University of California
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Department of Electrical Engineering
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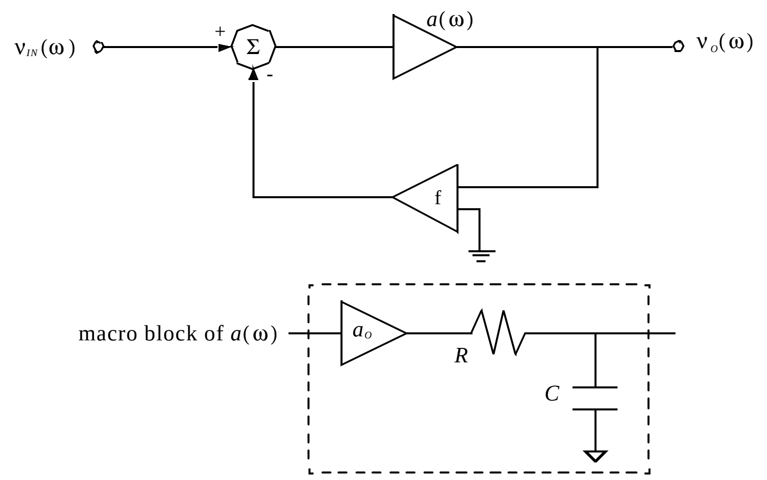
EECS140

Analog Circuit Design

Lectures on STABILITY

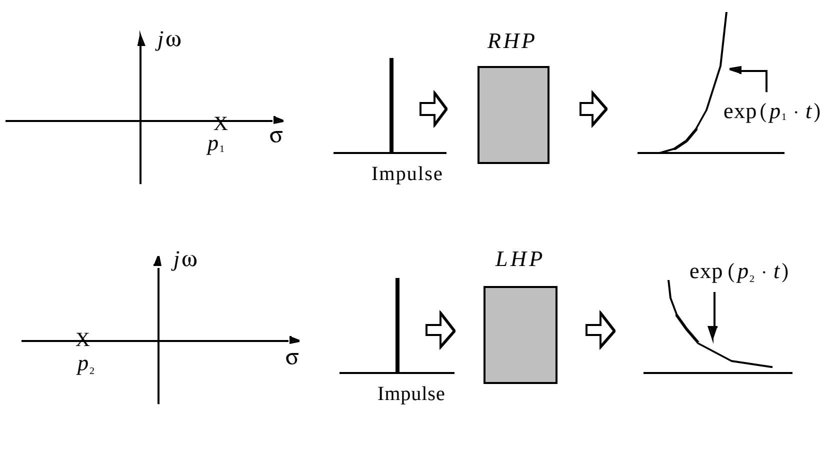
Effect of Feedback on Frequency Response

SB-1



Effect of Feedback on Frequency Response (Cont.)

SB-2



Effect of Feedback on Frequency Response (Cont.)

SB-3

Let $a(\omega)$ be a single pole response,

$$a(s) = \frac{a_o}{1 - \frac{s}{p_1}} \Leftrightarrow a(\omega) = \frac{a_o}{1 + j\frac{\omega}{\omega_{p1}}}$$

$$p_1 = -\omega_{p1}$$

$$\frac{v_{OUT}(s)}{v_{in}(s)} = A(s) = \frac{a(s)}{1 + a(s) \cdot f} = \frac{1}{f}\left(\frac{T(s)}{1 + T(s)}\right)$$

$$A(s) = \frac{\frac{a_o}{1 - \frac{s}{p_1}}}{1 + \frac{a_o}{1 - \frac{s}{p_1}} \cdot f} = \frac{a_o}{1 + a_o \cdot f}\left(\frac{1}{1 - \frac{s}{p_1 \cdot (1 + a_o \cdot f)}}\right)$$

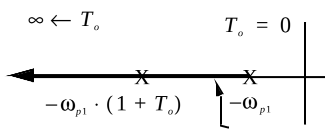
Effect of Feedback on Frequency Response (Cont.)

