various processes for repayment of loans
- calculate the fees and charges which apply to different options for borrowing money in order to make a purchase.

Terminology introduced in this unit

annuity
future value of an annuity
present value of an annuity.

Technology that may be used in support of this unit

- use the financial functions of a spreadsheet or graphing calculator to solve problems involving annuities
- use a graphing facility or graphing calculator to develop graphs to illustrate the growth of an annuity
- use spreadsheet software or a graphing calculator to create, model and analyse graphs and tables of loans
- use a graphing calculator to model financial situations.

Suggested applications and modelling tasks (FM5)

- At the end of 12 years, a netball club needs to replace goal posts and other club equipment. It is estimated the replacement cost will be $10 000. How much will the club need to deposit each year at 5% interest?
- Jo deposits $100 each month in her account for 10 years. How much does she accrue if the bank pays 7% interest compounded on a half-yearly basis?
- Superannuation problems.

- Use the table of future values of $1 of an ordinary annuity (see table below) to calculate the value of an ordinary annuity of $300 per month which is invested at the rate of 2% per month for 5 months.
- Use a spreadsheet to create the table.
- Calculate the number of periods needed to repay a specified loan, using ‘guess and refine’ methods.
- Investigate a ‘low-start’ home loan where the repayments are initially low because the principal/interest has been allowed to accrue or the interest rate is capped for an initial period of time. Assess and describe the negative aspects of these types of loans.

Future values of $1

	Interest Rate				
Period	**1%**	**2%**	**3%**	**4%**	**5%**
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

- Use a prepared spreadsheet to compare various home loans (see sample spreadsheet). The spreadsheet should allow the user to vary the amount borrowed, the rate, the repayment and the term of the loan.
- Draw a graph showing how the amount still owing is related to the term of the loan.
- Use the present value formula to calculate the monthly repayment on a home loan of $100 000 at 12% pa compounded monthly for 20 years. Hence calculate the total repayments and total interest over the term of the loan.

Loan Repayment Table - Version 1.0

Created by C. Buecher on 11 January 2000

Global Variables:

Initial Amount	$100,000	**Initial Interest Rate**	5%	**Term of Loan**	240
Increment	$5,000	**Increment**	1%	**(in months)**	

Calculations and Results:

Amount Borrowed	**Interest Rate p.a.** 5%	6%	7%	8%	9%	10%
$100,000	$659.96	$716.43	$775.30	$836.44	$899.73	$965.02
$105,000	$692.95	$752.25	$814.06	$878.26	$944.71	$1,013.27
$110,000	$725.95	$788.07	$852.83	$920.08	$989.70	$1,061.52
$115,000	$758.95	$823.90	$891.59	$961.91	$1,034.68	$1,109.77
$120,000	$791.95	$859.72	$930.36	$1,003.73	$1,079.67	$1,158.03
$125,000	$824.94	$895.54	$969.12	$1,045.55	$1,124.66	$1,206.28
$130,000	$857.94	$931.36	$1,007.89	$1,087.37	$1,169.64	$1,254.53
$135,000	$890.94	$967.18	$1,046.65	$1,129.19	$1,214.63	$1,302.78
$140,000	$923.94	$1,003.00	$1,085.42	$1,171.02	$1,259.62	$1,351.03
$145,000	$956.94	$1,038.83	$1,124.18	$1,212.84	$1,304.60	$1,399.28
$150,000	$989.93	$1,074.65	$1,162.95	$1,254.66	$1,349.59	$1,447.53
$155,000	$1,022.93	$1,110.47	$1,201.71	$1,296.48	$1,394.58	$1,495.78
$160,000	$1,055.93	$1,146.29	$1,240.48	$1,338.30	$1,439.56	$1,544.03
$165,000	$1,088.93	$1,182.11	$1,279.24	$1,380.13	$1,484.55	$1,592.29
$170,000	$1,121.92	$1,217.93	$1,318.01	$1,421.95	$1,529.53	$1,640.54

FM6: Depreciation

The focus of this unit is to investigate situations involving the depreciation of an asset over time.

Outcomes addressed

A student:

H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H5 makes predictions about the behaviour of situations based on simple models

H8 makes informed decisions about financial situations

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- modelling depreciation by using appropriate graphs, tables and functions
- using formulae for depreciation:
  a) the straight line method
  $S = V_0 - Dn$, where S = salvage (current) value of asset, D = amount of depreciation apportioned per period, V_0 = purchase price of the asset, and n = total number of periods
  b) the declining balance method
  $S = V_0(1 - r)^n$, where S is the salvage value after n periods, V_0 is the purchase price of the asset and r is the percentage interest rate per period, expressed as a decimal
- preparing tables of values and hence developing graphs of against n for different values of r
  Note: these are examples of exponential decay (see AM3, AM4)
- comparing the results obtained through each method
- using the above formulae to create and compare depreciation tables
- calculating tax deductions based on depreciation of assets.

Terminology introduced in this unit

asset
declining balance method
depreciation
salvage value
straight line method.

Technology that may be used in support of this unit

- use a spreadsheet to model the depreciation of an asset over time
- use technology to graph data for a depreciating asset and apply an equation of best fit to the data. Use this equation to predict the value of the asset in the future.

