A-B-Cs of Sun-Synchronous Orbit Mission Design

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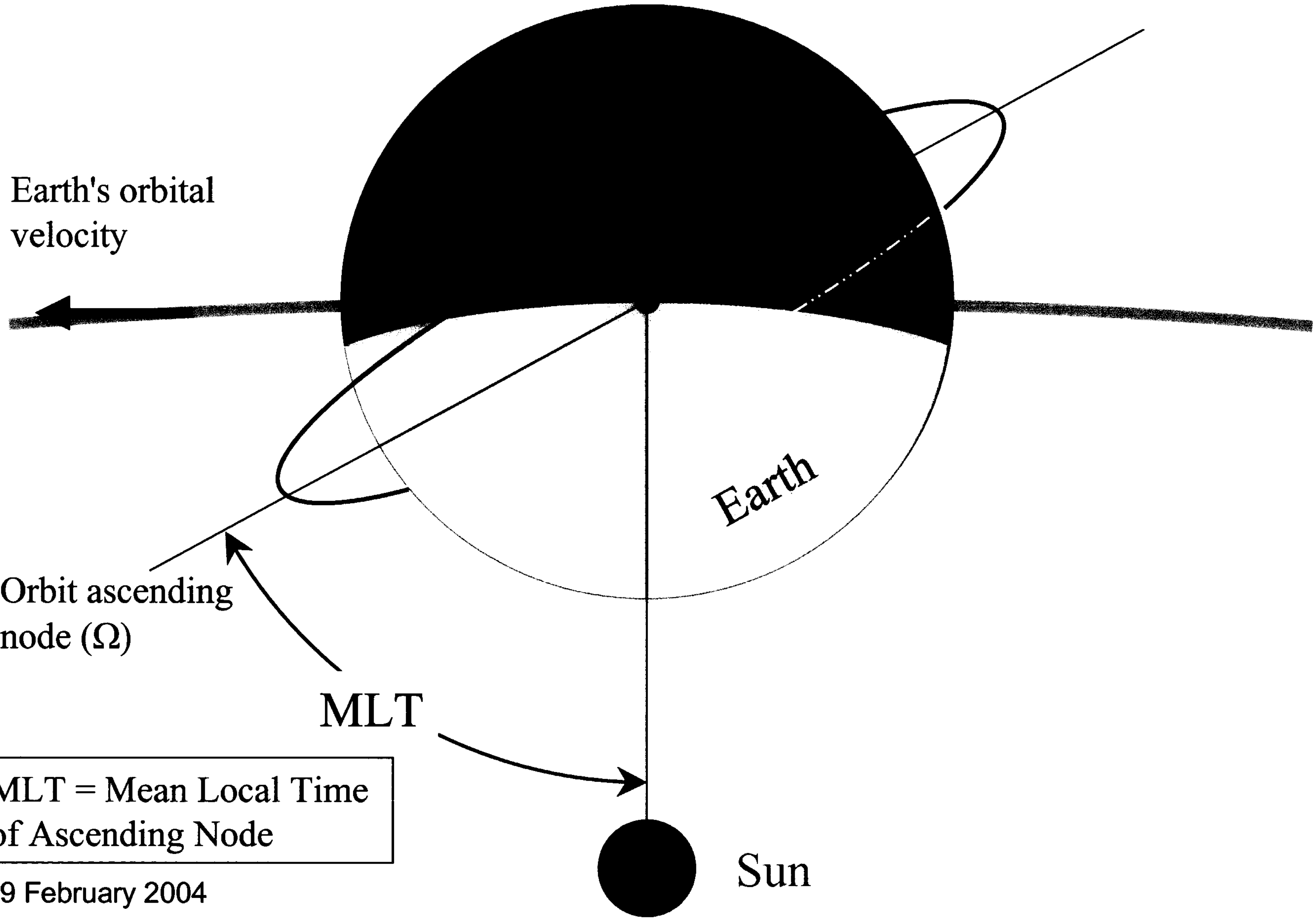
The Perturbation due to a Non-Spherical Earth

- The equation for the precession/regression of the line of nodes of an earth orbiter is common known and given as:
- It describes the rate and direction the orbit plane moves along the earth's equator
- It's derived in many standard astrodynamics/celestial mechanics textbooks
  - Battin, Danby, Escobal
- It is described and applied in many other books as well
  - Brown, Wertz, Bate
- This paper is about a unique and special application of this equation