SB-4

Let $T_o = a_o \cdot f$

$$A(s) = \frac{a_o}{1+T_o} \cdot \left(\frac{1}{1 - \frac{s}{p_1 \cdot (1+T_o)}} \right)$$

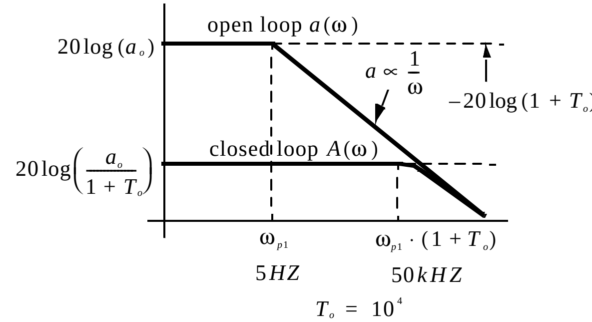
Pole is at $p_1 \cdot (1+T_o) \Rightarrow -\omega_{p1} \cdot (1+T_o)$



Root Locus - motion of poles as loop Gain is increased.

Effect of Feedback on Frequency Response (Cont.)

SB-5



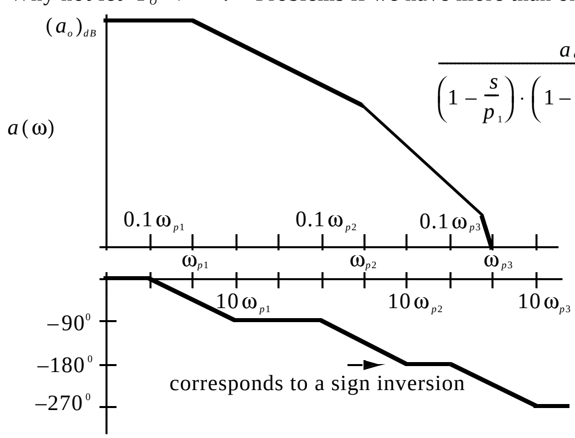
Gain reduction by negative feedback reduces Gain by $\left(\frac{1}{1+T_o}\right)$ and increases bandwidth by $(1+T_o)$

Effect of Feedback on Frequency Response (Cont.)

SB-6

Why not let $T_o \to \infty$? Problems if we have more than one pole.

$$\frac{a_o}{\left(1-\frac{s}{p_1}\right)\cdot\left(1-\frac{s}{p_2}\right)\cdot\left(1-\frac{s}{p_3}\right)}$$

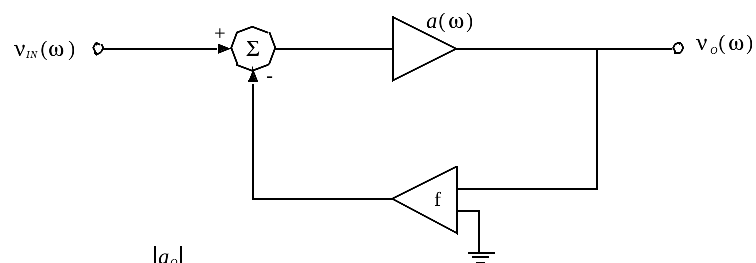


At the frequency $(10\omega_{p2})$ the pahse shift is 180^0 or negative feedback at DC is now positive feedback.

Effect of Feedback on Frequency Response (Cont.)

SB-7

Lets look at the motion of a single pole with positive feedback :



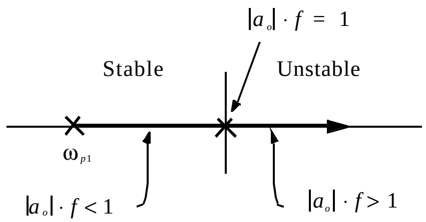
$$a(\omega) = -\frac{|a_o|}{1-\frac{s}{p_1}} \qquad p_1 = -\omega_{p1}$$

$$A(s) = \frac{a_o}{1+T_o} \cdot \left[\frac{s}{p_1 \cdot (1 - |a_o| \cdot f)} \right]$$

Effect of Feedback on Frequency Response (Cont.) SB-8

Since,

$$T_o = -|a_o| \cdot f$$



pole is at $-\omega_{p1} \cdot (1 - |a_o| \cdot f)$

If T < –1 or (1+T) < 0 the circuit is unstable

Effect of Feedback on Frequency Response (Cont.) SB-9

The condition for stability of a multipole response is the Nyquist Criteria.

$$A(s) = \frac{a(s)}{1 + a(s) \cdot f} = \frac{a(s)}{1 + T(s)}$$

Simple Version :

If $|Tj\omega| > 1$ at the frequency where the phase of $T(j\omega) = -180^0$,

then the circuit is unstable.