Suggested applications and modelling tasks (FM6)

- Students could collect price data for a depreciating asset over time and use technology to determine the depreciation rate. For example, students could collect twelve prices (one per month) of a particular model and make of car over an eight year period. Students could determine the median price for each year and then create a scatter plot of the median price versus the year. The current year's price could be taken as the car's initial value. By changing the value of r in the equation, students could determine the graph that best fits the scatter plot and so determine r, the depreciation rate. This could be achieved on a graphing calculator by letting X represent r and Y represent V.
- Zheng purchases a computer on March 1 for $5000. Calculate the amount that can be used as a tax deduction in this financial year if the declining balance method is used and the depreciation rate is 30% pa.
- Read and interpret published tables of item costs, eg the NRMA publishes these tables for motor vehicles.
- Students may compare graphs of the depreciation of an item. The value of the item would be read from the vertical axis and time read from the horizontal axis.

9.2 Data Analysis

DA5: Interpreting sets of data

The principal focus of this unit is the use of data displays, measures of location and measures of spread to summarise and interpret one or more sets of data.

Outcomes addressed

A student:

H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H4 analyses representations of data in order to make inferences, predictions and conclusions

H5 makes predictions about the behaviour of situations based on simple models

H9 develops and carries out statistical processes to answer questions which she/he and others have posed

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- identifying measures of location as mean and median
- identifying measures of spread as range, interquartile range and standard deviation
- investigating outliers in small data sets and their effects on the mean, median and mode
- describing the general shape of a graph or display which represents a given data set, eg in terms of smoothness, symmetry or number of modes
- making judgements about the data based on observed features of the display such as shape and skewness
- displaying data in double (back-to-back) stem-and-leaf plots
- displaying data in two box-and-whisker plots drawn on the same scale
- displaying two sets of data on a radar chart
- preparing an area chart to illustrate and compare different sets of data over time (see example at end of unit)
- using multiple displays to describe and interpret the relationships between data sets
- interpreting data presented in two-way table form, eg male/female versus exercise/no exercise
- comparing summary statistics from two sets of data.

Terminology introduced in this unit

area chart outlier skewness.

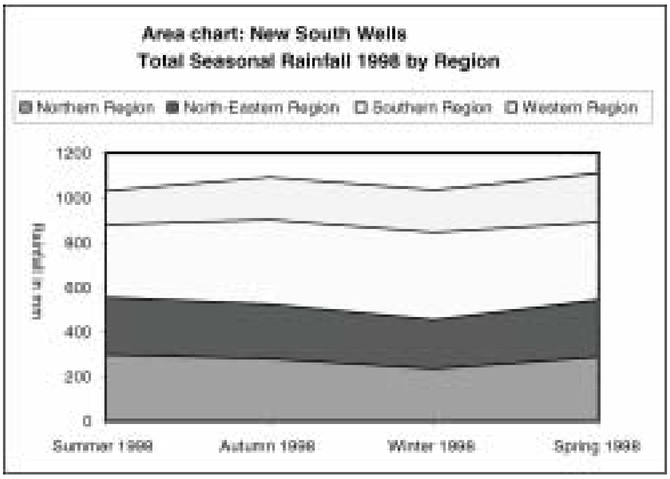
Technology that may be used in support of this unit

- use of a spreadsheet to create frequency tables and calculate mean and standard deviation
- calculate mean and standard deviation through the use of inbuilt statistical functions of a calculator.

Suggested applications and modelling tasks (DA5)

- Prepare a stem-and-leaf plot from collected data. Use it to decide whether clustering is present, whether the shape of the display indicates any skewness of scores or if there is any other tendency in the data.
- Students could use summary statistics and statistical displays to compare related pairs of data sets to decide, informally, whether they could have been drawn from the same population or whether they appear to be from different populations. The following (indicative) pairs of data sets could be considered:
  - home scores versus away scores in national sporting competitions
  - ages of Oscar-winning male actors versus ages of Oscar-winning female actors
  - blood pressure of males versus that of females
  - heights of male versus heights of female students (restricted age range)
  - customer waiting times on two different days of the week at a fast-food outlet
  - monthly rainfall in mm for different cities or regions, eg on average does it rain more in Melbourne than in Sydney? Is it a significant difference? Students could discuss what is meant by ‘significant’ in this context.
  - population by age bands for different countries or regions.
- Investigate population pyramid histograms.
- Given data on a particular population in a table such as the one shown below, answer questions like ‘What percentage of the women surveyed in this exercise are smokers?’ ‘What percentage of smokers surveyed in this exercise are women?’ Discuss the difference between these questions. Under what circumstances would the percentages be the same?

	men	women	**TOTALS**
smokers	4027	4426	8453
non-smokers	8321	7462	15 783
TOTALS	12 348	11 888	**24 236**



DA6: The normal distribution

In this unit, students will apply the properties of the standard normal distribution to the solution of real problems.

Outcomes addressed

A student:

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H4 analyses representations of data in order to make inferences, predictions and conclusions

H5 makes predictions about the behaviour of situations based on simple models

H9 develops and carries out statistical processes to answer questions which she/he and others have posed

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- describing the z-score (standardised score) corresponding to a particular score in a set of scores as a number indicating the position of that score relative to the mean
- using the formula $z = \frac{x - \bar{x}}{s}$ to calculate z-scores, where s is the standard deviation ($s = \sigma_n$ for a population, $s = \sigma_{n-1}$ for a sample)
- using calculated z-scores to compare scores from different data sets
- identifying the properties of data that are normally distributed, ie
  - the mean, median and mode are equal
  - if represented by a histogram, the resulting frequency graph is 'bell shaped'
- using collected data to illustrate that, for normally distributed data:
  - approximately 68% of scores will have z-scores between –1 and 1
  - approximately 95% of scores will have z-scores between –2 and 2
  - approximately 99.7% of scores will have z-scores between –3 and 3
- using these measures to make judgements in individual cases.

