

- The radiation density S decays with one over r-square with distance. This is known as the $\frac{1}{r^2}$ -law.
- This law is true for point sources even if they do not radiate in all directions equally, i.e. non-isotropic sources. We can also say, that the power density is one fourth when we double the distance.
- Note, that E- and H-field only decay with $\frac{1}{r}$,

Example:

A cell phone radiates with a transmitter power of 500mW. The antenna is isotropic. What is the power density in $\frac{W}{m^2}$ in 1cm, 1m und 1km distance?

$$S(1cm) = \frac{0.5 \text{ W}}{4\pi 0.01^2 m^2} = 397 \frac{W}{m^2}$$

$$S(1m) = 39.7 \frac{mW}{m^2}$$

$$S(1km) = 39,7 \frac{nW}{m^2}$$

How much is S in the distance of the geostationary orbit.

$$S(36000km) = 3 \cdot 10^{-14} \frac{W}{m^2}$$

The gain $G(\phi, \theta)$ of an antenna is defined as the radiation density $S(r, \phi, \theta)$ in direction ϕ, θ compared to the radiation density $S_0(r)$ of an isotropic radiator in the same distance r :

$$G(\phi, \theta) = \frac{S(r, \phi, \theta)}{S_0(r)}$$

The gain does not depend on the distance r . Often the gain is given in dB. i.e.

$$G_{dB} = 10 \log G$$

Since we refer the radiated power to the power of an isotropic antenna this unit is sometimes called dBi

Example:

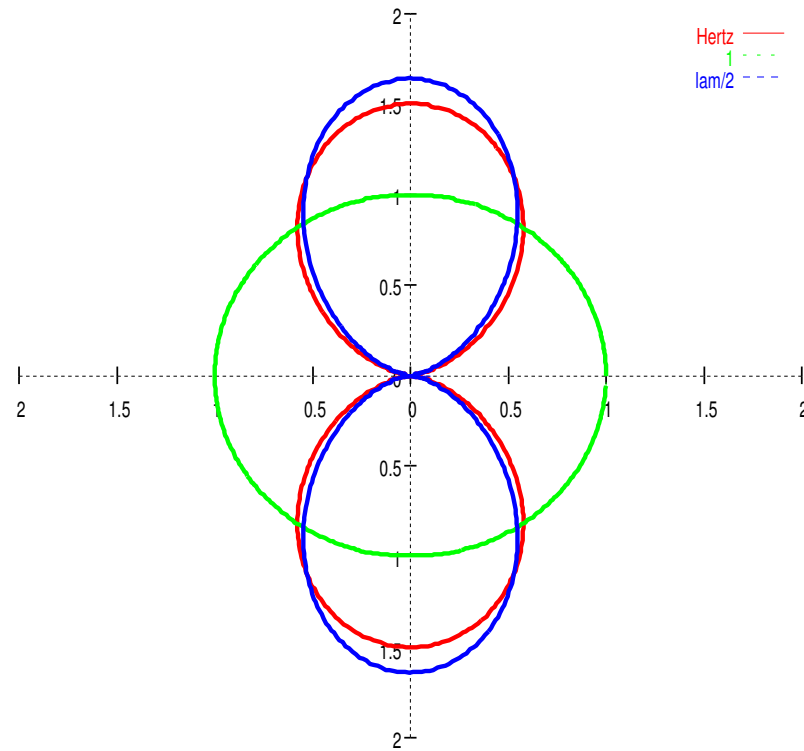
The gain of a Hertzian dipole and a $\lambda/2$ -Dipole is:

$$G(\phi, \theta) = 1,5 \sin^2 \theta \quad \text{Hertzian dipole}$$

$$G(\phi, \theta) = 1,64 \frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin^2 \theta} \quad \frac{\lambda}{2} \text{ - Dipole}$$

The gain is constant in ϕ -direction, and varies in θ -direction. The diagrams for all three antennas are shown here

Antenna Gain



Often the expression gain is used for the maximum Gain achievable, that is

$$G_{max} = \max(G(\phi, \theta))$$

Example:

Gain of a Hertzian Dipole $G_{dB,max} = 10 \log 1.5 = 1.76$ dBi

Gain of a $\lambda/2$ -Dipole $G_{dB,max} 1.64 = 2.15$ dBi

If the gain G is written without direction (ϕ, θ) we can assume that the writer refers to the the maximum gain.

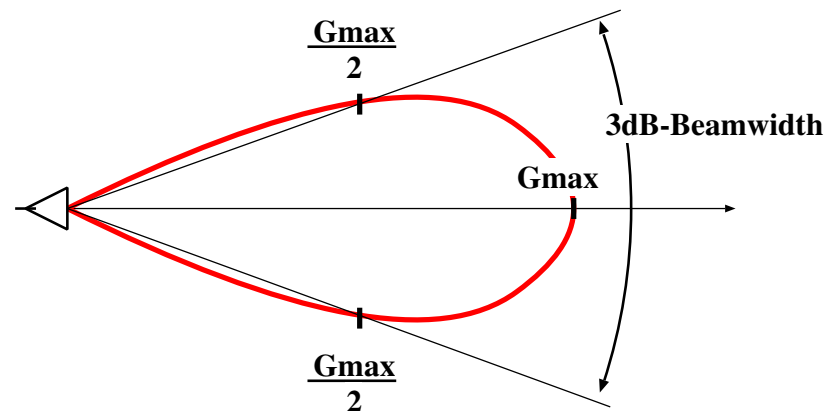
- The directivity D of an antenna is equal to the ratio of the maximum power flux density to the average value of an isotropic antenna excluding all losses in the antenna.
- The gain of an antenna includes all losses which occur inside the antenna due to copper losses and mismatch etc.
- The ratio between the gain and the directivity is called the antenna efficiency factor k .

$$G = kD$$

This number is one for lossless antennas and G is equal to D .

- In other words, the gain refers to the power fed to the antenna whereas the directivity refers to the total radiated power. Nehmen wir verlustfreie Antennen an, so gilt $D = G$.

- The half-power-beamwidth is defined as angular beamwidth between the points where the directivity reaches half its maximum (3 dB point).
- The beam width can be different in horizontal and vertical direction.



Example:

3dB-Beamwidth of the Hertzian Dipole:

$\sin^2 \theta = 0.5$, it follows $\theta = 45^\circ$

hence $\theta_{BW} = 90^\circ$, because we measure from 3dB-point to 3dB-point.

The gain and beamwidth of an antenna are related approximately by

$$D_0 = \frac{4\pi}{\Theta_{BW}\Phi_{BW}}$$

where θ_{BW} is the 3dB-beamwidth in θ -direction and Φ_{BW} in Φ -direction. This approximation is known as Kraus' equation. This approximation is only valid for high-gain antennas with no side lobes.

The EIRP (Effective Isotropic Radiated Power) is defined as the transmit power we have to use on an isotropic antenna in order to achieve the same power density as the power density of the examined antenna in main radiation direction.

The EIRP is not a real existing power but a value for calculations. Die EIRP is related to the real fed power of the antenna by

$$EIRP = G_{max} \cdot P_S$$

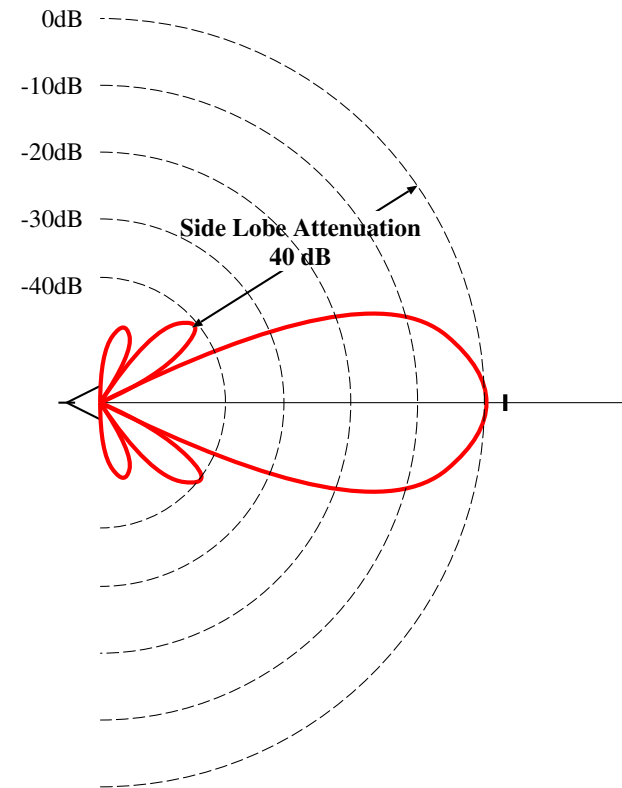
where P_S is the transmit power.