$$T(j\omega) = T(s)|_{s = j\omega}$$

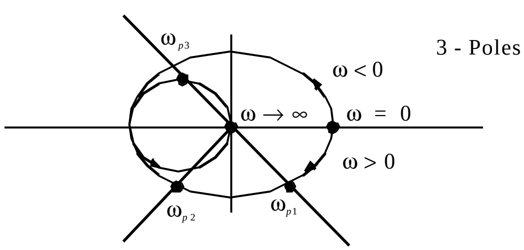
$$\theta_{T(j\omega)} = arc\tan\left[\frac{Im\{T(j\omega)\}}{Re\{T(j\omega)\}}\right]$$

Effect of Feedback on Frequency Response (Cont.) SB-10

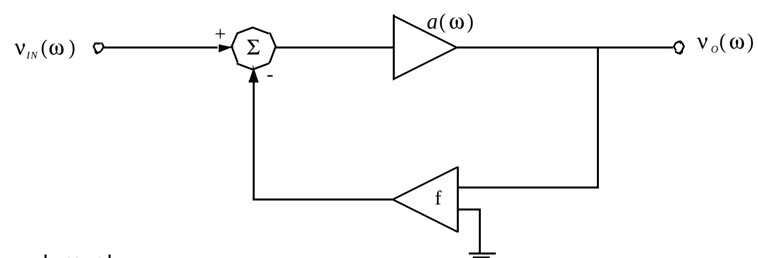
Complex Nyquist Criteria :

Plot $T(j\omega)$ on complex plane. As ω increases count number of times

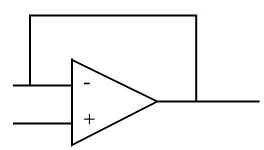
-1 is circled - even number means unstable (I Think).



Effect of Feedback on Frequency Response (Cont.) SB-11

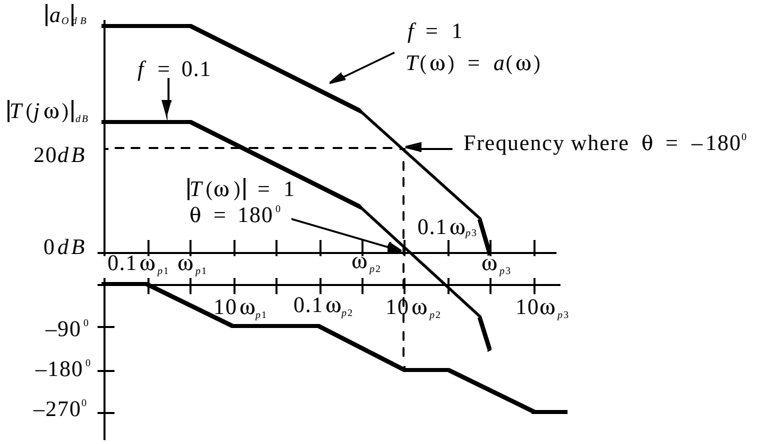


$$|T(j\omega)|_{180^\circ} < 1$$



Worst Case Stability Condition

Effect of Feedback on Frequency Response (Cont.) SB-12



Effect of Feedback on Frequency Response (Cont.) SB-13

PHASE MARGIN : Difference between the actual phase shift and -180^0

when $|T(\omega)| = 1$

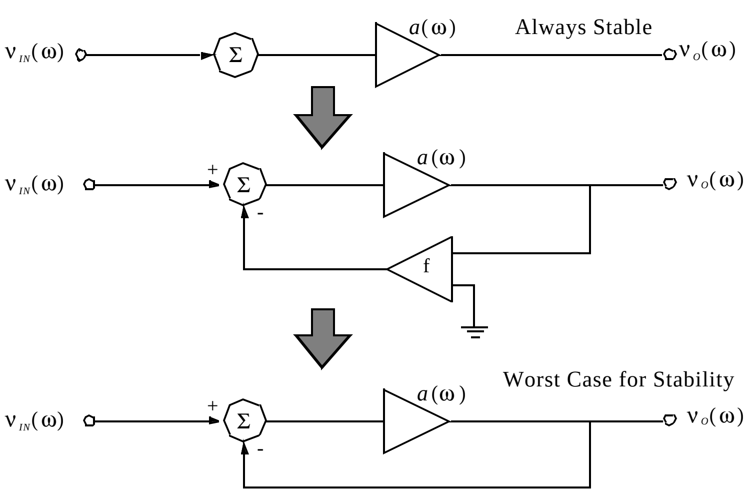
i.e. $\theta_m \equiv$ Phase Margin $= \theta[T(\omega)] - (-180^0)$

