Asteroid Masses and Densities

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Since 1989 both traditional groundbased techniques and modern spacecraft techniques have increased the number of asteroids with known masses from 4 to 24. At the same time, the shapes for 16 of these asteroids have been determined with sufficient precision to determine reliable volumes and bulk densities of these bodies. This review paper will look at the masses and densities that have been determined.

1. INTRODUCTION

Although knowledge of the masses and bulk densities of the asteroids is critical in assessing their composition, determining these quantities is a difficult task. Asteroid perturbations of the inner planets of the solar system constitute the largest insufficiently modeled perturbation of high-accuracy planetary ephemerides (*Standish,* 2000). Thus, improved knowledge of asteroid masses is required before planetary ephemerides can be improved. After 200 years of observations, masses have been determined for only 24 asteroids. The next several years should see a large increase in the number of asteroid mass and density determinations due to new methods of determining their masses.

The difficulty in determining asteroid masses lies in their small size. Even the largest main-belt asteroid, 1 Ceres, estimated to contain 30–40% of the mass of the main belt, is only 1% the mass of the Moon. Determining the mass of an asteroid requires observation of its gravitational effect on another body such as an asteroid satellite, or a perturbed body such as another asteroid or a spacecraft. Currently, 12 asteroids are known to have natural satellites (*Chapman et al.,* 1995; *Elliot et al.,* 2001; *Merline et al.,* 1999, 2000, 2001a,b, 2002; *Margot et al.,* 2001; *Veillet,* 2001). Two of these asteroids with satellites, 1998 WW_{31} and 2001 QT_{297}, are transneptunian objects. In 2000, 433 Eros became the first asteroid to be orbited by a spacecraft, *NEAR Shoemaker* (*Yeomans et al.,* 2000). Perturbations on *NEAR Shoemaker* were also used to determine the mass of 253 Mathilde (*Yeomans et al.,* 1998).

To first order, the perturbation of a test body can be estimated using the two-body ballistic particle model

$$\tan \frac{1}{2}\theta = \frac{G(m + M)}{v^2 b}$$

where θ is the angle of deflection in the center-of-mass frame of reference, m is the mass of one body, M is the mass of the other body, G is the gravitational constant, v is the relative velocity of the encounter, and b is the impact parameter (see Fig. 1).

Most asteroids have orbital planes near the ecliptic. Examination of the perturbing equation (see *Danby,* 1988, section 11.9) shows that a coplanar encounter will change only the semimajor axis and/or eccentricity, such that to a first approximation the major change to most perturbed asteroid orbits will be a change in these two elements. In addition, the perturbation is weak, so the change in orbital elements of the perturbed asteroid is small. Thus, the predominant observable for a perturbed asteroid is the cumulative change in its longitude as a function of time caused by a change in its semimajor axis.

Bulk densities are a function of only the mass and volume. If an asteroid's mass is known, determining the volume is equivalent to determining its bulk density. Except for the largest asteroids, their mean diameters give only rough approximations of their volumes because they do not have nearly spheroidal shapes.

2. EARLY MASS DETERMINATIONS

More than 150 years after the discovery of 4 Vesta in 1807, *Hertz* (1966) made the first asteroid mass determination by analyzing its perturbation on 197 Arete. Since Arete's orbital period is nearly 5/4 that of Vesta, it encounters Vesta every 18 years. The multiple encounters enhance the size of the perturbation of Vesta's mass on Arete, making the mass of Vesta easier to determine.

Between 1966 and 1989, masses were determined for three other asteroids, 1 Ceres (*Schubart,* 1970, 1971a,b, 1974; *Landgraff,* 1984, 1988; *Standish and Hellings,* 1989), 2 Pallas (*Schubart,* 1974, 1975; *Standish and Hellings,* 1989), and 10 Hygiea (*Scholl et al.,* 1987). During this time *Hertz* (1968) and *Standish and Hellings* (1989) redetermined the mass of Vesta. Aside from the masses determined by *Standish and Hellings* (1989), all these masses were determined using asteroid-asteroid perturbations.

