Mathematics B

Senior
Syllabus
2001

Mathematics B Senior Syllabus 2001

This syllabus is approved for general implementation until 2008, unless otherwise stated.

To be used for the first time with Year 11 students in 2002.

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1 RATIONALE

Mathematics is an integral part of a general education. It enhances both an understanding of the world and the quality of participation in a rapidly changing society. The range of career opportunities requiring an appropriate level of mathematical competence is rapidly expanding into such areas as health, environmental science, economics and management, while remaining crucial in such fields as the physical sciences, engineering, accounting, computer science and information technology areas. Mathematics is essential for widespread computational and scientific literacy, for the development of a more technologically skilled work force, for the development of problem-solving skills and for the understanding and use of data and information to make well considered decisions. It is valuable to people individually and collectively, providing important tools which can be used at personal, civic, professional and vocational levels.

At the personal level, the most obvious use of mathematics is to assist in making informed decisions in areas as diverse as buying and selling, home maintenance, interpreting media presentations and forward planning. The mathematics involved in these activities includes analysis, financial calculation, data description, inference, number, quantification and spatial measurement. The generic skills developed by mathematics are also constantly used at the personal level.

At the civic, professional and vocational levels, the generic skills, knowledge and application of mathematics underpin most of the significant activities in industry, trade and commerce, social and economic planning, and communication systems. In such areas, the concepts and application of functions, rates of change, total change and optimisation are very important. The knowledge and skills developed in Mathematics B are essential for all quantitative activities in the above areas. Higher-order thinking skills developed in problem solving are essential for further development in any quantitative areas. The demand for those who are skilled mathematically continues to rise, emphasising the need for schools to provide the opportunity for students to experience a thorough and well-rounded education in mathematical ideas, concepts, skills and processes.

Mathematics has provided a basis for the development of technology. In recent times, the uses of mathematics have increased substantially in response to changes in technology. The more technology is developed the greater the level of mathematical skill required. Students must be given the opportunity to appreciate and experience the power which has been given to mathematics by this technology. Such technology should be used to encourage students in understanding mathematical concepts, allowing them to "see" relationships and graphical displays, to search for patterns and recurrence in mathematical situations, as well as to assist in the exploration and investigation of real and life-like situations.

Mathematics B aims to provide the opportunity for students to participate more fully in life-long learning. It provides the opportunity for student development of:

- knowledge, procedures and skills in mathematics
- mathematical modelling and problem-solving strategies
- the capacity to justify and communicate in a variety of forms.

Such development should occur in contexts. These contexts should range from purely mathematical through life-like to real, from simple through intermediate to complex, from basic to more advanced technology usage, and from routine rehearsed through to innovative. Of importance is the development of student thinking skills, as well as student recognition and use of mathematical patterns.

The intent of Mathematics B is to encourage students to develop positive attitudes towards mathematics by approaches involving exploration, investigation, application of knowledge and skills, problem solving and communication. Students will be encouraged to mathematically model, to work systematically and logically, to conjecture and reflect, and to justify and communicate with and about mathematics. The subject is designed to raise the level of competence in the mathematics required for informed citizenship and life-long learning, to increase students' confidence in using mathematics to solve problems, and especially to provide a basis for a wide range of further studies.

Mathematics B provides opportunities for the development of the key competencies in situations that arise naturally from the general objectives and learning experiences of the subject. The seven key competencies are: collecting, analysing and organising information; communicating ideas and information; planning and organising activities; working with others and in teams; using mathematical ideas and techniques; solving problems; using technology. (Refer to *Integrating the Key Competencies into the Assessment and Reporting of Student Achievement in Senior Secondary Schools in Queensland*, published by QBSSSS in 1997.)

2 GLOBAL AIMS

Having completed the course of study, students of Mathematics B should:

- have significantly broadened their mathematical knowledge and skills
- be able to recognise when problems are suitable for mathematical analysis and solution, and be able to attempt such analysis or solution with confidence
- be aware of the uncertain nature of their world and be able to use mathematics to assist in making informed decisions in life-related situations
- have experienced diverse applications of mathematics
- have positive attitudes to the learning and practice of mathematics
- comprehend mathematical information which is presented in a variety of forms
- communicate mathematical information in a variety of forms
- be able to use justification in and with mathematics
- be able to benefit from the availability of a wide range of technologies
- be able to choose and use mathematical instruments appropriately
- be able to recognise functional relationships and applications.

3 GENERAL OBJECTIVES

3.1 INTRODUCTION

The general objectives of this course are organised into four categories:

- Knowledge and procedures
- Modelling and problem solving
- Communication and justification
- Affective.

3.2 CONTEXTS

The categories of Knowledge and procedures, Modelling and problem solving, and Communication and justification incorporate contexts of application, technology, initiative and complexity. Each of the contexts has a continuum for the particular aspect of mathematics it represents. Mathematics in a course of study developed from this syllabus must be taught, learned and assessed using a variety of contexts over the two years. It is expected that all students are provided with the opportunity to experience mathematics along the continuum within each of the contexts outlined below.

Application

Students must have the opportunity to recognise the usefulness of mathematics through its application, and the beauty and power of mathematics that comes from the capacity to abstract and generalise. Thus students' learning experiences and assessment programs must include mathematical tasks that demonstrate a balance across the range from life-related through to pure abstraction.

Technology

A range of technological tools must be used in the learning and assessment experiences offered in this course. This ranges from pen and paper, measuring instruments and tables through to higher technologies such as graphing calculators and computers. The minimum level of higher technology appropriate for the teaching of this course is a graphing calculator.

Initiative

Learning experiences and the corresponding assessment must provide students with the opportunity to demonstrate their capability when dealing with tasks that range from routine and well rehearsed through to those that require demonstration of insight and creativity.

Complexity

Students must be provided with the opportunity to work on simple, single-step tasks through to tasks that are complex in nature. Complexity may derive from either the nature of the concepts involved or from the number of ideas or techniques that must be sequenced in order to produce an appropriate conclusion.

3.3 OBJECTIVES

The general objectives for each of the categories in section 3.1 are detailed below. These general objectives incorporate several key competencies. The first three categories of objectives, Knowledge and procedures, Modelling and problem solving, and Communication and justification, are linked to the exit criteria in section 7.3.

3.3.1 Knowledge and procedures

The objectives of this category involve recalling and using results and procedures within the contexts of Application, Technology, Initiative and Complexity (see section 3.2).

By the conclusion of the course, students should be able to:

- recall definitions and results
- access and apply rules and techniques
- demonstrate number and spatial sense
- demonstrate algebraic facility
- demonstrate an ability to select and use appropriate technology such as calculators, measuring instruments and tables
- demonstrate an ability to use graphing calculators and/or computers with selected software in working mathematically
- select and use appropriate mathematical procedures
- work accurately and manipulate formulae
- recognise that some tasks may be broken up into smaller components
- transfer and apply mathematical procedures to similar situations.

3.3.2 Modelling and problem solving

The objectives of this category involve the uses of mathematics in which the students will model mathematical situations and constructs, solve problems and investigate situations mathematically within the contexts of Application, Technology, Initiative and Complexity (see section 3.2).

By the conclusion of the course, students should be able to demonstrate the category of modelling and problem solving through:

Modelling

- understanding that a mathematical model is a mathematical representation of a situation
- identifying the assumptions and variables of a simple mathematical model of a situation
- forming a mathematical model of a life-related situation
- deriving results from consideration of the mathematical model chosen for a particular situation
- interpreting results from the mathematical model in terms of the given situation
- exploring the strengths and limitations of a mathematical model.

Problem solving

- interpreting, clarifying and analysing a problem
- using a range of problem-solving strategies such as estimating, identifying patterns, guessing and checking, working backwards, using diagrams, considering similar problems and organising data
- understanding that there may be more than one way to solve a problem
- selecting appropriate mathematical procedures required to solve a problem
- developing a solution consistent with the problem
- developing procedures in problem solving.

Investigation

- identifying and/or posing a problem
- exploring a problem and from emerging patterns creating conjectures or theories
- reflecting on conjectures or theories making modifications if needed
- selecting and using problem-solving strategies to test and validate any conjectures or theories
- extending and generalising from problems
- developing strategies and procedures in investigations.

3.3.3 Communication and justification

The objectives of this category involve presentation, communication (both mathematical and everyday language), logical arguments, interpretation and justification of mathematics within the contexts of Application, Technology, Initiative and Complexity (see section 3.2).

Communication

By the conclusion of the course, students should be able to demonstrate communication through:

- organising and presenting information
- communicating ideas, information and results appropriately
- using mathematical terms and symbols accurately and appropriately
- using accepted spelling, punctuation and grammar in written communication
- understanding material presented in a variety of forms such as oral, written, symbolic, pictorial and graphical
- translating material from one form to another when appropriate
- presenting material for different audiences in a variety of forms (such as oral, written, symbolic, pictorial and graphical)
- recognising necessary distinctions in the meanings of words and phrases according to whether they are used in a mathematical or non-mathematical situation.

