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MOIRÉ FRINGES AS A MEANS OF ANALYZING STRAINS

By C. A. Sciammarella[1] and A. J. Durelli[2]

With discussion by Messrs. Stanley Morse; and C. A. Sciammarella and A. J. Durelli

SYNOPSIS

The objective of this paper is to show the application of moire fringes in the determination of strains in two-dimensional models. The moire pattern is interpreted as a function of displacements only from which strains in the Lagrangian and Eulerian descriptions are determined. This presentation is not limited to small deformations, and sets of formulas appropriate for every case are given here.

INTRODUCTION

In recent years, moire fringes have found increasing application in the field of stress analysis. Two main fields of application are the deflections of plates and the strains in two-dimensional problems. In this paper attention is focused on moire data from the latter.

In order to obtain strains, two different approaches have been followed, one that can be called "geometrical" and the other which consists in relating the fringes to the displacement field. The purpose of this paper is to present this last approach in the most general form.

The "geometrical" interpretation seems to have been originated in a paper published in Dutch in 1945 by D. Tollenard,[3] and was applied for the first time to the subject of the strain determination in 1952 by J. K. Kaczer and F. Kroupa[4] of the Physics Institute of the Charles University in Prague.

The basic idea behind the "geometrical approach" is the following. Moire fringes are formed by two interfering line screens. One is printed in the model and is subjected to deformations produced by the applied loads. The second screen applied on top of the first is used as reference or master. The moire pattern formation can be studied as the result of the intersections of the two above mentioned systems of lines. Knowing the distance between the master grid lines and measuring the distance between fringes, it is possible by geometric analysis of the intersections of the two systems of lines to compute the distance between the model grid lines at a point, and the corresponding change in direction. With these two data, normal and shear strains can be computed. The "geometrical" approach gives values of the strains that are the average values in a region limited by two fringes, because the applied formulas are valid for a homogeneous field.

Similar approaches are presented in other papers.[5,6,7] R. Bromley[8] obtains results similar to those previously referred to by using tensor notation. A different point of view in the analysis of the moire patterns was presented in 1948 by R. Weller and W. Shepphard.[9] These authors described the application of the moire fringes as a means of measuring displacements. In 1954, M. Dantu,[10] following the same lines, introduced the interpretation of the moire patterns in terms of the components of the displacements. Dantu's presentation is limited to the field of small strains and deformations. A broader point of view is presented herein, together with the corresponding set of formulas to be used in different cases.

FUNDAMENTAL PROPERTY OF THE MOIRE FRINGES

Two line screens are used in the strain analysis, the "model grid" and the "master grid." The distance between the grid lines is called pitch and is represented by p. By interference of the two screens a pattern of fringes is produced. The distance between fringes is called "fringe spacing" and has been assigned the symbol δ. Any line perpendicular to the master grid lines will be called principal section and a line parallel to the master grid lines, a secondary section. Model and master grids are assumed to be in the same plane.

Consider the interference of two line screens parallel to each other but of different pitch. It can be assumed that one is obtained from the other by uniform contraction or elongation. This mechanism of fringe formation is shown

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1 Assoc. Prof., Advanced Mechanics, Research Sect., Coll. of Engrg., Univ. of Florida, Gainesville, Fla.

2 Prof., Civ. Engrg., Catholic Univ. of America, Washington, D. C.

3 "Moire—Interferentieverschijnselen bij rasterdruk," by D. Tollenaar, Amsterdam Instituut voor Grafische Techniek, 1945.

4 "The Determination of Strains by Mechanical Interference," by J. Kaczer and F. Kroupa, Czechoslovak Journal of Physics, Vol. 1, 1952, p. 80.

5 "Untersuchungen zur Theorie der Doppelraster als Mittel zur Meszanzeige," by R. Lehman and A. Wiemer, Feingeratetechnik Heft, 1953, pp. 5-199.

6 "The Measurement of Plane Strains by a Photoscreen Method," by J. D. C. Crips, Proceedings, SESA, Vol. 15, No. 1, 1957.