Example:

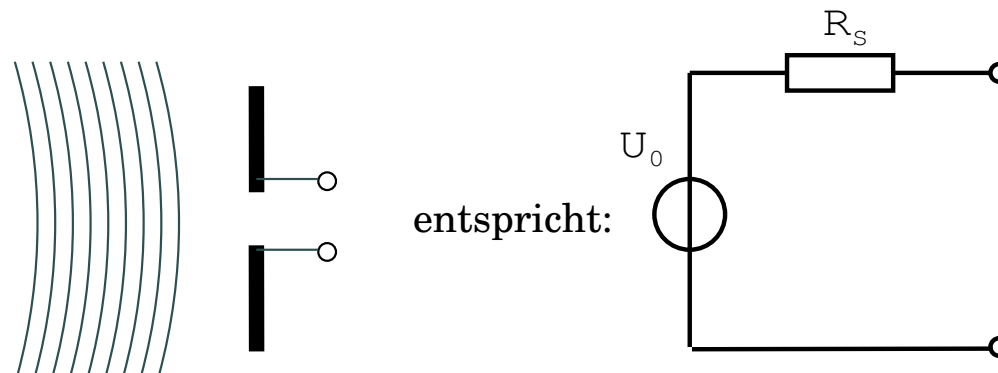
A mobile phone base station has 17 dBi Gain and a transmit power of 10 Watt. We get an EIRP of ca. 501 Watt.

- This means, that we have to transmit with 501 W if we replace the antenna by an isotropic antenna and still want to have the same radiation power density in the main direction.
- A typical value for EIRP of a fixed Satellite Service FSS is between 30 to 52 dBW.

- Large antennas with high gain inhibit usually more or less side lobes.
- These side lobes are secondary maxima of gain.
- This means that the antenna transmits and receives also in the direction of side lobes however attenuated.
- The attenuation of the side lobes is called side lobe attenuation usually given in Decibel.



- The conversion of guided waves into free waves can be converted back by receiving antennas.
- We call these antennas receiving antennas. They act like a current or voltage source with source impedance.

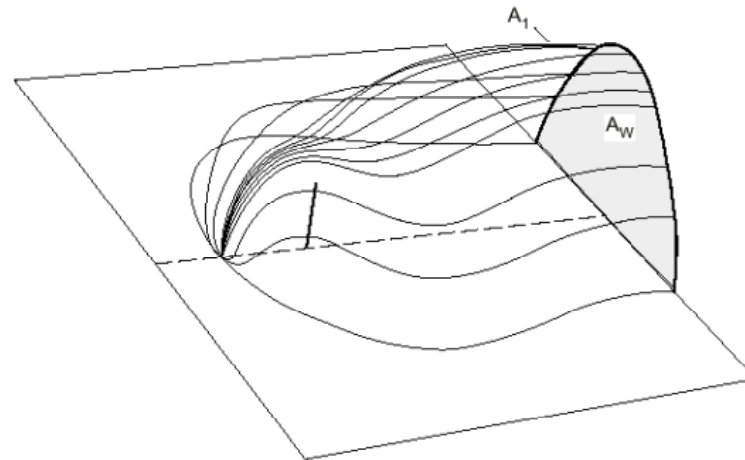


Assume we have the power flux density S_e at the receiving location. The antenna converts this to a received power of P_e . Then the antenna effectively covers an area of

$$A_e = \frac{P_e}{S_e}$$

usually similar to the geometric aperture of the antenna but is not equal to the physical aperture area.

The antenna area can also be defined for non-area antennas like wire antennas.



Interpretation of effective antenna area for dipole antenna: the field lines (and Poynting vector) are caught in the area A .

We can find the following relation between gain and effective antenna area

$$A_e = \frac{G_{max} \cdot \lambda^2}{4\pi} \quad (1)$$

Example:

Applied to the formerly mentioned antennas we get

$\frac{\lambda^2}{4\pi}$ for the isotropic Antenna

$1,5 \frac{\lambda^2}{4\pi}$ for the Hertzian Dipole

$1,64 \frac{\lambda^2}{4\pi}$ for the $\lambda/2$ -Dipole

These areas correspond to disc with the diameters $0,32 \lambda$, $0,39 \lambda$ und $0,41 \lambda$.

With the eqn. we can give the received power as a function of power flux density:

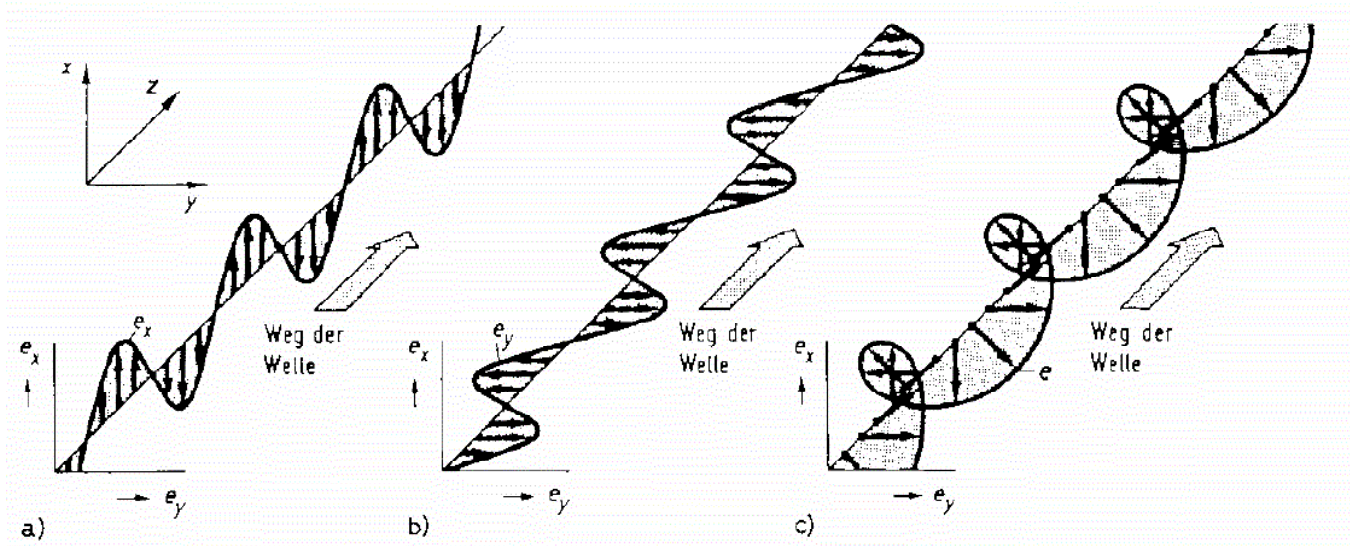
$$P_e = S_e A_e = S_e G \frac{\lambda^2}{4\pi} \quad (2)$$

We can now define an effective aperture efficiency

$$\eta = \frac{A_e}{A_{phy}} = \frac{\text{maximal eff. Antenna Area}}{\text{physical Area}}$$

This definition makes only sense for antennas with areas. The efficiency for parabolic dish antennas is around 90%.

