Mathematics C

Senior Syllabus 2001

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This syllabus is approved for general implementation until 2008, unless otherwise stated.

To be used for the first time with Year 11 students in 2002.

Floor 7, 295 Ann St Brisbane Queensland
PO Box 307 Spring Hill Qld 4004

Telephone: (07) 3864 0299
Fax: (07) 3221 2553
Email: office@qbssss.edu.au
Website: www.qbssss.edu.au

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1 RATIONALE

Mathematics is an integral part of a general education. It enhances both an understanding of the world and the quality of participation in a rapidly changing society.

Mathematics has been central to nearly all major scientific and technological advances. Many of the developments and decisions made in industry and commerce, in the provision of social and community services, and in government policy and planning, rely on the use of mathematics. Consequently, the range of career opportunities requiring and/or benefiting from an advanced level of mathematical expertise is rapidly expanding. For example, mathematics is increasingly important in health and life sciences, biotechnology, environmental science, economics, and business while remaining crucial in such fields as the physical sciences, engineering, accounting, computer science and the information technology areas.

Mathematics C aims to provide opportunities for students to participate more fully in life-long learning, to develop their mathematical potential, and to build upon and extend their mathematics. It is extremely valuable for students interested in mathematics. Students studying Mathematics C in addition to Mathematics B gain broader and deeper mathematical experiences that are very important for future studies in areas such as the physical sciences and engineering. They are also significantly advantaged in a wide range of areas such as finance, economics, accounting, information technology and all sciences. Mathematics C provides the opportunity for student development of:

- knowledge, procedures and skills in mathematics
- mathematical modelling and problem-solving strategies
- the capacity to justify and communicate in a variety of forms.

Mathematics students should recognise the dynamic nature of mathematics through the subject matter of Mathematics C which includes the concepts and application of matrices, vectors, complex numbers, structures and patterns, and the practical power of calculus. In the optional topics, students may also gain knowledge and skills in topics such as conics, dynamics, statistics, numerical methods, exponential and logarithmic functions, number theory, and recent developments in mathematics.

Mathematics has provided a basis for the development of technology. In recent times, the uses of mathematics have developed substantially in response to changes in technology. The more technology is developed, the greater is the level of mathematical skill required. Students must be given the opportunity to appreciate and experience the power which has been given to mathematics by this technology. Such technology should be used to help students understand mathematical concepts, allowing them to “see” relationships and graphical displays, to search for patterns and recurrence in mathematical situations, as well as to assist in the exploration and investigation of purely mathematical, real and life-like situations.

The intent of Mathematics C is to encourage students to develop positive attitudes towards mathematics by an approach involving exploration, investigation, problem solving and application in a variety of contexts. Of importance is the development of student thinking skills, as well as student recognition and use of mathematical structures and patterns. Students will be encouraged to model mathematically, to work systematically and logically, to conjecture and reflect, to prove and justify, and to communicate with and about mathematics. The subject is designed to raise the level of competence and confidence in using mathematics, through aspects such as analysis, proof and justification, rigour, mathematical modelling and problem solving. Such activities will equip students well in more general situations, in the appreciation of the power and diversity of mathematics, and provide a very strong basis for a wide range of further mathematics studies.

Mathematics C provides opportunities for the development of the key competencies in situations that arise naturally from the general objectives and learning experiences of the subject. The seven key competencies are: collecting, analysing and organising information; communicating ideas and information; planning and organising activities; working with others and in teams; using mathematical ideas and techniques; solving problems; using technology. (Refer to *Integrating the Key Competencies into the Assessment and Reporting of Student Achievement in Senior Secondary Schools in Queensland*, published by QBSSSS in 1997.)

2 GLOBAL AIMS

Having completed the course of study, students of Mathematics C should:

- be able to recognise when problems are suitable for mathematical analysis and solution, and be able to attempt such analysis or solution with confidence
- be able to visualise and represent spatial relationships in both two and three dimensions
- have experienced diverse applications of mathematics
- have positive attitudes to the learning and practice of mathematics
- comprehend mathematical information which is presented in a variety of forms
- communicate mathematical information in a variety of forms
- be able to benefit from the availability of a wide range of technologies
- be able to choose and use mathematical instruments appropriately
- be able to recognise functional relationships and dependent applications
- have significantly broadened their mathematical knowledge and skills
- have increased their understanding of mathematics and its structure through the depth and breadth of their study.

3 GENERAL OBJECTIVES

3.1 INTRODUCTION

The general objectives of this course are organised into four categories:

- Knowledge and procedures
- Modelling and problem solving
- Communication and justification
- Affective.

3.2 CONTEXTS

The categories of Knowledge and procedures, Modelling and problem solving, and Communication and justification incorporate contexts of application, technology, initiative and complexity. Each of the contexts has a continuum for the particular aspect of mathematics it represents. Mathematics in a course of study developed from this syllabus must be taught, learned and assessed using a variety of contexts over the two years. It is expected that all students are provided with the opportunity to experience mathematics along the continuum within each of the contexts outlined below.

Application

Students must have the opportunity to recognise the usefulness of mathematics through its application, and the beauty and power of mathematics that comes from the capacity to abstract and generalise. Thus students' learning experiences and assessment programs must include mathematical tasks that demonstrate a balance across the range from life-related through to pure abstraction.

Technology

A range of technological tools must be used in the learning and assessment experiences offered in this course. This ranges from pen and paper, measuring instruments and tables through to higher technologies such as graphing calculators and computers. The minimum level of higher technology appropriate for the teaching of this course is a graphing calculator.

Initiative

Learning experiences and the corresponding assessment must provide students with the opportunity to demonstrate their capability when dealing with tasks that range from routine and well rehearsed through to those that require demonstration of insight and creativity.

Complexity

Students must be provided with the opportunity to work on simple, single-step tasks through to tasks that are complex in nature. Complexity may derive from either the nature of the concepts involved or from the number of ideas or techniques that must be sequenced in order to produce an appropriate conclusion.

3.3 OBJECTIVES

The general objectives for each of the categories in section 3.1 are detailed below. These general objectives incorporate several key competencies. The first three categories of objectives, Knowledge and procedures, Modelling and problem solving, and Communication and justification, are linked to the exit criteria in section 7.3.

3.3.1 Knowledge and procedures

The objectives of this category involve the recall and use of results and procedures within **the contexts of application, technology, initiative and complexity**. (see section 3.2)

By the conclusion of the course, students should be able to:

- recall definitions and results
- access and apply rules and techniques
- demonstrate number and spatial sense
- demonstrate algebraic facility
- demonstrate an ability to select and use appropriate technology such as calculators, measuring instruments, geometrical drawing instruments and tables
- demonstrate an ability to use graphing calculators and/or computers with selected software in working mathematically
- select and use appropriate mathematical procedures
- work accurately and manipulate formulae
- recognise some tasks may be broken up into smaller components
- transfer and apply mathematical procedures to similar situations
- understand the nature of proof.

3.3.2 Modelling and problem solving

The objectives of this category involve the use of mathematics in which the students will model mathematical situations and constructs, solve problems and investigate situations mathematically within **the contexts of application, technology, initiative and complexity**. (see section 3.2)

By the conclusion of the course, students should be able to demonstrate the category of modelling and problem solving through:

Modelling

- understanding that a mathematical model is a mathematical representation of a situation
- identifying the assumptions and variables of a simple mathematical model of a situation
- forming a mathematical model of a life-related situation
- deriving results from consideration of the mathematical model chosen for the particular situation
- interpreting results from the mathematical model in terms of the given situation
- exploring the strengths and limitations of a mathematical model and modifying the model as appropriate.

Problem solving

- interpreting, clarifying and analysing a problem
- using a range of problem solving strategies such as estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems and organising data
- understanding that there may be more than one way to solve a problem
- selecting appropriate mathematical procedures required to solve a problem
- developing a solution consistent with the problem
- developing procedures in problem solving.

Investigation

- identifying and/or posing a problem
- exploring the problem and from emerging patterns creating conjectures or theories
- reflecting on conjectures or theories making modifications if needed
- selecting and using problem-solving strategies to test and validate any conjectures or theories
- extending and generalising from problems
- developing strategies and procedures in investigations.

3.3.3 Communication and justification

The objectives of this category involve presentation, communication (both mathematical and everyday language), logical arguments, interpretation and justification of mathematics within **the contexts of application, technology, initiative and complexity.** (see section 3.2)

Communication

By the conclusion of the course, students should be able to demonstrate communication through:

- organising and presenting information
- communicating ideas, information and results appropriately
- using mathematical terms and symbols accurately and appropriately
- using accepted spelling, punctuation and grammar in written communication
- understanding material presented in a variety of forms such as oral, written, symbolic, pictorial and graphical
- translating material from one form to another when appropriate
- presenting material for different audiences, in a variety of forms such as oral, written, symbolic, pictorial and graphical
- recognising necessary distinctions in the meanings of words and phrases according to whether they are used in a mathematical or non-mathematical situation.

Justification

By the conclusion of this course, the student should be able to demonstrate justification through:

- developing logical arguments expressed in everyday language, mathematical language or a combination of both, as required, to support conclusions, results and/or propositions
- evaluating the validity of arguments designed to convince others of the truth of propositions
- justifying procedures used
- recognising when and why derived results to a given problem are clearly improbable or unreasonable
- recognising that one counter example is sufficient to disprove a generalisation
- recognising the effect of assumptions on the conclusions that can be reached
- deciding whether it is valid to use a general result in a specific case
- recognising that a proof may require more than verification of a number of instances
- using supporting arguments, when appropriate, to justify results obtained by calculator or computer
- using different methods of proof.

3.3.4 Affective

Affective objectives refer to the attitudes, values and feelings which this subject aims at developing in students. **Affective objectives are not assessed for the award of exit levels of achievement**.

By the conclusion of the course, students should appreciate the:

- diverse applications of mathematics
- precise language and structure of mathematics
- uncertain nature of the world, and be able to use mathematics to assist in making informed decisions in life-related situations
- diverse and evolutionary nature of mathematics through an understanding of its history
- wide range of mathematics-based vocations
- contribution of mathematics to human culture and progress
- power and beauty of mathematics.

4 LANGUAGE EDUCATION

Language is the means by which meaning is constructed and shared and communication is effected. It is the central means by which teachers and students learn. Mathematics C requires students to use language in a variety of ways – mathematical, spoken, written, graphical, symbolic. The responsibility for developing and monitoring students' abilities to use effectively the forms of language demanded by this course rests with the teachers of mathematics. This responsibility includes developing students' abilities to:

- select and sequence information
- manage the conventions related to the forms of communication used in Mathematics C (such as short responses, reports, multi-media presentations, seminars)
- use the specialised vocabulary and terminology related to Mathematics C
- use language conventions related to grammar, spelling, punctuation and layout.

