Origins of the Equivalent Circuit Concept

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Abstract

The equivalent circuit concept derives from the Superposition Principle and Ohm's Law. Two forms of the equivalent circuit, the Thévenin equivalent and the Norton equivalent, distill any linear circuit into a source and an impedance. The development of these equivalents spans almost seventy-five years, with others than the eponymous people assuming equally important roles. This report describes the pertinent biographies of Helmholtz, Thévenin, Mayer, and Norton, and provides the relevant sections from their original papers on equivalent circuits.

Keywords: equivalent circuit, Helmholtz, Mayer, Norton, Thévenin

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1 Introduction

The theoretical foundations of linear circuit theory rest on Maxwell's theory of electromagnetism. In its more applied form, circuit theory rests on the key concepts of Kirchoff's Laws, impedance, Ohm's Law (in its most general sense by encompassing impedances), and the Principle of Superposition. From this foundation, *any* linear circuit can be solved: Given a specification of all sources in the circuit, a set of linear equations can be found and solved to yield any voltage and current in the circuit.

One of the most surprising concepts to arise from linear circuit theory is the *equivalent circuit*: No matter how complex the circuit, from the viewpoint of *any* pair of terminals, the circuit behaves as if it consisted *only* of a source and an impedance. Two equivalent circuit structures predominate: the Thévenin equivalent circuit and the Norton equivalent circuit (as they are known in the United States). As shown in figure 1, these circuits differ only in which kind of source — voltage source for the Thévenin equivalent and current source for the Norton. Because priority will be an issue in this paper, I use the terms "voltage-source" and "current-source" equivalents to describe them. From a narrow view, the equivalent circuit concept simplifies calculations in circuit theory, and brings to fore the ideas of input and output impedances. More broadly, the equivalent circuit notion means that a simpler but functionally equivalent form for complicated systems might exist. For example, this notion arises in queueing theory: The Chandy-Herzog-Woo theorem [1], sometimes known as Norton's Theorem, states that a complicated queueing system has an equivalent form in interesting situations.

1.1 Proof

The proof of the voltage- and current-source equivalent circuits is quite simple. Because of the Superposition Principle, the voltage appearing across any terminal pair in a circuit is a weighted linear combination of voltage contributions from all elements in the circuit. We separate this combination in two parts: that due to the independent sources and that due to the passive elements.

$$V = \underbrace{\sum_{\text{all sources } k} V_k}_{V_{\text{eq}}} + \underbrace{\sum_{\text{non-sources } l} Z_l I_l}_{-Z_{\text{eq}} I}$$

The first term represents the so-called *voltage-source equivalent source*. If we conceptually set all the sources to zero, this term vanishes and we are left with the contributions from the other elements in the circuit. If we attached a voltage source to the terminals equal to V, a current would flow and the zero-source circuit would appear to be an impedance. Thus, the expression for the terminal voltage for the complicated circuit can be reduced to the simple form[1]

$$V = V_{\text{eq}} - Z_{\text{eq}} I \ .$$

By solving this equation for I, $I = I_{\text{eq}} - V/Z_{\text{eq}}$, we can infer the current-source equivalent circuit, where the equivalent source I_{eq} equals $V_{\text{eq}}/Z_{\text{eq}}$. The equivalent impedances in the two equivalent circuits are the same.

1.2 Framing the story

This paper describes the development of the equivalent circuit notion. Its formal roots are Ohm's Law, Kirchoff's Laws, and the principle of superposition. Georg Simon Ohm (1789–1854) described his theory of conductors in his 1827 book [2]. Gustav Robert Kirchoff (1824–1887) described what have since become known as his laws in the 1840s. The Principle of Superposition

[1] The minus sign arises because of the way positive current flow is defined in figure 1.