7 "Use of Moire Effect to Measure Plastic Strains," by A. Vinckier and R. Dechaene, Transactions, ASME, No. 59-Met. 7.

8 "Two-Dimensional Strain Measurement by Moire," by R. Bromley, Proceedings, Physical Soc., Vol. 69, Part 3 B, 1956.

9 "Displacement Measurement by Mechanical Interferometry," by R. Weller and B. M. Shepard, Proceedings, SESA, Vol. 6, No. 1, 1948.

10 "Recherches Diverses d'Extensometrie et de Determination des Contraintes," by M. Dantu, Conference faite au GAMAC, le 22 fevrier, 1954.

in Fig. 1. A dark fringe will appear at the points where an opaque strip falls over a transparent strip. When two opaque strips coincide, there is a maximum of light intensity and one has a light fringe. If it is assumed that the model grid is the one with the largest pitch, a point P in the undeformed state has undergone a displacement equal to the pitch p of the master grid and is now at P′. Then the center line of the first light fringe indicates a displacement in the direction of the principal section equal to p; if the center of the light fringe is considered to the right of the first one, the displacement is equal to 2 p, and for the n th fringe, the displacement is np. It can be concluded that the moire pattern gives the relative displacement in the direction of the principal section referred to the final or deformed shape of the body. The pattern indicates that point P′ in the deformed state of the model has undergone a displacement p.