The learning of language is a developmental process. When writing, reading, questioning, listening and talking about mathematics, teachers and students should use the specialised vocabulary related to Mathematics C. Students should be involved in learning experiences that require them to comprehend and transform data in a variety of forms and, in so doing, use the appropriate language conventions. Some language forms may need to be explicitly taught if students are to operate with a high degree of confidence within mathematics.

Assessment instruments should use format and language that are familiar to students. They should be taught the appropriate language skills to interpret questions accurately and to develop coherent, logical and relevant responses. Attention to language education within Mathematics C should assist students to meet the language components of the exit criteria, especially the *Communication and justification* criterion.

5 ORGANISATION

5.1 INTRODUCTION

The **core topic**s are:

- Introduction to Groups
- Real and Complex Number Systems
- Matrices and Applications
- Vectors and Applications
- Calculus
- Structures and Patterns

The **option topics** are:

- Linear Programming
- Conics
- Dynamics
- Introduction to Number Theory
- Introductory Modelling with Probability
- Advanced Periodic and Exponential Functions
- School Option(s).

The core and option topics are discussed in detail in Section 6.

Throughout the course, certain previously learned mathematical knowledge and procedures will be required. While some have been identified and are listed under the heading Basic Mathematics in Appendix 1, others have been developed in Mathematics B. In designing the course sequence, provision should be allowed so that these aspects may be revised within topics as they are required throughout the course of study.

5.2 SELECTION OF TOPICS

The choice of option topics should be made so that they best suit the interests and needs of the particular cohort of students, the expertise and interests of the teaching staff and the resources of the school. This might mean that different choices are offered for different classes within the one cohort, or that the choices differ from year to year. If the school wishes to allow for this flexibility, the possibilities should be addressed within the work program.

5.3 TIME ALLOCATION

The minimum number of hours of timetabled school time including assessment for a course of study developed from this syllabus is 55 hours per semester.

Notional times are given for each core topic. These times are included as a guide and minor variations of these times may occur. **Approximately 30 hours** should be spent on each option. All options should be seen as equivalent with respect to level of difficulty.

5.4 SEQUENCING

After considering the subject matter and the appropriate range of learning experiences to enable the general objectives to be achieved, a **spiralling and integrated sequence** should be developed which allows students to see links between the different topics rather than seeing them as discrete units. For example, Introduction to groups can be used to provide a link between most of the core topics. It provides a thread which runs through the Real and complex number systems, Matrices and applications, Vectors and applications, and Structures and patterns.

The order in which the topics are presented in the syllabus is not intended to indicate a teaching sequence, but some topics include subject matter which is developed and extended in the subject matter of other topics. The school's sequence should be designed so that the subject matter is revisited and spiralled to allow students to internalise their knowledge before developing it further.

The following guidelines for the sequencing of subject matter should be referred to when developing a sequence for the course.

- No subject matter should be studied before the relevant prerequisite material.
- The sequences for Mathematics B and Mathematics C should be developed together to ensure that prerequisite material is covered appropriately.
- Subject matter across topics should be linked when possible.
- Sequencing may be constrained by a school's ability to provide physical resources.
- Time will be needed for maintaining previously learned mathematical knowledge and procedures.

5.5 TECHNOLOGY

The advantage of mathematics-enabled technology in the mathematics classroom is that it allows for the exploration of the concepts and processes of mathematics. Graphing calculators, for example, let students explore and investigate; they assist students with the understanding of concepts and they complement traditional approaches to teaching.

More specifically the mathematics-enabled technology allows students to tackle more diverse, life-related problems. Real life matrix application problems are more easily solved with this technology. This technology may be used in statistics to investigate larger data sets and rapidly produce a variety of graphical displays and summary statistics, thus freeing students to look for patterns, to detect anomalies in the data and to make informed comments. It must be used where numerical techniques are involved.

The minimum level of this technology emerging now is a graphing calculator. While student ownership of graphing calculators is not a requirement, *student access* to appropriate technology *is necessary* to enable students to develop the full range of skills required for successful problem solving during their course of study. Use of graphing calculators or computers will significantly enhance the learning outcomes of this syllabus.

6 TOPICS

6.1 INTRODUCTION

Each topic has a focus statement, subject matter and learning experiences which, taken together, clarify the scope, depth and emphasis for the topic.

Focus

This section highlights the intent of the syllabus with respect to the topic and indicates how students should be encouraged to develop their understanding of the topic.

Subject matter

This section outlines the subject matter to be studied in the topic. All subject matter listed in the topic must be included, however, the order in which it is presented is not necessarily intended to imply a teaching sequence.

Learning experiences

This section provides some suggested learning experiences which may be effective in using the subject matter to achieve the general objectives of the course. The numbers provided with the subject matter link to suggested learning experiences. Included are experiences which involve life-related applications of mathematics with both real and simulated situations, use of instruments and opportunities for modelling and problem solving. The listed learning experiences may require students to work individually, in small groups or as a class.

The learning experiences are suggestions only and are not prescriptive. Schools are encouraged to develop further learning experiences, especially those which relate to the school's location, environment and resources. Students should be involved in a variety of activities including those which require them to write, speak, listen or devise presentations in a variety of forms. A selection of learning experiences that students will encounter should be shown in the work program.

NB The learning experiences must provide students with the opportunity to experience mathematics along the continuum within each of the contexts.

Some of the key competencies, predominantly Using mathematical ideas and techniques, Solving problems and Using technology are to be found in the learning experiences within the topic areas. Opportunities are provided for the development of key competencies in contexts that arise naturally from the general objectives and learning experiences of the subject. The key competencies of: Collecting, analysing and organising information, Planning and organising activities, and Working with others and in teams also feature in some of the learning experiences.

6.2 CORE TOPICS

The order in which topics and items within topics are given should not be seen as implying a teaching sequence.

Introduction to groups (notional time 7 hours)

Focus

Students are encouraged to investigate the structures and properties of groups. It is **intended** that this introduction to groups should provide a **basis** for identifying the **common features** which are found in systems such as real and complex numbers, matrices and vectors.

Subject matter

Concepts of:

- closure
- associativity
- identity
- inverse (suggested learning experiences (SLEs) 1–8)
- definition of a group (SLEs 1–8)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. determine if the elements of a set form a group under a binary operation
2. determine the identity element and inverses in a group table
3. use a small Cayley table to determine whether a set of elements under a binary operation forms a group
4. investigate when the integers modulo *n* form groups under addition or multiplication
5. investigate groups formed by geometric transformations such as the reflections of a rectangle in its axes of symmetry and rotations of an equilateral triangle
6. construct a Cayley table and use it to identify subgroups (if any) such as the rotations of a square about its centre
7. find the element(s) which generate(s) the group in a group table
8. investigate the group structure of friezes, wall-papers or simple crystals by studying their symmetries under translations, rotations and reflections

Real and complex number systems (notional time 25 hours)

Focus

Students are encouraged to extend their knowledge of the real number system and to develop an understanding of the complex number system. Students should see the group structure within these systems as a link between the unfamiliar complex numbers and the familiar real numbers.

Subject matter

- structure of the real number system including:
  - rational numbers
  - irrational numbers (SLEs 2, 9, 10)
- simple manipulation of surds
- definition of complex numbers including standard and trigonometrical (modulus-argument) form (SLEs 1, 2)
- algebraic representation of complex numbers in Cartesian, trigonometric and polar form (SLEs 3, 4)
- geometric representation of complex numbers—Argand diagrams (SLE 4)
- operations with complex numbers including addition, subtraction, scalar multiplication, multiplication of complex numbers, conjugation (SLEs 1–8, 12)
- roots of complex numbers (SLE 6)
- use of complex numbers in proving trigonometric identities
- powers of complex numbers including de Moivre's Theorem
- simple, purely mathematical applications of complex numbers (SLEs 6, 7, 8, 11, 12)
- proof by mathematical induction (SLE 3)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. solve quadratic equations whose discriminant is negative
2. solve simple inequality statements such as $| z - a | > b$ in both the real and complex systems, and be able to give a verbal description of the meaning of the mathematical symbolism
3. use mathematical induction to prove De Moivre's Theorem
4. use polar forms to demonstrate multiplication and division of complex numbers
5. use geometry to demonstrate the effect of addition, subtraction and multiplication of complex numbers
6. solve simple equations of the form $z^n = w$ where n is an integer and w is a complex number
7. solve polynomial equations with real and complex coefficients (degree ≤ 3)
8. investigate the use of complex conjugates in the solution of polynomial equations with real coefficients
9. use a proof by contradiction to show that $\sqrt{2}$ is irrational
10. investigate some of the approximations to π which have been used
11. research areas in which complex numbers are used in life-related applications such as electric circuit theory, vibrating systems and aerofoil designs

12. investigate the group properties of matrices of the form

$$\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}$$

under both addition and multiplication; find interesting subsets of this class of matrices (known as quaternions); in particular, show that the eight matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix},$$
$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

form a group under multiplication

Matrices and applications (notional time 30 hours)

Focus

Students are encouraged to develop an understanding of the algebraic structure of matrices including situations where they form groups. Students should apply matrices in a variety of situations and use technology to facilitate the solution of problems involving matrices.

Subject matter

- definition of a matrix as data storage and as a mathematical tool (SLEs 1–7)
- dimension of a matrix
- matrix operations
  - addition
  - transpose
  - inverse
  - multiplication by a scalar
  - multiplication by a matrix (SLEs 1–7, 13, 14, 15)
- definition and properties of the identity matrix (SLEs 1, 3, 15)
- group properties of 2×2 matrices (SLE 5)
- determinant of a matrix (SLE 3)
- singular and non-singular matrices (SLE 1)
- solution of systems of homogeneous and non-homogeneous linear equations using matrices (SLEs 1, 6)
- applications of matrices in both life-related and purely mathematical situations (SLEs 1–12)
- relationship between matrices and vectors (SLEs 1, 6, 7, 12)

Suggested learning experiences

NB Many learning experiences in this topic are enhanced by the use of a calculator with matrix operations.