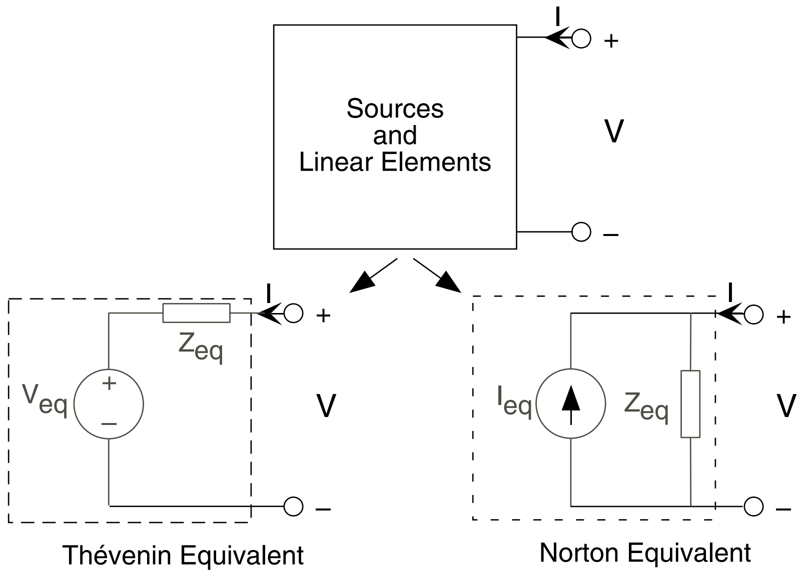

Figure 1: Thévenin's (voltage-source) equivalent circuit is shown at the left and Norton's (current-source) equivalent circuit at the right. The impedance Z_{eq} is the same in both cases and the source values are related to each other by $V_{\text{eq}} = Z_{\text{eq}} I_{\text{eq}}$.

was first clearly proclaimed by Hermann von Helmholtz (1821–1894) in his 1853 paper [3], in which he credits the result to his friend Émil du Bois-Reymond (1818–1896). In the same paper, Helmholtz derives the voltage source equivalent, and illustrates its application. Thirty years later, Léon Charles Thévenin (1857–1926), an engineer working for France's Postes et Télégraphes, published the same result [4, 5] apparently unaware of Helmholtz's work. In 1926, Edward Lawry Norton (1898–1983) wrote an internal Bell Laboratory technical report [6] that described in passing in the usefulness in some applications of using the current-source form of the equivalent circuit. In that same year, Hans Ferdinand Mayer (1895–1980) published the same result [7] and detailed it fully. Consequently, more than two people deserve credit for developing the equivalent circuit concept. In European countries, the equivalents are known by various combinations of these four person's names: Helmholtz-Thévenin, Helmholtz-Norton, Mayer-Norton, etc. As detailed subsequently, these people intertwine in interesting ways.

2 Helmholtz

Helmholtz was one of the nineteenth century's great scientists. Margeneau [9] describes him as "one of the last great universalists of science." His life is well documented; a detailed [10, 11] and numerous short biographies [12, for example] have been published, and his works have been collected [13]. He started his scientific career in electrophysiology. During his life, he refined the concept of the conservation of energy, invented the ophthalmoscope, brought physics and mathematics to the previously qualitative fields of physiological acoustics and optics, worked in hydrodymanics and electromagnetics, derived the wave equation that bears his name, and developed ideas in the philosophy of science. In 1853, Helmholtz was Associate Professor of Physiology at Königsburg. His 1853 publication *Some laws concerning the distribution of electric currents in conductors with applications to experiments on animal electricity* in Poggendorf's *Annalen* elaborated his note published the previous year [14]. In this paper, Helmholtz was concerned with

Hermann von Helmholtz 1821–1894	Léon Charles Thévenin 1857–1926	Hans Ferdinand Mayer 1895–1980	Edward Lawry Norton 1898–1983

Figure 2: Undated Helmholtz photograph taken late in his life. Thévenin's comes from the Suchet biography [8] and is also undated. Mayer's photograph was taken about 1940 and comes from the historical photograph collection provided on a CD by the SiemensForum (`http://www.siemens.de/siemensforum`). Norton's photograph has been provided by the AT&T Archives and is dated October 13, 1925.