if $\theta_m > 0$ then the amplifier is stable - typically $45^0 - 60^0$

$A = \frac{1}{f}$ more gain more stable

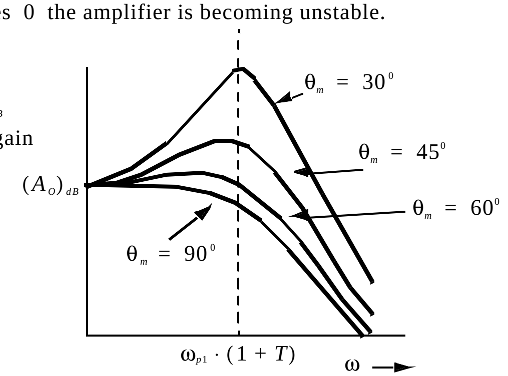
$R_{OUT} = \frac{r_o}{1 + T}$ higher R_{OUT} with more gain

Effect of Feedback on Frequency Response (Cont.) SB-14



Effect of Feedback on Frequency Response (Cont.) SB-15

As θ_m approaches 0 the amplifier is becoming unstable.



Effect of Feedback on Frequency Response (Cont.)

SB-16

$$A(\omega) = \frac{a(\omega)}{1 + a(\omega) \cdot f}$$

$$a(s) = \frac{N(s)}{D(s)}$$

$$A(s) = \frac{\frac{N(s)}{D(s)}}{1 + \frac{N(s)}{D(s)} \cdot f} = \frac{N(s)}{D(s) + N(s) \cdot f}$$

$N(s)$ ← zeros of a(s)

$D(s) + N(s) \cdot f$ ← poles of a(s)

if the feedback factor is frequency dependent, then,

$$f(s) = \frac{N_f(s)}{D_f(s)}$$

$$A(s) = \frac{N(s)D(s)}{D(s)D_f(s) + N(s)N_f(s)}$$

Compensation

SB-17

Compensation is the method in which an amplifier is modified so that it is stable.

One way is to decrease f (less feedback).

If ω_{180} is the frequency where,

$$\theta(a(\omega_{180^\circ})) = -180^\circ$$

then if,

$$f < \left|\frac{1}{a(\omega_{180^\circ})}\right|$$

then,

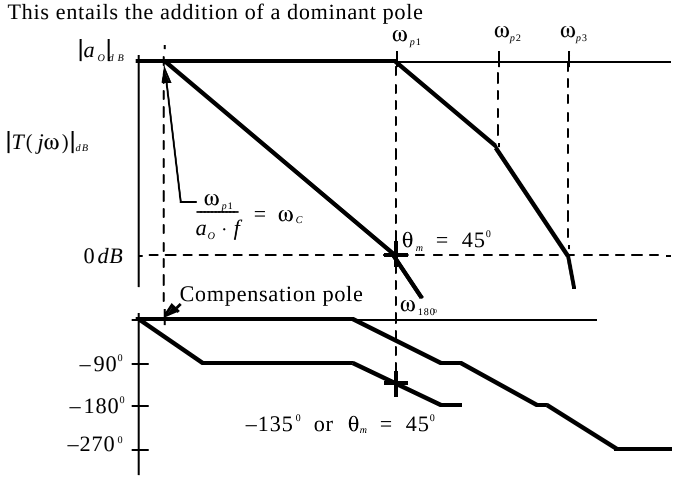
$$|T(\omega_{180^\circ})| = f \cdot |a(\omega_{180^\circ})|$$

and stability is ensured.

Narrowbanding for Compensation

SB-18

This entails the addition of a dominant pole



Narrowbanding for Compensation (Cont.)

SB-19

For,

$$\theta_m = 45°$$

add a compensation pole, ω_C at the fequency,

$$\frac{\omega_{P1}}{|a_o f|} = \omega_C$$

$$\omega_P = 1MHz$$

$$|a_o f| = 10^4$$

$$\omega_C = 100Hz$$

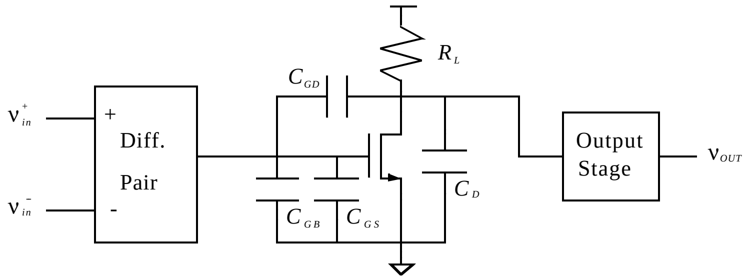
$-90°$ of phase shift from the new compensation pole.