$$\dot{\Omega} = -\frac{3}{2} J_2 \left(\frac{a_e}{p} \right)^2 n * \cos(i)$$

Eq. 1

Earth-Sun Geometry Schematic



MLT = Mean Local Time of Ascending Node

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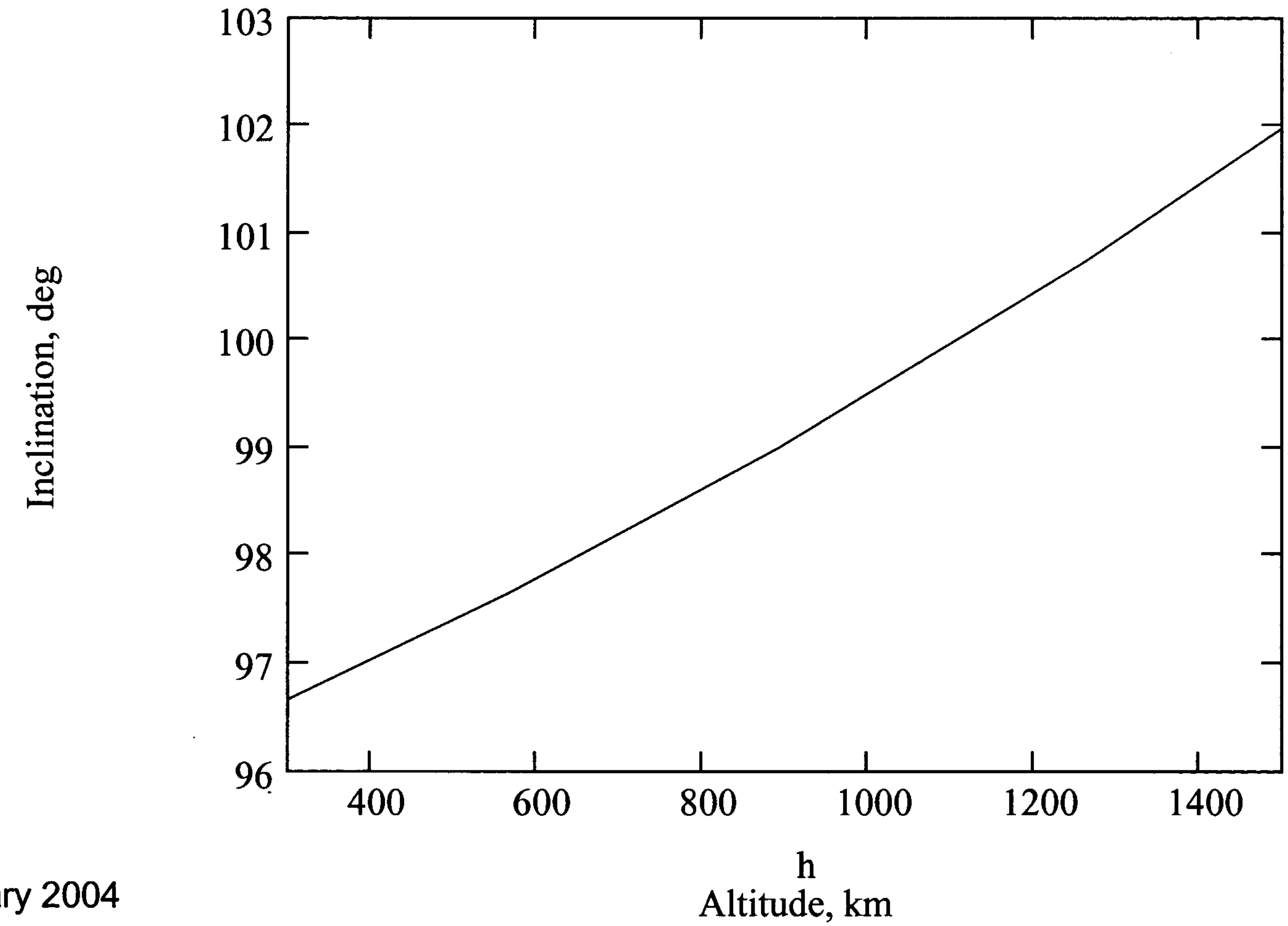
Defining a Sun-Synchronous Orbit

- A SS-O is characterized as a polar earth orbit that provides a consistent sun-lighting condition along the orbit's groundtrack (GT)
  - E.G., a satellite in a 01:30 p.m. SS-O making observations along the daylight side of the orbit will see lighting/shadowing conditions corresponding to an hour and a half pass the noon hour
  - For 10:30 a.m. SS-O, the lighting corresponds to an hour and a half before the noon hour
- The correct definition of a SS-O is an orbit where the node precesses at a rate equal to the earth's Mean Motion around the sun
  - I.E., $\dot{\Omega} = 360°/365.242199 = 0.9856$ deg/day
  - Assuming a circular orbit, this implies through Eq. 1 that by selecting the altitude, the inclination is automatically implied
- With this choice for $\dot{\Omega}$, the position of the node remains fixed with respect to the mean solar meridian and sun-lighting conditions as described above will result

Sun-Synchronous Condition: Inclination vs. Altitude (e = 0)