Polarisations: a) Linear-Vertical, b) Linear-Horizontal, c) Circular-lefthanded



Rod Antenna: Monopole or Dipole.

Dipoles

Folded dipole Radiation resistance between 200 und 300 Ω . Radiates like $\lambda/4$ -Dipole.

Slotantennas Radiation via slots in the surface of waveguides. Several Slots can be easily grouped to Antenna arrays

Low-gain antennas are used for telemetry on satellites, because we do not loose contact even if the satellite is misaligned.

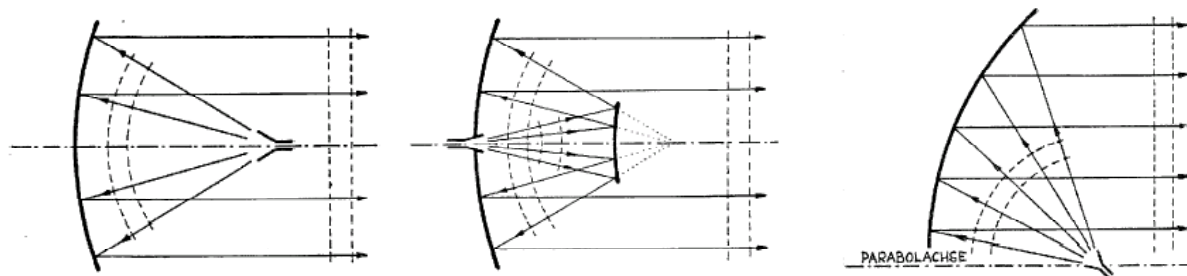
Antenna Parameter

Tabelle 1. Zusammenstellung grundlegender Antennenwerte

Antennenart	Darstellung, Belegung	Richtfaktor, Gewinn linear; (in dB)	wirksame Antennenfläche	effektive Höhe	Strahlungs-Widerstand	vertikales Richtdiagramm (3-dB-Bereich)	horizontales Richtdiagramm
isotrope Antenne	fiktiv	1; (0 dB)	$\frac{\lambda^2}{4\pi} = 0,08\lambda^2$	—	—		
Hertzscher Dipol, Dipol mit Endkapazität		1,5; (1,8 dB)	$\frac{3\lambda^2}{8\pi} = 0,12\lambda^2$	l	$80\left(\frac{\pi l}{\lambda}\right)^2 \Omega$		
kurze Antenne mit Dachkapazität auf leitender Ebene $h \ll \lambda$		3; (4,7 dB)	$\frac{3\lambda^2}{16\pi} = 0,06\lambda^2$	h	$160\left(\frac{\pi h}{\lambda}\right)^2 \Omega$		
kurze Antenne auf leitender Ebene $h \ll \lambda$		3; (4,7 dB)	$\frac{3\lambda^2}{16\pi} = 0,06\lambda^2$	$\frac{h}{2}$	$40\left(\frac{\pi h}{\lambda}\right)^2 \Omega$		
$\lambda/4$ -Antenne auf leitender Ebene		3,28; (5,1 dB)	$0,065\lambda^2$	$\frac{\lambda}{2\pi} = 0,16\lambda$	40Ω		
kurzer Dipol $l \ll \lambda$		1,5; (1,8 dB)	$\frac{3\lambda^2}{8\pi} = 0,12\lambda^2$	$\frac{l}{2}$	$20\left(\frac{\pi l}{\lambda}\right)^2 \Omega$		
$\lambda/2$ -Dipol		1,64; (2,1 dB)	$0,13\lambda^2$	$\frac{\lambda}{\pi} = 0,32\lambda$	73Ω		
λ -Dipol		2,41; (3,8 dB)	$0,19\lambda^2$	$\gg \lambda$	200Ω		
$\lambda/2$ -Schleifendipol		1,64; (2,1 dB)	$0,13\lambda^2$	$\frac{2\lambda}{\pi} = 0,64\lambda$	290Ω		
Schlitzantenne in Halbraum strahlend		3,28; (5,1 dB)	$0,26\lambda^2$	—	$\approx 500\Omega$		
kleiner Rahmen, n-Windungen, rechteckige Form		1,5; (1,8 dB)	$\frac{3\lambda^2}{8\pi} = 0,12\lambda^2$	$\frac{2\pi n A}{\lambda}$	$\frac{3100 n^2 (A/m)^2}{(\lambda/m)^4}$		
Spulenantenne auf langem Ferritstab $l \gg D$		1,5; (1,8 dB)	$\frac{3\lambda^2}{8\pi} = 0,12\lambda^2$	$\frac{\pi^2 n \mu_r D^2}{2\lambda}$	$19100 n^2 \mu_r^2 \left(\frac{D}{\lambda}\right)^4$		
Linie aus							

When we need strong focusing of the beam we have to use high gain antennas.

- The higher the gain the larger the antennas are.
- High gain antennas often use a reflector to focus the energy into one spot where the primary antenna is placed.
- The Figure shows commonly used reflector antennas.



Parabolantenna: a) Parabolantenna with central feed b) Cassegrain-Antenna with secondary reflector, c) Offset-Parabolantenna

Array antennas are fields of small antennas fed with signals which are well defined in amplitude and phase. With arrays we can also achieve high gains. An advantage of arrays is, that

- they can be electronically steered.
- the failure of one antenna has little effect on the radiation properties of the array.

Lovell Telescope

- Mass of the telescope 3200 tonnes
- Mass of bowl 1500 tonnes
- Height of elevation axis 50.5 metres
- Diameter of bowl 76.2 metres
- Maximum height above ground 89.0 metres
- Collecting area 4560 square metres



- Type Fixed reflector, movable feeds
- Diameter of reflector 1000 ft (304.8 m)
- Area of aperture 18 acres (73,000 square m)
- Shape of surface deg spherical cap
- Radius of curvature 870 ft (265 m)
- Surface accuracy 2.2 mm rms



27 antennas arranged in a huge Y pattern up to 36km (22 miles) across

- Each antenna is 25 meters (81 feet) in diameter
- they are combined electronically to give the resolution of an antenna 36km (22 miles) across,
- sensitivity equivalent to a dish 130 meters (422 feet) in diameter.

Very Large Array



- Early Warning for Star Wars
- The Golf Balls are 47 m in diameter
- The device can track missiles and compute the trajectories



- 100 m Diameter
- 3200 tons
- Area $A=7.850\text{m}^2$
- Resolution
- 35 Rad-seconds resolution at 1.3 cm wave length
- 6 min for tilting 90°



A wireless link consists of two antennas separated by a distance r of at least 10 wave lengths. The most important parameter of this link is the attenuation A of the link, das logarithmic ratio between transmit power P_t to received power P_r

$$A = \frac{P_t}{P_r}$$

The transmit power P_t generates a power flux density S_r at the receiver of

$$S_r = \frac{P_t G_t}{4\pi r^2}$$

plugged into equation 2 we get the received power

$$P_r = P_t G_t G_r \frac{\lambda^2}{(4\pi r)^2}$$

This equation is known as *Friis' Formula* . In logarithmic scale we get

$$A = \underbrace{-10\log G_t}_{\text{Gain Trans. Ant.}} - \underbrace{10\log G_r}_{\text{Gain Rec. Ant.}} + \underbrace{20\log\left(\frac{4\pi r}{\lambda}\right)}_{\text{Path Loss}}$$

The first two parts are the gains of the transmitting and receiving antenna. The last part is the so called free space loss or path loss. The free space loss is a function of λ .

