## Subgame Perfect Nash Equilibrium: An Example



The extensive form game above has 3 subgames: One is the whole game, since any game is a subgame of itself. There are two more subgames, one beginning after A and the other beginning after B.

The extensive form of the subgame beginning after A is:

and its normal form is:


The Nash Equilibrium of this subgame requires 2 to play D obtaining a payoff of 4 rather than 2 . It is shown in blue in the extensive forms.

The extensive form of the subgame beginning after B is:

and its normal form is:


This subgame has 2 Nash Equilibria. They are (R, E) shown in red and (L, F) shown in green. In the whole game, the possible strategies for player 1 are:

AL A, but if B were chosen, then choose L following 2's play.

AR A, but if B were chosen, then choose R following 2's play.

BL $\quad \mathrm{B}$ and L following 2's play.

BR $\quad B$ and $R$ following 2's play.

In the whole game, the possible strategies for player 2 are:
$C(A) E(B) \quad C$ if 1 plays $A$ and $E$ if 1 plays $B$.
$D(A) F(B) \quad D$ if 1 plays $A$ and $F$ if 1 plays $B$.
$C(A) F(B) \quad C$ if 1 plays $A$ and $F$ if 1 plays $B$.
$D(A) E(B) \quad D$ if 1 plays $A$ and $E$ if 1 plays $B$.

The normal form of this game is:

|  | A)E( | $\mathrm{D}(\mathrm{A}) \mathrm{F}(\mathrm{B})$ | $\mathrm{C}(\mathrm{A}) \mathrm{F}(\mathrm{B})$ | $\mathrm{D}(\mathrm{A}) \mathrm{E}(\mathrm{B})$ |
| :---: | :---: | :---: | :---: | :---: |
| AL | 3, 2 | 6, 4 | 3, 2 | 6, 4 |
| AR | 3, 2 | 6, 4 | 3, 2 | 6, 4 |
| BL | 2,1 | 8, 5 | 8, 5 | 2,1 |
| BR | 4, 6 | 3, 0 | 3, 0 | 4, 6 |

There are 5 Nash Equilibria:
(BR, C(A)E(B)), but this is not Subgame Perfect because C(A) is not played in the Nash Equilibrium in the subgame beginning after A .
(AL, $\mathrm{D}(\mathrm{A}) \mathrm{E}(\mathrm{B})$ ), but this is not Subgame Perfect because ( $\mathrm{L}, \mathrm{E}$ ) is not played in the subgame beginning after $B$.
(BL, $C(A) F(B)$ ), is not Subgame Perfect because $C(A)$ is not played in the Nash Equilibrium in the subgame beginning after A .
$(A R, D(A) E(B))$ is Subgame Perfect. It is shown by the thick lines below in the extensive form of the whole game, where blue shows the subgame of the game beginning after $A$ and red shows the Nash Equilibrium of the subgame beginning after B.

There is one other Subgame Perfect Nash Equilibrium. Find it, and show the moves it requires in a diagram like the one below.