Justification

By the conclusion of this course, the student should be able to demonstrate justification through:

- developing logical arguments expressed in everyday language, mathematical language or a combination of both, as required, to support conclusions, results and/or propositions
- evaluating the validity of arguments designed to convince others of the truth of propositions
- justifying procedures used
- recognising when and why derived results are clearly improbable or unreasonable
- recognising that one counter example is sufficient to disprove a generalisation
- recognising the effect of assumptions on the conclusions that can be reached
- deciding whether it is valid to use a general result in a specific case
- using supporting arguments, when appropriate, to justify results obtained by calculator or computer

3.3.4 Affective

Affective objectives refer to the attitudes, values and feelings which this subject aims at developing in students. **Affective objectives are not assessed for the award of exit levels of achievement.**

By the conclusion of the course, students should appreciate the:

- diverse applications of mathematics
- precise language and structure of mathematics
- diverse and evolutionary nature of mathematics and the wide range of mathematics-based vocations
- contribution of mathematics to human culture and progress
- power and beauty of mathematics.

4 LANGUAGE EDUCATION

Language is the means by which meaning is constructed and shared, and communication is effected. It is the central means by which teachers and students learn. Mathematics B requires students to use language in a variety of ways—mathematical, spoken, written, graphical, symbolic. The responsibility for developing and monitoring students' abilities to use effectively the forms of language demanded by this course rests with the teachers of mathematics. This responsibility includes developing students' abilities to:

- select and sequence information
- manage the conventions related to the forms of communication used in Mathematics B (such as short responses, reports, multimedia presentations, seminars)
- use the specialised vocabulary and terminology related to Mathematics B
- use language conventions related to grammar, spelling, punctuation and layout.

The learning of language is a developmental process. When writing, reading, questioning, listening and talking about mathematics, teachers and students should use the specialised vocabulary related to Mathematics B. Students should be involved in learning experiences that require them to comprehend and transform data in a variety of forms and, in so doing, use the appropriate language conventions. Some language forms may need to be explicitly taught if students are to operate with a high degree of confidence within mathematics.

Assessment instruments should use format and language that are familiar to students. They should be taught the language skills necessary to interpret questions accurately and to develop coherent, logical and relevant responses. Attention to language education within Mathematics B should assist students to meet the language components of the exit criteria, especially the criterion Communication and justification.

5 ORGANISATION

5.1 INTRODUCTION

The subject matter has been organised into seven topics, which are discussed in detail in section 6. All topics must be studied. The order in which topics are presented *does not imply* a teaching sequence. The topics are:

- Introduction to functions
- Rates of change
- Periodic functions and applications
- Exponential and logarithmic functions and applications
- Optimisation using derivatives
- Introduction to integration
- Applied statistical analysis

Throughout the course, certain fundamental knowledge and procedures are required. Some of these have been identified and listed under the heading “Maintaining basic knowledge and procedures” in appendix 1. Time should be provided to revise the fundamental knowledge and procedures within topics as they are required. Some relevant learning experiences are also listed in appendix 1. This maintenance takes time, and should be allowed for in designing the course sequence.

5.2 TIME ALLOCATION

The minimum number of hours of timetabled school time including assessment for a course of study developed from this syllabus is 55 hours per semester.

Notional times are given for each topic. These times are included as a guide, and minor variations of these times may occur.

5.3 SEQUENCING

After considering the subject matter and the appropriate range of learning experiences to enable the general objectives to be achieved, a *spiralling* and *integrated* sequence should be developed which allows students to see links between the different topics of mathematics included in the course rather than seeing them as discrete.

The order in which the topics are presented in the syllabus is not intended to indicate a teaching sequence, but some topics include subject matter that is developed and extended in the subject matter of other topics. The school’s sequence should be designed so that the subject matter is revisited and spiralled to allow students to internalise their knowledge before developing it further.

The following guidelines for the sequencing of subject matter should be referred to when developing a sequence for the course.

- No subject matter should be studied before the relevant prerequisite material has been covered.
- The sequencing of subject matter may depend on the importance placed by schools on students being able to make decisions about the mathematics suitable to their needs. For example, the Year 11 sections of the sequences for Mathematics A and Mathematics B may need to be developed together.

- The sequences for Mathematics B and Mathematics C should be developed together to ensure that prerequisite material is covered appropriately.
- Subject matter across topics should be linked when possible.
- Sequencing may be constrained by a school's ability to provide physical resources.
- Time will be needed for maintaining basic knowledge and procedures.

5.4 TECHNOLOGY

The advantage of mathematics-enabled technology in the mathematics classroom is that it allows for the exploration of the concepts and processes of mathematics. Graphing calculators, for example, let students explore and investigate; they assist students with the understanding of concepts, and they complement traditional approaches to teaching.

More specifically, the mathematics-enabled technology allows students to tackle more diverse, life-related problems. Real-life optimisation problems are more easily solved with this technology. It may be used in statistics to investigate larger datasets and rapidly produce a variety of graphical displays and summary statistics, thus freeing students to look for patterns, to detect anomalies in the data and to make informed comments.

The minimum level of higher technology appropriate for the teaching of this course is a graphing calculator. Although student ownership of graphing calculators is not a requirement, *student access* to appropriate technology *is necessary* to enable students to develop the full range of skills required for successful problem solving during their course of study. Use of graphing calculators or computers will significantly enhance the learning outcomes of this syllabus.

6 TOPICS

6.1 INTRODUCTION

Each topic has a focus statement, subject matter and suggested learning experiences which, taken together, clarify the scope, depth and emphasis for the topic.

Focus

This section highlights the intent of the syllabus with respect to the topic and indicates how students should be encouraged to develop their understanding of the topic.

Subject matter

This section outlines the subject matter to be studied in the topic. All subject matter listed in the topic must be included, but the order in which it is presented is not necessarily intended to imply a teaching sequence.

Learning experiences

This section provides some suggested learning experiences which may be effective in using the subject matter to achieve the general objectives of the course. The numbers provided with the subject matter link to suggested learning experiences. Included are experiences which involve life-related applications of mathematics with both real and simulated situations, use of instruments and opportunities for modelling and problem solving. The listed learning experiences may require students to work individually, in small groups or as a class.

The learning experiences are suggestions only and are not prescriptive. Schools are encouraged to develop further learning experiences, especially those which relate to the school's location, environment and resources. Students should be involved in a variety of activities including those which require them to write, speak, listen or devise presentations in a variety of forms. A selection of learning experiences that students will encounter should be shown in the work program.

N.B. The learning experiences must provide students with the opportunity to experience mathematics along the continuum within each of the contexts.

Some of the key competencies, predominantly Using mathematical ideas and techniques, Solving problems, and Using technology, are to be found in the learning experiences within the topic areas. Opportunities are provided for the development of key competencies in contexts that arise naturally from the general objectives and learning experiences of the subject. The key competencies of Collecting, analysing and organising information, Planning and organising activities, and Working with others and in teams, also feature in some of the learning experiences.

6.2 THE TOPICS

The order in which topics and items within topics are given should not be seen as implying a teaching sequence.

Introduction to functions (notional time 35 hours)

Focus

Students are encouraged to develop an understanding and appreciation of relationships between variables, be conversant with the three methods of representation (algebraic, graphical, numerical) and interrelate these methods in a variety of modelling situations, ranging from life-related to abstract. Emphasis should be placed on the recognition of functions, sketching, investigating shapes and relationships, and the general forms of functions. The use of technology is expected to assist students in these processes.

Subject matter

- concepts of function, domain and range (suggested learning experiences (SLEs) 1, 2, 3,4)
- mappings, tables and graphs as representations of functions and relations (SLEs 1, 2, 3, 4)
- graphs as a representation of the points whose coordinates satisfy an equation (SLEs 1, 4, 5, 7, 9, 16)
- distinction between functions and relations (SLEs 1, 2, 7, 14, 15)
- distinctions between continuous functions, discontinuous functions and discrete functions (SLEs 1, 3)
- practical applications of linear functions including:
  - direct variation
  - simple interest as an arithmetic progression
  - linear relationships between variables