Terminology introduced in this unit

normal distribution
standardised score
z-score.

Suggested applications and modelling tasks (DA6)

- Determine the proportion of individuals in a normal population that lies within one standard deviation of the mean.
- Packets of rice are each labelled as having a mass of 1 kg. The mass of these packets is normally distributed with a mean of 1.02 kg and a standard deviation of 0.01kg. Complete the following table:

Mass in kg	1.00	1.01	1.02	1.03	1.04
z-score			0	1	

  (a) What percentage of packets will have a mass less than 1.02 kg?
  (b) What percentage of packets will have a mass between 1.00 and 1.04 kg?
  (c) What percentage of packets will have a mass between 1.00 and 1.02 kg?
  (d) What percentage of packets will have a mass less than the labelled mass?
- A machine is set for the production of cylinders of mean diameter 5.00 cm with standard deviation 0.020 cm. Within what intervals will 99% of the diameters lie?
- If a cylinder, randomly selected from this production, has a diameter of 5.070 cm, what conclusion could be drawn?
- Given the frequency table of a familiar characteristic, students should analyse the data to determine whether or not it could be considered normally distributed.
- Students should investigate whether the results of a particular experiment would be normally distributed.
- Compare student performances in different tests to establish which is the ‘better’ performance.

DA7: Correlation

In this unit, students investigate the strength of association of data through examining a scatterplot of ordered pairs. Where appropriate, students find the equation of a line of fit and use the equation to make predictions.

Outcomes addressed

A student:

H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H4 analyses representations of data in order to make inferences, predictions and conclusions

H5 makes predictions about the behaviour of situations based on simple models

H9 develops and carries out statistical processes to answer questions which she/he and others have posed

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- plotting ordered pairs of data onto a scatterplot
- recognising from the scatterplot:
- whether the points appear to form a mathematical pattern
- whether the pattern appears to be linear
- establishing a median regression line to give a line of fit on a scatterplot with a ruler and pencil
- measuring the gradient of the line of fit drawn, with ruler and pencil
- noting the vertical intercept of the line of fit drawn
- establishing the equation of the resulting line of fit in form $y = mx + b$ (see AM2)
- using this equation to make predictions.

The remaining points relate to correlation. Students will not be required to calculate correlation coefficients.

- interpreting the strength of association using a given correlation coefficient
- interpreting the sign of a given correlation coefficient
- recognising that a high degree of correlation does not necessarily imply causality, eg there is a very high correlation between the sizes of one's left and right feet, but one does not cause the other.

Terminology introduced in this unit

causality
correlation
correlation coefficient
line of best fit
median regression line
scatterplot.

Technology that may be used in support of this unit

- use a graphing facility or graphing calculator to create a scatter plot from two sets of data and hence determine the equation of the line of best fit.

Suggested applications and modelling tasks (DA7)

- Students could collect data and model the following situations: (Consideration could include the strength of association and whether either variable can be used as a predictor of the other.)
  - hand span versus height
  - height versus weight
  - world sporting records versus time
  - time to run 400 metres versus resting heart rate
  - time to run 400 metres versus height
  - height versus age of children (aged from 0 to 12 years)
  - barometer readings versus maximum daily temperature
  - Mathematics test scores versus Science test scores (or English test scores).
- Plot a scatterplot from experimental data and interpret the diagram in terms of the theoretical situation.
- Evaluate reports and media articles in which information has been collected which suggests relationships between two sets of data, eg smoking and lung cancer.
- Discuss relationships between variables and whether these imply cause and effect, eg height and weight, the number of hotels and churches in rural towns, smoking and lung cancer, homework hours and test marks.

9.3 Measurement

M5: Further applications of area and volume

In this unit, the work commenced in M2: Applications of area and volume is extended to include surface area and volume of complex figures, and the use of approximations in calculating area and volume of irregular figures.

Outcomes addressed

A student:

H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H3 develops and tests a general mathematical relationship from observed patterns

H6 analyses two-dimensional and three-dimensional models to solve practical and mathematical problems

H7 interprets the results of measurements and calculations and makes judgements about reasonableness

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- calculating areas of ellipses, annuluses and parts of a circle (quadrant, sector) using appropriate formulae
- calculating areas of composite figures
- applying Simpson's rule over three equally spaced points, ie one application (problems involving five points should be treated using two applications)
- calculating external surface area of open (without top and/or bottom) and closed cylinders
- calculating surface area of spheres
- calculating volumes of composite solids
- determining errors in calculations resulting from errors in measurement.

Terminology introduced in this unit

annulus
ellipse
quadrant
sector.

Technology that may be used in support of this unit

- use of drawing tools to create 2D and 3D drawings.

Suggested applications and modelling tasks (M5)

- Design the shape and dimensions of a container that would have a given capacity, given the purpose and use of the container.
- Design cost-effective packaging. For example, groups of students are given four table-tennis balls and need to design a box to package them which minimises the use of material.
- As designers, students are told that they have been given a square piece of metal of side length 2 m from which to design an open rectangular water tank. The volume of water that the tank will hold will depend on the size of squares cut from each of the four corners of the square. Students choose a scale and make models of tanks and find the volume of water that they hold. They could graph results and determine when the volume is the greatest. What happens if the side of the original square is doubled?
- Students use Simpson’s rule to estimate the volume of a dam by calculating a series of equally spaced cross-sectional areas.
- Students calculate the cross-sectional area of a river, and hence calculate the flow-rate of the river.