determining from measurements of currents and voltages in muscle tissue the location of voltage sources (electromotive force generators) and the resulting current distribution. He described how the recent work of Kirchoff, Gauss, Ohm and others can help determine how what was then termed "animal electricity" flows. One of our primary characters, Hans Ferdinand Mayer, wrote a letter in 1950 [15] to the editor of *Electrical Engineering*, the non-technical publication of the AIEE. He was responding to a biography of Léon Thévenin that had been written the previous year in the same journal. He describes well Helmholtz's derivation; I provide Mayer's letter in full.

> With reference to the article "Leon Charles Thévenin" (*EE*, Oct '49, p 843–4), I would like to point out that the "Thévenin theorem" was published as early as 1853 by H. Helmholtz in Poggendorf's *Amalen* [sic] *der Physik und Chemie* (page 211), four years before Thévenin was born.
>
> On page 212 he first formulates the principle of superposition:
>
> > *If any system of conductors contains electromotive forces at various locations, the voltage (potential) at any point will be equal to the algebraic sum of the voltages (potentials), which any one of the electromotive forces would produce at this point independent of the others.* [Helmholtz's and Mayer's italics]
>
> Then he considers the case, where any two points of such a system (output terminals) are bridged by another conductor (load). He states, page 222, that no matter how complicated the system may be, it will behave with respect to the load as one single conductor of resistance, as calculated between these two points by Kirchof's rules, in series with an electromotive force, equal to the voltage between these two points before inserting the load.
>
> On page 223, he illustrates his theorem by the simple example (see Figure 3), where the system consists of two linear (lumped) conductors of resistance w_0 and w_1, in series with an electromotive source A. He then points out that, according to his theorem, the system with respect to a load w_2 can be replaced by an equivalent source, having the

electromotive force

$$s = A\frac{w_1}{w_0 + w_1}$$

and the interior resistance

$$w = \frac{w_0 \cdot w_1}{w_0 + w_1}$$

and consequently will drive a current in w_2

$$i_2 = \frac{s}{w + w_2}$$

In my opinion, this is a very clear formulation of what is now called "Thévenin's theorem."

Helmholtz not only considered the case of a system of "linear conductors" (lumped resistance) but also the general case of a space, filled with resistive material of different conductivity, and electromotive forces acting on the resistive medium (distributed resistances). He then states, *that if any two points at the surface of this space are connected by a load resistance, one can always replace the space by one lumped resistance in series with an electromotive force, and that this equivalent source will always drive the same current into the load as would the actual space source* [italicized in Helmholtz's original publication, but not in Mayer's letter].

I personally have no objection to calling this theorem "Thévenin's theorem," although it is called "Helmholtz's theorem" in other countries, but it is quite interesting that it was considered "new" in 1883, 30 years after Helmholtz's publication.

H.F. Mayer
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Mayer's summary implies little reasoning behind the basic result. In fact, Helmholtz used sophisticated mathematical and physical arguments to derive the result as well as ways of modeling current distributions in a distributed conductor such as muscle.

Though Thévenin was unaware of Helmholtz's result, others were not. Mayer of course knew of it in detail, and Wallot [16] references it in his 1932 German textbook. A description of Helmholtz's paper appears as a footnote on page 145 of the MIT course notes published in 1940 [17].

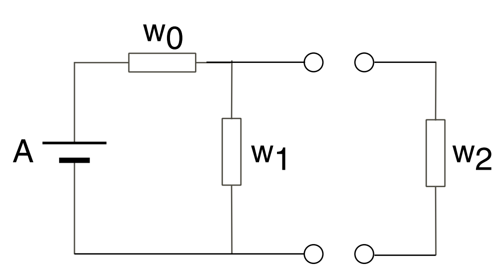

Figure 3: Redrawn replica of Mayer's figure (not present in Helmholtz's article) to help explain Helmholtz's derivation.