  (SLEs 8, 13, 14, 19, 20)
- practical applications of quadratic functions, the reciprocal function and inverse variation (SLEs 8, 9, 10, 12, 13,17, 18)
- relationships between the graph of $f(x)$ and the graphs of $f(x)+ a$, $f(x + a)$, $a f(x)$, $f(ax)$ for both positive and negative values of the constant a (SLE 5)
- general shapes of graphs of absolute value functions, the reciprocal function and polynomial functions up to and including the fourth degree (SLEs 4, 5, 6)
- algebraic and graphical solution of two simultaneous equations in two variables (to be applied only to linear and quadratic functions) (SLE 8)
- concept of the inverse of a function (SLE 12)
- composition of two functions (SLE 11)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. find the domain and range of functions in both mathematical and life-related contexts, given data in a variety of forms such as graphs, tables of values, mathematical expressions or descriptions of situations
2. use the vertical line test to determine if a relation is a function
3. draw graphs of step functions such as postal charges for packages against the weight of the packages, telephone charges per unit time against distance
4. examine the general shapes of polynomial functions of the type $y = x^n$, $n = 2, 3, 4$
5. using $f(x) = x^n$, for $n = 1$ *to* 4; investigate the relationships between the graph of $f(x)$ and the graphs of $f(x) + a$, $f(x + a)$, $a f(x)$ and $f(a x)$, by means of a graphing calculator and algebraic methods
6. investigate the number of times a straight line intersects the graph of a polynomial of degree n, $n = 1$ to 4
7. investigate the *shapes* of common relations which are not functions, for example, circles and ellipses
8. solve simultaneous equations which model life-related situations
9. locate the position algebraically, numerically and graphically of the highest point on a projectile path defined by a quadratic function
10. use quadratic functions in life-related situations such as: find the dose of a chemical needed to obtain a 50% kill of insects, if the percentage of insects killed, p, is related to the dose level, x, by the equation, $p = a + bx + cx^2$ where a, b and c are constants with values such as 2, -1 and 3
11. devise a procedure for producing the graph, table or algebraic expression for the sum of two functions $f(x) + g(x)$, and the composite function $f[g(x)]$ given graphs, tables or algebraic expressions of two functions, $f(x)$ and $g(x)$
12. use reciprocal relationships in practical contexts such as conversion from miles per gallon to litres per 100 kilometres
13. use a graphing calculator to investigate possible functions for data
14. use a graphing calculator to investigate the shapes of different functions
15. investigate the difficulties encountered in using a graphing calculator or computer software to draw graphs of relations which are not functions
16. derive the formula for the solution of a general quadratic equation
17. by approximating the surface area of a living being as a function of the square of its linear dimension, and the volume as a function of the cube of its linear dimension, investigate the limitations on the sizes of living beings in different environments
18. investigate how closely a quadratic function approximates (a) the shape of a hanging chain and (b) the curve of the cables of a suspension bridge; suggest an explanation for any difference between the two results
19. use linear functions in investigating break-even analysis
20. calculate the amount of simple interest generated over a given period using a graphing calculator or a suitable computer software package; plot these discrete values and generate a function which can be used to represent all values.

Rates of change (notional time 30 hours)

Focus

Students are encouraged to develop an understanding of average and instantaneous rates of change and of the derivative as a function. This understanding should be developed using both algebraic and graphical approaches. Students should be expected to apply the rules for differentiation and interpret the results. The use of technology is expected to assist students in these processes.

Subject matter

- concept of rate of change (SLEs 1, 2, 3, 4)
- calculation of average rates of change in both practical and purely mathematical situations (SLEs 1, 2, 3)
- interpretation of the average rate of change as the gradient of the secant (SLEs 1, 2)
- intuitive understanding of a limit (SLEs 1, 2, 3, 4)

N.B. Calculations using limit theorems are not required.

- definition of the derivative of a function at a point (SLEs 3, 4)
- derivative of simple algebraic functions from first principles (SLEs 3, 4)
- rules for differentiation including:

$\frac{d}{dp} p^{n}$ for rational values of n

$\frac{\mathrm{d}}{\mathrm{dr}}[k\,f(r)]$

$\frac{\mathrm{d}}{\mathrm{ds}}[f(s)+g(s)]$

$\frac{d}{dt}[f(t)\,g(t)]$

$\frac{d}{dx} f[g(x)]$

- evaluation of the derivative of a function at a point (SLEs 3, 4)
- interpretation of the derivative as an instantaneous rate of change (SLEs 1, 2, 5, 10)
- interpretation of the derivative as the gradient function (SLEs 1, 2, 4, 6)
- practical applications of instantaneous rates of change (SLEs 1–5, 7–11)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. determine average and instantaneous speeds from a distance–time graph
2. determine average and instantaneous accelerations from a velocity–time graph
3. use a numerical technique to estimate a limit or an average rate of change
4. graph a function and its gradient function; relate the features of each to the other

5. determine the instantaneous rate of change of a variable with respect to another variable in life-related situations given the mathematical model; such as:
   - the rate of population change with respect to time
   - the rate of change of resistance in a wire with respect to temperature
   - the rate of change of the surface area of an object with respect to volume
   - the rate of change of a cost function with respect to the number of items produced
6. find the equation of the tangent to a curve under various given conditions
7. compare the evaporation rate of water in open containers of varying cross-sections
8. discuss how instantaneous rates of change may be used to measure the sensitivity of the human body to various stimulants or sedatives
9. calculate the effect of small measurement errors in the calculation of a volume; relate this to a graph
10. investigate how the rate of change of air temperature varies during the daylight hours when the relationship is approximated by a quadratic
11. investigate the concept of marginal costs related to the derivative of a cost function

Periodic functions and applications (notional time 30 hours)

Focus

Students are encouraged to develop an understanding and appreciation of periodic functions, be conversant with the three methods of representation (algebraic, graphical, numerical) and interrelate these methods in a variety of modelling situations, ranging from life-related to abstract. Emphasis should be placed on the recognition of periodic functions, sketching, investigating shapes and relationships, and the general forms of periodic functions. The use of technology is expected to assist students in these processes. Trigonometric identities need not be developed beyond the Pythagorean identity.

Subject matter

- trigonometry including the definition and practical applications of the sine, cosine and tangent ratios (SLEs 1, 2)
- simple practical applications of the sine and cosine rules (the ambiguous case is not essential) (SLEs 1, 2)
- definition of a periodic function, the period and amplitude (SLEs 3, 4, 15)
- definition of a radian and its relationship with degrees (SLE 6)
- definition of the trigonometric functions sin, cos and tan of any angle in degrees and radians (SLEs 3, 6, 11)
- graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ (SLEs 3, 6, 10, 19, 20)
- significance of the constants A, B, C and D on the graphs of $y = A \sin(Bx+C)+D$, $y = A \cos(Bx+C) + D$ (SLEs 5, 9, 11)
- applications of periodic functions (SLEs 4, 5, 7, 10, 13, 14, 15, 18, 20, 21)
- Pythagorean identity $\sin^2 x + \cos^2 x = 1$ (SLE 8)
- solution of simple trigonometric equations within a specified domain (SLE 8)
- derivatives of functions involving $\sin x$ and $\cos x$ (SLEs 7, 10, 16)
- applications of the derivatives of $\sin x$ and $\cos x$ in life-related situations (SLEs 7, 10)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. use sine, cosine and tangent ratios to determine lengths/distances and magnitudes of angles in life-related situations such as:
   - guy ropes for tents or flagpoles
   - distances across rivers and valleys
2. use sine and cosine rules to solve triangles in two- and three-dimensional contexts and determine lengths/distances and the magnitude of angles in life-related situations such as:
   - the distance between two ships given the distances and bearings to a fixed point
   - the height of a tower given the direction and angles of inclination from two fixed locations of known distance apart
3. investigate the repetitive nature of daily temperature and human pulse
4. find the period, amplitude and frequency of periodic functions involving sine and cosine given their graphs and/or equations
5. find the period, amplitude and frequency of trigonometric functions which are used to model phenomena such as biorhythms, tide heights
6. using the concept of the unit circle:
   - evaluate the trigonometric functions sin, cos and tan of any angle in degrees and radians
   - explore the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$
7. investigate the periodic motion of a mass on the end of a spring; given the mathematical model of the displacement from a fixed position, find mathematical representations of its velocity and acceleration; solve these simple trigonometric equations to find displacement, velocity and acceleration at a given time during the periodic motion
8. find solutions of trigonometric equations for $-2\pi \le \theta \le 2\pi$ such as $2 \sin \theta = -0.7$, $2 \sin^2 \theta = \cos \theta$
9. investigate the effect of the constants A, B, C and D on the graphs of $y = A \sin (Bx+C) + D$, $y = A \cos (Bx+C) + D$ using graphing calculators
10. sketch the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ for any angle in degrees ($-720 \le x \le 720$) and in radians in the range of $-4\pi \le x \le 4\pi$
11. calculate the rate at which the water level is changing on a vertical marker given a sine function as a model of tide height
12. use the graph of a sinusoidal function to develop the corresponding algebraic form
13. investigate the periodic motion of a pendulum; mathematically model the motion of a pendulum using trigonometric equations
14. plot the tide heights at a specified point over a 24-hour period
15. explore the period, amplitude and frequency of periodic (oscillatory) phenomena including planetary motion, hormone cycles, ECGs, Halley's comet, reciprocating motion
16. develop the derivative of $\sin x$ from the graph of $\cos x$ and vice versa

17. plot, as a function of the date, the elapsed time between sunrise and sunset for capital cities in Australia
18. investigate the path of a point on a moving bicycle wheel
19. graph the slopes of the tangent of a sine curve
20. investigate daily electrical energy consumption over a period of time
21. use computer software or a graphing calculator to investigate more complicated periodic functions, for example,

$$\sin x + \tfrac{1}{2}\sin 2x + \tfrac{1}{3}\sin 3x \cdots$$

22. use a graphing calculator or computer package to investigate the superposition of two almost equal frequencies of sinusoidal waves to produce "beats", and identify the properties of the resulting function

Exponential and logarithmic functions and applications (notional time 35 hours)

Focus

Students are encouraged to develop an understanding and appreciation of exponential and logarithmic functions and the relationships between them. They should be conversant with the three methods of representation (algebraic, graphical, numerical). Emphasis should be placed on the application of these functions to solve problems in a range of life-related situations (e.g. finance and investment, growth and decay). The use of technology is expected to assist students in these processes.

It is not intended that a great emphasis be placed on simplification of expressions involving indices or logarithms.