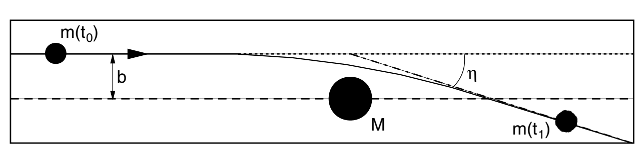

Fig. 1. Ballistic approximation of a small asteroid by a large one. From *Hilton et al.* (1996).

The masses of Ceres and Pallas were determined from their mutual perturbations, but they were not determined simultaneously. *Schubart* (1971a,b, 1974) determined Ceres' mass and subsequently determined Pallas' mass (*Schubart,* 1974, 1975). Unfortunately, Schubart's method of producing normal points from the observations of Ceres and Pallas resulted in masses for Ceres that were too high in both analyses: $(6.7 \pm 0.4) \times 10^{-10}$ $M_\odot$ (*Schubart,* 1971a,b) and $(6.0 \pm 0.7) \times 10^{-10}$ $M_\odot$ (*Schubart,* 1975). The masses subsequently determined for Pallas were $(1.3 \pm 0.4) \times 10^{-10}$ $M_\odot$ (Schubart, 1974) and $(1.1 \pm 0.2) \times 10^{-10}$ $M_\odot$ (*Schubart,* 1975).

Standish and Hellings (1989), using *Viking* lander radar ranging data from 1976 through 1981, simultaneously determined the masses of Ceres, Pallas, and Vesta from their perturbations of Mars. These ranging observations have an accuracy of 7 m over the period from 1976 through 1980 and 12 m from 1980 through 1981. The masses determined were $(5.0 \pm 0.2) \times 10^{-10}$ $M_\odot$ for Ceres (a 15% decrease from Schubart's mass), $(1.4 \pm 0.2) \times 10^{-10}$ $M_\odot$ for Pallas (a 30% increase), and $(1.5 \pm 0.3) \times 10^{-10}$ $M_\odot$ for Vesta (a 9% increase). Thus, the masses of the three largest asteroids were known with an uncertainty of $2–3 \times 10^{-11}$ $M_\odot$.

3. MODERN MASS DETERMINATIONS

Since 1989, masses have been determined for 20 additional asteroids, along with 13 new determinations for the mass of Ceres, 5 for Pallas, 7 for Vesta, and 1 for Hygiea. These recent mass determinations are summarized in Table 1.

Most of these mass determinations have been made using the classic method of observing the gravitational perturbation on a test asteroid. Although the stated uncertainties in the masses have also improved, the actual uncertainties from the classic method are more likely closer to the $2–3 \times 10^{-11}$ $M_\odot$ of *Standish and Hellings* (1989).

The Standish (personal communication, 2001) and *Pitjeva* (2001) masses for Ceres, Pallas, and Vesta were determined from their perturbations of Mars measured from *Viking* lander and *Mars Pathfinder* time delay radar observations.

Eleven of the mass determinations were made using methods previously unavailable. Although there were insufficient observations to determine an orbit for its satellite, 243 Ida I (Dactyl), the mass of 243 Ida was determined with an uncertainty of 15% by *Petit et al*. (1997) based on the constraint that Dactyl's orbit is stable.

The masses of 45 Eugenia (*Merline et al.,* 1999), 90 Antiope (*Merline et al.,* 2002), and 87 Sylvia (*Margot et al.,* 2001) were determined from observations of satellites using groundbased adaptive optics. Adaptive optics observations of 762 Pulcova suggest that it is not a single body, but is made up of two components orbiting each other, nearly in contact (*Merline et al.,* 2002).

The masses of 1999 KW_4, 2000 DP_{107}, and 2000 UG_{11} (*Margot et al.,* 2001) were determined using radar time delay-Doppler. Preliminary reduction of observations of 1999 KW_4 indicate that the motion of the primary about the center of mass is observable, making it possible to determine masses for both components of this binary asteroid.

The masses of 253 Mathilde (*Yeomans et al.,* 1998) and Eros (*Yeomans et al.,* 2000) are the first two asteroid masses determined by observing the perturbation of a spacecraft in the vicinity of the asteroid.

The Konopliv et al. (personal communication, 2002) mass of Vesta was determined from its perturbation of Eros and, indirectly, the *NEAR Shoemaker* spacecraft during a close approach (0.416 AU) between Vesta and Eros. The rather large distance of the encounter demonstrates both how large the perturbations of the largest asteroids can be and the sensitivity of spacecraft radar time-delay observations.

