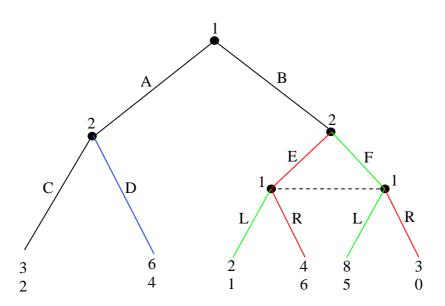
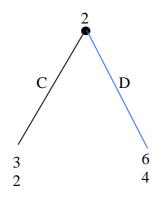
Subgame Perfect Nash Equilibrium: An Example

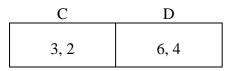


The extensive form game above has 3 subgames: One is the whole game, since any game is a subgame of itself. There are two more subgames, one beginning after A and the other beginning after B.

The extensive form of the subgame beginning after A is:

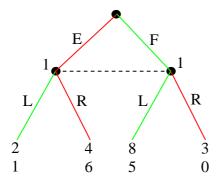


and its normal form is:



The Nash Equilibrium of this subgame requires 2 to play D obtaining a payoff of 4 rather than 2. It is shown in blue in the extensive forms.

The extensive form of the subgame beginning after B is:



and its normal form is:

| | Е | F | |
|---|------|------|--|
| L | 2, 1 | 8, 5 | |
| R | 4, 6 | 3, 0 | |

This subgame has 2 Nash Equilibria. They are (\mathbf{R}, \mathbf{E}) shown in red and (\mathbf{L}, \mathbf{F}) shown in green.

In the whole game, the possible strategies for player 1 are:

- AL A, but if B were chosen, then choose L following 2's play.
- AR A, but if B were chosen, then choose R following 2's play.
- BL B and L following 2's play.
- BR B and R following 2's play.

In the whole game, the possible strategies for player 2 are:

- C(A)E(B) C if 1 plays A and E if 1 plays B.
- D(A)F(B) D if 1 plays A and F if 1 plays B.
- C(A)F(B) C if 1 plays A and F if 1 plays B.
- D(A)E(B) D if 1 plays A and E if 1 plays B.

| | C(A)E(B) | D(A)F(B) | C(A)F(B) | D(A)E(B) |
|----|----------|----------|----------|----------|
| AL | 3, 2 | 6, 4 | 3, 2 | 6, 4 |
| AR | 3, 2 | 6, 4 | 3, 2 | 6, 4 |
| BL | 2, 1 | 8, 5 | 8, 5 | 2,1 |
| BR | 4, 6 | 3, 0 | 3, 0 | 4, 6 |

The normal form of this game is:

There are 5 Nash Equilibria:

(BR, C(A)E(B)), but this is not Subgame Perfect because C(A) is not played in the Nash Equilibrium in the subgame beginning after A.

(AL, D(A)E(B)), but this is not Subgame Perfect because (L, E) is not played in the subgame beginning after B.

(BL, C(A)F(B)), is not Subgame Perfect because C(A) is not played in the Nash Equilibrium in the subgame beginning after A.

 $(\mathbf{AR}, \mathbf{D}(\mathbf{A})\mathbf{E}(\mathbf{B}))$ is Subgame Perfect. It is shown by the thick lines below in the extensive form of the whole game, where blue shows the subgame of the game beginning after A and red shows the Nash Equilibrium of the subgame beginning after B.

There is one other Subgame Perfect Nash Equilibrium. Find it, and show the moves it requires in a diagram like the one below.