M6: Applications of trigonometry

This unit extends students' knowledge of trigonometry and area to include non-right- angled triangles. Problems to be solved will incorporate practical work with offset and radial surveys. Angles will be approximated to the nearest minute.

Outcomes addressed

A student:

H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H6 analyses two-dimensional and three-dimensional models to solve practical and mathematical problems

H7 interprets the results of measurements and calculations and makes judgements about reasonableness

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- solving problems using trigonometric ratios in one or more right-angled triangles
- using compass bearings (eight points only) and true bearings (three-figure bearings) in problem-solving related to maps and charts
- establishing the sine, cosine and tangent ratios for obtuse angles from a calculator
- determining the sign of the above ratios for obtuse angles
- preparing diagrams to represent given information
- using the sine rule to find lengths and angles

  $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

  Note: It is not intended that students study the ambiguous case of the sine rule.
- calculating area of a triangle using the formula

  $$A = \frac{1}{2}ab \sin C$$
- using the cosine rule to find lengths and angles

  $$c^2 = a^2 + b^2 - 2ab \cos C$$

  or

  $$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

- using appropriate trigonometric ratios and formulae in two-triangle problems where one triangle is right-angled and the diagram is given
- solving problems involving non-right-angled triangles
- selecting and using appropriate trigonometric ratios and formulae to solve problems
- conducting radial (both plane table and compass) surveys
- solving problems involving non-right-angled triangle trigonometry, Pythagoras' theorem and area in offset and radial surveys.

Terminology introduced in this unit

compass radial survey
cosine rule
offset survey
plane table radial survey
sine rule.

Suggested applications and modelling tasks (M6)

- Students estimate the area and perimeter of a part of the school grounds. They carry out a survey and complete a scale diagram, which they use to calculate the area and perimeter. Students present, compare and discuss their results. Would a different type of survey give different results?
- Use a large area, wool and 'Blu-tac' to demonstrate how to draw diagrams resulting from problems involving bearings. Compass points, angles and distances can be marked with chalk. The diagram is created with wool, which is anchored by 'Blu-tac' or plasticine. As each part of the problem is read and clarified, students discuss what to do and the wool is extended to the next position, where it is anchored.
- Carry out a radial survey of an irregular area and compare the result with an offset survey.
- Plan a walk or some other journey by reading and interpreting a map.
- Find gradients from contour lines on maps.
- Use navigational charts to plan routes and identify positions.
- Calculate the height of buildings and other structures.
- Investigate the derivation of the formulae used in this section.

M7: Spherical geometry

In this unit, geometry and trigonometry are applied to solve problems relating to the Earth as a sphere. Applications include locating positions on the Earth, using latitude and longitude, and calculating time differences.

Outcomes addressed

A student:

H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H6 analyses two-dimensional and three-dimensional models to solve practical and mathematical problems

H7 interprets the results of measurements and calculations and makes judgements about reasonableness

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- calculating arc lengths of a circle
- distinguishing between great and small circles
- using the Equator and the Greenwich Meridian as lines of reference for locations on the Earth's surface
- locating positions on the globe using latitude and longitude
- converting nautical miles (M) to kilometres and vice versa, given 1.852 km = 1 M
- calculating distances between two points on the same great circle in nautical miles and kilometres (radius of the Earth to be taken as 6400 km)
- defining 1 knot as a speed of 1 M per hour
- using time zones and the International Date Line in solving problems
- calculating time differences given the difference in longitudes
  (Apply 15° = 1 hour and 1°= 4 minutes time difference. Daylight-saving time is to be considered.)
- determining times for cities in different countries in related travel questions.

Terminology introduced in this unit

great circle
International Date Line
latitude
meridian of longitude
parallel of latitude

Greenwich Meridian
knot
longitude
nautical mile
small circle.

Suggested applications and modelling tasks (M7)

- Calculate the present time in all the capital cities of Australia.
- Investigate which States in Australia have daylight-saving, and if so, when it starts and finishes.
- Students collect data to enable them to discuss and present information on the popularity of daylight-saving.
- Students find three cities with the same time, say a given number of hours earlier and later than local time. These could be marked on a world map and displayed to reinforce the concept of time zones.
- Find the coordinates of the point on the Earth's surface that is at maximum distance from your current location.

9.4 Probability

PB3: Multi-stage events

The focus of this unit is on counting the number of outcomes for an experiment, or the number of ways in which an event may occur. The probability of particular outcomes may then be established. The formulae using factorial notation are not required.