$-45°$ from the second pole.

Pole Splitting

SB-20

It is better to use an existing pole rather than add another.



$$\frac{a_1}{1 + j\frac{\omega}{\omega_{p1}}} \qquad \frac{g_m R_L}{\left(1 + j\frac{\omega}{\omega_{p2}}\right)\cdot\left(1 + j\frac{\omega}{\omega_{p3}}\right)} \qquad \frac{a_2}{1 + j\frac{\omega}{\omega_{p4}}}$$

Pole Splitting (Cont.)

SB-21

Lets say ω_{p1} and ω_{p4} are given with,

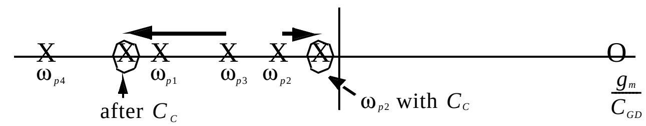
$\omega_{p4} \gg \omega_{p1}$

$C_{GD} \ll C_{GS}, C_D$

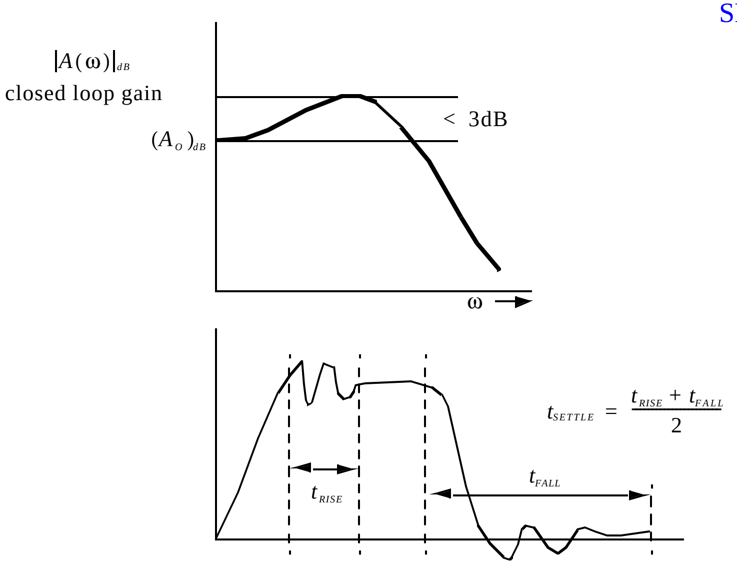
then,

$$\omega_{p2} = \frac{1}{R_{DIFF} C_{GS}} \qquad \omega_{p3} = \frac{1}{R_L C_D}$$

$$\omega_z = \frac{g_m}{C_{GD}}$$



SB-22



$$t_{SETTLE} = \frac{t_{RISE} + t_{FALL}}{2}$$

Pole Splitting (Cont.)

SB-23

If we add a compensation capacitor, C_C in parallel with C_{GD} :

$$\omega_{p2} = \frac{1}{R_{DIFF}\cdot(1 + g_m \cdot R_L)\cdot C_C}$$

$$\omega_{p3} = \frac{g_m}{C_{GS} + C_D}$$

Lets put numbers in :

$R_{DIFF} = 10 Meg\Omega$

$R_L = 5 Meg\Omega$

$C_{GS} = 0.1 pF$

$C_D = 0.1 pF$

$g_m = 10^{-3} Mhos$

$\omega_{p1} = 10 \cdot 10^6 \frac{rad}{sec}$

$\omega_{p4} = 100 \cdot 10^6 \frac{rad}{sec}$

$a_1 = 10^3$

$a_2 = 1$

Pole Splitting (Cont.)

SB-24

Before compensation, and with,

$$C_{GD} = 0$$

$$\omega_{p2} = \frac{1}{10^7 \cdot 10^{-13}} = 10^6 \frac{rad}{sec}$$

$$\omega_{p3} = \frac{1}{5 \cdot 10^6 \cdot 10^{-13}} = 2 \cdot 10^6 \frac{rad}{sec}$$

$$a(\omega) = \left(\frac{10^3}{1 + j\frac{\omega}{4 \cdot 10^6}}\right) \cdot \left(\frac{10^{-3} \cdot 5 \cdot 10^6}{\left(1 + j\frac{\omega}{10^6}\right) \cdot \left(1 + j\frac{\omega}{2 \cdot 10^6}\right)}\right) \cdot \left(\frac{1}{1 + j\frac{\omega}{10^8}}\right)$$

Pole Splitting (Cont.)