Subject matter

- index laws and definitions (SLE 1)
- definitions of a^x and $\log_a x$, for $a > 1$ (SLE 1)
- logarithmic laws and definitions (SLEs 1, 2, 16)
- definition of the exponential function e^x (SLEs 4, 6)
- graphs of, and the relationships between, $y = a^x$, $y = \log_a x$ for $a = e$ and other values of a (SLEs 3, 6, 12, 14, 17)
- graphs of $y = e^{kx}$ for $k \neq 0$ (SLEs 3, 6)
- solution of equations involving indices (SLEs 5, 8, 9, 11)
- use of logarithms to solve equations involving indices (SLEs 8, 9, 11)
- development of algebraic models from appropriate datasets using logarithms and/or exponents (SLEs 2, 5, 7)
- derivatives of exponential and logarithmic functions for base e (SLEs 3, 6, 7
- applications of exponential and logarithmic functions, and the derivative of exponential functions (SLEs 2, 5, 7, 10, 11, 13)
- applications of geometric progressions to compound interest including past, present and future values (SLEs 18, 19, 24)
- applications of geometric progressions to annuities and amortising a loan (SLEs 20, 21, 22, 23, 25, 26, 27, 29)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. use the identities $x = a^{\log_a x}$ and $y = \log_a(a^y)$ to investigate the logarithmic and index laws and definitions
2. investigate life-related situations that can be modelled by simple exponential functions, for example, applications of Newton's Law of Cooling, concentration against time in chemistry, carbon dating in archaeology and decrease of atmospheric pressure with altitude
3. use a graphing calculator or computer software to investigate the shapes of exponential and logarithmic functions and their derivatives
4. investigate the derivative of the function a^x and identify the significance of the exponential constant e
5. investigate the role of indices (or powers) in the establishment of formulae in financial matters such as compound interest, time required to repay a loan for given repayments and rate of interest
6. graph the derivative of a growth function or a decay function and interpret the result
7. investigate change such as radioactive decay, growth of bacteria, or growth of an epidemic, where the rate of change is proportional to the amount of material left or the current population size
8. consider that the proportion of a radioactive material remaining after time t has elapsed is e^{-kt}, where k is a positive constant; investigate the relationship between k and the half-life of the material
9. use logarithms to solve equations involving indices such as:
   - consider the probability of obtaining at least one head in n tosses of a fair coin, and then find how many tosses are required for this probability to be at least 0.9
   - consider the time taken for an investment to double for a given compound interest rate
10. consider that the difference between assuming that running time is proportional to distance and assuming that log (running time) is proportional to log (distance); interpret the value of the constant of proportionality in the second model; world record times for either male or female athletes may be of interest in this context.
11. investigate logarithmic scales, for example, decibels, Moh's scale of hardness, Richter scale and pH
12. plot the logarithms of some apparent growth functions, for example, car registrations over time, to produce a near-linear graph
13. investigate the time at which the quantity of the intermediate substance reaches a maximum in a simple two-step radioactive decay; i.e., the original substance decays to an intermediate substance which in turn decays to an inert substance
14. plot the logarithm of the population of Australia at censuses (a) from 1891 to 1933 (b) from 1947 to 1971 and (c) from 1971 to 1991; recognise that the linear tendencies of the plot indicate power/exponential relationships in the original

15. discuss the significance of constants such as 2.4 in advertisements about the size of Australian families
16. explain the workings of a slide rule
17. by graphing the logarithm of the distance of planets from the sun against the logarithm of the time of revolution about the sun, investigate the relationship between the variables
18. use a graph and/or a table of values created by a spreadsheet or a calculator to compare interest accrued and yearly balances for an investment over five years where (a) a flat rate applies, and (b) compound interest applies
19. develop the formula for compound interest; solve a range of financial problems involving this formula
20. construct a loan repayment schedule showing principal, interest charged and balance owing
21. develop and use formulae for the amount (future value) and present value of an annuity using geometric series
22. calculate mortgage repayments of a home loan given the interest rate and the term of the loan
23. calculate the term of the loan given the interest rate and the mortgage payment
24. calculate the effective interest rate given an annual nominal interest rate and the compounding period
25. use a spreadsheet or a calculator to examine the effect of changing the interest rate, term or repayment on a housing loan
26. prepare a list of charges, interest rates and conditions for investment in different commercial institutions; justify the selection of one of these to match a given financial scenario
27. investigate the use of tables by financial institutions in annuity calculations
28. discuss responses of newspaper financial columnists to financial questions
29. solve problems involving sinking funds designed for future purchases
30. investigate the effect of taxation on a variety of investment situations such as superannuation and other cash investments

Optimisation using derivatives (notional time 25 hours)

Focus

Students are encouraged to develop an understanding of the use of differentiation as a tool in a range of situations which involve the optimisation of continuous functions. This understanding should be developed using both algebraic and graphical approaches. The use of technology is expected to assist students in these processes.

Subject matter

- positive and negative values of the derivative as an indication of the points at which the function is increasing or decreasing (SLEs 10, 1*2*)
- zero values of the derivative as an indication of stationary points (SLE 4)
- concept of relative maxima and minima and greatest and least value of functions (SLE 2)
- relationships between the graph of a differentiable function, and the graph of the derivative of that function (SLE 10)
- methods of determining the nature of stationary points (SLEs 1–9, 13)
- greatest and least values of a function in a given interval (SLEs 1–9, 12, 13)
- recognition of the problem to be optimised (maximised or minimised) (SLEs 1, 2, 5, 6, 7, 8, 9, 13)
- identification of variables and construction of the function to be optimised (SLEs 1–9, 11)
- applications of the derivative to optimisation in life-related situations (SLEs 1, 2, 5, 6, 7, 8, 9)
- interpretation of mathematical solutions and their communication in a form appropriate to the given problem (SLEs 1, 2, 5, 6, 7, 8, 9)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. use life-related situations such as enclosing a rectangular area with a fixed length of fencing, to demonstrate the need for calculus to determine optimal values
2. interpret a table of data values as to the rate of increase, greatest and least values taken by a smoothly varying function
3. interpret a graph as to rates of increase or decrease of a function; relate these observations to the behaviour of the derivative
4. use zero values of the derivative to find local optima and points of horizontal inflection in curve sketching of simple functions (the solutions should not be dependent on the factor theorem)
5. solve shortest distance problems in two-dimensional geometry such as finding the shortest distance from a given point to a point on a given straight line through the origin (geometry of the straight line and circle is all that will be assumed)
6. investigate applications of shortest distance problems such as a water tap inside the boundary of a property is to be connected to a main which runs along the boundary of the property where the boundary is to be assumed straight; find the shortest length of pipe needed to connect the tap to the main
7. maximise areas subject to restrictions on their perimeters; it should *not* be assumed that formulae for areas and perimeters other than rectangles and circles will be known; elimination of constraint variables will involve only simple algebra
8. minimise perimeters subject to constraints on area; it should *not* be assumed that formulae for areas and perimeters other than rectangles and circles will be known; elimination of constraint variables will involve only simple algebra

9. investigate situations finding optimal quantities and/or optimal costs such as the optimal use of materials used for the manufacture of various containers of simple shapes
10. consider the flight path of a projectile launched at an arbitrary angle to the vertical from zero height, falling back to earth under gravity; neglect any forces other than gravity and ignore the curvature of the earth and its rotation; investigate the maximum height and maximum horizontal range for different launching angles and initial velocity (the required equations of trajectory should be given)
11. investigate examples of shortest travel time, including the path of a light ray upon reflection from a plane surface, and examples in which the velocity varies according to the mode of travel
12. investigate the effect on the maximum height and horizontal range if the projectile is launched at some height above the horizontal reference level
13. use computer software and graphing calculators in the investigation of optimal points and optimal values in life-related situations

Introduction to integration (notional time 25 hours)

Focus

Students are encouraged to develop an understanding of the concept of integration as a process by which a "whole" can be obtained from the summation of a large number of parts. This understanding should be developed using both numerical and analytical techniques, in life-related situations as well as in purely mathematical situations. The emphasis in the topic should be on the applications of integration rather than on developing a large repertoire of techniques. The use of technology is expected to assist students in these processes.