Outcomes addressed

A student:

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H3 develops and tests a general mathematical relationship from observed patterns

H4 analyses representations of data in order to make inferences, predictions and conclusions

H10 solves problems involving uncertainty using basic principles of probability

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- constructing and using a tree diagram to establish the sample space for a simple multi-stage event
- multiplying the number of choices at each stage to determine the number of outcomes for a multi-stage event
- establishing that the number of ways in which n different items can be arranged is $n(n-1)(n-2)\ldots\times 1$, eg the number of arrangements of 4 different items is $4\times 3\times 2\times 1 = 24$; the number of arrangements of 3 different items is $3\times 2\times 1 = 6$
- checking that these results are true by listing arrangements for small numbers of items
- establishing the number of *ordered* selections that can be made from a group of different items (small numbers only), eg if selecting *two particular positions* (such as captain and vice-captain) from a team of five people, the number of selections is $5\times 4 = 20$
- verifying these results by listing ordered selections (small numbers only)
- establishing the number of *unordered* selections that can be made from a group of different items (small numbers only), eg if selecting a *pair* of people to represent a team of five, the number of selections is half of the number of ordered selections
- verifying by listing unordered selections (small numbers only)
- using the formula for the probability of an event to calculate the probability that a particular selection will occur
- using probability tree diagrams to solve problems involving two-stage events.

Terminology introduced in this unit

ordered selection
tree diagram
unordered selection.
probability tree diagram
two stage events

Suggested applications and modelling tasks (PB3)

- Determine the total number of choices in a game in which six different numbers are chosen from 40.
- Use Venn diagrams to describe and interpret situations with intersecting sample spaces.
- Choosing cards from a pack of cards, with and without replacement, could be used to investigate two-stage experiments.
- Select a ‘triple-decker’ (three flavours) ice-cream from four different flavours: chocolate, strawberry, vanilla and butterscotch. If you are fussy about the order in which you eat the scoops, your choices are ordered. Show, with a list and a tree diagram, that there are 24 possible arrangements for your ice-cream. What is the probability that the chocolate scoop will be on the top? If you are happy to put all three scoops together in a tub and eat them all at once, how many different choices do you have?
- How many different ways are there of answering a four question True/False test? Check by listing the possible responses.
- In how many ways can the names of three candidates be listed on a ballot paper? If you are one of the candidates, what is the probability that your name will be at the top of the paper? Check by listing.
- A road toll of $1.50 is paid in coins at a toll gate. List the different ways in which you could pay this toll, assuming that any combination of $1, 50c, 20c and 10c coins may be used.
- A new model car is available with leatherette or cloth upholstery, and four exterior paint colours. There is also an option for air-conditioning, and an option for a CD player. If a car dealer wishes to display one car for each possible combination, how many cars would be needed?

PB4: Applications of probability

In this unit, students calculate expected outcomes from simple experiments and compare them with experimental results.

Outcomes addressed

A student:

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H4 analyses representations of data in order to make inferences, predictions and conclusions

H10 solves problems involving uncertainty using basic principles of probability

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- calculating the expected number of times a particular outcome would arise, given the number of trials of a simple experiment, by establishing the theoretical probability and multiplying by the number of trials
- comparing the above result with an experimental result
- calculating financial expectation by multiplying each financial outcome by its probability and adding the results together

  Note: A financial loss is regarded as negative.
- carrying out simulations to model events, eg tossing a coin with the outcomes representing the sex of the offspring
- drawing up a table (two-way table) to illustrate results gained on a test designed to determine the existence (in a particular case) of a phenomenon which has a low overall probability of occurrence, eg screening for medical conditions, or using a lie detector to indicate guilt or innocence
- interpreting the information in the table and making judgements about the conclusions established by the test.

Terminology introduced in this unit

expected outcome
financial expectation
simulation
two-way table.

Technology that may be used in support of this unit

- using a spreadsheet to simulate large numbers of trials
- using inbuilt graphing facilities of the spreadsheet to investigate the results of a simulation.

Suggested applications and modelling tasks (PB4)

- Determine whether it is better to buy 10 tickets in one lottery, or one ticket in each of 10 lotteries.
- Given the following options, most people would prefer (A)
- sure gain of $300
  (B) 30% chance of gaining $1000 and 70% chance of gaining nothing
- Use expectation to decide whether this preference is justified.
- Investigate different strategies for playing party games such as 'Greedy Pig'.
- Simulate the number of boys and girls in a large number of families to answer questions about the most likely distributions under different circumstances, eg if couples stopped having children as soon as the first girl was born, would there be more girls than boys in the long run?
- Paul plays a game involving the tossing of two coins. He gains $5 if they both show heads and $1 for a head and a tail, but loses $6 if they both show tails. What is his financial expectation from this game?

9.5 Algebraic Modelling

AM3: Algebraic skills and techniques

This unit develops algebraic skills and techniques that are used in work-related and everyday contexts. As far as possible, real contexts should be used to demonstrate the use of algebra in practical life.

Outcomes addressed

A student:

H2 integrates mathematical knowledge and skills from different content areas in exploring new situations

H3 develops and tests a general mathematical relationship from observed patterns

H7 interprets the results of measurements and calculations and makes judgements about reasonableness

H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- substituting into and evaluating algebraic expressions — linear, quadratic, cubic, as well as those involving square and cube roots, such as $r = \sqrt[3]{\frac{3V}{4\pi}}$
- adding and subtracting like terms
- multiplying and dividing algebraic terms and expressions
- changing the subject of equations and formulae involving linear and quadratic terms, eg make s the subject of $v^2 = u^2 + 2as$
- solving equations after substituting values, eg evaluate t when $d = 5t^2$ and d = 300
- solution of equations arising from practical situations by estimation and refinement, eg if x is the number of years required for an investment to double at 5% pa compound interest, then $1.05^x = 2$. Find the value of x by making an initial estimate and refining the result, using a calculator
- using positive and negative powers of ten as part of expressing measurements in scientific notation.