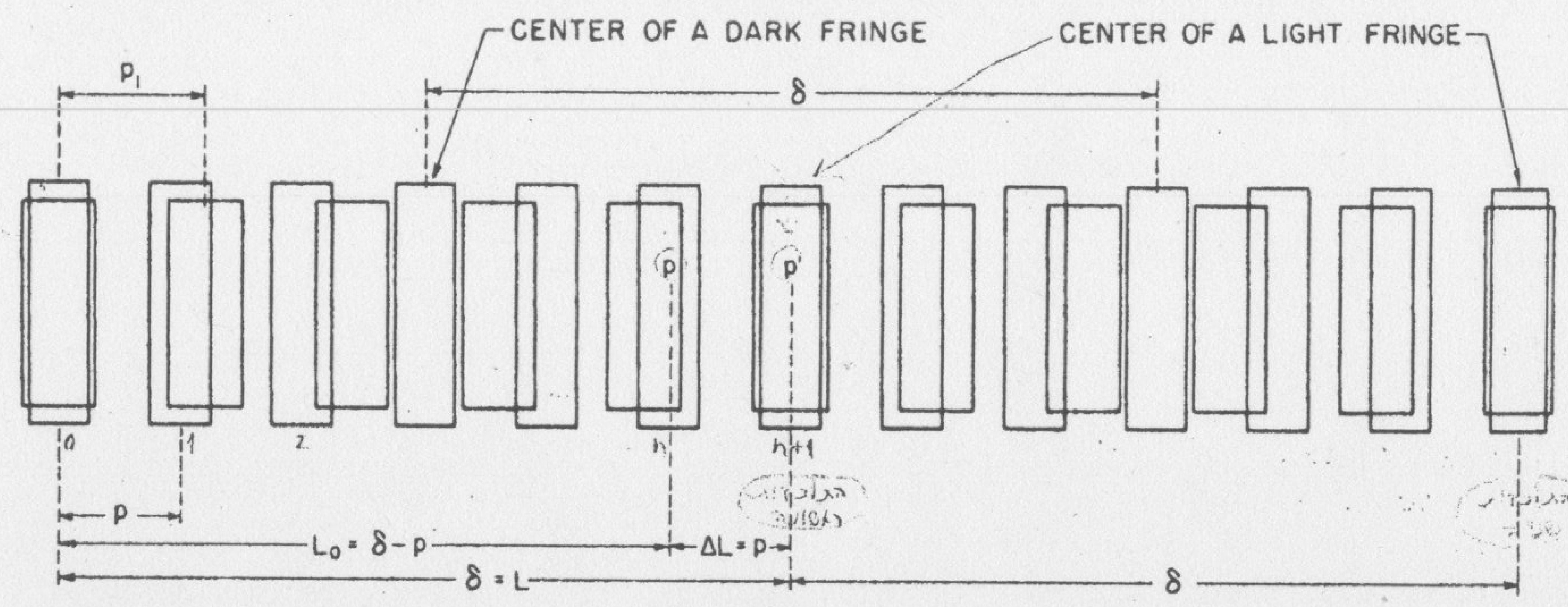

FIG. 1.—FORMATION OF MOIRE FRINGES IN A TENSILE SPECIMEN

In the mechanics of continua, the description of the displacements with the final coordinates of the points as independent variables is called Eulerian description. The so-called nominal reduction-area strain

$$ {}_{\epsilon}E = \frac{L - L_o}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

in which L_o is the initial length of the specimen and L, the final length, is consistent with the Eulerian description. Applying this definition to the pattern of Fig. 1

$$ {}_{\epsilon}E = \frac{p}{\delta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Consistent with the Lagrangian description is the so-called engineering or conventional strain

$$ {}_{\epsilon}L = \frac{L - L_o}{L_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

In terms of the moire data the engineering strain is

$$ {}_{\epsilon}L = \frac{p}{\delta - p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