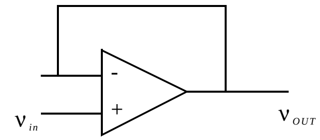
SB-25

Compensate this amplifier for the worst case,

$$f = 1$$

with,

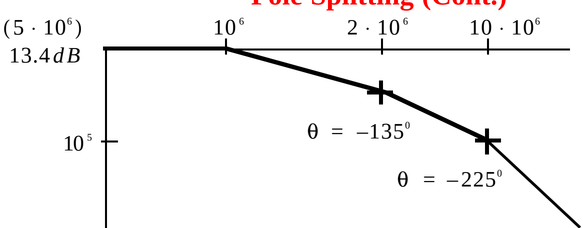
$$\theta_m = 45^0$$



Worst Case Stability Condition

Pole Splitting (Cont.)

SB-26



Somewhere between 2MHz and 10MHz,

$$\theta = -180^0$$

but the loop gain,

$$T » 1 \qquad T \sim 10^5$$

So how to compensate it?

Add C_c so that the gain at the first non-dominant pole (w_{p1}). since w_{p3} will move to a higer frequency and w_{p2} will move lower

Pole Splitting (Cont.)

SB-27

$$\omega_{P2} = \frac{\omega_{P1}}{5 \times 10^6} = \frac{10^7}{5 \times 10^6} = 2 \cdot \frac{rad}{sec}$$

Formula for ω_{P2} & ω_{P3} with C_C :

$$C_C » C_{GS}, C_D$$

$$\omega_{P2} = \frac{1}{R_{DIFF} \cdot (1 + g_m \cdot R_L) \cdot C_C}$$

$$\omega_{P3} = \frac{g_m}{C_{GS} + C_D}$$

$$\omega_Z = \frac{g_m}{C_C}$$

Pole Splitting (Cont.)

SB-28

$$C_C = 10pF$$

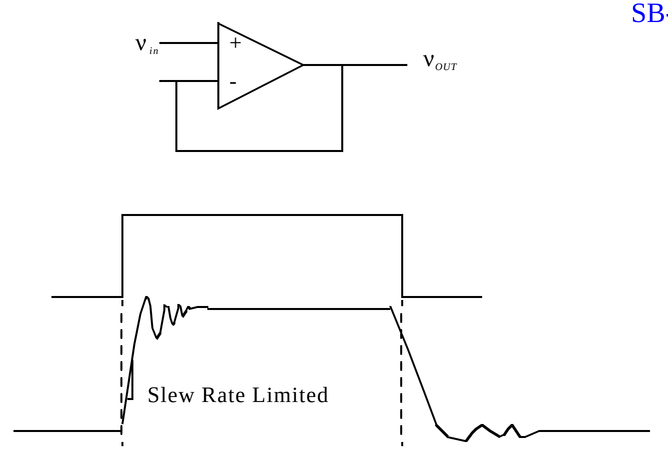
$$\omega_{P2} = \frac{1}{10^7 \times 5 \times 10^3 \times C_C}$$

$$\omega_{P3} = \frac{10^{-3}}{0.2 \times 10^{-12}} = 5 \times 10^9 \cdot rad/\sec = 5000 Mrad/\sec$$

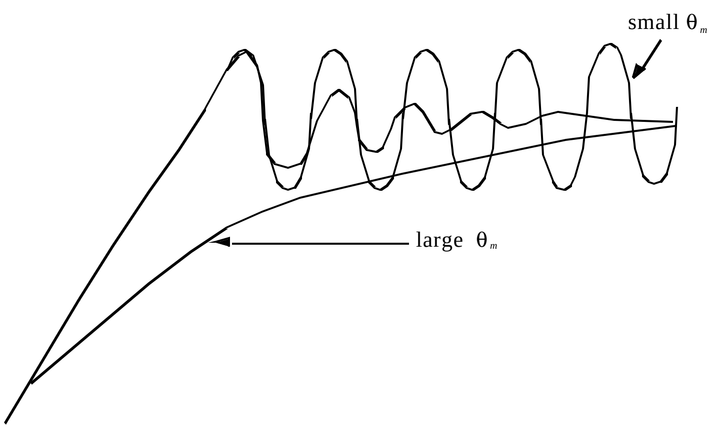
$$\omega_Z = \frac{10^{-3}}{10^{-11}} = 10^8 \cdot \frac{rad}{\sec}$$

So by adding a 10pF capacitor this circuit is made stable.