Suggested applications and modelling tasks (AM3)

- Washing removes 20% of a deep stain at each wash. Show that three washes remove about half of the stain. How many washes are needed to reduce a stain to 5% of its original amount?
- My photocopier enlarges by 150%. Can I produce a picture about twice the size of an original? Explain how this could be done.
- The distance that an object falls (d metres) in a certain time (t seconds) is approximated by $d = 5t^2$. Create a table of values for this function manually, and compare this result with that obtained through the use of a spreadsheet. How long does it take for an object to fall 300 m?

AM4: Modelling linear and non-linear relationships

This unit focuses on the examination of practical problems that can be modelled algebraically.

Outcomes addressed

A student:

H1 appreciates the importance of mathematics in her/his own life and its usefulness in contributing to society
H2 integrates mathematical knowledge and skills from different content areas in exploring new situations
H3 develops and tests a general mathematical relationship from observed patterns
H5 makes predictions about the behaviour of situations based on simple models
H11 uses mathematical argument and reasoning to evaluate conclusions drawn from other sources, communicating his/her position clearly to others.

Students learn and acquire the following skills, knowledge and understanding

- generating tables of values and graphing linear functions with pencil and paper
- interpretation of the point of intersection of the graphs of two linear functions drawn from practical contexts, eg 'breakeven' points (see applications)
- generating tables of values and graphing quadratic functions of the form $y = ax^2 + bx + c,\ x \geq 0$ with pencil and paper
- noting that different forms of an expression produce identical graphs, eg $y = (x-2)^2 + 3,\ y = x^2 - 4x + 7$
- using a quadratic graph to find maximum and minimum values in practical contexts
- generating tables of values and graphing cubic, exponential and hyperbolic functions with pencil and paper. Functions to be restricted to the following forms – cubic: $y = ax^3,\ x \geq 0$; exponential: $y = b\left(a^x\right),\ x \geq 0$; hyperbolic $y = \frac{a}{x},\ x > 0$
- recognition that, for $a > 1$, $y = b(a^x)$ represents exponential growth and, for $0 < a < 1$, it represents exponential decay (see FM2, FM6)
- development of equations such as $y = ax^2$, $h = at^3$ from descriptions of situations in which one quantity varies directly as the power of another
- development of an equation such as $y = \frac{a}{x}$ from a description of a situation in which one quantity varies inversely with another
- subsequent evaluation of a in the equations shown in the above two points, given one pair of variables, and using the resulting formula to find other values of the variables
- using algebraic functions as models of physical phenomena
- recognising the limitations of models when interpolating and/or extrapolating.

Terminology introduced in this unit

exponential function
extrapolate/interpolate
hyperbolic function
quadratic functions.

Technology that may be used in support of this unit

use graphing software (or a graphing calculator) to:

- graph linear, quadratic, cubic and exponential functions
- find graphically a point of intersection using ‘zooming’ techniques
- identify ordered pairs on a graph by ‘tracing’
- determine the equation of a line of best fit on a scatterplot.

Suggested applications and modelling tasks (AM4)

- Distance/velocity/time graphs and problems.
- Is the approximation method ‘Double and add 30° for converting from Celsius to Fahrenheit always close? The formula for converting degrees Celsius to degrees Fahrenheit could be graphed, along with the formula arising from the ‘rule of thumb’. This could also be investigated with a spreadsheet and/or a graphing calculator.
- Create graphs to demonstrate the solution of simultaneous linear equations is the location of ‘breakeven’ points, eg in graphs of costs and income, or in finding the time when a newly installed appliance starts to save money for a household.
- Sketch at least ten rectangles that have the same perimeter. Record length versus area in a table. Plot the resulting function and use the graph to determine the shape with maximum area.
- Investigate models of population growth.
- Plot a function showing the growth of a population of bacteria.
- Investigate compound interest as the time period shortens.
- Investigate via technology the effect of adding/subtracting a constant term on the graph of a function.
- On the Earth, the formula $d = 5t^2$ can be used to express the distance (d metres) that an object falls in t seconds, if air resistance can be ignored. Investigate equivalent expressions for other bodies, eg on the moon, the equation is $d = 0.8t^2$.
- Solve problems where two quantities vary in a related way. For example, the number of eggs used in a recipe for a particular cake varies with the square of the diameter of the tin, for tins with constant depth. If two eggs are used in a recipe for a 15 cm diameter tin, how many eggs would be used if the diameter of the tin is 35 cm?