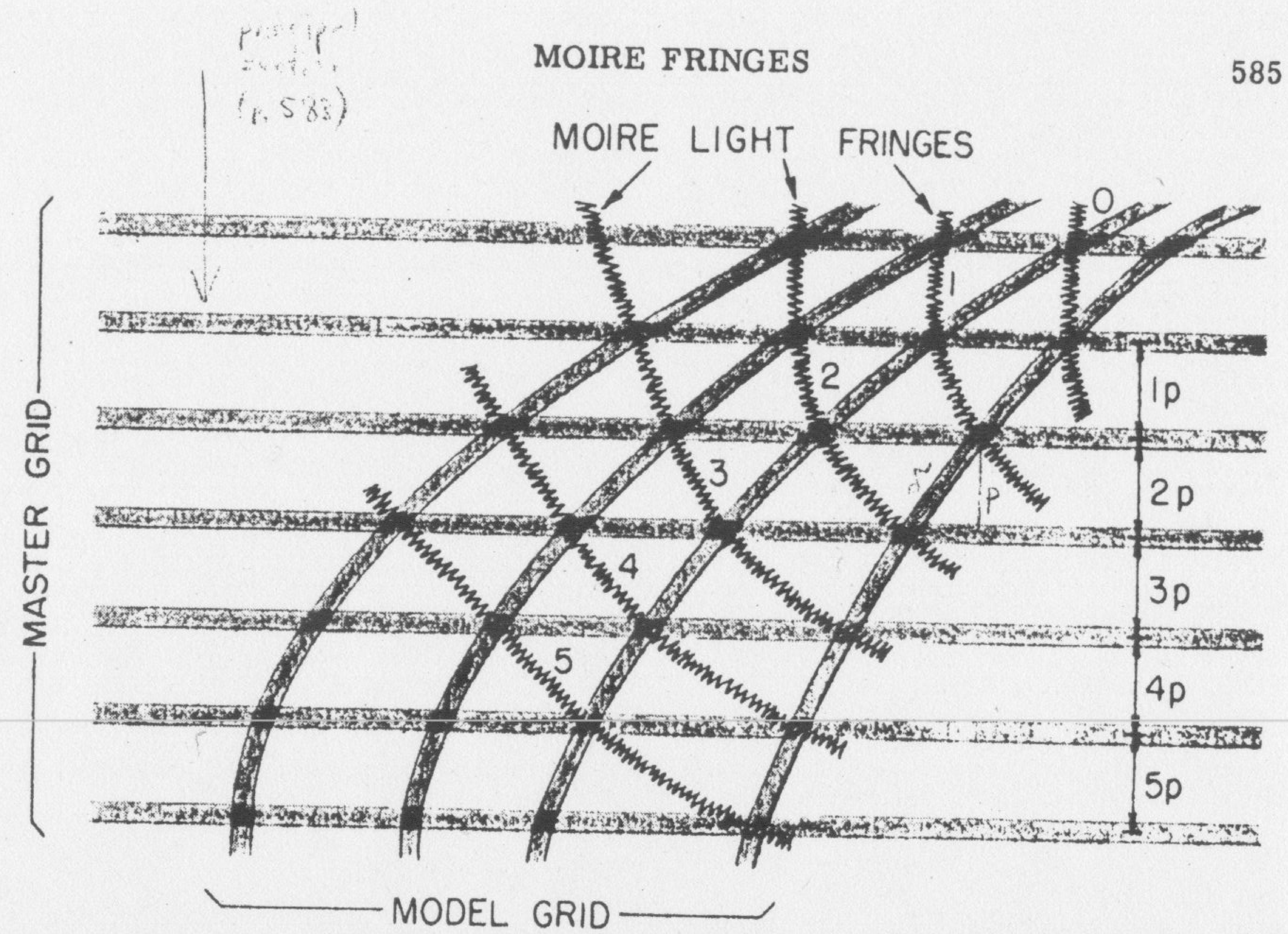

FIG. 2.—MOIRE FRINGES ARE THE LOCI OF THE POINTS PRESENTING THE SAME RELATIVE DISPLACEMENT IN THE DIRECTION NORMAL TO THE MASTER GRID