$$A = -G_t - G_r + L_p$$

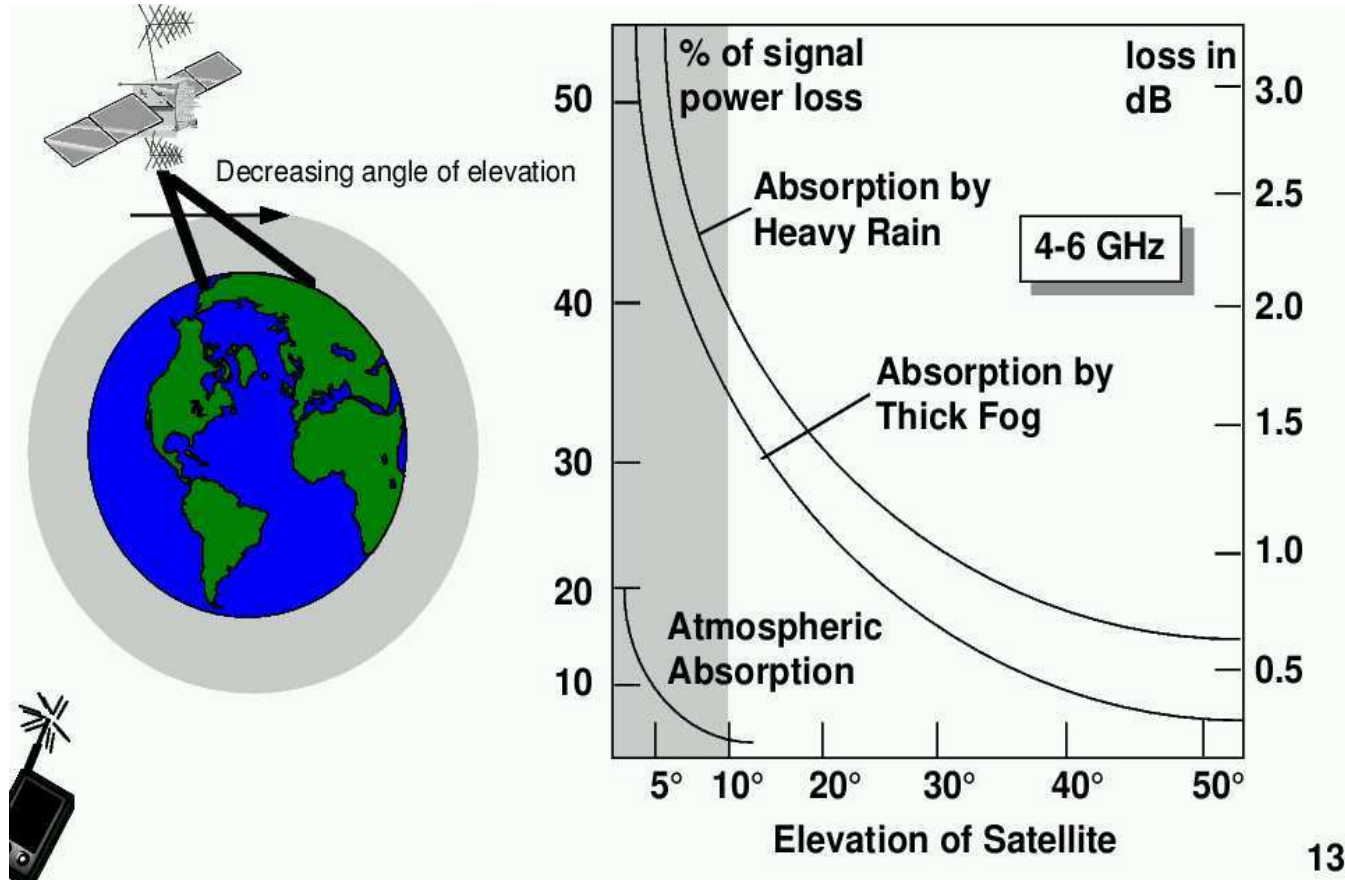
This formula is in real life more complicated. In addition to the free space loss we get also losses

- atmospheric and rain losses L_a
- losses inside the transmitting antenna L_{ta}
- losses inside the receiving antenna L_{ra}
- polarisation mismatch L_{pol}
- losses due to pointing error $L_{pointing}$

