# luca belcastro <br> ... about "MAR IBIS" <br> for violin, viola, cello and piano 

## INTRODUCTION

## I - GOLDEN SECTION

Euclid, the Greek mathematician of about 300 BC , wrote the Elements which is a collection of 13 books on Geometry. It starts from basic definitions called axioms or "postulates" (self-evident starting points). The propositions involve constructing geometric figures using a straight edge and compasses only so that we can only draw straight lines and circles.
In Book 6, Proposition 30, Euclid shows how to divide a line in mean and extreme ratio which we would call "finding the golden section $G$ point on the line".


Euclid used this phrase to mean the ratio of the smaller part of this line, $G B$ to the larger part $A G$ is the same as the ratio of the larger part, $A G$, to the whole line $A B$ :

$$
G B: A G=A G: A B
$$

The geometrical construction to find the point $G$ proposed by Euclid is:


This is the mathematical solution:

If we let the line $A B$ have unit length and $A G$ have length $g$ (so that $G B$ is then just 1-g) then the definition means that:

$$
\begin{aligned}
& (1-g): g=g: 1 \\
& g^{2}+g=1
\end{aligned}
$$

So there is only one positive number which when added to its square give 1 . It is $0,6180339887 \ldots$

It seems that this ratio had been of interest to earlier Greek mathematicians, especially Pythagoras (580 BC 500 BC ) and his "school". For the armony of the proportion it became more and more interesting in the Middle Ages, Renaissaince and so; it was used in Architecture and studied in the nature, in the physics, etc.

## II - FIBONACCI

Leonardo of Pisa, named Fibonacci, which means "the son of Bonacci" was son of a kind of customs officer in the North Africa; so he grew up with a North African education under the Moors and later travelled extensively around the Mediterranean coast. He would have met with many merchants and learned of their systems of doing arithmetic. He soon realised the many advantages of the "Hindu-Arabic" system over all the others.

## Fibonacci's Rabbits

The original problem that Fibonacci investigated (in the year 1202) was about how fast rabbits could breed in ideal circumstances.
Suppose a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits. Suppose that our rabbits never die and that the female always produces one new pair (one male, one female) every month from the second month on. The puzzle that Fibonacci posed was...

How many pairs will there be in one year?

1. At the end of the first month, they mate, but there is still one only 1 pair.
2. At the end of the second month the female produces a new pair, so now there are 2 pairs of rabbits.
3. At the end of the third month, the original female produces a second pair, making 3 pairs in all.
4. At the end of the fourth month, the original female has produced yet another new pair, the female born two months ago produces her first pair also, making 5 pairs.

The number of pairs of rabbits at the start of each month is $1,1,2,3,5,8,13,21,34,55,89,144,233, \ldots$
This is knew as Fibonacci Numbers.
We can note that every number of this series is the sum of the two previous numbers.

## Fibonacci Numbers and the Golden Section

If we take the ratio of two successive numbers in Fibonacci's series, (1, 1, 2, 3, 5, 8, 13, 21, ...), we will find the following series of numbers:
$1 / 1=1 \quad 1 / 2=0,5 \quad 2 / 3=0,667 \quad 3 / 5=0,6 \quad 5 / 8=0,625 \quad 8 / 13=0,615 \quad 13 / 21=0,619 \quad 21 / 34=0,618 \quad 34 / 55=0,618$
The ratio seems to be settling down quickly to 0,618, the Golden Section.

I use the Golden Section to define the macro and micro formal structure of my compositions, individuating different points in successive levels. Es.:


## III - MATHEMATICAL COMBINATIONS

If we do a mathematical combination of 2 numbers, we have only 2 possibilities.
If we do a mathematical combination of $3,4,5,6$ number, we have respectively 6 possibilities $(3 \times 2 \times 1), 24$ possibilities $(4 \times 3 \times 2 \times 1)$, 120 possibilities ( $5 \times 4 \times 3 \times 2 \times 1$ ), 720 possibilities $(6 \times 5 \times 4 \times 3 \times 2 \times 1$ ).

I use these mathematical combinations to organize the sounds in my compositions; this allows to have a series of the same sounds with always different sequences. I use also a key number, changing the possibilities' order, to have other different series with the same sounds.
If the sounds are more than 6 I utilize a second level of combinations defining a hierarchy.

## PRESENTATION

In the chamber composition "mar ibis"for violin, viola, cello and piano, the carrying "melodic line", which covers all the score, is entrusted to the three string instruments so as to create a continuous echo of repetitions dilated in the time; during the development of this musical idea, nearly like underlining, the piano takes part with a percussive character.
This "melodic line", which every time presents itself again in a different combination of the notes that compose it, is interrupted from two other different elements, which are in contrast one with the other, nearly puntuation of the main structure; the first one is characterized by the harmonic of the strings amplified from the piano (momentary suspensions), the second one by the solo cello with the heel of the bow, therefore with determined character.
These three elements are organized so as to follow an own independent way, even if inserted in a rigid formal structure, in fact the sections characterized from them present different length and duration, but always proportioned and calculated on the total duration of the composition.
In spite of his strict formal structure, the work researches, without an apparent regoular rythm also, a "natural" movement.