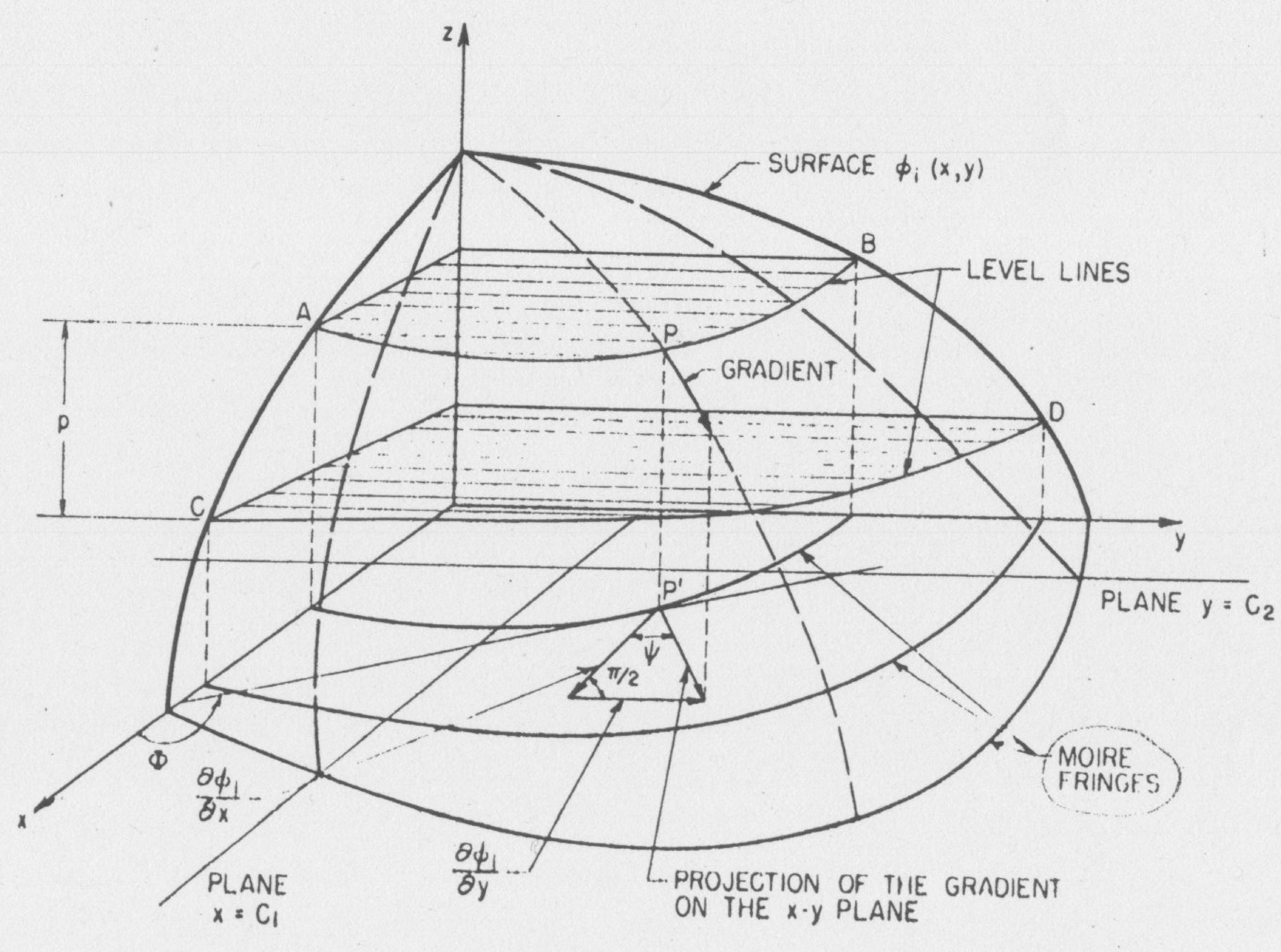

FIG. 3.—SURFACE OF THE COMPONENTS OF DISPLACEMENTS ϕ_i (x,y)

It can be easily shown that

$$\epsilon_L = \frac{\epsilon_E}{1 - \epsilon_E} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

As shown in Fig. 2, the Moire fringes are the loci of the points presenting the same relative displacement in the direction normal to the master grid. Call zero the first fringe to the right in Fig. 2, one the second fringe, two the third fringe, and so on. Assuming that the points of fringe zero do not experience any displacement, the fringe one is produced by the displacement p of the points of the model grid in the direction of the principal section, fringe two is produced by the displacement 2 p, and the nth fringe by the displacement n p. Then the moire fringes are the loci of points with a relative displacement in the direction of the principal section which is equal to an integer number times the pitch of the master grid. These displacements are given with respect to the deformed or final shape of the model. Each fringe is characterized by a parameter. This parameter is arbitrary since we are considering relative displacements. This parameter is called "order of the fringes" and is assigned the letter n.

In the following, a cartesian system of axes x and y is used as a reference system. The component of the displacement parallel to the x-axis is given the symbol u and the component of the displacement parallel to the y-axis is assigned the symbol v.

The component of displacement of a point in a two-dimensional continuous medium parallel to a reference direction is given by a function of two variables $\phi_i(x,y)$. Here i can take the values 1 or 2, 1 if the reference direction is the x-axis, 2 if the reference direction is the y-axis. The prior mentioned function has the following geometric interpretation. The function $z = \phi_i(x,y)$ in cartesian coordinates represents a surface (Fig. 3). This surface can be represented by the projection of its contour lines on the x-y plane. These lines are intersection of the surface with planes of equation z = n p. The resulting lines are given by

$$\phi_i(x,y) = k \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which k is a constant. This is also the property of the moire fringes.

To determine strains from displacements, it is necessary to compute the derivatives of the displacements. The rate of change of the surface of displacements $\phi_i(x,y)$ at a point is given by the gradient shown in Fig. 3. Because this gradient is also a function of x, y, it can be represented by its projection in the x-y plane.

The partial derivatives of $\phi_i(x,y)$ are given by

$$\frac{\partial \phi_i}{\partial x} = \text{grad}\, \phi_i(x,y) \cos \psi \quad \ldots\ldots\ldots\ldots (7a)$$

and

$$\frac{\partial \phi_i}{\partial y} = \text{grad}\, \phi_i(x,y) \sin \psi \quad \ldots\ldots\ldots\ldots (7b)$$

in which ψ is the angle between the x-axis and grad ϕ_i. The tangent of ψ is given by (Fig. 3)

$$\tan \psi = \frac{\dfrac{\partial \phi_i}{\partial y}}{\dfrac{\partial \phi_i}{\partial x}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