Subject matter

- definition of the definite integral and its relation to the area under a curve (SLEs 1–7)
- the value of the limit of a sum as a definite integral (SLEs 1–7)
- definition of the indefinite integral (SLEs 1–8)
- rules for integration including (SLEs 1–8)

  $\int a\,f(x)dx$

  $\int[f(x) \pm g(x)]dx$

  $\int f(ax+b)dx$
- indefinite integrals of simple polynomial functions, simple exponential functions, *sin (ax + b), cos (ax + b)* and $\dfrac{1}{a\,x+b}$ (SLEs 1–5, 7, 8)
- use of integration to find area (SLEs 1, 2, 4, 5, 7, 8, 11)
- practical applications of the integral (SLEs 1, 3, 4, 5, 7, 8, 10, 12, 16)
- trapezoidal rule for the approximation of a value of a definite integral numerically (SLEs 4, 5, 6, 10, 13, 14, 15)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. use integration to calculate the areas of regions by finding the area under the curve for suitably chosen functions including functions which intersect the x axis within the given interval
2. use integration to calculate the area enclosed by two intersecting curves
3. investigate the motion of a falling body in terms of its displacement and velocity as functions of time neglecting air resistance
4. from a velocity time function (or graph) determine a distance or displacement function (or graph); interpret the result
5. from an acceleration time function (or graph) determine a velocity function (or graph); interpret the result
6. apply the trapezoidal rule to integrals of known values and compare the approximate solutions with the exact solutions; investigate the variation in accuracy with the number of strips chosen
7. calculate the volume of simply shaped objects by summing the volumes of a set of thin slabs of simple geometry (the use of the formula for volumes of solids of revolution is not intended)
8. determine the function Q given a rate of growth or decay of some quantity Q such as quantity of bacteria, size of epidemic, drug concentration, population size as a simple function of time or Q alone; interpret
9. determine the volume of water that could be contained in a trough of given length and parabolic cross-section
10. find as many ways as you can of calculating the volume of objects such as a touch football or a loaf of bread
11. investigate the Monte Carlo technique for definite integrals and area
12. investigate the method for finding the volume of timber in a tree trunk by using several functions to describe the shape of the trunk; a typical taper equation to describe a tree could consist of a quadratic equation for the base, a linear equation for the main part of the trunk, and a second quadratic equation for the tip of the tree
13. calculate the distance travelled in a car by taking speedometer readings at regular intervals and then using these readings in a numerical formula; check the accuracy of the result by using the tripmeter.
14. calculate the approximate volume of fill to be removed in the construction of a road cutting by approximating the cross-sectional area using a numerical method
15. investigate the accuracy of the trapezoidal rule applied as a composite rule with strip widths chosen according to the different slopes of the integrand
16. investigate consumer surplus and producer surplus in relation to supply and demand functions

Applied statistical analysis (notional time 25 hours)

Focus

Students are encouraged to develop a working knowledge of the concepts involved in describing, summarising, comparing and modelling data, and of some elementary concepts in using data to estimate probabilities and parameters, and to answer simple questions. Students are encouraged to develop skills in interpreting and commenting on data in context. It is expected that calculators (or computers) will be used routinely for calculations and graphical displays.

Note that the introduction to hypothesis testing does not require the treatment of critical regions, levels, confidence intervals, type 1 and 2 errors.

Subject matter

- identification of variables and types of variables and data (continuous and discrete); practical aspects of collection and entry of data (SLEs 1, 2, 4, 5, 6, 14, 15, 16, 23)
- choice and use in context of appropriate graphical and tabular displays for different types of data including pie charts, barcharts, tables, histograms, stem-and-leaf and box plots (SLEs 1, 2, 3, 14)
- use of summary statistics including mean, median, standard deviation and interquartile distance as appropriate descriptors of features of data in context (SLEs 1, 2, 3, 7, 11, 12, 13, 15, 16)
- use of graphical displays and summary statistics in describing key features of data, particularly in comparing datasets and exploring possible relationships (SLEs 1, 2, 3, 11, 12, 13, 14, 15)
- use of relative frequencies to estimate probabilities; the notion of probabilities of individual values for discrete variables and intervals for continuous variables (SLEs 5, 6, 9, 16, 17, 18)
- probability distribution and expected value for a discrete variable (SLEs 6, 17, 18)
- identification of the binomial situation and use of tables or technology for binomial probabilities (SLEs 5, 6, 7, 8)
- introduction of the concept of hypothesis testing through assessing whether data are consistent with a stated value of a proportion in life-related problems, using binomial probabilities (SLEs 8, 19)
- concept of a probability distribution for a continuous random variable; notion of expected value and median for a continuous variable (SLEs 3, 9, 13, 18, 21)
- the normal model and use of standard normal tables (SLEs 10, 20, 22)

Suggested learning experiences

The following suggested learning experiences may be developed as individual student work, or may be part of small group or whole class activities.

1. organise a set of real data into an understandable form using a variety of approaches such as summary statistics and graphical displays
2. given a set of data, produce a concise summary of the main information in the data, referring to graphical displays and summary statistics

3. use graphical displays on the same scale to give an effective visual comparison between two or more datasets, and comment on general comparative features, making allowance for variation
4. identify the effect of different sampling situations in pursuit of a random sample e.g. Gallup poll compared with a phone-in poll
5. identify situations with events that could be assumed to be equally likely such as birthdays, month of birth, male/female births
6. identify discrete variables and estimate probabilities of their values from data and/or model probabilities from assumptions; for example, the number of girls in families of two or three children by listing probabilities
7. examine a number of situations for which the binomial is appropriate and use tables of binomial probabilities; for an example with $n = 3$ or 4, write down, from the binomial tables, the probability of each value of the variable, calculate the expected value and relate it to the example
8. assess the strength of evidence in data about the value of a proportion or probability in binomial situations such as:
   - are births equally likely to be boys or girls? observation: number of female births in a random sample of birth notices in newspapers
   - do teenagers have a preference between, for example, cats and dogs as pets? observation: number who prefer cats to dogs in a random sample of teenagers
   - In both cases, identify the situation (binomial) and the hypothesis to be tested (p = 0.5). (Observe the number of females/catlovers out of n. Use binomial tables to find the probability of getting this or more extreme values if p is indeed 0.5. If this probability is small (for example, less than 0.01 or 0.02), evidence is against $p = 0.5$. If this probability is not negligible (for example, greater than 0.1 or 0.07) then $p = 0.5$ cannot be ruled out.)
9. consider situations such as choosing a point at random on a line of given length, or the arrival of a bus at random in a given interval of time, to obtain the uniform (rectangular) distribution for the continuous variable denoting the position of the point or the arrival time of the bus where equal sub-intervals of time are equally likely; represent probabilities by areas
10. consider a number of life-related situations where a normal distribution may be assumed; standardise variables and use standard normal tables
11. examine the use of summary statistics in, for example, newspapers, articles, TV programs such as weather reports and advertisements, government reports
12. examine reports by the Real Estate Institute (e.g. house prices in different areas) and explain their choice of measure of central tendency
13. compare the effects of an outlier on a variety of summary statistics
14. use a computer database to store, sort and graphically display data
15. discuss different sampling situations and possible difficulties and sources of bias; e.g, due to such things as poor questionnaire design, a lack of random sampling or to practical difficulties such as survey interviewer influence
16. discuss why it is easier to estimate parameters such as the proportion of women who work full-time rather than the proportion of full-time workers who are women

17. use the tabled information given in the newspaper about previous Gold Lotto draws to determine whether the numbers are drawn at random; i.e. whether the numbers follow a uniform probability distribution model
18. ask a group of people to try to generate random numbers between, say, 0 and 50, and use graphical displays to investigate how successful they were; e.g. use a histogram to check rectangular shape, and plot numbers in order of generation to check on trends or patterns
19. select an issue that involves the collection of binary data (e.g. students prefer working in groups to working alone; or cricket followers prefer the radio commentary to the TV commentary); identify the statistical hypothesis; collect or use given data and prepare a report using the results and relevant probability statements
20. use a statistical model with known mean, standard deviation and a normal distribution, to simulate a sample of data; e.g. create a sample of “DDT levels in cows’ milk” by assuming that the DDT level, in standard concentration units, is a random normal variable with a mean of 2.3 and a standard deviation of 0.23; calculate the sample statistics and compare them with the parameters used to create the data (the normally distributed random components could be generated using a table of random normal numbers or a computer package)
21. use a computer package to simulate a number of samples of data from the same distribution and look at the variation in the summary statistics that are estimating population parameters
22. compare probabilities obtained from normal tables with those estimated from data that looks nearly normal
23. examine data collected by a survey or by observation or an experiment to check for recording or measurement error; to decide how to handle non-compliant responses or observations; and to prepare data for entry.

7 ASSESSMENT

7.1 INTRODUCTION

The purpose of assessment is to make judgments about how well students meet the general objectives of the course. In designing an assessment program, it is important that the assessment tasks, conditions and criteria are compatible with the general objectives and the learning experiences. Assessment then, both formative and summative, is an integral and continual aspect of a course of study. The distinction between formative and summative assessment lies in the purpose for which that assessment is used.

Formative assessment is used to provide feedback to students, parents, and teachers about achievement over the course of study. This enables students and teachers to identify the students' strengths and weaknesses so that, by informing practices in teaching and learning, students may improve their achievement and better manage their own learning. The formative techniques used should be similar to summative assessment techniques, which students will meet later in the course. This provides students with experience in responding to particular types of tasks under appropriate conditions. It is advisable that each assessment technique be used formatively before it is used summatively.

Summative assessment, while also providing feedback to students, parents, and teachers, provides information on which levels of achievement are determined at exit from the course of study. It follows, therefore, that it is necessary to plan the range of assessment instruments to be used, when they will be administered, and how they contribute to the determination of exit levels of achievement (see section 7.8). Students' achievements are matched to the standards of exit criteria, which are derived from the general objectives of the course (see section 3). Thus, summative assessment provides the information for certification at the end of the course.

7.2 PRINCIPLES OF ASSESSMENT

The Board's policy on assessment requires consideration to be given to the underlying principles below when devising an assessment program. These principles are to be considered together and not individually in the development of an assessment program.