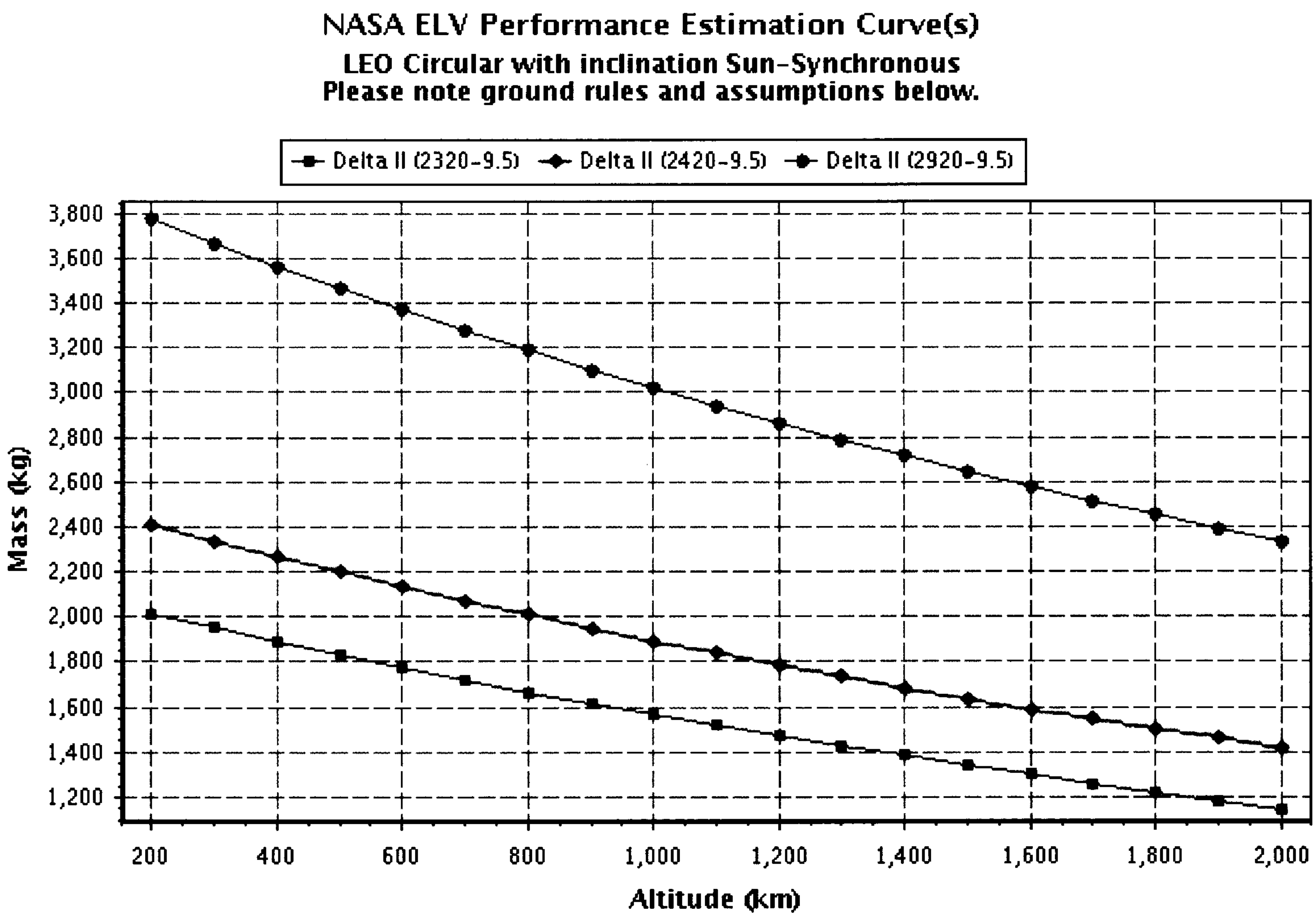
Reckoning Time

- Understanding how time is reckoned is a complex subject
  - Sidereal time
  - Apparent solar time
  - Mean solar time
- Sidereal time is based on the earth's rotation rate relative to the stars/vernal equinox and is not useful for reckoning time since it loses approximately 4 minutes per day measured in mean solar time
- Apparent solar time is inconvenient since the sun's motion is not regular with respect to the background stars and can vary >16 min per day
  - Obliquity of the ecliptic, i.e., sun's change in declination
  - Elliptic earth orbit, i.e., The Equation of Time
- Only mean solar time based on the mean sun crossing the mean solar meridian is consistent with 86400 seconds per day
- The MLT for SS-Os is also based on the mean solar meridian

System Engineering the Mission Design

Stated Science Requirements/ Desires	Motivating Objective or Instrument Characteristic	Traceable Orbit Characteristic/Parameter
Limitation on the range to a target; viewing angle constraints	Instrument sensitivity, resolution, field of view/swath-width, allowable elongation/ distortion over a footprint, etc.	Orbit altitude
Number and distribution of targets to be observed (for discrete targets)	Unique geographic targets to be measured	Orbit altitude, inclination; groundtrack grid density; groundtrack tied point to achieve over-flight of specific lat/lon
Area coverage to be provided (for continuous targets)	Percentage of earth's surface to be accessible for observation	Orbit inclination, altitude
Frequency with which targets/areas are to be sampled	Allowable time interval before a repeat observation is possible	Orbit altitude
Sun-lighting conditions to be provided (for optical measurements)	Consistent sun shadows for targets	Orbit nodal position and/or nodal Mean Local Time; orbit inclination
Seasonal considerations of observations	Visual access to Antarctica (for example) during Antarctic summer	Orbit nodal position and/or nodal Mean Local Time; orbit inclination
Overall duration/period of time necessary to measure some phenomenon through it life-cycle	Life expectancy for instruments, system, mission life	Orbit altitude

Delta ELV Performance to SS-O Altitude



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Orbit Parameters for SS-Os
with an Integer Number of Revs in One-Day

- Five solutions, corresponding to 12, 13, 14, 15, & 16 revs per day, exist over a range of altitudes desired for low earth SS-Os
- The equatorial altitude for these solutions ranges between 250 and 1680 km
- These solutions have coarse GT grids, i.e., >2500 km between adjacent groundtracks
  - η is the angle subtended from the nadir direction to the adjacent groundtrack
- Although interesting orbits, they provide only localized coverage

Revs per Day, #	**Orbital Period, sec**	**Equator Altitude, km**	**Dist betw/ Adj GTs, km**
12	7200.00	1680.86	3339.59 (η=51.5°)
13	6646.15	1262.09	3082.69 (η=56.1°)
14	6171.43	893.79	2862.50 (η=61.1°)
15	5760.00	566.89	2671.67 (η=66.7°)
16	5400.00	274.42	2504.69 (η=72.7°)