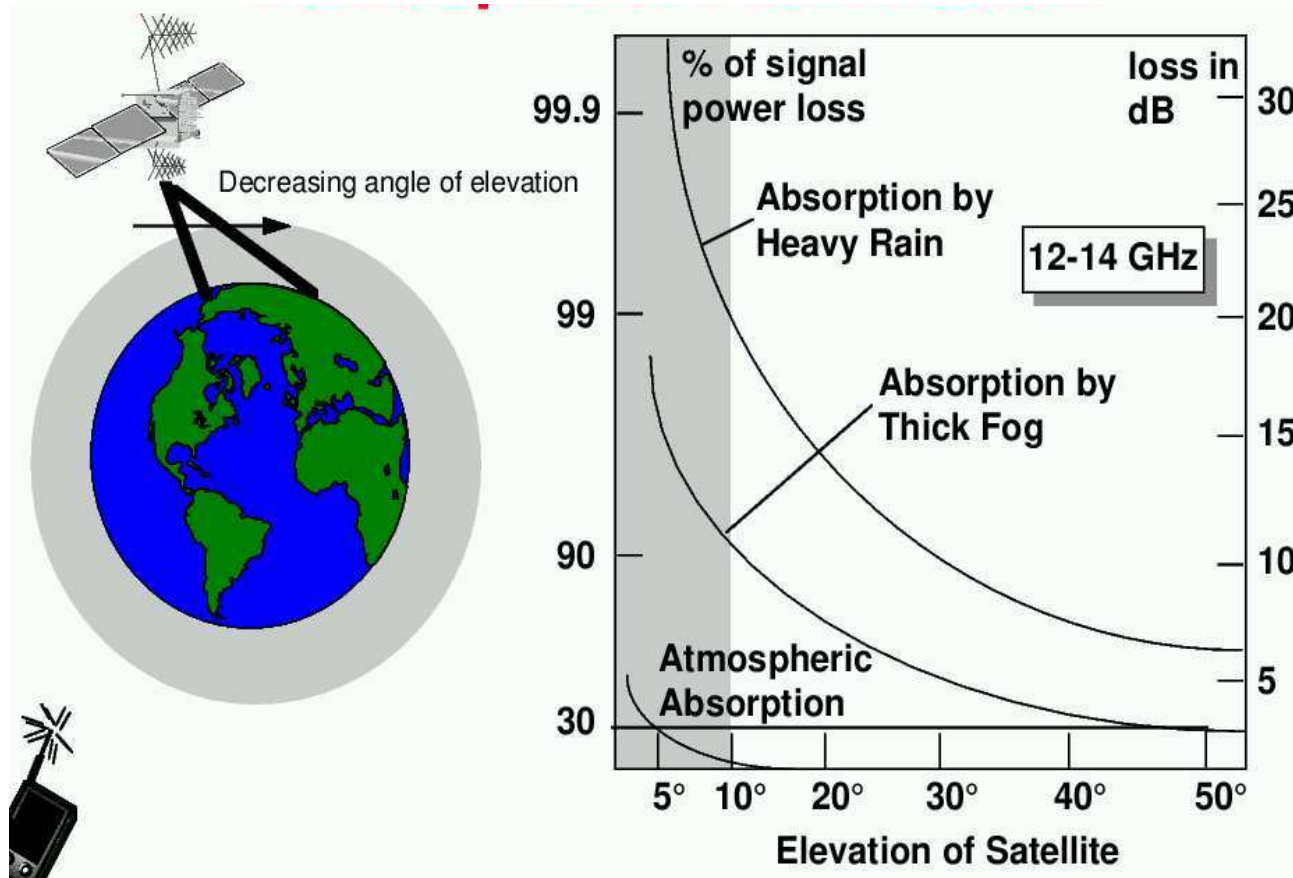
That means our formula becomes

$$P_e = \frac{P_s G_r G_t}{L_p L_a L_{ta} L_{ra} L_{pol} L_{point}}$$

Atmospheric Attenuation

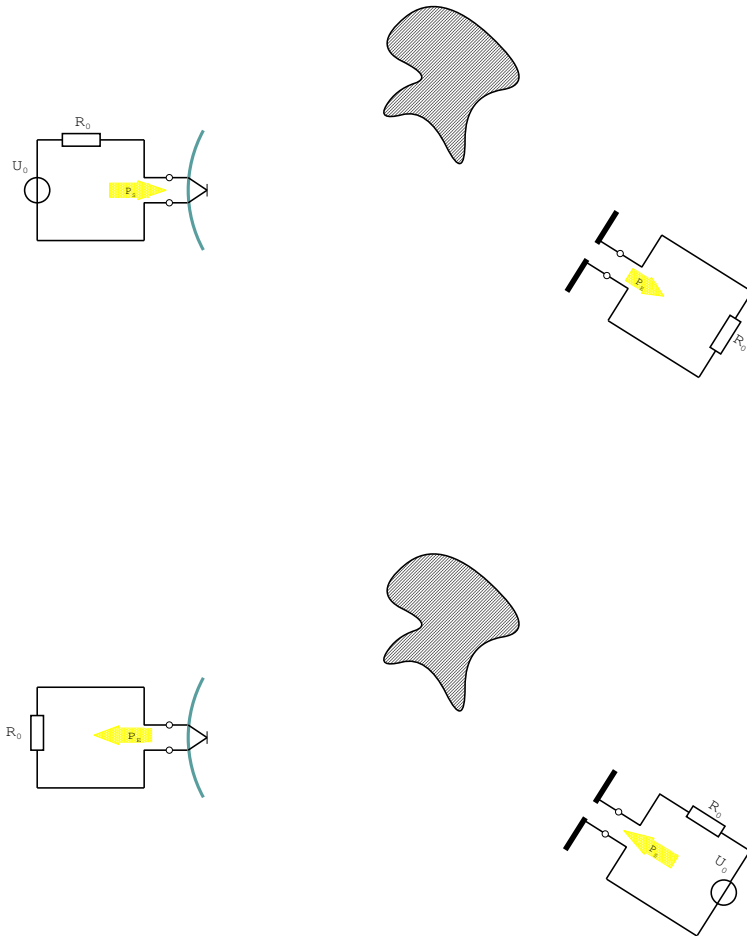


Atmospheric Attenuation



Reziprocität

Reziprocität: The ratio of available transmit power P_S and received power P_E is in both directions the same.



The Link Budget calculates the Power losses and gains through the complete RF link, it includes:

- Transmitter Power P_t
- Transmission losses in the Transmitter through lines etc L_{tl}
- Tx Antenna Gain G_t
- Antenna Pointing Loss $L_{point,t}$
- Free Space Loss L_p
- Atmospheric Loss (Clouds, Vapour etc.) L_{atm}
- Rx Antenna Pointing loss $L_{point,r}$
- Polarisation Loss L_{pol}
- Reception Losses in cables etc. L_{rl}
- Noise Temperature Contribution L_N
- Other Losses L_{other}

Example:

Given is a satellite in geo orbit (40000km to earth station), transmits with 2 W, Tx-Antenna-Gain is 17 dB at 11 GHz

Calculate:

- a) Flux density on earth surface
- b) Power received by antenna with effective aperture of 10m^2
- c) Received power when Earth station antenna has gain of 52.3 dB

Solution:

a) Flux density is

$$S = \frac{P_t}{4\pi r^2} = \frac{2\text{W} \cdot 10^{-1.7}}{4\pi(4 \cdot 10^7 \text{m})^2} = 4.97 \cdot 10^{-15} \frac{\text{W}}{\text{m}^2} \hat{=} -143\text{dBW}$$

Or in dB by

$$EIRP = (P_t + G_t)\text{dBW} = 20\text{dBW}$$

with Path Loss:

$$L_P = 20\log\left(\frac{4\pi r}{1}\right) = 20\left(\frac{4\pi 40000 \cdot 10^3}{1}\right) = 174\text{dBm}^2$$

Hence

$$S = 20\text{dBW} - 174\text{dB} = -154\text{dBW}/\text{m}^2$$

Power received by Antenna with $A_{eff} = 10\text{m}^2$

$$P_r = S \cdot A_{eff} = 4.97 \cdot 10^{-14}\text{W}$$

c) Received Power when Antenna has 53 dB Gain at 11 GHz in dB:

$$P_r = \underbrace{P_t + G_t}_{EIRP} + G_r + L_P = 20\text{dBW} + 53\text{dB} + L_P$$

$$\text{with } L_P = 20 \log\left(\frac{4\pi r}{\lambda}\right) = 205.3\text{dB}$$

$$P_r = -133\text{dBW}$$

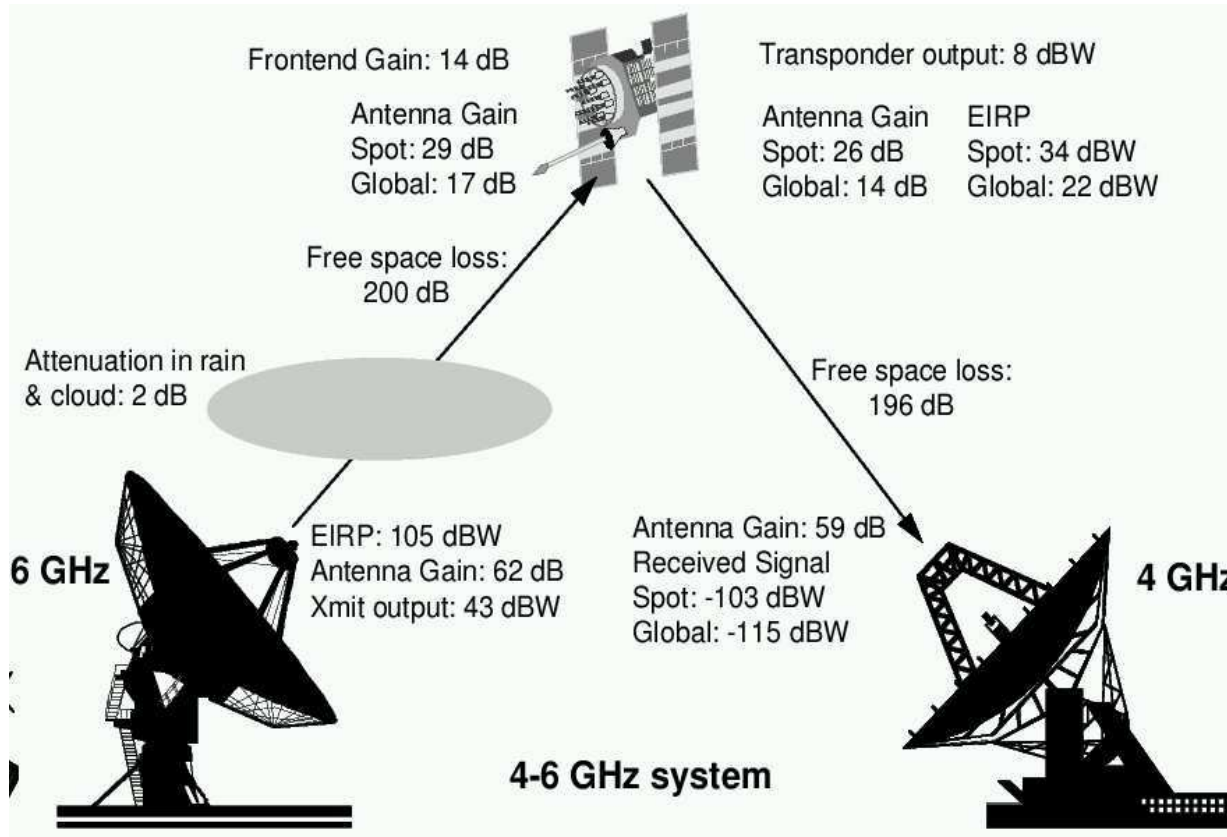
- Satellite has limited electrical power, 1 to 500 W
- Antenna on satellite is limited, fixed <3m, folding ant <10 m
- Received must be small according to FCC requirements, <-100dBW/m²