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. solve linear equations by using matrices and Gaussian elimination; solution of equations involving more than three variables will involve the use of graphing calculators.
2. investigate transition probability matrices in situations such as, recording over a period of time, the changes in major weather conditions as stated by newspaper or TV weather reports (fine, showers, cloudy, clearing); construct the matrix describing the probabilities that one condition will be followed by each different condition; given today's weather find the most probable sequence of weather conditions in the near future
3. use matrices to encode and decode messages
4. demonstrate the use of the transformation matrices (rotation, reflection, dilation) as an application of 2×2 matrices to geometric transformations in the plane
5. consider subsets of 2×2 matrices forming a group under addition or multiplication.
6. show that the change of frame of reference used in Newtonian mechanics,

   $$\begin{bmatrix} x' \\ t' \end{bmatrix} = \begin{bmatrix} 1 & -v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix}$$

   $x' = x - vt$, $t' = t$, can be written using matrices
7. use input-output (Leontief) matrices in economics
8. investigate the use of matrices in dietary problems in health
9. research the use of Leslie matrices in ecology
10. investigate the use of matrices in dominance problems such as in predicting the next round results (rankings) for the national netball competition
11. investigate the use of matrices in game strategies
12. investigate the application of matrices in formulating a mathematical model for a closed economic system
13. research nilpotent matrices where the matrix, **A**, is nilpotent if it has the property $\mathbf{A}^2 = \mathbf{O}$, (**O** is the zero matrix)
14. research idempotent matrices where the matrix, **A**, is idempotent if it has the property $\mathbf{A}^2 = \mathbf{A}$

15. investigate the group properties of matrices of the form

$$\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}$$

under both addition and multiplication; find interesting subsets of this class of matrices (known as quaternions); in particular, show that the eight matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

form a group under multiplication

Vectors and applications (notional time 30 hours)

Focus

Students are encouraged to develop an understanding of vectors as entities which can be used to describe naturally occurring systems. They should also understand that the meaning of a vector comes from the situation and the model being considered. Students should become aware of the links between vectors and matrices. The emphasis should be on those vectors which describe situations involving magnitude and direction.

Subject matter

(a) For vectors as a one-dimensional array

- definition of a vector
- relationship between vectors and matrices (SLE 1)
- operations on vectors including:
  - addition
  - multiplication by a scalar
- scalar product of two vectors (SLE 1)
- simple life-related applications of vectors (SLEs 1, 3)

(b) For vectors describing situations involving magnitude and direction

- definition of a vector (see Appendix 2)
- relationship between vectors and matrices (SLE 2)
- two and three dimensional vectors and their algebraic and geometric representation (SLEs 3, 4, 6)
- operations on vectors including: (SLEs 3–5, 9, 10, 13)
  - addition
  - multiplication by a scalar
- scalar product of two vectors (SLEs 2, 8)
- vector product of two vectors (SLE 7)
- unit vectors

- resolution of vectors into components acting at right angles to each other (SLEs 3–5, 11, 12)
- calculation of the angle between two vectors
- applications of vectors in both life-related and purely mathematical situations(SLEs 1–13)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. show that the cost of weekly shopping is the scalar product of the shopping list vector and the unit cost vector
2. show that if $\boldsymbol{x}$ is a column vector of order n

$$\boldsymbol{x}^T\boldsymbol{x} = \sum_{i=1}^{n} x_i^2$$

3. use addition and subtraction in life-related situations such as the effect of current flow on a boat; consider the concept of relative velocity
4. use resolution of vectors to consider the equilibrium of a body subject to a number of coplanar forces acting at a point
5. use vectors in three dimensions such as the placement of a TV aerial mast and its wire supports on a roof of a building
6. solve problems from geometry using vectors such as proving the concurrency of (a) the medians and (b) the bisectors of the internal angles of a triangle
7. use the vector product to calculate areas in situations such as the calculation of the area of a suspended triangular shade canopy
8. use scalar product to solve problems in situations such as the evaluation of work as a product of force and displacement
9. investigate the use of vectors in surveying
10. investigate the effect of wind on wind propelled craft
11. investigate the way medical staff use vectors to put a broken bone in suitable traction; consider the weights and angles of the ropes that are needed
12. investigate the forces exerted by the hip, knee and ankle joints in pushing and pulling a bicycle pedal
13. use addition of vectors to see how the apparent motion of planets in the solar system depends on the frame of reference chosen

Calculus (notional time 30 hours)

Focus

Students are encouraged to extend their knowledge of analytical and numerical techniques of integration. Students should also gain further experience in applying differentiation and integration to both life-related and purely mathematical situations. They should appreciate the importance of differential equations in representing problems involving rates of change.

Subject matter

- integrals of the form

  $$\int \frac{f'(x)}{f(x)}\,dx$$

  $$\int f[g(x)].g'(x)\,dx$$

- simple integration by parts (SLE 4)
- development and use of Simpson's rule (SLEs 5, 6, 7, 9, 10, 11, 12, 15)
- approximating small changes in functions using derivatives (SLEs 1, 2)
- life-related applications of simple, linear, first order differential equations with constant coefficients (SLEs 3, 8, 13, 14)
- solution of simple, linear, first order differential equations with constant coefficients (SLEs 3, 6, 8, 10, 13, 14, 15)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. investigate life related situations where small changes in calculated quantities due to small errors in measurements can be approximated using derivatives, such as the tolerance in the volume of a soft drink can, produced by small errors in the diameter or height
2. find approximate solutions to equations by using small changes from equations with exact solutions such as an approximate value of $\sqrt[3]{28}$
3. investigate life-related situations that can be modelled by simple differential equations such as growth of bacteria, cooling of a substance
4. use integration by parts to evaluate expressions such as

   $$\int e^{ax} \sin bx\,dx \quad and \quad \int t^2 e^t\,dt$$

5. use Simpson's rule to evaluate definite integrals where the indefinite integral cannot be found such as

   $$\int_0^{\pi} \frac{\sin x}{x}\,dx$$

6. compare the values of areas of simple shapes determined from known rules with approximations determined from Simpson's rule such as the area of a quadrant of a circle of radius 4 compared with the area represented by

   $$\int_0^4 \sqrt{16 - x^2}\,dx$$

   evaluated using Simpson's rule
7. use Simpson's rule with discrete data in situations such as the volume of fill to be removed in the construction of a road cutting

8. investigate the motion of falling objects, where resistance is proportional to the velocity, by considering the differential equation

$$m\frac{dv}{dt} = mg - kv$$

9. compare the accuracy of numerical techniques with analytical results for selected integrals
10. verify integrals in integral tables by differentiation of the result
11. show that Simpson's Rule is exact for polynomials of degree three or less
12. investigate the varying volumes for the earth obtained when its shape is assumed to be (a) a sphere, and (b) an ellipsoid
13. find an expression for the pressure, P, as a function of altitude in an isothermal atmosphere where the rate of decrease of atmospheric pressure with increasing altitude is proportional to the density of the air, ρ, pressure, density and temperature, T, are related by $P = R\rho T$, and R is a constant
14. find an expression for the amount of the desired product Pu_{239} present, as a function of time after start up in a breeder reactor where U_{238} is converted to Pu_{239} at a constant rate and Pu_{239} is converted to Pu_{240} at a rate proportional to the amount of Pu_{239} present
15. use tables of integrals or computer software to evaluate a given integral

Structures and patterns (notional time 30 hours)

Focus

Students are encouraged to develop their ability to recognise and use structures and patterns in a wide variety of situations. They should appreciate the value of symmetries and patterns in making generalisations to explain, simplify or extend their mathematical understanding. Justification of results is important and, where appropriate, results should be validated inductively or deductively. It is **not** intended that a great emphasis be placed on repetitive calculations in arithmetic progressions, geometric progressions, permutations or combinations.

Subject matter

- sum to infinity of a geometric progression (SLEs 1, 2)
- purely mathematical and life-related applications of arithmetic and geometric progressions (SLEs 10, 11, 12)
- sequences and series other than arithmetic and geometric (SLEs 3, 4, 16)
- permutations and combinations and their use in purely mathematical and life-related situations (SLEs 7, 8, 9, 14, 15)
- recognition of patterns in well known structures including Pascal's Triangle and Fibonacci sequence (SLEs 5, 6, 13)
- applications of patterns (SLEs 1–11, 17)
- use of the method of finite differences (SLEs 4, 18)
- proof by induction (SLE 4)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. establish the formula for the sum to n terms of a geometric progression, and hence the formula for the sum to infinity of a geometric progression; verify the formula by mathematical induction
2. recognise geometric progressions in many different algebraic forms such as $2, 4p^2, 8p^4, 16p^6 \ldots$; determine the general term, sum of n terms, sum to infinity of such sequences
3. use finite differences in determining polynomial coefficients for polynomials of degree ≤ 3
4. use finite difference methods to establish the formula in situations such as the sum of the first n positive integers, the sum of the first n squares, the sum of the first n cubes; use the principle of mathematical induction to prove the formula obtained
5. search for patterns in Pascal's Triangle and verify any claims algebraically or otherwise, such as ${}^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$
6. investigate patterns in Fibonacci numbers such as:

   $f_1 + f_2 + f_3 + \ldots + f_n = f_{n+2} - 1$

   $f_1 + f_3 + f_5 + \ldots + f_{2n-1} = f_{2n} - 1$

   $$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix} \textit{ for } n \geq 1 \textit{ given } f_0 = 0$$
7. investigate permutations and combinations which arise in games of chance such as, in poker, the number of hands containing two aces, the number of hands that are a full house
8. apply counting techniques to investigate problems in situations such as:
   - the amount of wool eaten by the offspring of one female moth who lays 300 eggs if each larva eats 15 milligrams of wool, two-thirds of eggs die, fifty per cent of remaining eggs are female and there are five generations per year
   - the number of different sequences possible in 4 months, when a patient must receive H-insulin for 2 months and P-insulin for the other 2 months in a medical study
9. investigate the use of the inclusion-exclusion principle in counting cases in situations such as determine how many individuals were interviewed in a survey of the eating habits of teenagers if 10 ate pizzas, 5 ate pies, 2 ate both and 7 ate neither; consider the effect of adding hamburgers as a third option
10. investigate the use of an arithmetic progression in situations such as:
    - the calculation of the length of batten material required for tiling a hip roof
    - the calculation of the total number of potatoes required for a "potato race" over a given distance if the distance between potatoes is a specified constant
11. use geometric progressions in situations involving the sum to infinity
12. investigate logarithmic spirals and polar curves which occur in nature such as in nautilus shells
13. investigate the occurrence of Fibonacci numbers in nature such as in spirals in sunflowers, pineapples and other plants

14. given a cube and six different colours, determine how many different ways the cube can be painted so that each face is a different colour; extend to other regular solids
15. apply the pigeonhole principle to solve problems in situations such as showing there are at least 2 in a group of 8 people whose birthdays fall on the same day of the week in any given year
16. use series expansions for e^x, $\sin x$ and $\cos x$ to illustrate :
    derivatives of e^x, $\sin x$ and $\cos x$
    $e^{ix} = \cos x + i \sin x$
17. use symmetries to find the order of groups of rotations such as rotations of squares, equilateral triangles
18. verify the outcomes of finite differences by using graphing calculators

6.3 OPTION TOPICS

Linear programming (notional time 30 hours)

Focus

Students are encouraged to develop an understanding of the methodology of Linear programming and to see how it is used to solve problems in life-related situations. They should appreciate that graphical techniques of solution have limited applicability, but that other techniques can be used for the solution of problems with a large number of variables. The use of technology is expected to assist students in these processes.