SB-29

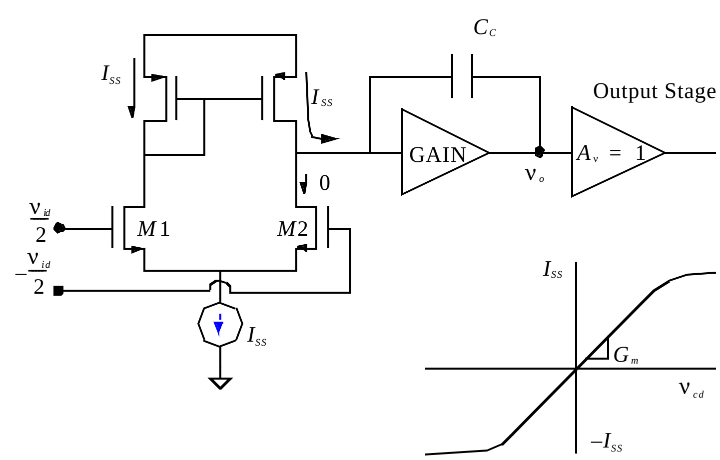


SB-30



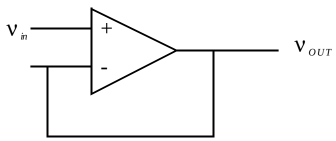
Slew Rate & Compensation Miller Op Amp

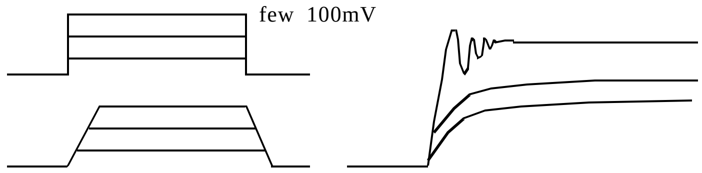
SB-31



Slew Rate & Compensation Miller Op Amp (Cont.)

SB-32





Slew Rate Volts/µsec 10 low

20 – 50 med

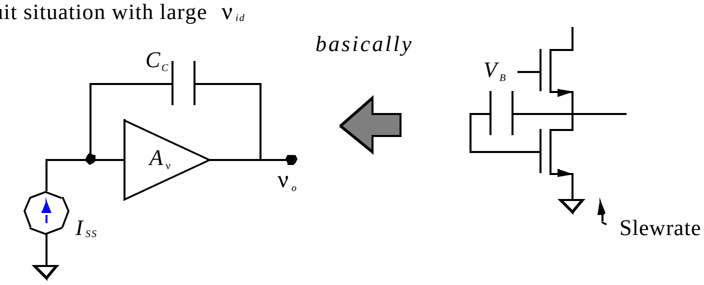
100 high

EECS140 ANALOG CIRCUIT DESIGN LECTURES ON STABILITY

Slew Rate & Compensation Miller Op Amp (Cont.)

SB-33

Circuit situation with large v_{id}



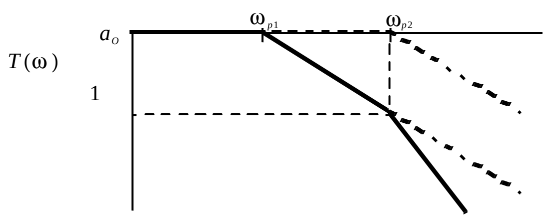
$$I_{SS} = -C_C \cdot \frac{dv_o}{dt} \quad \text{or} \quad \frac{dv_o}{dt} = -\frac{I_{SS}}{C_C} = \text{slew rate}$$

$$v_o = \frac{1}{C} \cdot \int I_{SS} \cdot dt = \frac{I_{SS}}{C} \cdot t \quad \leftarrow \text{linear with time}$$

EECS140 ANALOG CIRCUIT DESIGN LECTURES ON STABILITY

Slew Rate & Compensation Miller Op Amp (Cont.)