SS-Os with Finer GT Grids

- Next consider SS-Os which repeat their GT in two-days:
  - If solutions exist for all integers between 12 and 16 for the one-day repeat, then solutions exist for integers between 24 and 32 for the two-day repeat:

    24, 25, 26, 27, 28, 29, 30,31, 32
  - Apparently 9 possible solutions
- A quick calculation shows that solutions for 24, 26, 28, 30, and 32 are degenerate with 12,... 16 for one-day repeats, i.e., they have identically the same periods, with the integer solutions for the one-day repeat
  - Therefore, there are only four new solutions for the two-day repeat, but these have 25, 27, 29, and 31 revs, thereby decreasing the spacing between nodes at the expense of increasing the re-visit time ("access")

Notation for Solutions with *R* Revs in *D* Days

- Extending the previous reasoning, repeat GT orbits can be found for 3, 4, 5, 6, 7, … and so forth days
- Example: Thee-day repeat orbits

  36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48
- Removing the degenerate solutions yields 8 unique solutions which repeat their GT in three-days:

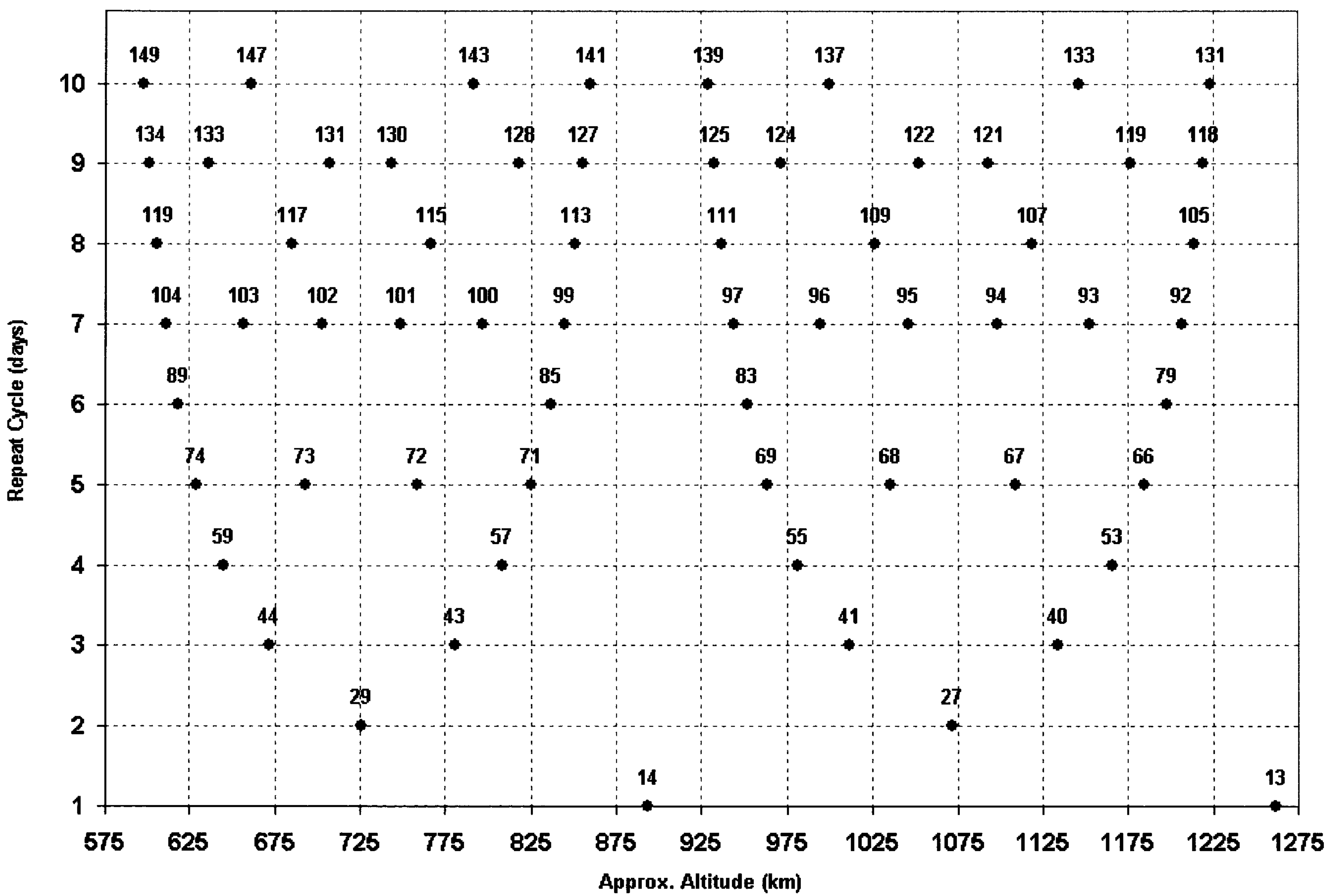
  37, 38, 40, 41, 43, 44, 46, 47
- A convenient notation for one of these solutions is:

  3D43R = 3-day repeat in exactly 43 revs or

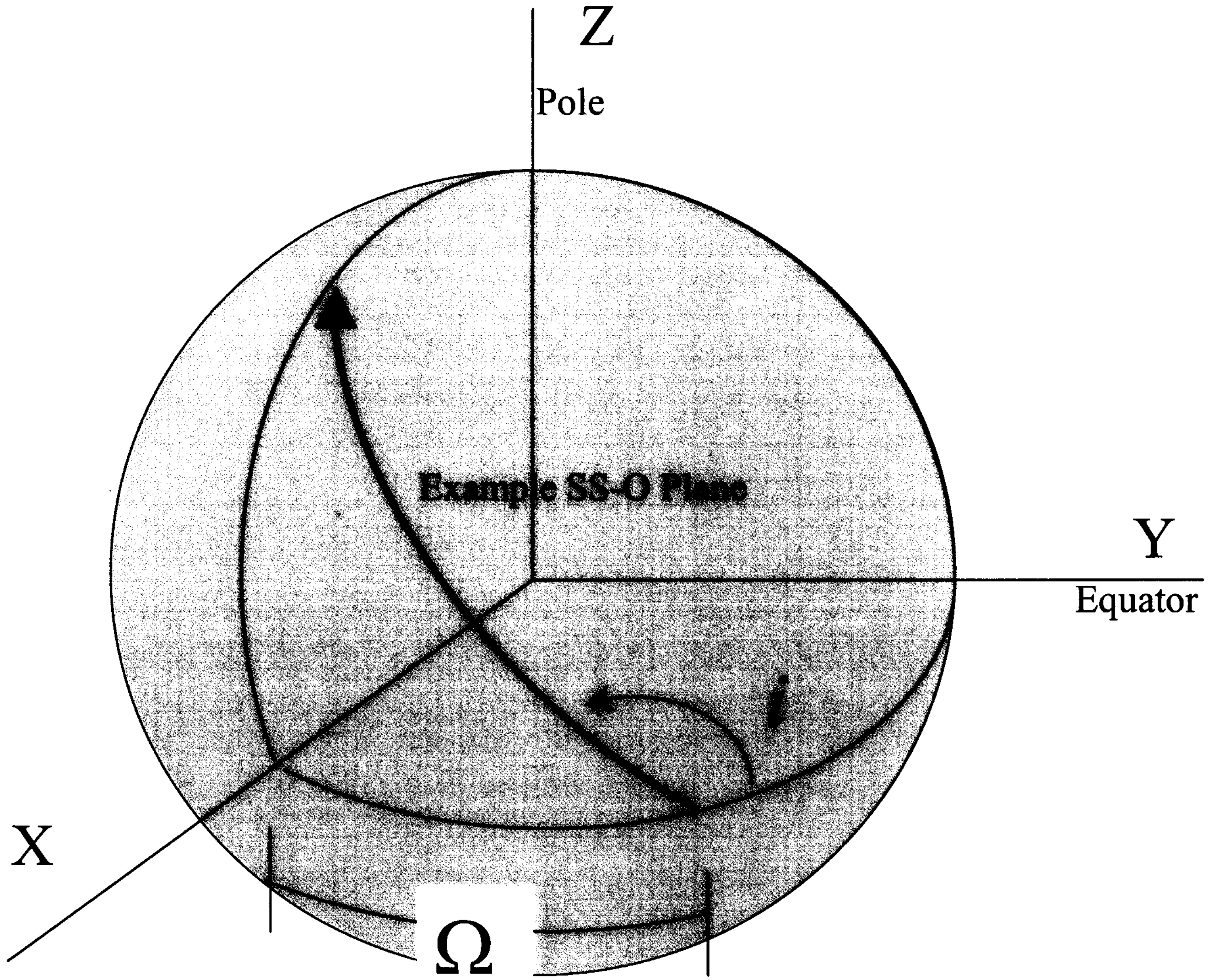
  3D47R = 3-day repeat in exactly 47 revs and so forth
- Another example: Seven-day repeat orbit solutions =>