3 Thévenin

Biographies about Thévenin were published in 1926 [18], the year of his death, and again in 1949 [8]. Léon Charles Thévenin was born in Meaux, France (located some twenty miles from Paris) on March 30, 1857. He graduated from the École Polytechnique in 1876 (the year the telephone was developed by Bell) and, in 1878, joined the France's national electrical communication company Postes et Télégraphes, where he spent all of his career. He retired in 1914 to his family home in Meaux, and died in Paris on September 21, 1926, two months before Mayer and Norton independently described their extension of his and Helmholtz's result.

In 1882, he was appointed to teach courses for training inspectors in the engineering department because of his credentials (he successfully passed license examinations in mathematics and physical sciences upon graduation from the Polytechnique) and his apparent interest in teaching. In developing and teaching his courses, he found novel ways of explaining known results and new techniques as well, the equivalent circuit being one of them. The year 1883 marked publication of at least four papers [4, 19–21] in *Annales Télégraphiques*, the second of which [4] described what he thought was his new equivalent circuit result. Excited by his result, Thévenin wanted to report it to the French Academy of Sciences. According to Suchet [8], Thévenin asked a colleague, the mathematical physicist Aimée Vaschy (1857–1899), to comment on the paper. Vaschy thought the result incorrect. Thévenin consulted others, and varied opinions were offered. Eventually his previously published paper [4] was published virtually verbatim[2] in *Compte Rendu* [5] in the same year. The following translation of Thévenin's paper shows that he used an elegant approach to prove his theorem.

ELECTRICITY. — *On a new theorem of dynamic electricity*
Note by Mr. **L. Thévenin**

Theorem. — Assuming any system of linear interconnected ([1]) conductors, and containing some electromotive forces $E_1, E_2, \ldots, E_n$ distributed in any way, one considers two points A and A' belonging to the system and actually having the potentials V and V'. If the points A and A' are connected by a wire ABA' having resistance r, not having an electromotive force, the potentials at the points A and A' take on different values of V and V', but the current i flowing in the wire is given by the formula $i = \frac{V-V'}{r+R}$, in which R represents the *resistance of the primitive system, measured between the points* A *and* A' *considered as electrodes* [Thévenin's italics].

Thus, Ohm's law applies, not only to simple electric motor circuits that have well-defined poles, like a battery or a DC machine, but to any network of conductors that one would consider such as an electric motor at arbitrary poles, given that the electromotive force is, in each case, equal to the *pre-existing* [Thévenin's italics] potential difference at two points chosen for poles.

This rule, *which has not been mentioned before today* [italics not present in the original], is very useful in certain theoretical calculations. From a practical viewpoint, it permits immediate evaluation, by two easily obtained experimental means, of the current that flows in a given branch attached to any network of conductors, without being otherwise preoccupied with the detailed constitution of the network.

To show the theorem, we suppose that one introduces in the conductor ABA' an electromotive force $-E$, equal and opposite to the potential difference $V - V'$. Clearly, no other current flows through the conductor ABA'. Thus, the system of electromotive

[2] The only difference is the footnote appears in the text in the *Annales Télégraphiques* version.

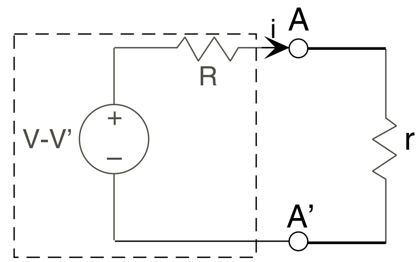

Figure 4: Circuit derived from Thévenin's proof of his theorem. No figure appears in his short paper.

forces $-E$, E_1, $E_2, \ldots, E_n$ give instead a new distribution of currents, among which is one where the current through ABA' is null.