Subject matter (SLEs 1–14)

- recognition of the problem to be optimised (maximised or minimised)
- identification of variables, parameters and constraints
- construction of the linear objective function and constraints with associated parameters
- graphing linear functions associated with the constraints and identification of the regions defined by the constraints
- recognition that the region bounded by the constraints gives the feasible (possible) solutions
- recognition that different values of the objective function in two variables can be represented by a series of parallel lines
- use of a series of parallel lines to find the optimal value of the objective function in two variables (parallel or rolling ruler, graphical method)
- observation that the feasible region is always convex, and thus the optimal solutions occur at an edge or a corner point of the feasible region
- interpretation of mathematical solutions and their communication in a form appropriate to the given problem
- relationship between algebraic and geometric aspects of problems with constraints in two and three dimensions
- use of the simplex algorithm to solve life-related problems in which maximal solutions are required, and all constraints place positive, upper bounds on the variables.

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. take a life-related problem given in English, formulate it into a linear programming problem, solve by the graphical method. and interpret the solutions in terms of the original English problem
2. investigate how linear programming is used to assist management decisions in areas such as manufacturing, transport, primary industries and environmental management
3. consider optimal solutions of simple problems such as balancing diets
4. use a computer graphing package to graph linear functions
5. use parallel rulers to identify optimal solutions
6. change parameters or constraints in a given problem and investigate the effect on optimal solutions
7. consider the composition of a fleet of vehicles necessary to do a particular job at minimum cost
8. solve a problem involving the allocation of two crops to the areas available on a farm in order to optimise profit when there are constraints on the labour and finances available; consider the allocation of three crops
9. investigate the design of an optimal-sized solar-powered home which is to be competitive in the marketplace; constraints will include the size of solar cells, living area, cost of storage batteries and total cost of construction
10. use a computer to find optimal solutions when the variables must have integer values
11. research the history of linear programming
12. formulate a two-dimensional linear programming problem and write it in matrix form
13. research techniques used for solving linear optimisation problems when the variables are restricted to integer values (integer programming)
14. explain how the simplex algorithm systematically examines the vertices of the feasible region to determine the optimal solution.

Conics (notional time 30 hours)

Focus

Students are encouraged to extend their knowledge of coordinate geometry in two dimensions. They are encouraged to appreciate the interrelationships that exist between areas of mathematics. These relationships should be illustrated by applying coordinate geometry and complex numbers to conics.

Subject matter

- concept of a locus, directrix and focal point (SLEs 1–17)
- circle as a locus in:
  - Cartesian form $x^2 + y^2 = a^2$
  - parametric form x = a cos θ , y = a sin θ
  - complex number form $|z| = s$ (SLEs 1–6, 8, 11, 17)
- definition of eccentricity e
- ellipse as a locus in:
  - Cartesian form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
  - parametric form $x = a\cos\theta,\ y = b\sin\theta$
  - complex number form $|z - p| + |z - q| = s$ where $s > |p - q|$
  - polar form $r = \frac{d}{1 + e\cos\theta}, 0 < e < 1$ (SLEs 3, 5 ,7 ,9 , 10, 11, 14, 16, 17)
- hyperbola as a locus in:
  - Cartesian form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $xy = c$
  - parametric form $x = \frac{a}{\cos\theta}, y = b \tan\theta; x = t, y = \frac{c}{t}$
  - complex number form $||z - p| - |z - q|| = s$; where 0 < s < $|p - q|$
  - polar form $r = \frac{d}{1 + e\cos\theta}, e > 1$ (SLEs 3, 5, 10, 11, 15, 17)
- parabola as a locus in:
  - Cartesian form $y^2 = 4ax$
  - parametric form $x = at^2, y = 2at$
  - polar form $r = \frac{d}{1 + e\cos\theta}, e = 1$
    (SLEs 1, 2, 3, 5, 8, 10, 11, 12, 13, 17)

    in all cases above a, b, c, d, p, q and s are constants
- simple applications of conics (SLEs 1–17)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. introduce the concept of locus using situations such as the path followed by:
   the handle on an opening door
   a boat sailing equidistant from two fixed lights
   a netballer moving to remain equidistant from a goal post and a sideline
2. derive the Cartesian forms for the circle and parabola
3. find the Cartesian equations of conics after a given translation has been applied
4. find the centre and radius of a circle given its Cartesian equation

5. sketch conics given the Cartesian form showing directrices, focuses, asymptotes, and axes of symmetry as appropriate
6. convert the complex number form for a circle to the corresponding Cartesian form using both algebraic and geometric methods
7. convert the complex number form for an ellipse to the corresponding Cartesian form using both algebraic and geometric methods; situations should be limited to those where both focuses lie on the same coordinate axis
8. find the equation of a tangent to a circle or a parabola given its parametric form
9. investigate how to construct the elliptical hole required to be cut out of a sloping roof to fit a vertical cylindrical vent
10. use a graphing calculator to plot a conic whose form is given parametrically
11. find the equation of tangents, chords, and normals to conics whose equations are given in Cartesian form
12. investigate why parabolic reflectors are used in astronomical telescopes, hand-held torches and microwave repeater stations
13. investigate the shape on the ground of the leading edge of the sonic boom produced by a supersonic aircraft flying at high altitudes
14. research the use of elliptic reflectors in the treatment for kidney stones
15. research how hyperbolas were used in the Omega navigation system
16. investigate how the properties of ellipses are used in whispering rooms
17. use a dynamic geometry package to investigate locus behaviour

Dynamics (notional time 30 hours)

Focus

Students are encouraged to develop an understanding of the motion of objects which are subjected to forces. The approach used throughout this topic should bring together concepts from both vectors and calculus.

Subject matter

- derivatives and integrals of vectors (SLEs 1, 2, 3, 13)
- Newton's laws of motion in vector form applied to objects of constant mass (SLEs 2–15)
- application of the above to:
  - straight line motion in a horizontal plane with variable force
  - vertical motion under gravity with and without air resistance
  - projectile motion without air resistance
  - simple harmonic motion (derivation of the solutions to differential equations is not required)
  - circular motion with uniform angular velocity

  (SLEs 4–12, 14, 15)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. given the position vector of a point as a function of time such as **r** (t) = t **i** + t^2 **j** + sin t **k** determine the velocity and acceleration vectors
2. given the displacement vector of an object as a function of time, by the processes of differentiation, find the force which gives this motion
3. given the force on an object as a function of time and suitable prescribed conditions, such as velocity and displacement at certain times, use integration to find the position vector of the object
4. investigate the motion of falling objects such as in situations in which:
   resistance is proportional to the velocity, by considering the differential equation
   $$m\frac{dv}{dt} = mg - kv$$
   resistance is proportional to the square of velocity, by considering the differential equation
   $$m\frac{dv}{dt} = mg - kv^2$$
   where k is a positive constant
5. model vertical motion under gravity alone; investigate the effects of the inclusion of drag on the motion
6. develop the equations of motion under Hooke's law; verify the solutions for displacement by substitution and differentiation; relate the solutions to simple harmonic motion and circular motion with uniform angular velocity
7. from a table of vehicle stopping distances from various speeds, calculate (a) the reaction time of the driver and (b) the deceleration of the vehicle, which were assumed in the calculation of the table
8. model the path of a projectile without air resistance, using the vector form of the equations of motion starting with **a** = -g **j** where upwards is positive
9. use the parametric facility of a graphing calculator to model the flight of a projectile
10. investigate the flow of water from a hose held at varying angles, and model the path of the water
11. investigate the motion of a simple pendulum with varying amplitudes
12. investigate the angle of lean required by a motorcycle rider to negotiate a corner at various speeds
13. use the chain rule to show that the acceleration can be written as
    $$a = v\frac{dv}{dx} = \frac{d}{dx}(\frac{1}{2}v^2)$$
    if the velocity, v, of a particle moving in a straight line is given as a function of the distance, x

14. investigate the speed required for a projectile launched vertically to escape from the earth's gravitational field, ignoring air resistance but including the variation of gravitational attraction with distance
15. use motion detectors to investigate problems e.g. rolling a ball down a plank

Introduction to number theory (notional time 30 hours)

Focus

Students are encouraged to extend their knowledge of the properties of integers, and to appreciate the usefulness of apparently abstract mathematics. They are also encouraged to gain an understanding of the power of congruence in solving problems involving integers.