  7D85R, 7D86R, 7D87R, … 7D109R, 7D111R

Discrete Sun-Synchronous Repeat Groundtrack Orbits vs. Altitude



Earth-Centered Coordinate Frame



Ω = MLT

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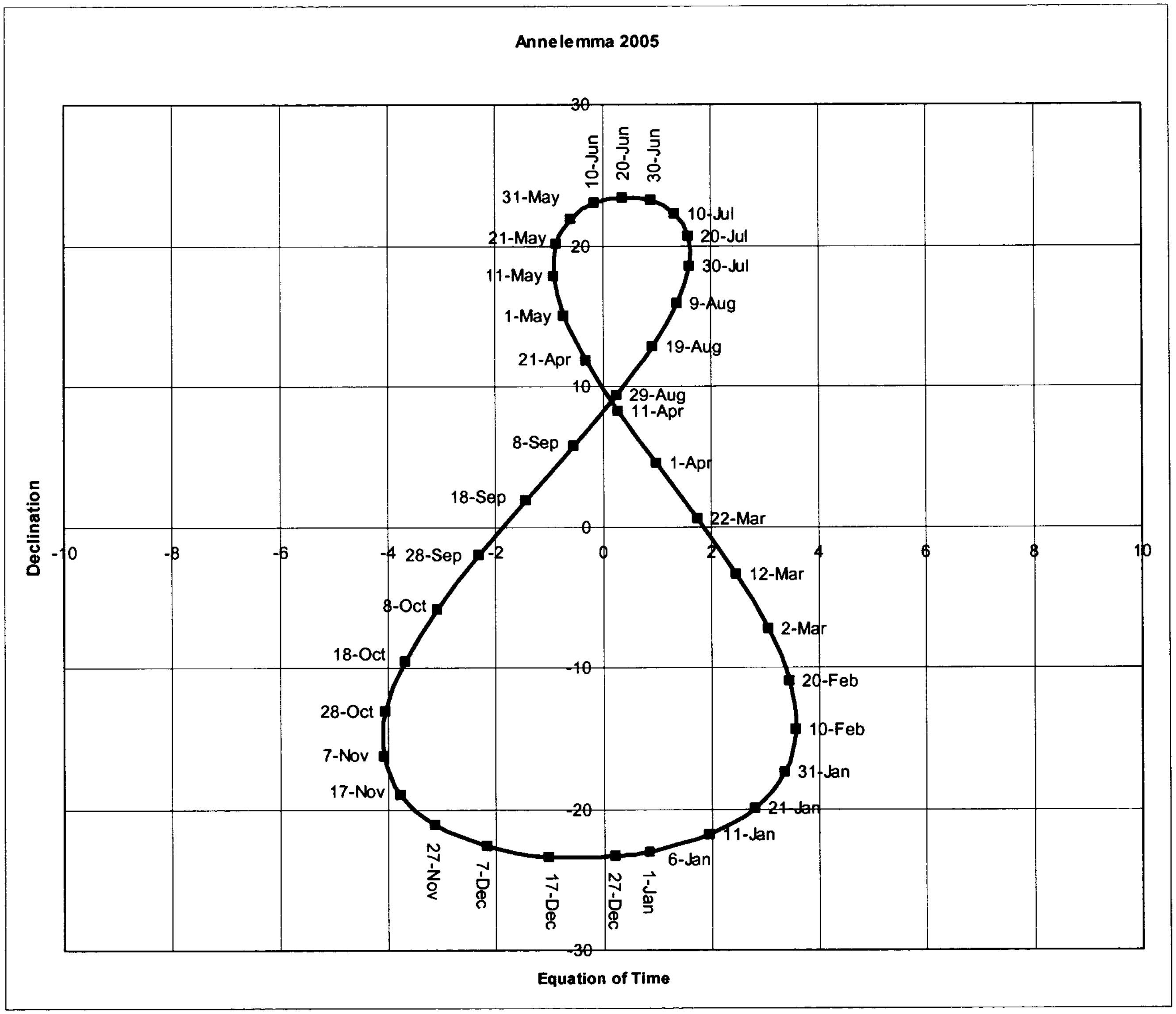
Analemma in Declination – Equation of Time Space