- The surface area of a sphere can be approximated by $S = 3D$, where D is the diameter. Compare results from a number of spheres with those obtained using $S = \pi D^2$. (Note: S and D are in the same basic unit, eg if D is in cm, S will be in cm^2).
- Investigate reaction times and stopping distances for cars travelling at different speeds. For example, a car travelling at v km/h will travel 0.7v m during the 2.5 seconds it takes the driver to react to a dangerous situation. For a car with excellent brakes on a dry level road, the total distance travelled by a car once the driver begins to realize that there is a need to stop can be expressed by $d = 0.01v^2 + 0.7v$, if there is no skidding .
- Cubic models can be used to estimate the mass of objects. For example, could you lift a cubic metre of cork? An expression for the mass in grams, M, of a cube of cork is $M = 0.25x^3$, where x is the side length of the cube in centimetres.
- Hyperbolic models can be used to find how much each person contributes when a cost is shared. For example, a household has $306 in bills. Make a table and a graph to show how much each person pays if there are 2, 3, 4 or 5 people contributing equally to pay these bills.
- The time in seconds taken to travel 100 m is inversely related to the speed in kilometres per hour. Students could illustrate this with a table, a graph and a formula.
- An exponential expression such as $M = 1.5(1.2)^x$ can be used to describe the mass M kg of a baby orang-utang at age x months. This model applies for a limited time, up to x = 6.
- The size of the world's population in billions can be approximated by the expression $y = 2.486(1.02)^x$, where x is the number of years since 1950 and 2.486 billion was the size of the population in 1950. This model could be checked against recent data. Similar expressions can be developed in different countries and cities. If a city has a declining population, for example a decline of 5% pa from an initial population of 100 000, the equation is $y = 100\ 000(0.95)^x$.
- Review models of depreciation, inflation and appreciation.
- Graph and compare the functions $y = 1000(1.05)^x$ and $y = 1000(0.95)^x$. What might these graphs represent?

10 Course Requirements

The *General Mathematics Stage 6 Syllabus* includes a Preliminary course of 120 hours (indicative time) and an HSC course of 120 hours (indicative time).

The General Mathematics Preliminary course is constructed on the assumption that students have achieved the outcomes in the core of the Standard Mathematics course for the School Certificate, together with the recommended options: *Trigonometry* and *Further Algebra*. Completion of the Preliminary course is a prerequisite for the study of the HSC course.

Students may not study General Mathematics in conjunction with any other Mathematics course in Stage 6.

11 Post-school Opportunities

The study of General Mathematics Stage 6 provides students with knowledge, understanding and skills that form a valuable foundation for a range of courses at university and other tertiary institutions.

In addition, the study of General Mathematics Stage 6 assists students to prepare for employment and full and active participation as citizens. In particular, there are opportunities for students to gain recognition in vocational education and training. Teachers and students should be aware of these opportunities.

Recognition of Student Achievement in Vocational Education and Training (VET)

Wherever appropriate, the skills and knowledge acquired by students in their study of HSC courses should be recognised by industry and training organisations. Recognition of student achievement means that students who have satisfactorily completed HSC courses will not be required to repeat their learning in courses in TAFE NSW or other Registered Training Organisations (RTOs).

Registered Training Organisations, such as TAFE NSW, provide industry training and issue qualifications within the Australian Qualifications Framework (AQF).

The degree of recognition available to students in each subject is based on the similarity of outcomes between HSC courses and industry training packages endorsed within the AQF. Training packages are documents that link an industry's competency standards to AQF qualifications. More information about industry training packages can be found on the National Training Information Service (NTIS) website (www.ntis.gov.au).

Recognition by TAFE NSW

TAFE NSW conducts courses in a wide range of industry areas, as outlined each year in the TAFE NSW Handbook. Under current arrangements, the recognition available to students of mathematics in relevant courses conducted by TAFE is described in the HSC/TAFE Credit Transfer Guide. This guide is produced by the Board of Studies and TAFE NSW and is distributed annually to all schools and colleges. Teachers should refer to this guide and be aware of the recognition that may be available to their students through the study of General Mathematics
Stage 6. This information can be found on the TAFE NSW website (www.tafensw.edu.au/mchoice).

Recognition by other Registered Training Organisations

Students may also negotiate recognition into a training package qualification with another Registered Training Organisation. Each student will need to provide the RTO with evidence of satisfactory achievement in General Mathematics Stage 6 so that the degree of recognition available can be determined.

12 Assessment and Reporting

12.1 Requirements and Advice

The information in this section of the syllabus relates to the Board of Studies' requirements for assessing and reporting achievement in the Preliminary and HSC courses for the Higher School Certificate.

Assessment is the process of gathering information and making judgements about student achievement for a variety of purposes.

In the Preliminary and HSC courses those purposes include:
- assisting student learning
- evaluating and improving teaching and learning programs
- providing evidence of satisfactory achievement and completion in the Preliminary course
- providing the Higher School Certificate results.

Reporting refers to the Higher School Certificate documents received by students that are used by the Board to report both the internal and external measures of achievement.

NSW Higher School Certificate results will be based on:
- an assessment mark submitted by the school and produced in accordance with the Board's requirements for the internal assessment program
- an examination mark derived from the HSC external examinations.

Results will be reported using a course report containing a performance scale, with bands describing standards of achievement in the course.

The use of both internal assessment and external examinations of student achievement allows measures and observations to be made at several points and in different ways throughout the HSC course. Taken together, the external examinations and internal assessment marks provide a valid and reliable assessment of the achievement of the knowledge, understanding and skills described for each course.

Standards Referencing and the HSC Examination

The Board of Studies will adopt a standards-referenced approach to assessing and reporting student achievement in the Higher School Certificate examination.

The standards in the HSC are:

- the knowledge, skills and understanding expected to be learned by students — the *syllabus standards*
- the levels of achievement of the knowledge, skills and understanding — the *performance standards*.

Both *syllabus standards* and *performance standards* are based on the aims, objectives, outcomes and content of a course. Together they specify what is to be learned and how well it is to be achieved.

Teacher understanding of standards comes from the set of aims, objectives, outcomes and content in each syllabus together with:

- the performance descriptions that summarise the different levels of performance of the course outcomes
- HSC examination papers and marking guidelines
- Samples of students' achievement on assessment and examination tasks.