Subject matter

- primes, composites and the Fundamental Theorem of Arithmetic (SLEs 1, 6, 10, 12, 14–16, 19–22)
- divisors, Euclidean algorithm, lowest common multiples (LCM) and greatest common divisors (GCD), (SLEs 2, 9, 13, 17–19, 23)
- modular arithmetic (SLEs 3, 5, 8, 11, 24)
- congruence including simple simultaneous congruence (SLE 7)
- simple Diophantine equations (SLE 4)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. consider conditions on the integers a, b and r under which $ax + by = r$ is solvable for the integers x and y
2. develop tests for divisibility by 2, 2^n, 3, 3^2, 5, 5^n, 7, 11, 13, 1001
3. investigate the existence of inverses under multiplication modulo n
4. solve some simple Diophantine equations
5. use modular arithmetic to prove/disprove propositions about integers
6. research Fermat, Mersenne, perfect, abundant and/or amicable numbers
7. research the use of congruence in ciphering methods
8. investigate the period of the operation $x_{n+1} = ax_n + b \bmod c$ for generating random numbers
9. research interesting facts about numbers and create problems for others to solve, e.g: "The sum of the divisors of 7^3 is a square number; can you find any other numbers that fit this condition?" (400 is one such number)
10. investigate facts about prime numbers
11. investigate the way modular arithmetic is used to calculate the date of Easter
12. from a table of prime numbers, multiply the number of prime numbers less than an integer, n, by the logarithm of n, and compare the result with n; do this for several values of n
13. investigate greatest common divisors among pairs of Fibonacci numbers

14. investigate for which integers (a, b) a number of the form a^b - 1 can be a prime
15. investigate for which integers (a, b) a number of the form a^b + 1 can be a prime
16. investigate prime numbers of the form $a^2 + b$, for various values of b
17. investigate Pythagorean triples
18. investigate the representation of numbers by sums of squares
19. explain Fermat's Little Theorem and/or the Chinese Remainder Theorem
20. research Goldbach's Conjecture
21. research Fermat's Last Theorem
22. research different proofs that the number of primes is infinite
23. in ancient Egypt calculations were done using only rational numbers with 1 as the numerator; show that any rational number p/q can be rewritten as the sum of not more than p distinct rational numbers, each of which has 1 as the numerator
24. investigate the use of modular arithmetic in the construction of Latin squares in the design of experiments

Introductory modelling with probability (notional time 30 hours)

Focus

Students are encouraged to develop an understanding of the key mathematical concepts and tools that underlie the areas of modelling with probability. Students are encouraged to develop interpretative and problem-solving skills in applying operations of a Boolean nature, the basic probability rules, and conditional probability, and in identifying and formulating stochastic models. The approach should include the role of data in estimating probabilities and parameters, and in investigating stochastic and deterministic models.

Subject matter

- events and set operations, including translation to and from word descriptions (SLEs 1, 2, 4)
- logical circuits and truth tables, including similarities and contrasts to set operations (SLEs 2, 3, 4)
- working with probabilities of events (SLEs 1, 4, 5, 6)
- independence; system/circuit reliability (SLEs 6, 7, 8)
- conditional probability and the law of total probability (SLEs 9, 10, 11)
- use of transition probability matrices in simple Markov chain models (SLEs 12, 13)
- estimating conditional probabilities in simple Markov chain models (SLEs 6, 9, 12, 13)
- random numbers—the uniform distribution (SLEs 14–18)
- random numbers versus the chaotic behaviour of a simple non-linear system such as the logistic map (SLEs 16, 17, 18)
- the exponential distribution (SLEs 19, 20)
- use data to compare and explore the suitability of an exponential model or a uniform model (SLEs 19, 20)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. explore the correspondence between word descriptions and set notations of events—for example, each queue has exactly 2 people; there are at least 4 people over the two queues
2. express the event that logical circuits involving series and parallel components are operational using "and", "exclusive or" and "negation" operations
3. investigate truth tables for situations such as political statements
4. investigate the correspondences between set operations and the operations of "and", "exclusive or" and "negation"
5. from the three probability axioms establish the key probability rules
6. use data to estimate probabilities of simple and complex events
7. use independence to establish the reliability of a variety of communication configurations involving series and parallel components
8. investigate the role of independence in situations such as the Challenger disaster
9. use data from surveys and the percentages quoted in media outlets to explore and define conditional probability
10. use processes over time and processes ranging from manufacturing to medical to epidemiology, to establish and explore the law of total probability
11. *investigate the roles of independence and conditional probability in famous law cases and royal commissions in Australia and similar investigations in the USA—for example, those involving forensic evidence such as the Splatt case of South Australia
12. investigate transition probability matrices in situations such as recording, over a period of time, changes in the weather as stated by newspaper or TV weather reports (fine, showers, cloudy, clearing); construct the matrix describing the probabilities that one condition will be followed by each different condition; given today's weather, find the most probable sequence of the weather in the near future
13. *use data and simple Markov chain modelling to develop models with estimated probabilities for a range of situations such as signals, weather, psychology, savings, traffic, or insurance
14. establish the model for choosing a point at random on a line of given length
15. collect and investigate human data on random number generation
16. *collect and explore data from the random number generators of various computer packages
17. *research mathematical models for pseudo-random number generation
18. *investigate graphical methods for comparing data from stochastic and deterministic sources; for example, compare (pseudo-) random numbers with the numbers produced by the sequence $x_n = kx_{n-1}(1 - x_{n-1})$ for different values of k
19. *collect data on such variables as the length of phone calls, the times between trucks, the response time on the internet; explore the key features of the data

20. *obtain the expected value and median of the uniform distribution and of the exponential distribution; use data to estimate parameters of the uniform and exponential models for data, and explore the suitability of the two models

**Particularly suitable for assignment/project learning experiences*

Advanced periodic and exponential functions (notional time 30 hours)

Focus

Students are encouraged to extend their knowledge of trigonometric and exponential functions. They are also encouraged to investigate various ways these can be combined to model life-related situations.

Subject matter

- definitions of secant, cosecant, cotangent (SLE 12)
- expansions of $\sin(x \pm y)$, $\cos(x \pm y)$, (SLEs 1, 3, 4, 22)
- life-related applications of the sine and cosine functions (SLEs 2, 5–8, 13, 14, 15, 16, 18, 21)
- general shapes of graphs of $y = e^{ax}\sin bx$ and $y = e^{ax}\cos bx$ where a is both positive and negative
- applications of $e^{ax}\sin bx$ and $e^{ax}\cos bx$ (SLEs 4, 17, 19)
- logistic curve $y = \dfrac{A}{1 + Ce^{-kt}}$ (where A, C and k are constants) as a model of many natural systems (SLEs 9, 10, 11, 20)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small-group or whole-class activities.

1. express $y = a\cos x + b\sin x$ in the form $A\cos(x + C)$ or $A\sin(x + D)$ and graph the curve
2. the unmodulated waveform from a radio or TV transmitter can be described by $x = a \sin bt$; this is changed when broadcasting music; for AM broadcasting the constant a is modified to $a + c \sin ft$; for FM broadcasting the constant b is modified to $b + c \sin ft$; use a computer package to plot these waveforms, and relate the parameters involved to the frequency of transmission, and the pitch and loudness of the sound
3. investigate the use of rotation matrices to show the identities for $\sin(x \pm y)$, $\cos(x \pm y)$
4. use complex numbers to evaluate

   $$\int e^{ax} \sin bx \, dx \text{ or } \int e^{ax} \cos bx \, dx$$
5. investigate musical notes as a combination of sine functions
6. investigate the period of a pendulum for large angles
7. investigate the motion of a water wheel or ferris wheel as an example of a mathematical model based on a sinusoidal function

8. investigate the motion of an orbiting satellite as an example of a situation that can be modelled by a sinusoidal function
9. investigate systems which follow the logistic curve, e.g. biological growth with environmental constraints, the spread of disease or rumour, death of individuals with increasing dose rates of a toxic substance, concentration of end product in a chemical reaction
10. verify that the differential equation
    $\frac{dy}{dt} = \frac{k}{A} y(A - y)$ where $y = \frac{A}{1+C}$ at $t = 0$
    has the logistic equation $y = \frac{A}{1+Ce^{-kt}}$ as its solution
11. use Australian Census data for your city, town or shire to fit the logistic curve for population growth
    $p(t) = \frac{L}{1+Ce^{-r(t-t_0)}}$
    where $p(t)$ is the population at time t, t_0 is some convenient starting date, and L, C and r are parameters to be estimated
12. use a computer or a graphing calculator to investigate the graphs of sec x, cosec x and cot x, and relate these to the graphs of sin x, tan x and cos x
13. use a computer or a graphing calculator to draw the graphs of
    $y = \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \ldots$
    $y = \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots$
    $y = \cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \ldots$
    $y = \sin x + \frac{1}{2^3}\sin 2x + \frac{1}{3^3}\sin 3x + \ldots$
14. large amplitude waves sometimes appear on the ocean with no apparent cause; they arise from the superposition of a number of waves of different amplitudes and wavelengths; use computer software or a graphing calculator to investigate the result of adding various sine waves with different wavelengths and amplitudes
15. investigate the periodic patterns which are traced out by a point on the circumference of a circle which rolls around the inside or outside of a larger circle; use polar coordinates to write an expression for the curve
16. identify some oscillations with increasing amplitude in physical situations such as the Tacoma Bridge collapse
17. use a video of a mass oscillating on the end of a spring to see how closely the position of the mass follows an expression of the form $e^{at}\sin bt$, where a is negative
18. some surfers believe that ocean waves occur in “sets” of seven, with the seventh wave being “larger” than the preceding six waves; use a computer or graphing calculator to find functions whose graphs represent these sets of waves

19. study the vertical motion of a car on its springs; use a video to see whether the motion is the sum of two exponential functions, Ae^{at} and Be^{bt}, where *a* and *b* are negative, or whether it consists of a decaying oscillation of the form $e^{at} \sin bt$, where *a* is negative
20. all the heavy elements were formed in a supernova explosion which preceded the formation of the solar system; assuming that the two major isotopes of uranium, U_{238} and U_{235}, were then formed in equal amounts, use their present relative abundances and half-lives to estimate the time at which the supernova explosion took place (a suitable unit for time is a thousand million years)
21. investigate how the description of periodic phenomena in electric circuits uses complex numbers
22. develop multiple angle trigonometric formulae using de Moivre's Theorem.

School option(s) (notional time 30 hours per option)

Schools may develop option(s) of their own choice subject to the guidelines below:

- each option must be consistent with the rationale and global aims of the course
- each option offered to a student must have subject matter substantially different from any other subject matter that the student will meet in Mathematics B and Mathematics C
- each option should contain subject matter ensuring a level of challenge and integration comparable to that provided by the other optional topics
- in addition to specifying the subject matter, the work program will need to indicate a focus and state the range of likely learning experiences for each option
- each option should assist in providing the student with experiences in a balance of branches of mathematics
- although the use of computer packages is encouraged, the study of computer programming language(s) is not appropriate as a school option.

7 ASSESSMENT

7.1 INTRODUCTION

The purpose of assessment is to make judgments about how well students meet the general objectives of the course. In designing an assessment program, it is important that the assessment tasks, conditions and criteria are compatible with the general objectives and the learning experiences. Assessment then, both formative and summative, is an integral and continual aspect of a course of study. The distinction between formative and summative assessment lies in the purpose for which that assessment is used.

Formative assessment is used to provide feedback to students, parents, and teachers about achievement over the course of study. This enables students and teachers to identify the students' strengths and weaknesses so that, by informing practices in teaching and learning, students may improve their achievement and better manage their own learning. The formative techniques used should be similar to summative assessment techniques, which students will meet later in the course. This provides students with experience in responding to particular types of tasks under appropriate conditions. It is advisable that each assessment technique be used formatively before it is used summatively.