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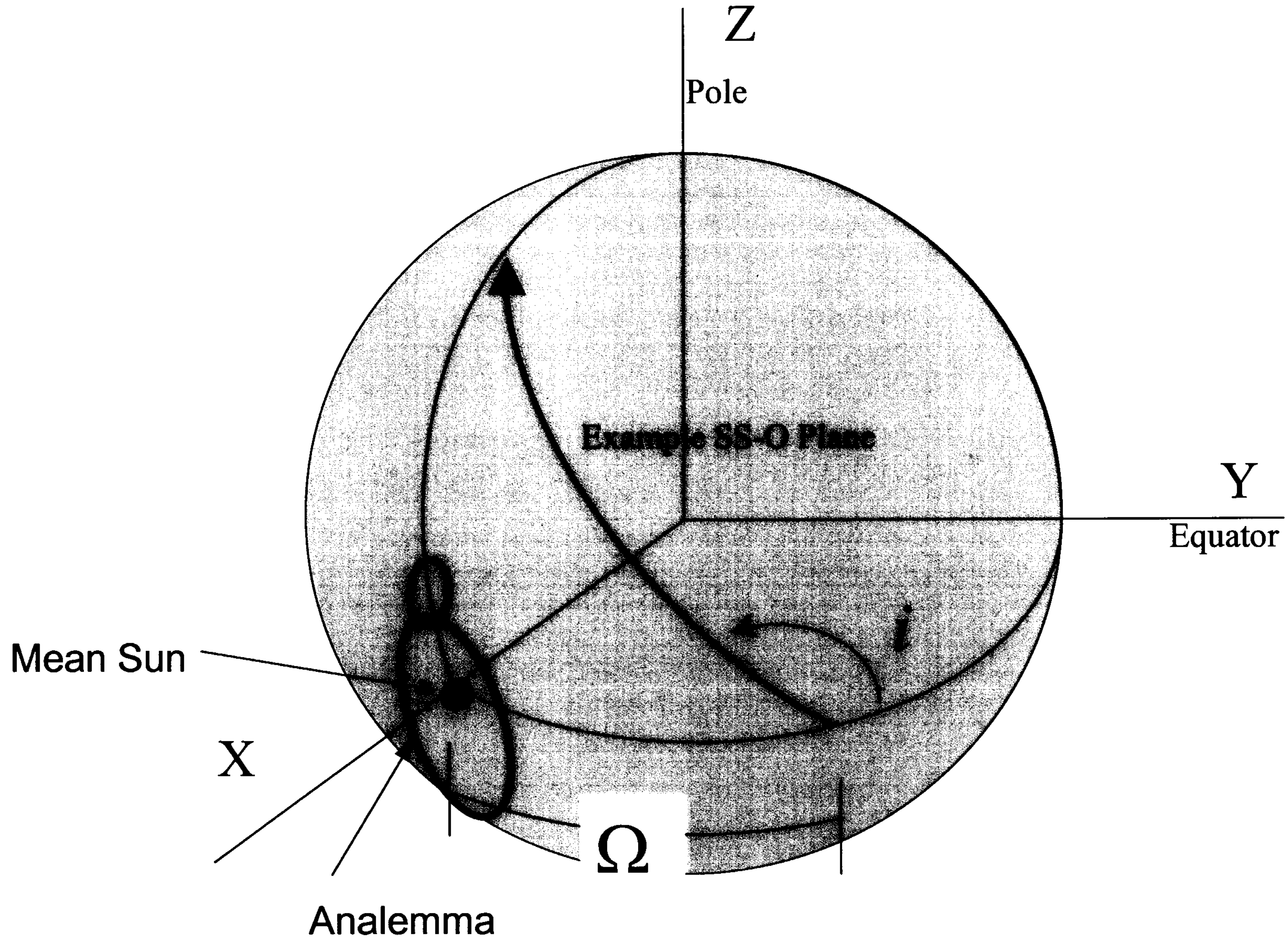
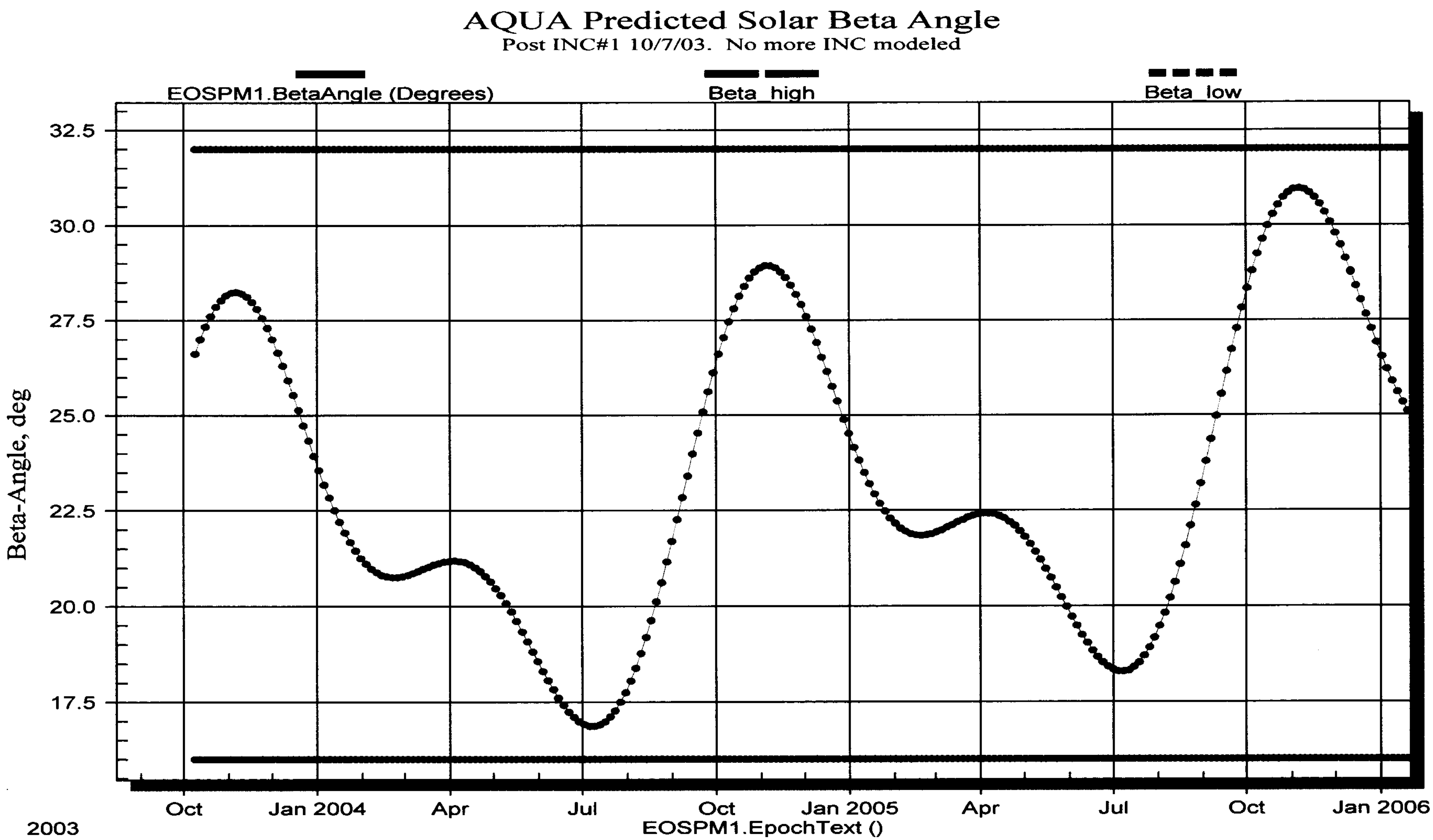
Mean Sun at Coordinates = (0,0)