SB-34



$\theta_m = 45°$

$f = 1$

$a_o = a_{DIFF} \cdot a_g$

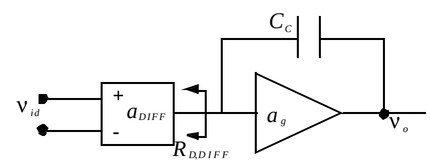
Location of compensation pole :

$$\omega_C = \frac{\omega_{P2}}{a_o}$$

EECS140 ANALOG CIRCUIT DESIGN LECTURES ON STABILITY

Slew Rate & Compensation Miller Op Amp (Cont.)

SB-35



$$a_{DIFF} = g_m \cdot R_{DDIFF}$$

$$\omega_C = \frac{1}{R_{Ddiff} \cdot a_g \cdot C_C} = \frac{\omega_{P2}}{a_o} = \frac{\omega_{P2}}{a_{DIFF} \cdot a_g}$$

$$\frac{\omega_{P2}}{a_{DIFF}} = \frac{1}{R_{DDIFF} \cdot C_C} = \frac{\omega_{P2}}{g_m \cdot R_{DDIFF}}$$

$C_C = \frac{g_m}{\omega_{P2}}$ The size of the compensation depends only on g_m & ω_{P2}

Slew Rate & Compensation Miller Op Amp (Cont.) SB-36

$$g_m = \frac{2 \cdot I_{DS}}{V_{DSAT}}$$

$$V_{DSAT} = \frac{2 \cdot I_{DS}}{g_m}$$

$$g_{m1}\Big|_{I_{DS} = \frac{I_{SS}}{2}}$$

$$\frac{dv_o}{dt} = \frac{I_{SS}}{C_C} = \frac{I_{SS}}{g_{m1}} \cdot \omega_{P2} = \frac{\frac{I_{SS}}{2} \cdot 2}{g_{m1}} \cdot \omega_{P2}$$

$$\boxed{\text{Slewrate} = \frac{dv_o}{dt} = V_{DSAT1} \cdot \omega_{P2}}$$

$V_{DSAT} = 0.1V$

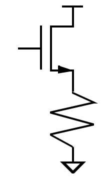
$\omega_{P2} = 10MHz \cdot 2\pi$ $\qquad SR = 6.3V/\mu\text{sec}$

Slew Rate & Compensation Miller Op Amp (Cont.) SB-37

How to increase slew rate :

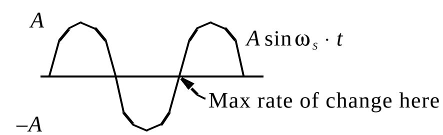
Increase $V_{DSAT1} \Rightarrow$ More current, smaller $\frac{W}{L}$

Increase ω_{P2}



$$\frac{g_m}{1 + g_m R_S}$$

Slew rate limits max change :



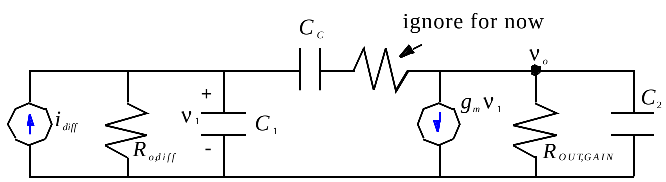
$A = 2V \qquad \omega_s = 10^6 \cdot 2\pi$

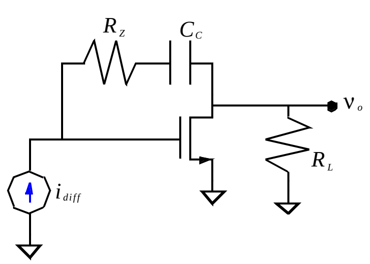
$SR = 13V/\mu\text{sec}$

$$\frac{dV_{SIG}}{dt} = \omega_s \cdot A\cos\omega_s \cdot t$$

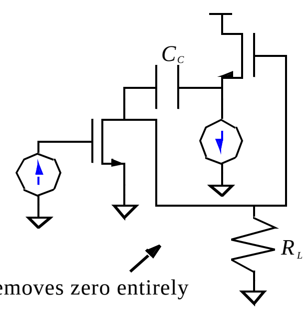
Max Value $\omega_s \cdot A$

MOS Miller Amp - Right Half Plane Zero SB-38





also



removes zero entirely

MOS Miller Amp - Right Half Plane Zero (Cont.) SB-39

$$\omega_z = \frac{1}{C_C \cdot \left(\frac{1}{g_m} - R_z\right)}$$