Underlying principles of assessment

- Exit achievement levels are devised from student achievement in all areas identified in the syllabus as being mandatory.
- Assessment of a student's achievement is in the significant aspects of the course of study identified in the syllabus and the school's work program.
- Information is gathered through a process of continuous assessment.
- Exit assessment is devised to provide the fullest and latest information on a student's achievement in the course of study.
- Selective updating of a student's profile of achievement is undertaken over the course of study.
- Balance of assessment is a balance over the course of study and not necessarily a balance over a semester or between semesters.

Mandatory aspects of the syllabus

Judgement of student achievement at exit from a school course of study must be derived from information gathered about student achievement in those aspects identified in a syllabus as being mandatory. The assessment program, therefore, must include achievement of the general objectives of the syllabus.

Significant aspects of the course of study

Significant aspects refer to those areas included in the course of study, determined by the choices permitted by the syllabus, and seen as being particular to the context of the school and to the needs of students at that school. These will be determined by the choice of learning experiences appropriate to the location of the school, the local environment and the resources selected.

The significant aspects of the course must reflect the objectives of the syllabus.

Achievement in both mandatory and significant aspects of the course must contribute to the determination of the student's exit level of achievement.

The assessment of student achievement in the significant aspects of the school course of study must not preclude the assessment of the mandatory aspects of the syllabus.

For Mathematics B, the significant aspects are the subject matter of each topic.

Continuous assessment

This is the means by which assessment instruments are administered at suitable intervals and by which information on student achievement is collected. It requires a continuous gathering of information and the making of judgements in terms of the stated criteria and standards throughout the two-year program of study.

Levels of achievement must be arrived at by gathering information through a process of continuous assessment at points in the course of study appropriate to the organisation of the learning experiences. They must not be based on students' responses to a single assessment task at the end of a course or instruments set at arbitrary intervals that are unrelated to the developmental course of study.

Fullest and latest information

Judgements about student achievement made at exit from a school course of study must be based on the fullest and latest information available.

'Fullest' refers to information about student achievement gathered across the range of general objectives. 'Latest' refers to information about student achievement gathered from the latest period in which the general objectives are assessed.

Fullest and latest information consists of both the most recent data on developmental aspects together with any previous and not superseded data. Decisions about achievement require both to be considered in determining the student's level of achievement.

The information used to determine a student's exit level of achievement is to be the 'fullest and latest' available. The 'fullest' refers to the collection of assessment information that covers the full range of objectives. The 'latest' refers to information obtained through selective updating.

The assessment instruments for summative purposes are used to determine a student's exit level of achievement. Any formative assessment of knowledge, processes and skills through the program of study becomes a learning experience for the student, whose achievement should therefore benefit when similar assessment techniques are applied for summative purposes.

Selective updating

Selective updating is related to the developmental nature of the two-year course of study. It is the process of using later information to supersede earlier information. Information about student achievement should, therefore, be updated continually when objectives and criteria are revisited.

As the criteria are treated at increasing levels of complexity, assessment information gathered at earlier stages of the course may no longer be typical of student achievement. The information should therefore be selectively updated to reflect student achievement more accurately. Selective updating operates within the context of continuous assessment.

A student profile should be maintained to allow the selective updating of student data. By increasing the amount of information available, the student profile more accurately indicates overall student achievement.

Balance

Balance of assessment is a balance over the course of study and not necessarily a balance within a semester or between semesters. The assessment program must ensure an appropriate balance over the course of study as a whole.

Appropriate balance is established through determining suitable variety, quantity and timing in the assessment conditions, criteria and techniques.

7.3 EXIT CRITERIA

Student achievement will be judged on the following three exit criteria:

- Knowledge and procedures
- Modelling and problem solving
- Communication and justification.

The exit criteria reflect the categories of general objectives, and have been defined in section 3.3 of the syllabus.

7.4 ASSESSMENT REQUIREMENTS

A school's assessment plan must enable students to demonstrate their achievement across the full range of the general objectives in the first three categories in section 3.3.

The assessment plan must be designed to cover the continuum of each of the contexts.

All three exit criteria must be adequately represented in assessment data to enable the overall quality of a student's achievement in each criterion to be determined.

Some points which must be taken into account in assessment are given below:

- Assessment instruments must provide students with the opportunity to demonstrate achievement in the general objectives along the continuum within each of the contexts (see section 3.2).
- In a well balanced plan there should be many items that allow information to be collected on more than one criterion. It is not appropriate to set items which collect information only on Communication and justification.
- Information on student achievement in Knowledge and procedures and Communication and justification may be obtained from items assessing student achievement in Modelling and problem solving.
- It is not appropriate to record on a profile separate information on each aspect of a criterion.
- Information on student achievement in each criterion may be provided by a global consideration of the student response to a task or set of tasks. Such information must be supported by comparison with task-specific descriptors based on the general objectives of the syllabus.

7.4.1 Special consideration

Guidance about the nature and appropriateness of special consideration and special arrangements for particular students may be found in the Board's policy statement on special consideration: *Special Consideration: Exemption and Special Arrangements in Senior Secondary School-Based Assessment*. This statement also provides guidance on responsibilities, principles and strategies that schools may need to consider.

To enable special consideration to be effective for students so identified, it is important that schools plan and implement strategies in the early stages of an assessment program and not at the point of deciding levels of achievement. The special consideration might involve alternative teaching approaches, assessment plans and learning experiences.

7.5 ASSESSMENT TECHNIQUES

It is expected that appropriate technology use will be incorporated in assessment tasks.

A balanced assessment plan that has validity in assessing achievement across the full range of the general objectives includes a variety of assessment techniques such as those described below.

Extended modelling and problem-solving tasks

This form of assessment may require a response that involves mathematical language, graphs and diagrams, and could involve a significant amount of conventional English. It will typically be in written form, a combination of written and oral forms, or multimedia forms.

The activities leading to an extended modelling and problem-solving task could be done individually and/or in groups, and the extended modelling and problem-solving task could be prepared in class time and/or in students' own time.

Reports

A report is typically an extended response to a task such as:

- an experiment in which data are collected, analysed and modelled
- a mathematical investigation
- a field activity
- a project.

A report could comprise such forms as:

- a scientific report
- a proposal to a company or organisation
- a feasibility study.

The activities leading to a report could be done individually and/or in groups and the report could be prepared in class time and/or in students' own time. A report will typically be in written form, or a combination of written and oral multimedia forms.

The report will generally include an introduction, analysis of results and data, conclusions drawn, justification, and when necessary, a bibliography, references and appendices.

Supervised tests

Supervised tests commonly include tasks requiring quantitative and/or qualitative responses. Supervised tests could include a variety of items such as:

- multiple-choice questions
- questions requiring a short response:
  - in mathematical language and symbols
  - in conventional written English, ranging in length from a single word to a paragraph
- questions requiring a response including graphs, tables, diagrams and data
- questions requiring an extended answer where the response includes:
  - mathematical language and symbols
  - conventional written English, more than one paragraph in length
  - a combination of the above.

Assessment tasks other than tests must be included at least twice each year and should contribute significantly to the decision making-process in each criterion.

7.5.1 Authorship of tasks

In order to attest that the response to a task is genuinely that of the student, procedures such as the following are suggested:

- the teacher monitors the development of the task by seeing plans and a draft of the student's work
- the student produces and maintains documentation of the development of the response
- the student acknowledges all resources used; this will include text and source material and the type of assistance received
- the school develops guidelines and procedures for students in relation to both print and electronic source materials/resources, and to other types of assistance that have been sought.

7.6 RECORDING INFORMATION

Information on student achievement in each criterion may be recorded in various ways. However, the methods of recording and the frequency with which records will be updated must be clearly outlined in the work program.

7.7 DETERMINING EXIT LEVELS OF ACHIEVEMENT

On completion of the course of study, the school is required to award each student an exit level of achievement from one of the five categories:

Very High Achievement

High Achievement

Sound Achievement

Limited Achievement.

Very Limited Achievement.

The school must award an exit standard for each of the three criteria (Knowledge and procedures; Modelling and problem solving; Communication and justification), based on the principles of assessment described in this syllabus. The criteria are derived from the general objectives described in section 3.3. The minimum standards associated with the three exit criteria are described in section 7.7. When teachers are determining a standard for each criterion, the extent to which the qualities of the work match the descriptors *overall* should strongly influence the standard awarded.

For Year 11, particular standards descriptors may be selected from the matrices in table 2 and/or adapted to suit the task. These standards are used to inform the teaching and learning process. For Year 12 instruments, students should be provided with opportunities to understand and become familiar with the expectations for exit. The exit standards are applied to the summative body of work selected for exit.

Of the seven key competencies, the five that are relevant to this subject[1] are embedded in the descriptors in table 2. Elements of some of the key competencies are embedded within the standards associated with the exit criteria. The key competencies of "Using mathematical ideas and techniques" and "Using technology" are to be found within the "Knowledge and procedures" criterion. The "Modelling and problem-solving" criterion involves elements from "Collecting, analysing and organising information", "Using mathematical ideas and techniques", and "Solving problems", whereas "Collecting, analysing and organising information", "Communicating ideas and information", and "Solving problems" are involved in the "Communication and justification" criterion.