- C-Band 6/4GHz : Bandwidth 500-1000 MHz
- Ku-Band 14/11GHz: Bandwidth 1000-1500 MHz
- Ka-Band 20/30GHz: Bandwidth 3000 MHz

- FCC Regulations
- Availability
- Antenna Dimensions
- Rain Attenuation
- Noise Considerations
- Equipment Availability
- Costs

- Sketch a link path
- Think carefully about the system of interest
- Roll up large contributors first: Transmitted Power, Antenna gains, path loss
- Comment the link budget with all units eg. dB, W, dBm
- Most important is $(S/N)_{min}$: The minimal received Power-to-Noise Ratio under worst conditions
- $(S/N)_{min}$ depends on the operation modes such as
 - Modulation schemes
 - Desired Quality of operation
 - Coding gain
 - Channel Bandwidth
 - Thermal noise of Receiver

Typical Link

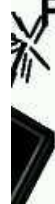


Rain Effect

- Rainfall introduces attenuation by absorption and scattering of signal energy
- Absorptive attenuation introduces noise
 - » A dB rain attenuation yields power loss ratio of $10^{A/10}$
 - » Effective noise temperature of rain $T_{\text{Rain}} = T_A (1 - 1/A)$
 - » T_A is a measured quantity between 270 and 290 K
- Suppose “clear sky” C/N is 20 dB, effective noise temperature is 400 K, apparent absorber temperature is 280 K, rain attenuation exceeds 1.9 dB 0.1% of time, how does this effect C/N?
 - » 1.9 dB = 1.55:1 power loss (i.e., $10^{1.9/10} = 1.55$)
 - » $T_{\text{rain}} = 280(1 - 1/1.55) = 99.2$ K
 - » $400 + 99.2 = 499.2$ K
change in noise power is $499.2 - 400 = 0.96$ dB
(= $10 \log (499.2/400)$)
 - » C/N = $20 - 1.9 - 0.96 = 17.14$ dB

Typical Up-Link

	4/6 GHz	12/14 GHz	20/30 GHz	12/14 GHz
Transmitter power, dBw	35	25	20	20
Transmitter system loss, dB	-1	-1	-1	-1
Transmitter antenna gain, dB	55	46	76	62
Atmospheric loss, dB	0	-0.5	-2	-0.5
Free space loss, dB	-200	-208	-214	-208
Receiver antenna gain, dB	20	46	53	60
Receiver system loss, dB	-1	-1	-1	-1
Received power, dBw	-92	-93.5	-69	-68.5
Noise temperature, °K	1000	1000	1000	1000
Received b/w, MHz	36	36	350	36
Noise, dBw	-128	-128	-118	-128
Received SNR, dB	36	34.5	49	59.5
Loss in bad storm, dB	2	10	25	10
Received SNR in bad storm, dB	34	24.5	24	49.5



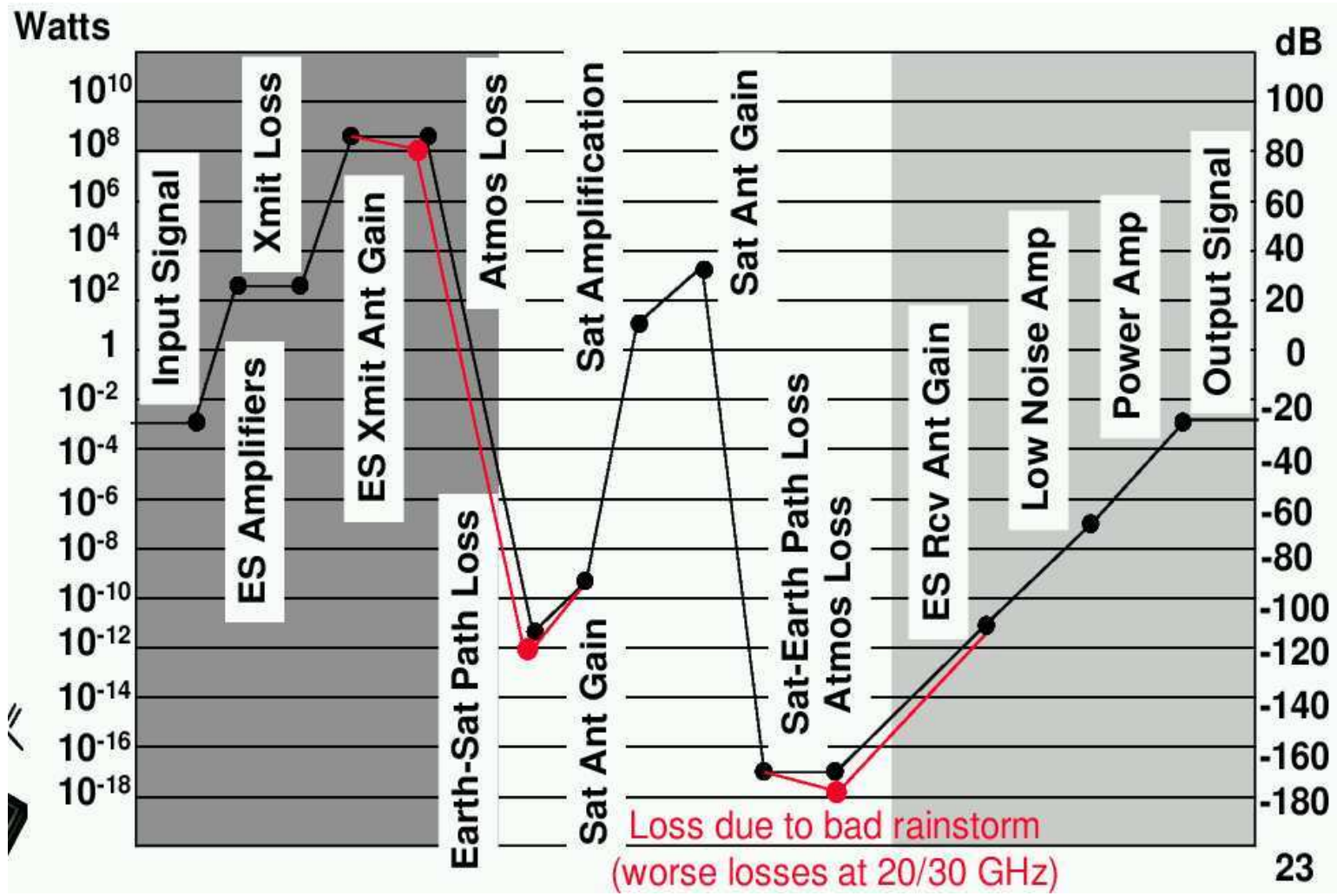
DBS, receive-only ES
1.8 m, 9 m satellite antenna

Typical Down-Link

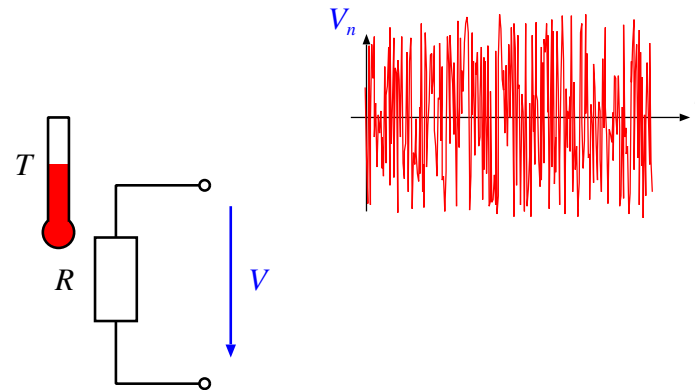
	4/6 GHz	12/14 GHz	20/30 GHz	12/14 GHz
Transmitter power, dBw	18	20	8	10
Transmitter system loss, dB	-1	-1	-1	-1
Transmitter antenna gain, dB	16	44	49	58
Atmospheric loss, dB	-197	-206	-210	-206
Free space loss, dB	0	-0.6	-2	-0.6
Receiver antenna gain, dB	51	44	72	44
Receiver system loss, dB	-1	-1	-1	-1
Received power, dBw	-114	-100.6	-85	-96.6
Noise temperature, °K	250	1000	250	1000
Received b/w, MHz	36	36	350	36
Noise, dBw	-131	-128	-121	-128
Received SNR, dB	17	27.4	36	31.4
Loss in bad storm, dB	2	10	25	10
Received SNR in bad storm, dB	15	17.4	11	21.4