We suppose now that, in the same conductor, one introduces, at the same time with the first, a second electromotive force $+E$, equal to the potential difference $V - V'$ and in the same sense. By virtue of the principle of the independence of simultaneous electromotive forces, the force $+E$ gives birth to a new current distribution, that simply superimposes in the preceding one. Among these new currents, the one flowing through the conductor ABA' is precisely the sought current i, because the effect of the forces $+E$ and $-E$, equal and opposite, cancel each other. The resulting current i is only due to the force $+E = V - V'$, whose consequence is in the branch r, one can, by calling R a certain resistance, write, according to Ohm's Law, $i = \frac{V-V'}{r+R}$. Moreover, the significance of the quantity R immediately appears; it is the resistance of a wire that can replace the primitive network of conductors between the points A and A', without the undisturbed flow due to a constant electrical source that would exist in the branch r before it was modified. The quantity R has a precise physical significance, and one can call it the resistance of the primitive network measured between the points A and A' considered like electrodes. The statement of the theorem results immediately from this definition.

([1])In such a way that the end of each is connected to at least a second conductor.

Figure 4 may help the reader understand what Thévenin's model was. Thévenin's derivation is correct, and certainly provides more engineering insight than Helmholtz's physics-based approach.

The history of Bell Labs claims that one its employees Hammond Hayes realized in 1885 that important theoretical work in electrical systems was being done in Europe, and that "advancement[s] in electrical theory abroad . . . undoubtedly came to Hayes' attention." [22, p. 888] That history lists Thévenin's result as one of those advancements. Despite Vaschy's initial reaction, he played an important role in making Thévenin's result widely known. In 1890, Vaschy published *Traité d'Électricité et de Magnétisme* [23], a well-written, definitive, two-volume treatise on theoretical and applied electromagnetism. On page 153 of Volume I he presents Thévenin's theorem much as Thévenin had in his paper, associates his name with it, and references both of Thévenin's 1883 papers; Vaschy does not mention Helmholtz. Also note that Helmholtz was alive when Thévenin's paper appeared and when Vaschy's treatise was published. In the 1940 edition of the circuit-theory text written by Timbie and Bush [24], the authors state on page 40 that

This general theorem was originally proposed by Thévenin in 1883, but it has not been in general use until recently. However, the engineers of the American Telephone and Telegraph Company have used it since about 1904.

Indeed, as late as 1926, Thévenin's and Helmholtz's result was not generally known. In that year, a physicist at the National Bureau of Standards rederived it [25].[3] Thévenin's theorem was described in the Electrical Engineers' Handbook published in 1936 [26].

4 Mayer

Hans Ferdinand Mayer was born on October 23, 1985 in Pforzheim, Germany, which is located halfway between Stuttgart and Karlsruhle. After receiving a leg wound in his first action in World War I (1914), he studied physics and mathematics at the Technische Hochschule in Stuttgart and then went on to the University of Heidelberg to become a research assistant to Philipp Lenard (1862–1947), a Nobel Prize winner in physics (1905). He received his doctorate in 1920, with his dissertation concerning the interaction of slow electrons with moelcules. He continued working as a research assistant for Lenard until 1922, and then joined Hause-Siemens. He became Director of Siemens Research Laboratory in 1936. Except for interludes during and after World War II, he worked for Siemens until his retirement in 1962. He published 25 technical papers during his life and secured over 80 patents. He received an honorary doctorate from the Technische Hochschule in Stuttgart in 1956, the Gauss-Weber Medal from the University of Göttingen and the Philipp Reis award from the German Post Office in 1961, and the Ring of Honor from the VDE in 1968 [27]. Mayer died on October 16, 1980 in Munich.