Summative assessment, while also providing feedback to students, parents, and teachers, provides information on which levels of achievement are determined at exit from the course of study. It follows, therefore, that it is necessary to plan the range of assessment instruments to be used, when they will be administered, and how they contribute to the determination of exit levels of achievement (see section 7.8). Students' achievements are matched to the standards of exit criteria, which are derived from the general objectives of the course (see section 3). Thus, summative assessment provides the information for certification at the end of the course.

7.2 PRINCIPLES OF ASSESSMENT

The Board's policy on assessment requires consideration to be given to the underlying principles below when devising an assessment program. These principles are to be considered together and not individually in the development of an assessment program.

Underlying principles of assessment

- Exit achievement levels are devised from student achievement in all areas identified in the syllabus as being mandatory.
- Assessment of a student's achievement is in the significant aspects of the course of study identified in the syllabus and the school's work program.
- Information is gathered through a process of continuous assessment.
- Exit assessment is devised to provide the "fullest and latest" information on a student's achievement in the course of study.
- Selective updating of a student's profile of achievement is undertaken over the course of study.
- Balance of assessment is a balance over the course of study and not necessarily a balance over a semester or between semesters.

Mandatory aspects of the syllabus

Judgement of student achievement at exit from a school course of study must be derived from information gathered about student achievement in those aspects identified in a syllabus as being mandatory. The assessment program, therefore, must include achievement of the general objectives of the syllabus.

Significant aspects of the course of study

Significant aspects refer to those areas included in the course of study, determined by the choices permitted by the syllabus, and seen as being particular to the context of the school and to the needs of students at that school. These will be determined by the choice of learning experiences appropriate to the location of the school, the local environment and the resources selected.

The significant aspects of the course must reflect the objectives of the syllabus.

Achievement in both mandatory and significant aspects of the course must contribute to the determination of the student's exit level of achievement.

The assessment of student achievement in the significant aspects of the school course of study must not preclude the assessment of the mandatory aspects of the syllabus.

For Mathematics C, the significant aspects are the subject matter of each topic.

Continuous assessment

This is the means by which assessment instruments are administered at suitable intervals and by which information on student achievement is collected. It requires a continuous gathering of information and the making of judgements in terms of the stated criteria and standards throughout the two-year program of study.

Levels of achievement must be arrived at by gathering information through a process of continuous assessment at points in the course of study appropriate to the organisation of the learning experiences. They must not be based on students' responses to a single assessment task at the end of a course or instruments set at arbitrary intervals that are unrelated to the developmental course of study.

Fullest and latest information

Judgements about student achievement made at exit from a school course of study must be based on the fullest and latest information available.

'Fullest' refers to information about student achievement gathered across the range of general objectives. 'Latest' refers to information about student achievement gathered from the latest period in which the general objectives are assessed.

Fullest and latest information consists of both the most recent data on developmental aspects together with any previous and not superseded data. Decisions about achievement require both to be considered in determining the student's level of achievement.

The information used to determine a student's exit level of achievement is to be the fullest and latest available." The fullest" refers to the collection of assessment information that covers the full range of objectives. "The latest" refers to information obtained through selective updating.

The assessment instruments for summative purposes are used to determine a student's exit level of achievement. Any formative assessment of knowledge, processes and skills through the program of study becomes a learning experience for the student, whose achievement should therefore benefit when similar assessment techniques are applied for summative purposes.

Selective updating

Selective updating is related to the developmental nature of the two-year course of study. It is the process of using later information to supersede earlier information. Information about student achievement should, therefore, be updated continually when objectives and criteria are revisited.

As the criteria are treated at increasing levels of complexity, assessment information gathered at earlier stages of the course may no longer be typical of student achievement. The information should therefore be selectively updated to reflect student achievement more accurately. Selective updating operates within the context of continuous assessment.

A student profile should be maintained to allow the selective updating of student data. By increasing the amount of information available, the student profile more accurately indicates overall student achievement.

Balance

Balance of assessment is a balance over the course of study and not necessarily a balance within a semester or between semesters. The assessment program must ensure an appropriate balance over the course of study as a whole.

Appropriate balance is established through determining suitable variety, quantity and timing in the assessment conditions, criteria and techniques.

7.3 EXIT CRITERIA

Student achievement will be judged on the following three exit criteria:

- Knowledge and procedures
- Modelling and problem solving
- Communication and justification.

The exit criteria reflect the categories of general objectives, and have been defined in section 3.3 of the syllabus.

7.4 ASSESSMENT REQUIREMENTS

The assessment plan must enable students to demonstrate their achievement across the full range of the general objectives in the first three categories in section 3.3.

The assessment plan must be designed to cover the continuum of each of the contexts.

All three criteria must be adequately represented in assessment data to enable the overall quality of a student's achievement in each criterion to be determined.

Some points which must be taken into account in assessment are given below:

- Assessment instruments must provide students with the opportunity to demonstrate achievement in the general objectives along the continuum within each of the contexts. (see section 3.2)
- In a well-balanced plan there should be many items that allow information to be collected on more than one criterion. It is not appropriate to set items which collect information only on Communication and justification.
- Information on student achievement in Knowledge and procedures, and Communication and justification may be obtained from items assessing student achievement in Modelling and problem solving.
- It is not appropriate to record on a profile separate information on each aspect of a criterion.
- Information on student achievement in each criterion may be provided by a global consideration of the student response to a task or set of tasks. Such information must be supported by comparison with task-specific descriptors based on the general objectives of the syllabus.

7.4.1 Special consideration

Guidance about the nature and appropriateness of special consideration and special arrangements for particular students may be found in the Board's policy statement on special consideration: *Special Consideration: Exemption and Special Arrangements in Senior Secondary School-Based Assessment* (30 May 1994). This statement also provides guidance on responsibilities, principles and strategies that schools may need to consider.

To enable special consideration to be effective for students so identified, it is important that schools plan and implement strategies in the early stages of an assessment program and not at the point of deciding levels of achievement. The special consideration might involve alternative teaching approaches, assessment plans and learning experiences.

7.5 ASSESSMENT TECHNIQUES

It is expected that the use of appropriate technology will be incorporated in assessment tasks.

A balanced assessment plan that has validity in assessing achievement across the full range of the general objectives includes a variety of assessment techniques such as those described below.

Extended modelling and problem-solving tasks

This form of assessment may require a response that involves mathematical language, graphs and diagrams, and could involve a significant amount of conventional English. It will typically be in written form, a combination of written and oral forms, or multimedia forms.

The activities leading to an extended modelling and problem-solving task could be done individually and/or in groups, and the extended modelling and problem-solving task could be prepared in class time and/or in students' own time.

Reports

A report is typically an extended response to a task such as:

- an experiment in which data is collected, analysed and modelled
- a mathematical investigation
- a field activity
- a project.

A report could comprise such forms as:

- a scientific report
- a proposal to a company or organisation
- a feasibility study.

The activities leading to a report could be done individually and/or in groups and the report could be prepared in class time and/or in students' own time. A report will typically be in written form, or a combination of written and oral multimedia forms.

The report will generally include an introduction, analysis of results and data, conclusions drawn, justification, and, when necessary, a bibliography, references and appendices.

Supervised tests

Supervised tests commonly include tasks requiring quantitative and/or qualitative responses. Supervised tests could include a variety of items such as:

- multiple-choice questions
- questions requiring a short response:
  - in mathematical language and symbols
  - in conventional written English, ranging in length from a single word to a paragraph
- questions requiring a response including graphs, tables, diagrams and data
- questions requiring an extended answer where the response includes:
  - mathematical language and symbols
  - conventional written English, more than one paragraph in length
  - a combination of the above.

Assessment tasks other than tests must be included at least twice each year and should contribute significantly to the decision making-process in each criterion.

7.5.1 Authorship of tasks

In order to attest that the response to a task is genuinely that of the student, procedures such as the following are suggested:

- the teacher monitors the development of the task by seeing plans and a draft of the student's work
- the student produces and maintains documentation of the development of the response
- the student acknowledges all resources used; this will include text and source material and the type of assistance received
- the school develops guidelines and procedures for students in relation to both print and electronic source materials/resources, and to other types of assistance (including human) that have been sought.

7.6 RECORDING INFORMATION

Information on student achievement in each criterion may be recorded in various ways. However, the methods of recording and the frequency with which records will be updated must be clearly outlined in the work program

7.7 DETERMINING EXIT LEVELS OF ACHIEVEMENT

On completion of the course of study, the school is required to award each student an exit level of achievement from one of the five categories:

Very High Achievement

High Achievement

Sound Achievement

Limited Achievement.

Very Limited Achievement.

The school must award an exit standard for each of the three criteria (Knowledge and procedures, Modelling and problem solving, Communication and justification), based on the principles of assessment described in this syllabus. The criteria are derived from the general objectives and are described in section 3.3. The minimum standards associated with the three exit criteria are described in section 7.7. When teachers are determining a standard for each criterion, the standard awarded should be informed by how the qualities of the work match the descriptors overall.

For Year 11, particular standards descriptors may be selected from the matrices in table 2 and/or adapted to suit the task. These standards are used to inform the teaching and learning process. For Year 12 instruments, students should be provided with opportunities to understand and become familiar with the expectations for exit. The exit standards are applied to the summative body of work selected for exit.

Of the seven key competencies, the five that are relevant to this subject[1] are embedded in the descriptors in table 2. Elements of some of the key competencies are embedded within the standards associated with the exit criteria. The key competencies of "Using mathematical ideas and techniques", and "Using technology", are to be found within the "Knowledge and procedures" criterion. The "Modelling and problem solving" criterion involves elements from "Collecting, analysing and organising information", "Using mathematical ideas and techniques", and "Solving problems", whereas "Collecting, analysing and organising information", "Communicating ideas and information", and "Solving problems", are involved in the Communication and justification criterion.

When standards have been determined in each of the three criteria, table 1 is used to determine the exit level of achievement, where *A* represents the highest standard and *E* the lowest.