DBS, receive-only ES
1.8 m, 9 m satellite antenna

Power Diagram



- Noise N is a signal which is superimposed on our signal C disturbs our signal.
- Performance of system is calculated by Signal-to-noise Ratio C/N
 - typically $C/N > 10$ dB is required
- Noise comes from
 - noise picked up by the antenna
 - receiver noise
 - noise, generated in the receiver



A resistor R at Temperature T generates an available noise power (matched load with R) of

$$P_n = \frac{V_n^2}{4R} = kTB$$

where $k = 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$ (Boltzman's constant)

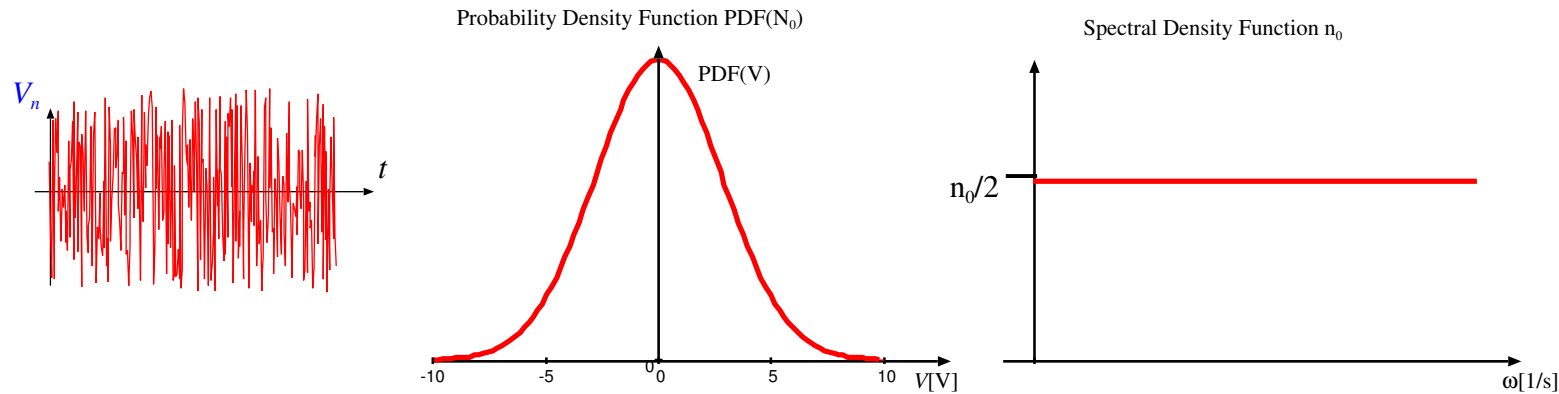
The power can be represented in terms of the power spectral density

$$S_n(\omega) = \frac{P_n}{3B} = \frac{kT}{2} = \frac{n_0}{2}$$

with $n_0 = kT$

This is known as the two-sided power spectral density. having frequency terms from $-B$ to B .

The spectrum is constant, i.e. we have *white noise*.

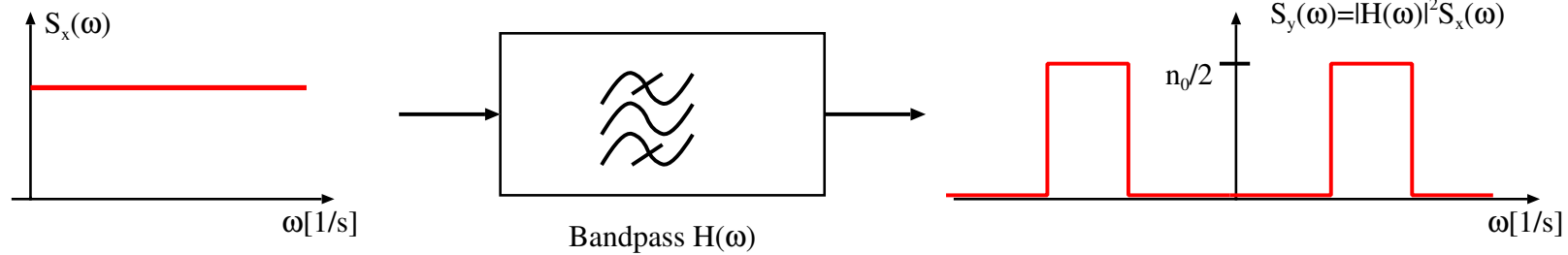


The noise amplitudes follow a Gaussian distribution,

$$f_n(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-n^2/2\sigma^2}$$

where σ^2 is the variance of the Gaussian noise. This noise distribution is known as white Gaussian noise. σ describes the “average” effective voltage of the noise signal.

Spectral Density Function



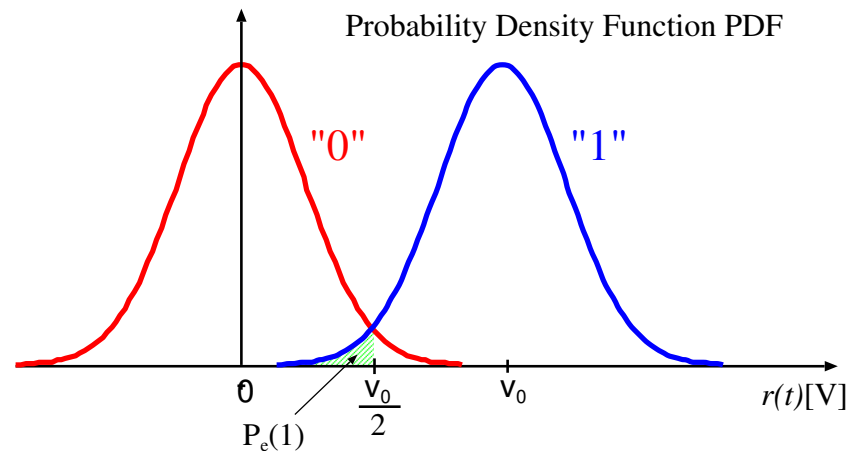
For band limited noise we get the total noise power of

$$P_n = \Delta f n_o$$

Consider a signal $s(t)$ with two states 1 and 0, represented by the voltage v_0 and 0.

A signal larger than $\frac{v_0}{2}$ is detected as 1 below as 0.