Short biographies of Mayer are provided in [27–29], with [27] listing his publications. As recognized as he was for his technical work, Mayer's personal life perhaps had more impact. As described in [28, 30, 31], Mayer secretly leaked to the British in November 1939 all he knew of Germany's warfare capabilities, particularly concerning electronic warfare. Because he represented Siemens as a technical expert in negotiations with companies outside Germany, he had the opportunity to travel widely about Europe. While in Oslo, Norway, he typed and mailed a two-page report of what he knew and mailed it to the British Embassy in Oslo. Because Mayer wrote it anonymously, the British, led by Reginald Jones, had to determine the report's accuracy. Jones found what became known as the Oslo Report to be a technically knowledgeable person's description of what he/she knew (although it contains some errors) [31]. Only after the war did Jones determine that Mayer was the "Oslo Person." Mayer did not even tell his family of his role in the Oslo Report until 1977 [30]. He requested that his contribution not be made known until after his and his wife's death. Jones described Mayer's contributions in 1989 [28] and a newspaper feature appeared that same year [30]. During the war, Mayer continued working at Siemens, until he was arrested in 1943 by the Gestapo for listening to the BBC and speaking out against the Nazi regime.[4] He was saved from execution by his doctoral advisor Lenard, despite Lenard being a strong supporter of the Nazis (he first met Hitler in 1926) and being anti-Semitic to the extreme (so much so he could not believe any Jew's physics, Einstein in particular). Mayer was put into the Dachau concentration camp, and later moved into four others during the remaining years of the war. After the war, he joined the electronics research effort at Wright-Patterson Air Force Base located in Dayton. Ohio, which at the time was the U.S. Air Force's primary research laboratory. He left the laboratory in 1947 to become Professor of Electrical Engineering at Cornell University [32]. It is during this

[3] Interestingly, this paper references Helmholtz's work [3] for the superposition principle. Wenner apparently did not read 10 pages further or did not understand Helmholtz's result.

[4] The Nazis were never aware of the Oslo Report.

time he wrote his letter describing Helmholtz's role in developing equivalent circuits [15]. After the Federal Republic was established in 1949 and Siemens was returning to its pre-war prominence, he returned to Germany in 1950 to work with Siemens in Munich.

In November 1926, Mayer published a paper [7] that describes the conversion of the voltage-source equivalent circuit to a parallel combination of a current source and the equivalent impedance (figure 5). This paper makes no reference to Helmholtz or Thévenin; in fact, he refers to both forms as "Ersatzschema" (equivalent circuits). He derives the current-source equivalent circuit by simply noting that it has the same terminal behavior as the voltage-source equivalent (similar to the proof given earlier in this paper). His concern was finding the equivalent circuit for the output of electronic amplifiers. Mayer is perhaps the first to point out that the equivalent voltage and current source values equal the open-circuit voltage and short-circuit current respectively. His paper is about two and one-half pages long, with about a page of it an editor's comment. His portion is divided into five numbered sections, two of which are translated here.

> 1. [A review of vacuum tube amplifiers and how the voltage-source equivalent reflects their characteristics.]
>
> 2. Consider first the simple case in Fig. 1 [referring to Abb. 1 in this paper's figure 5], where an electromotive source E with an internal resistance R_i is connected to an external resistance R_a. Such an arrangement is perfectly equivalent from the viewpoint of R_a to that shown in Fig. 2 [Abb. 2 in figure 5], where the electromotive source is replaced by a current source $J = \frac{E}{R_i}$, because in both cases the same voltage V results. From the viewpoint of R_a, the circuit it is attached to can be characterized two different ways: either, as in Fig. 1 [Abb. 1], as a electromotive source E and an internal resistance R_i or, as in Fig. 2 [Abb. 2], as a current source J and an internal conductance $G_i = 1/R_i$. As with the electromotive source E in Fig. 1 [Abb. 1], the current source J in Fig. 2 [Abb. 2]does not depend on outside resistances [loads]. The source value E is identical to the open-circuit voltage V and J equals the short-circuit current.

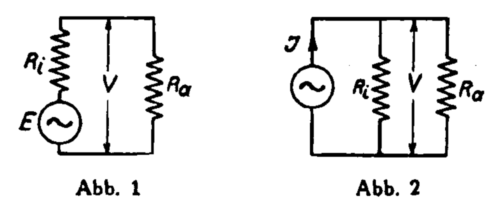

Figure 5: Reproduction of the critical figure from Mayer's paper.