[1] KC1: Collecting, analysing and organising information; KC2: Communicating ideas and information; KC5: Using mathematical ideas and techniques; KC6: Solving problems; KC7: Using technology

Table 2: Minimum requirements for exit levels

VHA	Standard *A* in any two exit criteria and no less than a *B* in the remaining criterion
HA	Standard *B* in any two exit criteria and no less than a *C* in the remaining criterion
SA	Standard *C* in any two exit criteria, **one of which must be the Knowledge and procedures criterion**, and no less than a *D* in the remaining criterion
LA	Standard *D* in any two exit criteria, **one of which must be the Knowledge and procedures criterion**
VLA	Does not meet the requirements for Limited Achievement

.

Table 1: Minimum standards associated with exit criteria

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: Knowledge and procedures	The **overall quality** of a student's achievement across the full range within the contexts of application, technology and complexity, and across topics, **consistently demonstrates**: • accurate recall, selection and use of definitions, results and rules • appropriate use of technology • appropriate selection and accurate and proficient use of procedures • effective transfer and application of mathematical procedures.	The **overall quality** of a student's achievement across a range within the contexts of application, technology and complexity, and across topics, **generally demonstrates**: • accurate recall, selection and use of definitions, results and rules • appropriate use of technology • appropriate selection and accurate use of procedures.	The **overall quality** of a student's achievement in the contexts of application, technology and complexity, **generally demonstrates**: • accurate recall and use of basic definitions, results and rules • appropriate use of some technology • accurate use of basic procedures.	The **overall quality** of a student's achievement in the contexts of application, technology and complexity, **sometimes demonstrates**: • accurate recall and use of some definitions, results and rules • appropriate use of some technology.	The **overall quality** of a student's achievement **rarely demonstrates** knowledge and use of procedures.

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: Modelling and problem solving	The **overall quality** of a student's achievement across the full range within each context, and across topics **generally demonstrates** mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations, identifying assumptions and variables • selecting and using effective strategies • selecting appropriate procedures required to solve a wide range of problems • appropriate synthesis of procedures and strategies; …*and* in some contexts and topics **demonstrates** mathematical thinking which includes: • synthesis of procedures and strategies to solve problems • initiative and insight in exploring the problem • exploring strengths and limitations of models • refining a model • extending and generalising from solutions.	The **overall quality** of a student's achievement across a range within each context, across topics, **generally demonstrates** mathematical thinking which includes: • interpreting, clarifying and analysing a range of situations, identifying assumptions and variables • selecting and using effective strategies • selecting appropriate procedures required to solve a range of problems; …*and* in some contexts and topics **demonstrates** mathematical thinking which includes appropriate synthesis of procedures and strategies.	The **overall quality** of a student's achievement in all contexts **generally demonstrates** mathematical thinking which includes: • interpreting and clarifying a range of situations • selecting strategies and/or procedures required to solve problems.	The **overall quality** of a student's achievement **sometimes demonstrates** mathematical thinking which includes following basic procedures and/or using strategies required to solve problems.	The **overall quality** of a student's achievement **rarely demonstrates** mathematical thinking which includes following basic procedures and/or using strategies required to solve problems.

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: Communication and justification	The **overall quality** of a student's achievement across the full range within each context **consistently demonstrates**: • accurate and appropriate use of mathematical terms and symbols • accurate and appropriate use of language • collection and organisation of information into various forms of presentation suitable for a given use or audience • use of mathematical reasoning and proof to develop logical arguments in support of conclusions, results and/or propositions • recognition of the effects of assumptions used • evaluation of the validity of arguments • justification of procedures.	The **overall quality** of a student's achievement across a range within each context **generally demonstrates**: • accurate and appropriate use of mathematical terms and symbols • accurate and appropriate use of language • collection and organisation of information into various forms of presentation suitable for a given use or audience • use of mathematical reasoning and proof to develop simple logical arguments in support of conclusions, results and/or propositions • justification of procedures.	The **overall quality** of a student's achievement in some contexts **generally demonstrates**: • accurate and appropriate use of basic mathematical terms and symbols • accurate and appropriate use of basic language • collection and organisation of information into various forms of presentation • use of some mathematical reasoning to develop simple logical arguments in support of conclusions, results and/or propositions.	The **overall quality** of a student's achievement **sometimes demonstrates** evidence of the use of the basic conventions of language and mathematics.	The **overall quality** of a student's achievement **rarely demonstrates** use of the basic conventions of language or mathematics.

Contexts are explained in section 3.2.

7.8 REQUIREMENTS FOR VERIFICATION FOLIOS

A verification folio is a collection of a student's responses to assessment instruments on which the level of achievement is based. Each folio should contain a variety of assessment techniques demonstrating achievement in the three criteria, Knowledge and procedures, Modelling and problem solving, and Communication and justification, over the range of topics. This variety of assessment techniques is necessary to provide a range of opportunities from which students may demonstrate achievement.

In the verification folio requirements for the subject, the minimum and maximum number of assessment instruments are stipulated. Schools must ensure that the verification folios presented in October contain all summative assessment instruments and corresponding student responses upon which judgments about interim levels of achievement have been made to that point.

It is necessary that a student's achievement in the three criteria is monitored throughout the course so that feedback in terms of the criteria is provided to the student. The verification folio is an ideal medium for students and teachers to monitor progress throughout the course.

For verification purposes, schools must submit student folios which contain:

- student achievement data profiled in the three exit criteria
- the student responses to *all* summative assessment instruments (in the case of non-written responses, the minimum requirement will be a student criterion sheet or sheets completed by the teacher along with supporting material provided by the student.)
- a minimum of four instruments from Year 12 with at least one of these being a report, extended modelling and problem-solving task, or similar.

A verification folio must consist of a minimum of 4 to a maximum of 10 pieces of summative work. These should represent a range of assessment techniques (see section 7.5) and provide adequate information on which to substantiate the school's judgments regarding student achievement in each criterion.

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8 DEVELOPING A WORK PROGRAM

The work program is a formal expression of the school's interpretation of this syllabus. It has three primary functions. First, it provides guidance to the teachers of the subject as to the nature and requirements of the Mathematics C course at the school. Second, it provides similar guidance to the school's students, and their parents, in relation to the subject matter to be studied and how achievement of the syllabus objectives will be assessed. Third, it provides a basis for accreditation by the Board for the purposes of including students' results for the subject on their Senior Certificates.

The school's work program should be a document which does not require reference to other documents to be understood. The work program must contain the following components.

Table of contents Facilitates the readability of the document; pages must be numbered.

Rationale Provides justification for including the subject in the school curriculum. The rationale may be derived principally from the syllabus statement but should also include information on the school's philosophy, student population, resources and any other factors which may influence the decisions made in designing a course of study to cater for the special characteristics of the school and its students.

Global aims Statements of the long-term achievements, attitudes and values that are to be developed by the students studying the subject, but which are not directly assessed by the school. These should include the global aims listed in this syllabus.

General objectives As indicated in this syllabus.

Contexts As indicated in this syllabus.

Course organisation Course organisation provides:

- a summary of the spiralling and integrated sequence developed by the school to give an overview of the topics
- details of the sequence, indicating:
  - the subject matter to be taught in each unit of work (the subject matter listed in the syllabus is the minimum to be included)
  - time allocations for each unit.

If the school wishes to offer different topics to different groups of students, details should be included here.

The sequence must be developed in accordance with Section 5 of this syllabus.

Technology A commitment to use higher technologies such as graphing calculators and/or computers with appropriate software, or other developing technologies.

Language statement As indicated in this syllabus.

Educational equity A statement must be included by the school indicating that due consideration has been given to the issues associated with the educational equity statement in this syllabus.

Learning Experiences A commitment to use a variety of learning experiences which are appropriate to the age group concerned, and consistent with the objectives of this syllabus.

A sample of a sequence of work must be included in the work program. This should be of sufficient length to demonstrate that the general approach by the school is consistent with the intent of this syllabus. It should show how the choice of subject matter is to be taught, and the variety of learning experiences that will provide opportunities for students to achieve the general objectives within the contexts of the course.

Assessment Including:

- an assessment plan which:
  - provides a balanced assessment program in which techniques other than formal written tests or examinations are to be included at least twice each year, constitute a meaningful contribution to exit levels of achievement
  - clearly indicates the summative tasks—data gathered as a result of implementing the assessment plan should allow fullest and latest information to determine exit levels of achievement
  - allows sufficient information to be available for the recommendation of interim levels of achievement; for example, for monitoring and verification purposes
  - while allowing for flexibility within the school, contains sufficient information to show that there will be enough data gathered to enable valid judgments to be made on student achievement in each of the three criteria
  - indicates the criteria associated with tasks and the conditions of implementation of tasks; a general statement on the conditions of implementation of assessment will suffice;
    (An example of such a statement is "To ensure validity and reliability of assessment instruments, this school will use a variety of implementation conditions for assessment of student achievement e.g. assignments undertaken in class time, assignments undertaken at home, and group projects. The conditions appropriate to the actual instruments will be supplied at monitoring and verification.")
  - avoids over-assessment.
- a commitment to include as much assessment as possible from Semester 4 for each of the three criteria in the verification submission
- an example of the individual student profile to be included in the verification folio
- how assessment data are combined to reach an overall standard in each criterion (a completed student profile may help to clarify the explanation); fullest and latest information is not obtained by an arbitrary 'weighting' of semesters or by using Semester 1 assessment instruments as summative in a well-sequenced course
- the procedure for awarding exit levels of achievement which are consistent with the criteria and standards of this syllabus (section 7); the school should indicate a commitment to ensure that the achievement of students identified as near a threshold (either above or below) is checked against the verbal descriptors of the syllabus before a level of achievement is awarded.

9 EDUCATIONAL EQUITY

Equity means fair treatment of all. In developing work programs from this syllabus, schools are urged to consider the most appropriate means of incorporating the following notions of equity.

Schools need to provide opportunities for all students to demonstrate what they know and what they can do. All students, therefore, should have equitable access to educational programs and human and material resources. Teachers should ensure that the particular needs of the following groups of students are met: female students; male students; Aboriginal students; Torres Strait Islander students; students from non–English-speaking backgrounds; students with disabilities; students with gifts and talents; geographically isolated students; and students from low socioeconomic backgrounds.

The subject matter chosen should include, where appropriate, the contributions and experiences of all groups of people. Learning contexts and community needs and aspirations should also be considered when selecting subject matter. In choosing suitable learning experiences teachers should introduce and reinforce non-racist, non-sexist, culturally sensitive and unprejudiced attitudes and behaviour. Learning experiences should encourage the participation of students with disabilities and accommodate different learning styles.

It is desirable that the resource materials chosen recognise and value the contributions of both females and males to society and include the social experiences of both sexes. Resource materials should also reflect the cultural diversity within the community and draw from the experiences of the range of cultural groups in the community.