Earth-Centered Coordinate Frame



Aqua Solar Beta-Angle Prediction



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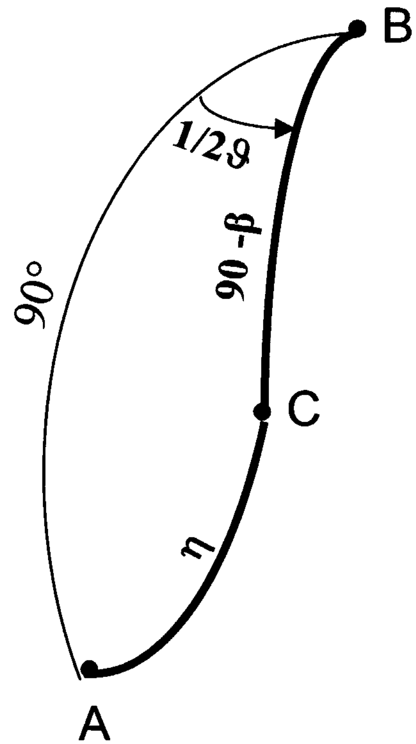
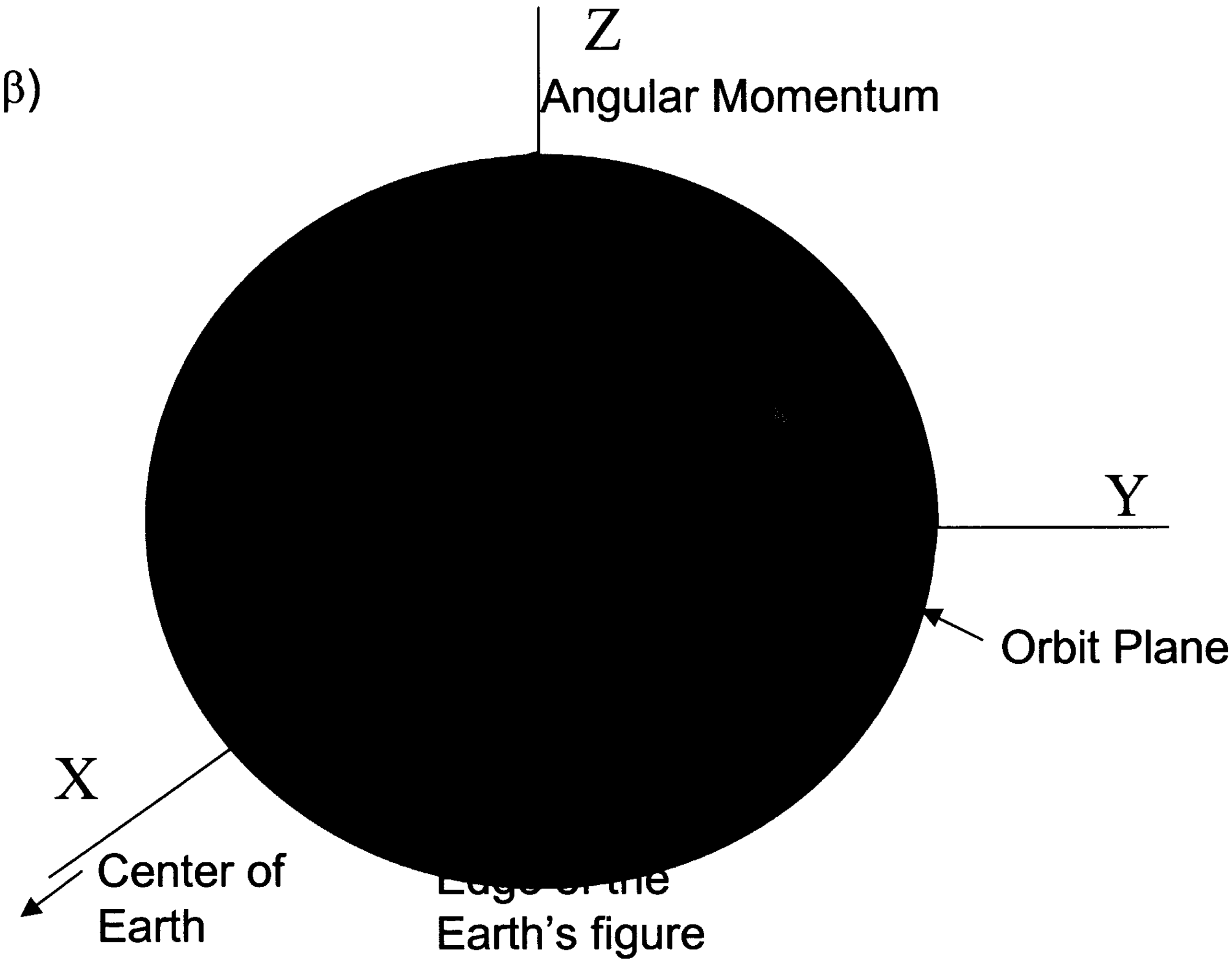
Orbit Plane Geometry for Computing the Time Spent in Shadow

Given Spherical Triangle A-B-C, the Law of Cosines gives:

$\cos(\vartheta/2)=\cos(\eta)/\cos(\beta)$





Summary

- SS-Os are defined as low earth orbits which have a nodal precession rate equal to the earth's Mean Motion
  - There is a unique coupling between orbit altitude and inclination to achieve this precession rate as implied by Eq. 1
  - This precession has the effect of making the position of the nodes with respect to the mean sun remain fixed (to first order)
- SS-O altitudes can be selected to provide a repeat groundtracks in an integer number of revs in an integer number of days
- Sun-lighting conditions on the orbit and for observations made from the orbit can be determined by selecting the MLT
- The paper provides several simple algorithms that enable the calculation of altitude (hence inclination) and MLT to satisfy common mission requirements

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- Roger R. Bate, et al, *Fundamentals of Astrodynamics*, Dover Publications, Inc., 1971