## AN ANALYSIS OF "MARI"

In order to exemplify the formal scheme and the elements present in "mar ibis", we can analyse "mari" for solo cello, nearly a preparatory "studio".

## I - THE 3 ELEMENTS PRESENT IN "MARI"

## A - The main "melodic line" and the "rhythmic pulsation"

In order to obtain the main "melodic line" I created a "string" using the mathematical combination of 5 sounds (with key 5):
$C(1) F(2) G(3) A(4)$ plus a repetition one octave higher of $C$ (5)
To define the duration of every note of this "string" I decided that every new $1 / 4$ begins from the note after every $F$ and finishes at the following $F$; this allows to alternate groups with a various number of notes and with total value of $1 / 4$ and avoiding therefore a regular pulsation.

After to have created the "string" for the lenght of 57 measures of $4 / 4$, I subdivided it in 4 parts using the GS of II level:

$$
\begin{array}{lllll}
0 & 21.75 & 35.25 & 48.75 & 57
\end{array}
$$

Now:

- the melodic line alternates stops on high $C$ to that one on one of the other sounds of the "string" - $F(1) \quad G(2)$ A (3) - using the mathematical combination of the 3 sounds (with key 5):

123132213231312321 become 312231321132213123
Es. $\quad C$ (3) $C$ (1) $C$ (2) $C$ (2) ... etc.

Before every stop on $C$ there is a figuration that it begins at the end of the previous held note and comprises others $2 C$ and the other notes included in that part of "string".

To give this line a direction (getting near the figurations in the time), the durations of the stops will be shortened: the held notes will comprise 8-6-4-2 repetitions for the $C$ and 4-3-2-1 for the others; the change will happen at the beginning of every one of the 4 parts of GS of II levelf.

After every stop on $F(1)$ the line is interrupted until the repetition of $F$ included (eliminating therefore $1 / 4$ at the end of every fragment); so 13 fragments of varied duration are created. The total number of measures becomes therefore $53,75(57-3,25)$.

From the beginning of the IV GS of II level of every fragment, a F sharp is inserted to double held notes; if in this GS it is present a figuration, the F sharp is interrupted and it is resumed on the successive held note from its II GS of II level.

In order to obtain the "rhythmic pulsation" I created a "rhytmic string" derived from the position that the letters that compose the word "violoncello" occupy in the Italian alphabet (because I decided to only use irregular groups until 6 notes, the numbers after 12 have been reduced adding their ciphers):

| from | 20 | 9 | 13 | 10 | 13 | 12 | 3 | 5 | $10(10)$ | 13 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| to | 2 | 54 | 4 | 55 | 4 | 66 | 3 | 5 | 55 | 4 |

(number of pulsations every 1/4)

Numbers $3,5,9,10,12,13$, and 20 are erased every 21 pulsations.
This "rhytmic string", that repeats itself equal every 3 measures and half for all the duration of the "melodic line", it is used only under held notes, from their II GS of II level

## B - ... with the heel of the bow

The previous "rhytmic string" is used also for the element "with the heel of the bow".
The sounds used are:
A sharp (1) B (2) C sharp (3) D (4)

The "chords" (detached when they are between two pauses) become "figures", using the mathematical combination of the 4 sounds with key 5, from the beginning of the II GS of II level of every fragment.
$C$ - "G flat - B"
Alternation of $G b$ (flat) and $B$ in $G S$ towards the high register and harmonics, with suspension character.

## II - THE ORGANIZATION OF THE 3 ELEMENTS IN "MARI"

Everyone of the 13 fragments of the "melodic line" is GS of 13 greater sections.
Juxtaposing these sections, the 53,75 total measures of fragments become 87.
8 measures are introduced between the two sections before and after the IV GS of II level $(74,25)$ of 87 (between the 9th and the 10th section).
The total measures become therefore 95 .
(These sections begin to the measure: $0 / 7 / 19.5 / 27.25 / 38.25 / 41.5 / 51.25 / 59.25 / 64=====79 / 81.75 /$ 86.5 / 91.75)

The total 95 measures are divided in GS of III level.
This create 8 sections to which I bind together, alternated, the two elements $B$ and $C$; this creates therefore a background that it will be audible only in absence of the "melodic line".

## III - CHANGES AND TRANSFORMATIONS FOR "MAR IBIS"

The element A "melodic line" is entrusted to the viola; alternatively violin (one octave higher) and cello (one octave lower) create a continuous echo to the figurations with doubled duration, leaving from the first $C$ of every figuration. In this way the 13 fragments are dilated in time.
The rhytmic pulsation of the pizzicato of "mari" becomes the percussive element in augmented fourth of the piano, in amplification,

Also the element $C(G b-B)$, assigned to the violin, has echoes by the other instruments.
Because of these echoes, nearly suspensions of the time, the total measures become 118.

From the II GS of II level of 118 (45), the strings play the figurations of the "melodic line" in "pizzicato". The fragments in "pizzicato" are the V, IX and XII; that is respectively, leaving from the end, the II, the III from the previous one and the IV from the previous one. This in order to increase the directionality already given by the getting near of the figurations of $A$, like previously seen, during the piece.

The element $B$ with the heel of the bow of the cello remains unvaried.