When standards have been determined in each of the three criteria, table 1 is used to determine the exit level of achievement, where *A* represents the highest standard and *E* the lowest.

[1] KC1: collecting, analysing and organising information; KC2: communicating ideas and information; KC5: using mathematical ideas and techniques; KC6: solving problems; KC7: using technology.

Table 1: Minimum requirements for exit levels

VHA	Standard *A* in any two exit criteria and no less than a *B* in the remaining criterion
HA	Standard *B* in any two exit criteria and no less than a *C* in the remaining criterion
SA	Standard *C* in any two exit criteria, **one of which must be the Knowledge and procedures criterion**, and no less than a *D* in the remaining criterion
LA	Standard *D* in any two exit criteria, **one of which must be the Knowledge and procedures criterion**
VLA	Does not meet the requirements for Limited Achievement

Table 2: Minimum standards associated with exit criteria

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: Knowledge and procedures	The **overall quality** of a student's achievement across the full range within the contexts of application, technology and complexity, and across topics, **consistently demonstrates**: • accurate recall, selection and use of definitions, results and rules • appropriate use of technology • appropriate selection, and accurate and proficient use of procedures • effective transfer and application of mathematical procedures	The **overall quality** of a student's achievement across a range within the contexts of application, technology and complexity, and across topics, **generally demonstrates**: • accurate recall, selection and use of definitions, results and rules • appropriate use of technology • appropriate selection and accurate use of procedures.	The **overall quality** of a student's achievement in the contexts of application, technology and complexity **generally demonstrates**: • accurate recall and use of basic definitions, results and rules • appropriate use of some technology • accurate use of basic procedures.	The **overall quality** of a student's achievement in the contexts of application, technology and complexity **sometimes demonstrates**: • accurate recall and use of some definitions, results and rules • appropriate use of some technology.	The **overall quality** of a student's achievement **rarely demonstrates** knowledge and use of procedures.

[This table continues on the next two pages.]

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion: Modelling and problem solving	The **overall quality** of a student's achievement across the full range within each context, and across topics **generally demonstrates mathematical thinking** which includes: • interpreting, clarifying and analysing a range of situations, identifying assumptions and variables • selecting and using effective strategies • selecting appropriate procedures required to solve a wide range of problems • appropriate synthesis of procedures and strategies; … *and* in some contexts and topics **demonstrates** mathematical thinking which includes: • synthesis of procedures and strategies to solve problems • initiative and insight in exploring the problem • exploring strengths and limitations of models • extending and generalising from solutions.	The **overall quality** of a student's achievement across a range within each context, and across topics, **generally demonstrates mathematical thinking** which includes: • interpreting, clarifying and analysing a range of situations, identifying assumptions and variables • selecting and using effective strategies • selecting appropriate procedures required to solve a range of problems; … *and* in some contexts and topics **demonstrates** mathematical thinking which includes appropriate synthesis of procedures and strategies.	The **overall quality** of a student's achievement in all contexts **generally demonstrates mathematical thinking** which includes: • interpreting and clarifying a range of situations • selecting strategies and/or procedures appropriate to problems.	The **overall quality** of a student's achievement **sometimes demonstrates mathematical thinking** which includes following basic procedures and/or using basic strategies.	The **overall quality** of a student's achievement **rarely demonstrates mathematical thinking** which includes following basic procedures and/or using basic strategies.

	Standard *A*	Standard *B*	Standard *C*	Standard *D*	Standard *E*
Criterion; Communication and justification	The **overall quality** of a student's achievement across the full range within each context **consistently demonstrates**: • accurate and appropriate use of mathematical terms and symbols • accurate and appropriate use of language • collection and organisation of information into various forms of presentation suitable for a given use or audience • use of mathematical reasoning to develop logical arguments in support of conclusions, results and/or propositions • justification of procedures and conclusions • recognition of the effects of assumptions used • evaluation of the validity of arguments.	The **overall quality** of a student's achievement across a range within each context **generally demonstrates**: • accurate and appropriate use of mathematical terms and symbols • accurate and appropriate use of language • collection and organisation of information into various forms of presentation suitable for a given use or audience • use of mathematical reasoning to develop simple logical arguments in support of conclusions, results and/or propositions • justification of procedures	The **overall quality** of a student's achievement in all contexts **generally demonstrates**: • accurate and appropriate use of basic mathematical terms and symbols • accurate and appropriate use of basic language • collection and organisation of information into various forms of presentation; • use of some mathematical reasoning to develop simple logical arguments in support of conclusions, results and/or propositions.	The **overall quality** of a student's achievement **sometimes demonstrates** evidence of the use of the basic conventions of language and mathematics.	The **overall quality** of a student's achievement **rarely demonstrates** use of the basic conventions of language or mathematics.

Contexts are explained in section 3.2.

7.8 REQUIREMENTS FOR VERIFICATION FOLIOS

A verification folio is a collection of a student's responses to assessment instruments on which the level of achievement is based. Each folio should contain a variety of assessment techniques demonstrating achievement in the three criteria, Knowledge and procedures; Modelling and problem solving; and Communication and justification, over the range of topics. This variety of assessment techniques is necessary to provide a range of opportunities from which students may demonstrate achievement.

In the verification folio requirements for the subject, the minimum and maximum number of assessment instruments are stipulated. Schools must ensure that the verification folios presented in October contain all summative assessment instruments and corresponding student responses upon which judgments about interim levels of achievement have been made to that point.

It is necessary that a student's achievement in the three criteria is monitored throughout the course so that feedback in terms of the criteria is provided to the student. The verification folio is an ideal medium through which students and teachers can monitor progress throughout the course.

For verification purposes, schools must submit student folios which contain:

- student achievement data profiled in the three exit criteria
- the student responses to all summative assessment instruments (in the case of non-written responses, the minimum requirement will be a student criterion sheet or sheets completed by the teacher along with supporting material provided by the student)
- a minimum of four instruments from Year 12 with at least one of these being a report, extended modelling and problem-solving task, or similar.

A verification folio must consist of a minimum of 4 to a maximum of 10 pieces of summative work. These should represent a range of assessment techniques (see section 7.5) and provide adequate information on which to substantiate the school's judgments regarding student achievement in each criterion.

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8 DEVELOPING A WORK PROGRAM

The work program is a formal expression of the school's interpretation of this syllabus. It has three primary functions. First, it provides guidance to the teachers of the subject as to the nature and requirements of the Mathematics B course at the school. Second, it provides similar guidance to the school's students, and their parents, in relation to the subject matter to be studied, and how achievement of the syllabus objectives will be assessed. Third, it provides a basis for accreditation by the Board for the purposes of including students' results for the subject on students' Senior Certificates.

The school's work program should be a document which does not require reference to other documents to be understood. The work program must contain the following components.

Table of contents Facilitates the readability of the document; pages must be numbered.

Rationale Provides justification for including the subject in the school curriculum. The rationale may be derived principally from the syllabus statement but should also include information on the school's philosophy, student population, resources and any other factors which may influence the decisions made in designing a course of study to cater for the special characteristics of the school and its students.

This may be an appropriate place for the school to provide any additional information such as whether students are able to study both Mathematics A and Mathematics B, and whether any Board-registered subjects in mathematics are offered.

Global aims Statements of the long-term achievements, attitudes and values that are to be developed by the students studying the subject, but which are not directly assessed by the school. These should include the global aims listed in this syllabus.

General objectives As indicated in this syllabus.

Contexts As indicated in this syllabus. The contexts must be incorporated in the sample of the sequence of work as required in the "Learning experiences" section.

Course organisation Course organisation provides:

- a summary of the spiralling and integrated sequence developed by the school to give an overview of the topics
- details of the sequence, indicating:
  - the subject matter to be taught in each unit of work (the subject matter listed in the syllabus is the minimum to be included)
  - time allocations for each unit.

The sequence must be developed in accordance with Section 5 of this syllabus.

Technology A statement of commitment to use higher technologies such as graphing calculators and/or computers with appropriate software, or other developing technologies.

Language statement As indicated in this syllabus.

Educational equity A statement must be included by the school indicating that due consideration has been given to the issues associated with the educational equity statement in this syllabus.

Learning Experiences A commitment to use a variety of learning experiences which are appropriate to the age group concerned, and consistent with the objectives of this syllabus.

A sample of a sequence of work must be included in the work program. This should be of sufficient length to demonstrate that the general approach by the school is consistent with the intent of this syllabus. It should show how the choice of subject matter is to be taught, and the variety of learning experiences that will provide opportunities for students to achieve the general objectives within the contexts of the course.