12.2 Internal Assessment

The internal assessment mark submitted by the school will provide a summation of each student's achievements measured at points throughout the course. It should reflect the rank order of students and relative differences between students' achievements.

Internal assessment provides a measure of a student's achievement based on a wider range of syllabus content and outcomes than may be covered by the external examination alone.

The assessment components, weightings and task-requirements to be applied to internal assessment are identified on page 84 of this document. They ensure a common focus for internal assessment in the course across schools, while allowing for flexibility in the design of tasks. A variety of tasks should be used to give students the opportunity to demonstrate outcomes in different ways and to improve the validity and reliability of the assessment.

12.3 External Examination

In General Mathematics Stage 6, the external examinations include written papers to be marked externally. The specifications for the examination in General Mathematics Stage 6 are on page 86 of this document.

The external examination provides a measure of student achievement in a range of syllabus outcomes that can be reliably measured in an examination setting.

The external examination and its marking and reporting will relate to syllabus standards by:

- providing clear links to syllabus outcomes
- enabling students to demonstrate the levels of achievement outlined in the course performance scale
- applying marking guidelines based on established criteria.

12.4 Board Requirements for the Internal Assessment Mark In Board Developed Courses

For each course the Board requires schools to submit an assessment mark for each candidate.

The collection of information for the HSC internal assessment mark must not begin prior to the completion of the Preliminary course.

The Board requires that the assessment tasks used to determine the internal assessment mark must comply with the components, weightings and types of tasks specified in the table on page 84 of this document.

Schools are required to develop an internal assessment program which:

- specifies the various assessment tasks and the weightings allocated to each task
- provides a schedule of the tasks designed for the whole course.

The school must also develop and implement procedures to:

- inform students in writing of the assessment requirements for each course before the commencement of the HSC course
- ensure that students are given adequate written notice of the nature and timing of assessment tasks
- provide meaningful feedback on students' performance in all assessment tasks
- maintain records of marks awarded to each student for all assessment tasks
- address issues relating to illness, misadventure and malpractice in assessment tasks
- address issues relating to late submission and non-completion of assessment tasks
- advise students in writing if they are not meeting the assessment requirements in a course and indicate what is necessary to enable the students to satisfy the requirements
- inform students about their entitlements to school reviews and appeals to the Board
- conduct school reviews of assessments when requested by students
- ensure that students are aware that they can collect their Rank Order Advice at the end of the external examinations at their school.

12.5 Assessment Components, Weightings and Tasks

Preliminary Course

The suggested components, weightings and tasks for the Preliminary Course are detailed below.

<table>
<tr><th>Assessment Components</th><th>Weighting</th><th>Tasks might include</th></tr>
<tr><td>Knowledge and skills</td><td>40</td><td rowspan="3">• examination-style questions
• assignments
• oral or written reports
• samples of students’ work
• practical investigations or projects
• students’ written explanation of problem solutions
• practical tasks such as measurement activities</td></tr>
<tr><td>Applications</td><td>60</td></tr>
<tr><td>Marks</td><td>100</td></tr>
</table>

HSC Course

Up to 30% of the internal assessment in General Mathematics Stage 6 may be based on the Preliminary course.

<table>
<tr><th>Assessment Components</th><th>Weighting</th><th>Tasks might include</th></tr>
<tr><td>Knowledge and skills</td><td>40</td><td rowspan="3">• examination-style questions
• assignments
• oral or written reports
• samples of students’ work
• practical investigations or projects
• students’ written explanation of problem solutions
• practical tasks such as measurement activities</td></tr>
<tr><td>Applications</td><td>60</td></tr>
<tr><td>Marks</td><td>100</td></tr>
</table>

While the allocation of weightings to the various tasks set for the HSC course is left to individual schools, the percentages allocated to each assessment component must be maintained. For each component, the assessment may be spread over more than one task.

12.6 Summary of Internal and External Assessment

Internal Assessment	Weighting	External Assessment	Weighting
Whole course assessment using a range of tasks as indicated on p 84 of this document.	100	A written examination consisting of: Whole course assessment using a range of items	100
	100		100

12.7 HSC External Examination Specifications

The written examination in General Mathematics will consist of an examination paper of $2\frac{1}{2}$ hours duration (plus 5 minutes reading time).

Section I *(22 marks)*

- There will be TWENTY-TWO multiple-choice questions of equal value.
- All questions are **compulsory**.

Section II *(78 marks)*

- There will be SIX questions based mainly on the topics prescribed for the HSC course. No more than 30% of the examination will be based on the Preliminary course. Questions based on the Preliminary course can also be asked when they lead in to questions based on the HSC course. Marks from these lead-in questions will not be counted in the 30% Preliminary allowance.
- All questions are **compulsory**.
- All questions will be worth 13 marks.
- Each question will consist of a number of parts requiring free-response answers.

The formula sheet that accompanies this syllabus will be included with the examination paper.

12.8 Reporting Student Performance Against Standards

Student performance in an HSC course will be reported against standards on a course report. The course report includes a performance scale for the course describing levels (bands) of achievement, an HSC mark located on the performance scale, an internal assessment mark and an examination mark. It will also show, graphically, the statewide distribution of examination marks of all students in the course.

Each band on the performance scale (except for Band 1) includes descriptors that summarise the attainments typically demonstrated in that band.

The distribution of marks will be determined by students' performance against the standards and not scaled to a predetermined pattern of marks.