To this signal we add a Gaussian noise signal $n(t)$. Our outcome of the sum signal $r(t) = s(t) + n(t)$ is shown here

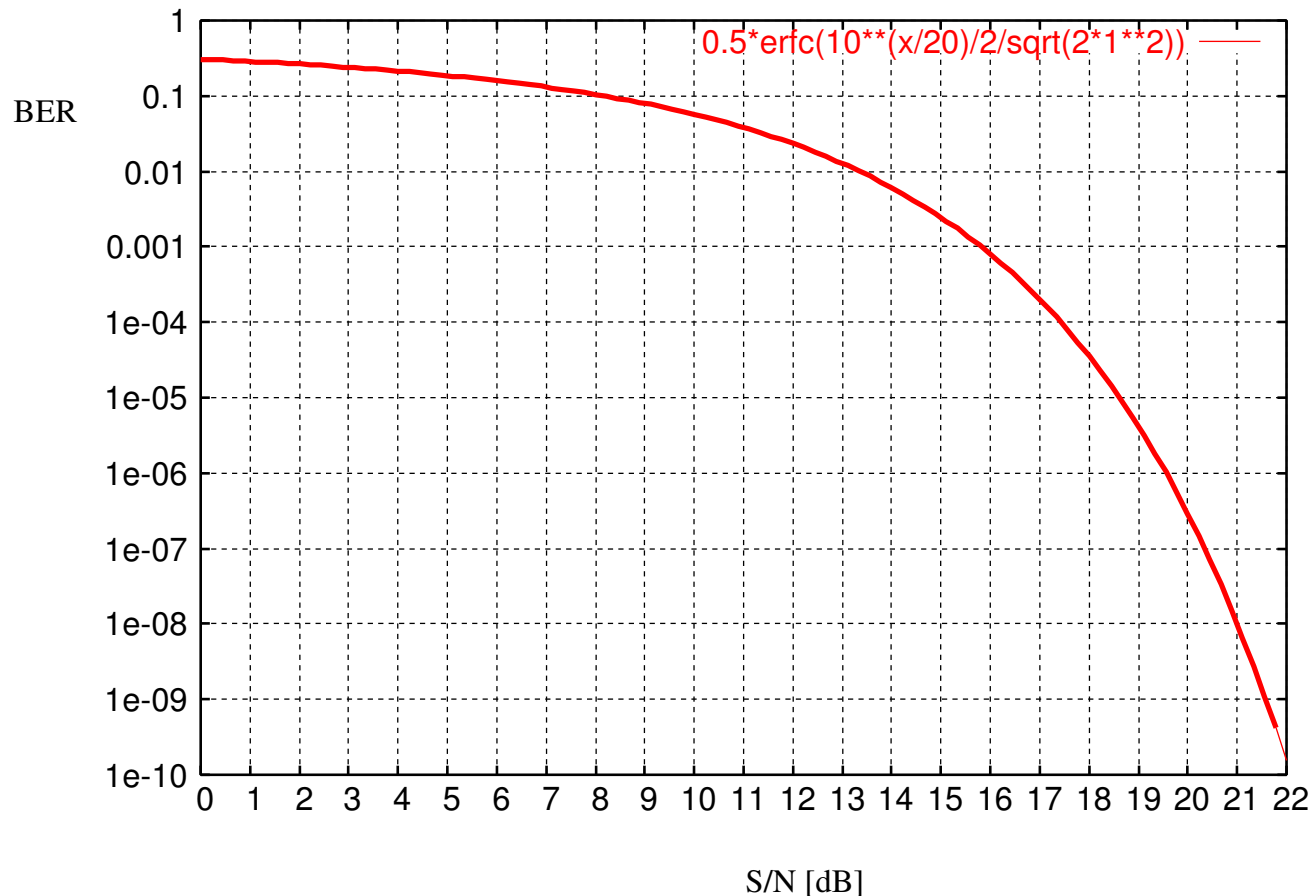


The probability of a 1 detected as a zero is

$$P_e(1) = \frac{1}{2} \operatorname{erfc}(x_0) = \frac{1}{2} \operatorname{erfc}\left(\frac{v_0}{2\sqrt{2}\sigma}\right)$$

where erfc is the complementary error function

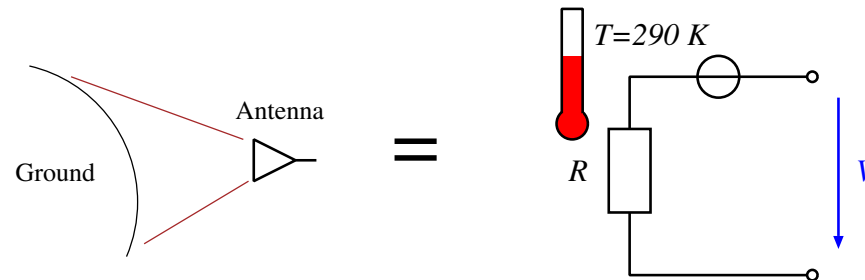
With the signal-to-noise ratio of v_0/σ we can plot the the probability of a Bit error



Error Probabilities of 10^{-6} to 10^{-8} are often desired in practice.

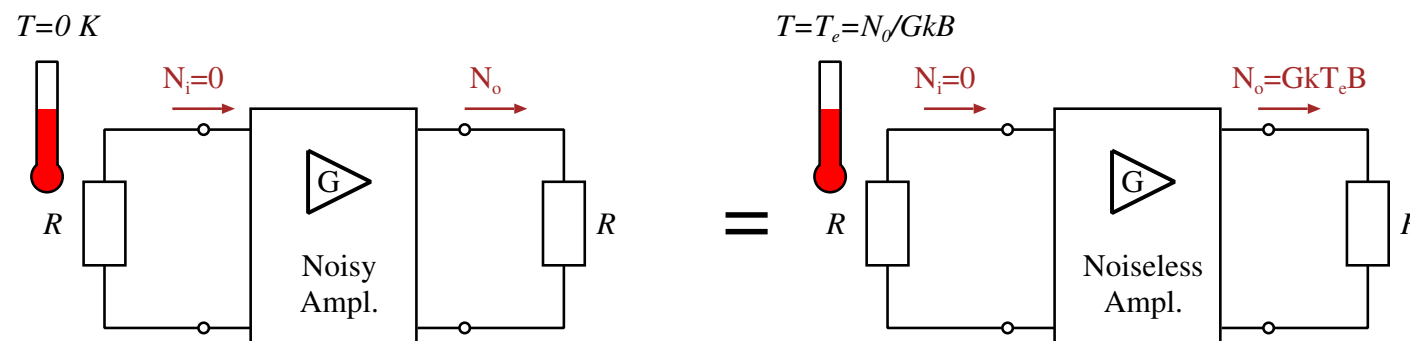
An Antenna pointed to the ground acts like a Resistor at Temperature

$$T_0 = 290\text{K}$$



- Cosmic Noise (stars and interstellar matter)
 - decreases with frequency (negl. above 1 GHz)
 - Certain parts of sky have hot sources
- Sun $T_{Sun} \approx 12000 f^{\frac{3}{4}} \text{K}$
 - Point Antenna away
- Moon= Black Body with $T_{moon} \approx 200 - 300 \text{K}$
- Earth Surface $T \approx 270 \text{K}$
- Propagation Medium: rain, vapour, etc
 - Noise reduces with elevation angle
- Man made noise

Microwave components generate noise. The noise can be characterised by an equivalent noise figure



The equivalent noise temperature t_e of an amplifier is the temperature to be added to the input resistance temperature, in order to represent the noise inside the amplifier

The noise figure of a two port is defined as as the ratio of the input S/N to the output S/N

$$F = \frac{S_i/N_i}{S_o/N_o} \geq 1$$

Noise figure and noise temperature are related by

$$T_e = (F - 1)T_0$$

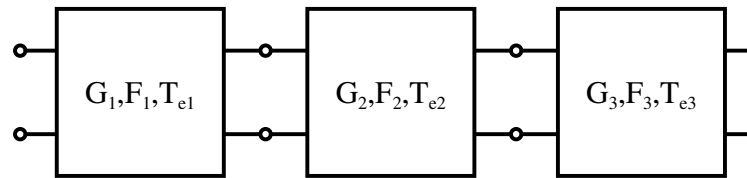
where T_0 is defined as 290 K reference

A line with loss L has a noise figure of

$$F = 1 + \frac{T_e}{T_0} = 1 + (L - 1) \frac{T}{T_0}$$

At room temperature $T = T_0$ the noise figure is simply $F = L$.

Noise Figure of Cascaded Components



$$F_{tot} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_2} + \dots$$

or using Temperatures

$$T_{e,tot} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_2} + \dots$$

Example Receiver Noise

TBD