Assessment Including:

- an assessment plan which:
  - provides a balanced assessment program in which techniques other than formal written tests or examinations are to be included at least twice each year, constitute a meaningful contribution to exit levels of achievement
  - clearly indicates the summative tasks—data gathered as a result of implementing the assessment plan should allow fullest and latest information to determine exit levels of achievement
  - allows sufficient information to be available for the recommendation of interim levels of achievement; for example, for monitoring and verification purposes
  - while allowing for flexibility within the school, contains sufficient information to show that there will be enough data gathered to enable valid judgments to be made on student achievement in each of the three criteria
  - indicates the criteria associated with tasks and the conditions of implementation of tasks; a general statement on the conditions of implementation of assessment will suffice;

    (An example of such a statement is "To ensure validity and reliability of assessment instruments, this school will use a variety of implementation conditions for assessment of student achievement e.g. assignments undertaken in class time, assignments undertaken at home, and group projects. The conditions appropriate to the actual instruments will be supplied at monitoring and verification.")
  - avoids over-assessment.
- a commitment to include as much assessment as possible from Semester 4 for each of the three criteria in the verification submission
- an example of the individual student profile to be included in the verification folio
- how assessment data are combined to reach an overall standard in each criterion (a completed student profile may help to clarify the explanation); fullest and latest information is not obtained by an arbitrary 'weighting' of semesters or by using Semester 1 assessment instruments as summative in a well-sequenced course
- the procedure for awarding exit levels of achievement which are consistent with the criteria and standards of this syllabus (section 7); the school should indicate a commitment to ensure that the achievement of students identified as near a threshold (either above or below) is matched to the verbal descriptors of the criteria of the syllabus before a level of achievement is awarded.

9 EDUCATIONAL EQUITY

Equity means fair treatment of all. In developing work programs from this syllabus, schools are urged to consider the most suitable means of incorporating the following notions of equity.

Schools need to provide opportunities for all students to demonstrate what they know and what they can do. All students, therefore, should have equitable access to educational programs and human and material resources. Teachers should ensure that the particular needs of the following groups of students are met: female students; male students; Aboriginal students; Torres Strait Islander students; students from non–English-speaking backgrounds; students with disabilities; students with gifts and talents; geographically isolated students; and students from low socioeconomic backgrounds.

The subject matter chosen should include, where appropriate, the contributions and experiences of all groups of people. Learning contexts and community needs and aspirations should also be considered when selecting subject matter.

In choosing suitable learning experiences teachers should, where possible, introduce and reinforce non-racist, non-sexist, culturally sensitive and unprejudiced attitudes and behaviour. Learning experiences should encourage the participation of students with disabilities and accommodate different learning styles.

It is desirable that the resource materials chosen recognise and value the contributions of both females and males to society and include the social experiences of both sexes. Resource materials should also reflect the cultural diversity within the community and draw from the experiences of the range of cultural groups in the community.

Efforts should be made to identify, investigate and remove barriers to equal opportunity to demonstrate achievement. This may involve being proactive in finding out about the best ways to meet the special needs, in terms of learning and assessment, of particular students.

The variety of assessment techniques in the work program should allow students of *all* backgrounds to demonstrate their knowledge and skills in a subject in relation to the criteria and standards stated in this syllabus. The syllabus criteria and standards should be applied in the same way to all students.

Teachers may find the following resources useful for devising an inclusive work program:

Australian Curriculum, Assessment and Certification Authorities 1996, *Guidelines for Assessment Quality and Equity* 1996, Australian Curriculum, Assessment and Certification Authorities,, available through QBSSSS, Brisbane.

Department of Education, Queensland 1991, *A Fair Deal: Equity Guidelines for Developing and Reviewing Educational Resources*, Department of Education, Brisbane.

Department of Training and Industrial Relations 1998, *Access and Equity Policy for the Vocational Education and Training System*, DTIR, Brisbane.

[Queensland] Board of Senior Secondary School Studies 1994, *Policy Statement on Special Consideration*, QBSSSS, Brisbane.

[Queensland] Board of Senior Secondary School Studies 1995, *Language and Equity: A discussion paper for writers of school-based assessment instruments*, QBSSSS, Brisbane.

[Queensland] Board of Senior Secondary School Studies 1995, *Studying Assessment Practices: A resource for teachers in schools*, QBSSSS, Brisbane.

APPENDIX 1

MAINTAINING BASIC KNOWLEDGE AND PROCEDURES

Basic knowledge and procedures

The following knowledge and procedures will be required throughout the course and must be learned or maintained as required:

- metric measurement including measurement of mass, length, area and volume in practical contexts
- calculation and estimation with and without instruments
- rates, percentages, ratio and proportion
- simple interest
- basic algebraic manipulations
- identities, linear equations and inequalities
- gradient of a straight line
- equation of a straight line
- plotting points using Cartesian coordinates
- solutions of a quadratic equation
- graphs of quadratic functions
- tree diagrams as a tool for defining sample spaces and estimating probabilities
- the summation notation: $\sum_{i=1}^{n} x_i$.

Maintaining mathematical procedures

The following learning experiences are included as suggestions.

Suggested learning experiences

- prepare a seminar revising prerequisite material; for example, revising differentiation before integration
- solve a new problem requiring previously learnt skills
- help other students in a lower year with their mathematics
- work without a calculator for a time
- complete a test given to a lower year, have another student mark it and then discuss anomalies
- design some questions to help others revise a topic
- visualisation of simple three-dimensional figures
- each member of a group writes a summary of a topic, then students share the summaries
- interpretation and drawing of scale drawings and plans
- practical applications of volume and surface area of regular shapes

APPENDIX 2

EXPLANATION OF SOME TERMS

Amortising a loan
The repayment of a debt with constant repayments at fixed intervals.

Annuities
The accumulation of fixed payments at fixed intervals over a period of time.

Frequency plot
A diagrammatic presentation of the frequency distribution of the observations; for example, bar chart, pie chart, histogram, frequency polygon or an ogive.

Justification of procedures
Justification of procedures may include:

- providing evidence (words, diagrams, symbols, etc.) to support processes used
- stating a generic formula before using specifically
- providing a reasoned, well-formed, logical sequence within a response.

Mathematical model
Any representation of a situation which is expressed in mathematical terms. It should be noted that models may be as simple as expressing simple interest as:

$$I = \frac{\Pr t}{100}$$

or showing the relationship between two variables as a scattergram.

Median boxplot (box-and-whisker plot)
A graphical presentation of some main features of a dataset. The simplest version of a box plot is formed by drawing a box extending from the lower to the upper quartiles, marking the median within that box, and drawing lines (called whiskers) from the box to the smallest and largest data points.

There are slight variations in the possible ways of identifying the median and quartiles of data: these variations make very little difference except for small or sparse datasets. One technique is illustrated in the following example:

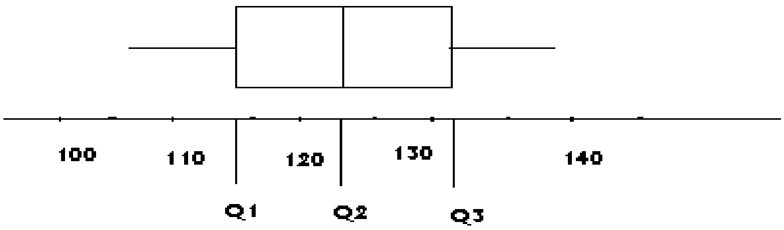
Example: Consider the following 18 systolic blood pressures (bp)

110, 130, 108, 125, 111, 122, 126, 119, 114,
134, 120, 132, 134, 130, 107, 137, 120, 136

These data are ordered and numbered from the smallest to largest below.

x:	107	108	110	111	114	119	120	120	122	125	126	130	130	132	134	134	136	137
order:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

The median, Q2, is taken to be 123.5. The lower quartile, Q1, is the median of the lower 9 observations, viz. 114, and the upper quartile, Q3, is the median of the upper 9 observations, viz. 132. Thus there are exactly four observations below Q1, between Q1 and Q2, between Q2 and Q3, and above Q3.



With this technique, datasets with 16, 17, 18 or 19 observations all have exactly four observations below Q1, above Q3 and between Q1, Q2, Q3.

Note 1: A more informative version of the box plot, particularly with larger datasets, that is also often provided by statistical computer packages, takes the whiskers out to the last data points within a certain distance of the quartiles and then marks individual *data points beyond the whiskers.*

Note 2: A box plot can be presented vertically or horizontally.

Outlier

An extreme value in the observations, for example, an observation which lies beyond the box in the box-and-whisker plot, or a point which is well away from the line of best fit.

Stemplot (stem-and-leaf plot)

An exploratory technique that simultaneously ranks the data and gives an idea of the distribution. Example: The following 16 average daily temperatures have been recorded to the nearest degree Celsius:

31 21 35 30 22 23 9 24

13 41 30 21 29 24 18 28

2 | 1 represents 21

Preliminary stem-and-leaf plot of the temperatures:

Unit = 1

0	9
1	3, 8
2	1, 2, 3, 4, 1, 9, 4, 8
3	0, 0, 1, 5
4	1

Preliminary stem-and-leaf plot of the temperatures:
Unit = 1

0	9
1	3, 8
2	1, 1, 2, 3, 4, 4, 8, 9
3	0, 0, 1, 5
4	1

Summary statistics
Characteristics which describe the sample of observations, for example, the mean, median or standard deviation.

Trapezoidal rule
The area under the curve above the x-axis between the limits $x = a$ and $x = b$, can be approximated by dividing the interval $[a, b]$ into n sub-intervals of equal length, $l = \frac{|b - a|}{n}$, and using the formula:

$$A \approx l \ [\frac{1}{2} f(a) + f(a + l) + (f(a + 2l) + \ldots + f(a + (n - 1)l) + \frac{1}{2} f(b)]$$

Variation
The way in which the observations differ (vary) from each other, often measured by the standard deviation or range.