> 3. [A reinterpretation of section 2 in terms of vacuum tube amplifiers.]
>
> 4. [An example of a parallel loading circuit showing that the current source equivalent makes calculations simpler.]
>
> 5. The reciprocity between the networks shown in Fig. 1 and 2 [Abb. 1 and 2] can be extended to any network. Assume a network has resistances $R_1, \ldots, R_n$ and equivalent electromotive sources $E_1, \ldots, E_n$. If one wants to calculate the current flowing through any resistance, then the following way proves fruitful: First imagine a particular resistance is isolated and calculate the open-circuit voltage occurring in its place. The remaining network can be replaced by one source, its electromotive force

> equal to E, and its internal resistance is the resistance of the network seen from the particular resistance's viewpoint if all the electromotive forces $E_1, \ldots, E_n = 0$.
>
> The other equivalent source of Fig. 2 [Abb. 2] would be found by replacing the particular resistance by a short circuit and measuring the current J that flows. The network from the particular resistance's viewpoint can be replaced by a single current source having a value equal to the short-circuit current J. The internal conductance G of this equivalent network is that seen from the particular resistance if all the electromotive sources to zero. The relationship between the resistance R and the conductance G is $R \cdot G = 1$.
>
> [The paper concludes with a long remark by the editor Hermann Schulz showing that the equivalent extends to complex amplitude sources and impedances. He also shows how the Norton equivalent can be usefully applied in a two-port example.]

Mayer's description is quite clear and contains all the central ideas found in modern presentations of equivalent circuits. Wallot's 1932 textbook describes the current-source equivalent and references Mayer [16]. To my knowledge, no textbook written by American authors mentions Mayer.

5 Norton

No biography was ever written about Norton; what follows was obtained from the AT&T Archives. Detailed information about Norton can be found at the author's web site.

Edward Lawry Norton was born on July 29, 1898 in Rockland, Maine. He served as a radio operator in the U.S. Navy between 1917 and 1919. He first attended the University of Maine, then transferred to M.I.T. and received his S.B. degree (electrical engineering) in 1922. He then joined the Western Electric Company (the predecessor to Bell Telephone Laboratories) and received his masters' degree in electrical engineering from Columbia University in 1925. He remained with Bell Labs all of his career, retiring in 1963. He died on January 28, 1983 at the King James Nursing Home in Chatham, New Jersey.

During his forty-one year career at Bell Labs, he wrote only three technical papers [33–35], none of which concerned or mentioned the equivalent circuit that bears his name today. During his career he obtained 18 patents,[5] which also contain no mention of his equivalent circuit. He wrote 92 technical reports during his career, and in one of these, *Design of Finite Networks for Uniform Frequency Characteristic* dated November 3, 1926, a short paragraph describing the current-source equivalent appears [6].

> The illustrative example considered above gives the solution for the ratio of the input to output current, since this seems to be of more practical interest. An electric network usually requires the solution for the case of a constant voltage in series with an output impedance connected to the input of the network. This condition would require the equations of the voltage divided by the current in the load to be treated as above. It is ordinarily easier, however, to make use of a simple theorem which can be easily proved, that the effect of a constant voltage E in series with an impedance Z and the network is the same as a current $I = \frac{E}{Z}$ into a parallel combination of the network and the impedance Z. If, as is usually the case, Z is a pure resistance, the solution of this case reduces to the case treated above for the ratio of the two currents, with

[5] A typed biography form in the AT&T archives dated July 20, 1954 states that he had 19 patents. A handwritten biography form dated two years later states "approximately twenty." Only 18 could be found in the U.S. Patent and Trademark Office records.

> the additional complication of a resistance shunted across the input terminals of the network. If Z is not a resistance the method still applies, but here the variation of the input current $\frac{E}{Z}$ must be taken into account.

During the time of this technical report, Norton was working on circuit design and on electrical models for phonographs [36]; he is mentioned in the paper [37, Footnote 7] that resulted from this work. Note that Norton's technical report is dated the same *month* as Mayer's publication.