Efforts should be made to identify, investigate and remove barriers to equal opportunity to demonstrate achievement. This may involve being proactive in finding out about the best ways to meet the special needs, in terms of learning and assessment, of particular students. The variety of assessment techniques in the work program should allow students of **all** backgrounds to demonstrate their knowledge and skills in a subject in relation to the criteria and standards stated in this syllabus. The syllabus criteria and standards should be applied in the same way to all students.

Teachers may find the following useful for devising an inclusive work program.

Australian Curriculum, Assessment and Certification Authorities 1996, *Guidelines for Assessment Quality and Equity* 1996, Australian Curriculum, Assessment and Certification Authorities, available through QBSSSS, Brisbane.

Department of Education, Queensland 1991, *A Fair Deal: Equity guidelines for developing and reviewing educational resources*, Department of Education, Brisbane.

Department of Training and Industrial Relations 1998, *Access and Equity Policy for the Vocational Education and Training System*, DTIR, Brisbane.

[Queensland] Board of Senior Secondary School Studies 1994, *Policy Statement on Special Consideration*, QBSSSS, Brisbane.

[Queensland] Board of Senior Secondary School Studies 1995, *Language and Equity: A discussion paper for writers of school-based assessment instruments*, QBSSSS, Brisbane.

[Queensland] Board of Senior Secondary School Studies 1995, *Studying Assessment Practices: A resource for teachers in schools*, QBSSSS, Brisbane.

APPENDIX 1
BASIC MATHEMATICS

The following knowledge and procedures will be required throughout the course and must be learned or maintained as required:

- metric measurement including measurement of mass, length, area and volume in practical contexts
- calculation and estimation with and without instruments
- rates, percentages, ratio and proportion
- identities, linear equations and inequations
- the gradient of a straight line
- plotting points using Cartesian coordinates
- basic algebraic manipulations
- the equation of a straight line
- the formula for the zeros of a quadratic equation
- completing the square in a quadratic
- absolute value
- summation notation:

  $$\sum_{i=1}^{n} x_i \, .$$

APPENDIX 2 EXPLANATION OF SOME TERMS

Congruence

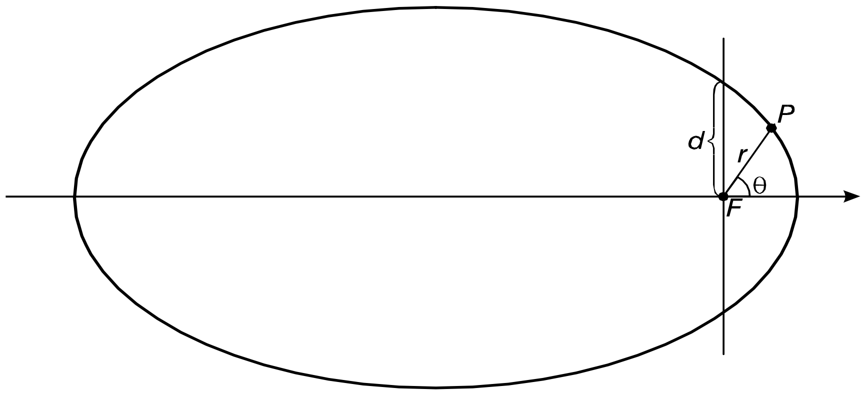
A congruence is any statement of the form $a \equiv b \bmod n$, read as "a is congruent to b modulo n", and means that $a - b$ is divisible by n, where all of a, b and n are integers; for example, $13 \equiv -2 \bmod 5$.

Conics, polar form

The position of a point P on a conic can be expressed using polar coordinates, (r,θ), with one focus at the origin, as

$$\frac{d}{r} = 1 + e \cos\theta$$

where e is the eccentricity of the conic, and d is as shown in the following diagram (for the classically minded, d is called the *semi latus rectum*).



Diophantine equation

An equation in more than one variable, with integer coefficients, where integer solutions (normally non-negative) are required. A Diophantine equation may have no solutions, a finite number of solutions, or infinitely many solutions. For example:

$3x + 6y = 16$ has no solutions;

$3x + 5y = 23$ has two positive solutions, (1,4) and (6,1);

$x^2 + y^2 = z^2$ has infinitely many solutions.

Euclidean algorithm

A procedure for determining the greatest common divisor of two integers, x and y, without factorising them. Denote the quotient and remainder at each division by q_i and r_i respectively, then

$x = yq_1 + r_1$

$y = r_1 q_2 + r_2$

$r_1 = r_2 q_3 + r_3$

$r_2 = r_3 q_4 + r_4$

. . . . . .

$r_{n-2} = r_{n-1} q_n$.

The last non-zero remainder is the greatest common divisor of *x* and *y*.

Exponential distribution

This is a continuous distribution whose probability density function is given by

$$f(x) = \eta e^{-\eta x} \qquad 0 < x$$

where η is a non-negative parameter.

It is the distribution of the time until the next event in a memoryless process, that is a process in which events are occurring at random at an average rate of η per unit time.

Fundamental theorem of arithmetic

Any positive integer can be factorised into the product of prime numbers in only one way, apart from the order in which the factors occur.

Jusification of procedures

Justification of procedures may include:

- providing evidence (words, diagrams, symbols, etc) to support processes used
- stating a generic formula before using specifically
- providing a reasoned, well formed, logical sequence within a response.

Law of total probability

This is the term often given to the following procedure for obtaining the probability of a complex event through knowledge of how it depends on each of a set of simple events:

Consider an event B and consider a partition of the whole sample space into a set of events $A_1, \ldots, A_k$, that are disjoint and whose union is the whole space, i.e. whose union covers every possibility. If we know the probabilities of all the events A_i, and the conditional probabilities of B given A_i, then we can find the probability of B by noting that

$$\begin{aligned} \Pr(B) &= \Pr(BA_1) + \Pr(BA_2) + \ldots.. + \Pr(BA_k) \\ &= \Pr(B|A_1)\Pr(A_1) + \ldots\ldots + \Pr(B|A_k)\Pr(A_k) \end{aligned}$$

Leontief matrices

Row entries give amount of input of different products required to contribute to the production of one unit of column product. Post-multiplication by a gross-product vector then gives the internal consumption vector—the amount of products of different types required to produce the given output amounts. That is, gross-product (output) vector (**x**) and input-output matrix (**A**) gives the internal consumption as **Ax**. Note that consumer demand equals **x** - **Ax**.

Linear first order differential equations with constant coefficients

These are equations of the form

$$p\frac{dy}{dx} = ky + b$$

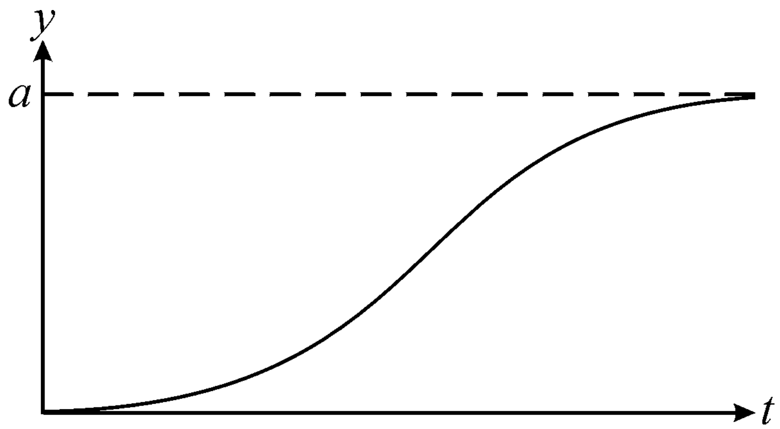
where *p, k*, and *b* are constants.

Linear objective function

The function to be optimised in linear programming.

Logistic curve, logistic growth

Many natural systems may be modelled by a sigmoidal curve in which the rate of change in *y*, with respect to *t* say, begins slowly and builds rapidly before finally tapering off.



The logistic growth equation

$$\frac{dy}{dt} = ry(a - y).$$

describes such a system where *r* is the growth rate and *a* may be thought of as the "carrying capacity" of the environment if *y(t)* is taken to be a population.

$$y(t) = \frac{a}{1 + ke^{-rt}}.$$

Note that a is the asymptotic limit of *y* and that the maximum rate of change occur at

$(t, y) = (\frac{\ln k}{ar}, \frac{a}{2})$, the point of inflection in the logistic curve.

Logistic map

This is a discrete recurrence relationship that corresponds to the differential equation for the logistic function. The discrete logistic recurrence relationship on (0, 1) is given by $x_{n+1} = cx_n(1 - x_n)$, where c is some constant < 4. For different values of c the sequence of values produced by this recurrence relationship exhibits different behaviour.

Mathematical model

Any representation of a situation which is expressed in mathematical terms. It should be noted that models may be as simple as expressing simple interest as $I = \frac{Prt}{100}$ or showing the relationship between two variables as a scattergram.

Scalar product

There are two types of scalar product, corresponding to the two types of vectors discussed below.

The scalar product of vectors of the data structure type is the real number obtained by taking the sum of the products of corresponding components of the two vectors. It is related to the product of matrices.

For vectors expressed in terms of direction and magnitude, the scalar product, or "dot" product, is the product of the magnitudes of the two vectors and the cosine of the angle between them.

$$x \cdot y = |x||y|\cos\theta$$

It is also equal to the sum of the products of corresponding components of the two vectors.

Simplex algorithm

Most life-related applications involve more than two variables. The simplex algorithm is a general procedure used in linear programming to find the optimal solution for situations involving two or more variables by systematically examining the vertices of the feasible region and stopping when the optimum has been found.

Simpson's Rule

A formula for numerical integration.

$$\int_a^{a+2h} f(x)dx \approx \frac{1}{3}h[f(a) + 4f(a+h) + f(a+2h)]$$

By using Simpson's Rule on adjoining intervals a more general form can be obtained.

$$\int_a^{a+2nh} f(x)dx \approx \frac{1}{3}h[f(a) + 4f(a+h) + 2f(a+2h) + 4f(a+3h) + 2f(a+4h) + \ldots$$
$$+4f(a+(2n-1)h) + f(a+2nh)]$$

Uniform distribution

This is the simplest of the continuous distributions and is sometimes also called the rectangular distribution because its probability density function is a rectangular shape. If a point is chosen at random from the interval (a, b), then the probability density function of the position of the point, X, is given by

f(x) =	1/(b-a),	$a < x < b$
	0,	elsewhere.