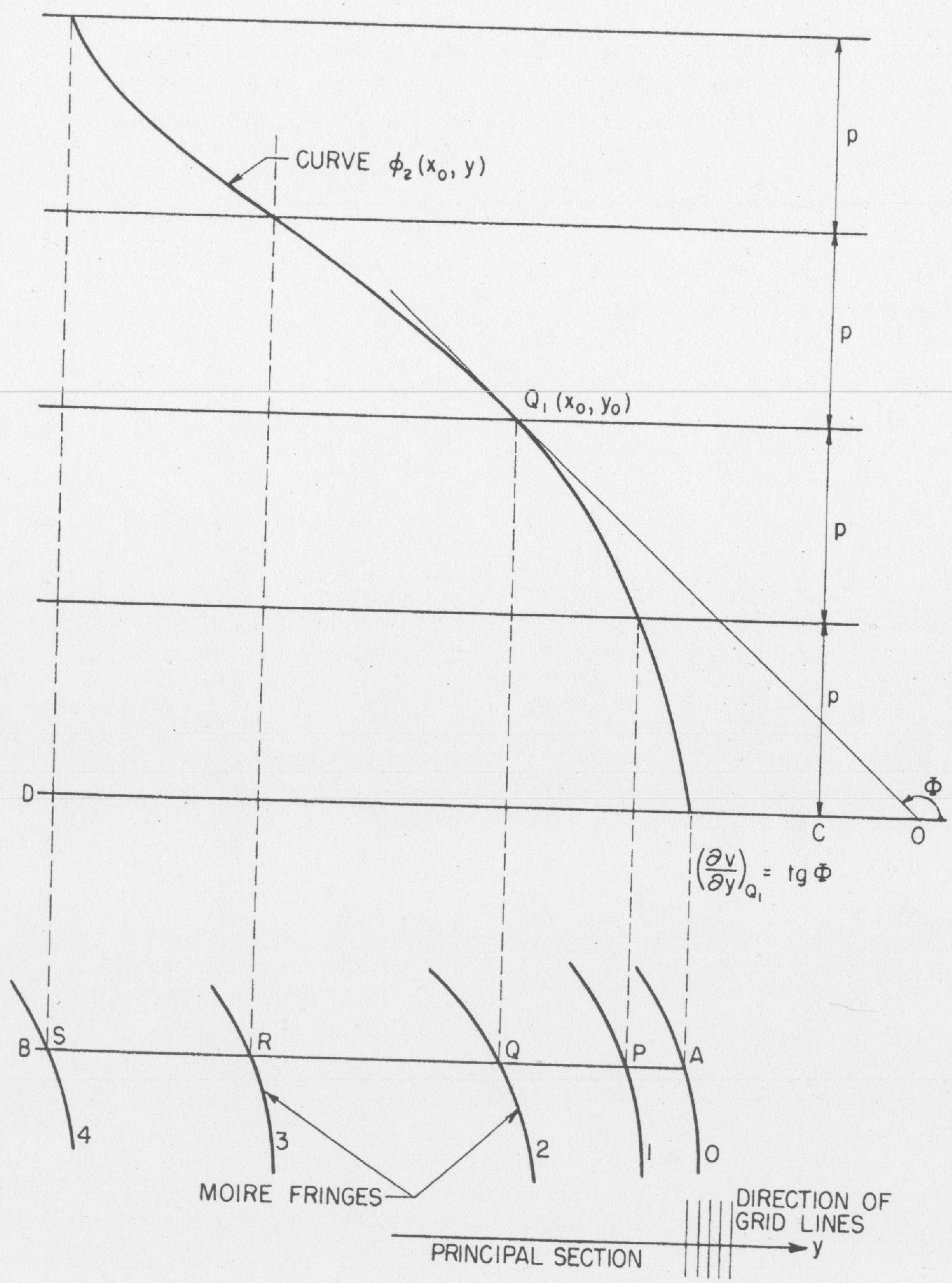

FIG. 4.—CONSTRUCTION OF THE INTERSECTION CURVE OF THE SURFACE $\phi_2(x,y)$ WITH THE PLANE $x = x_0$