Because of Norton's lack of publications, it appears that Norton preferred working behind the scenes. As described in the history of Bell Labs [38, p. 210], this reticence belied his capabilities.

> Norton was something of a legendary figure in network theory work who turned out a prodigious number of designs armed only with a slide rule and his intuition. Many anecdotes survive. On one occasion T.C. Fry called in his network theory group, which included at that time Bode, Darlington and R.L. Dietzold among others, and told them: "You fellows had better not sign up for any graduate courses or other outside work this coming year because you are going to take over the network design that Ed Norton has been doing single-handed."

He applied his deep knowledge of circuit analysis to many fields, and after World War II he worked on Nike missile guidance systems [39]. Norton became a Fellow of the Acoustical Society of America and a Fellow of the Institute of Radio Engineers in 1961.

How Norton's name became associated with the equivalent circuit that bears his name is murky. It is not mentioned (but Thévenin is) in the 1929 classic book [40, pp. 55–56] by Shea, who worked at Bell Laboratories. The Timbie and Bush textbook edition from 1940 mentions Thévenin but not Norton or his equivalent; their 1951 edition does [41]. Smith's 1949 textbook clearly describes the current-source equivalent but without reference [42]. The 1940 publication of the book derived from teaching the first course in circuit analysis at MIT mentions the voltage-source equivalent but not the current-source equivalent [17]; however, on page 145 a tantalizing footnote describes both Thévenin's and Helmholtz's contributions and ends with

> The theorem is actually more general than stated for the special case of resistance [sic] networks. It can be stated in terms of current source and conductance if desired.

6 Conclusions

As frequently occurs in science and engineering, the name associated with a law or concept may not have been the first or even primary responsible person. Helmholtz clearly originated the voltage-source equivalent. I can only presume that his paper's title (concentrating on animal electricity) and its publication early in his career meant it was not read by electrical scientists despite Helmholtz's eventual scientific stature. Helmholtz's biography [10, 11] describes the critical portion of his paper, but makes no allusion to Thévenin or its engineering importance. That said, in Europe, Helmholtz's name is associated with the equivalent circuit (any Web search will reveal this fact). Mayer developed the current-source version more fully and more publicly than did Norton. However, Mayer's work was published in a somewhat obscure German technical journal. I have not been able to determine just how Norton's name became associated with the current-source equivalent. Again, in Europe, both people are associated with it.

The current-source equivalent did not occur to early electrical scientists because of the seeming impossibility of a current source existing. An ideal current source will produce a specified current no matter what is attached to it, be the attached element an open circuit (which means it produces a controlled current into free space) or a short circuit (current flows through an ideal wire without dissipating any more heat than when attached to a non-zero impedance). Only later did Norton and Mayer realize that the current-source equivalent was easier to use in theoretical work in certain situations (e.g., when the load consists of a parallel combination). We now recognize that it also provides more insight into the circuit when the equivalent impedance is larger than the load, in which case the current flow is approximately constant across variations in load impedance.

Despite the current-source equivalent being taught to all electrical engineers in their first course, Norton labored in relative obscurity. He was very well known by those at Bell Laboratories. Telephone calls to now-retired Bell Laboratories researchers I know revealed that all recalled him and confirmed his stature at the Lab. Presumably one or several of Norton's colleagues credited the current-source equivalent to him some time before 1950.

Hans Ferdinand Mayer deserves more recognition in the United States than he has now. His contribution was published and, because journal publication delays usually exceed those of a technical report, presumably discovered the utility of the current-source equivalent earlier than Norton. His journal, *Telegraphen- und Fernsprech-Technik*, was then and now not well-known; this fact certainly contributed to an unawareness of his work. His wartime actions were courageous but not well-known in the United States. Perhaps future textbooks should follow Mayer's suggestion: Credit the voltage-source equivalent to Thévenin. I would suggest that the current-source equivalent be named the Mayer-Norton equivalent.

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