Future possible asteroid missions, such as *Dawn* (*Russell et al.,* 2001), *Muses-C* (*Fujiwara et al.,* 2001), and *Hera* (*Sears et al.,* 2000), will return information on the masses, volumes, and densities of the asteroids they encounter. Space missions, however, are limited because of the expense required for the manufacture, launch, and monitoring of a spacecraft.

4. LIMITATIONS TO DETERMINING ASTEROID MASSES FROM THE PERTURBATIONS OF ASTEROIDS

The correlation between the *a priori* mass used for a third asteroid, such as Pallas, can significantly change the mass determined for an asteroid, such as Ceres. Figure 2 shows the mass of Ceres determined by several authors using different test asteroids. Except for *Standish and Hellings* (1989) and *Hilton* (1999), all the mass determinations since 1989 have used approximately the same *a priori* mass for Pallas. Generally, the masses determined agree with each other to within 1×10^{-11} $M_\odot$. Hilton showed that using exactly the same data but a different *a priori* mass for Pallas, a significantly different mass is obtained for Ceres. The mass determined is independent of the asteroid used as a test body, but varies linearly with the mass adopted for Pallas. In this particular case, the correlation between the masses of Ceres and Pallas is caused by the similarity in the mean motions and mean longitudes of Ceres and Pallas ($0° \leq |\lambda_{Ceres} - \lambda_{Pallas}| \leq 35°$ between 1801 and 2001).

More recent mass determinations such as *Michalak* (2001) and *Goffin* (2001) have included analyses of how changing the *a priori* mass of other asteroids in the model can change the value of the mass to be determined. Goffin's analysis shows that that the high inclination and eccentricity of Pallas is sufficient to discriminate between the its perturbation and that of Ceres on a test asteroid, in disagreement with Hilton.

Long-range encounters with multiple medium-sized asteroids (diameter ~50–150 km) induces noise in the orbit of the perturbed asteroid in a mass determination. The individual perturbations may be insignificant, but the encounters are numerous enough that their combined effect can

TABLE 1. Recent asteroid mass determinations.

Asteroid	Mass $M_\odot$	Method of Determining Mass	Reference
1 Ceres	$(4.7 \pm 0.3) \times 10^{-10}$	asteroid perturbation	*Goffin* (1991)
	$(4.80 \pm 0.08) \times 10^{-10}$	asteroid perturbation	*Sitarski et al.* (1992)
	$(4.8 \pm 0.2) \times 10^{-10}$	asteroid perturbation	*Williams* (1992)
	$(4.62 \pm 0.07) \times 10^{-10}$	asteroid perturbation	*Sitarski and Todorovic-Juchniewicz* (1992)
	$(5.0 \pm 0.2) \times 10^{-10}$	asteroid perturbation	*Viateau and Rapaport* (1995)
	$(4.71 \pm 0.09) \times 10^{-10}$	asteroid perturbation	*Carpino and Knežević* (1996)
	$(4.26 \pm 0.09) \times 10^{-10}$	asteroid perturbation	*Kuzmanoski* (1996)
	$(4.79 \pm 0.04) \times 10^{-10}$	asteroid perturbation	*Viateau and Rapaport* (1997a)
	$(4.76 \pm 0.02) \times 10^{-10}$	asteroid perturbation	*Viateau and Rapaport* (1998)
	$(4.39 \pm 0.04) \times 10^{-10}$	asteroid perturbation	*Hilton* (1999)
	$(4.70 \pm 0.04) \times 10^{-10}$	asteroid perturbation	*Michalak* (2000)
	$(4.76 \pm 0.02) \times 10^{-10}$	Mars perturbation	E. M. Standish (personal communication, 2001)
	$(4.81 \pm 0.01) \times 10^{-10}$	Mars perturbation	*Pitjeva* (2001)
2 Pallas	$(1.59 \pm 0.05) \times 10^{-10}$	asteroid perturbation	*Hilton* (1999)
	$(1.2 \pm 0.3) \times 10^{-10}$	asteroid perturbation	*Michalak* (2000)
	$(1.17 \pm 0.03) \times 10^{-10}$	asteroid perturbation	*Goffin* (2001)
	$(1.08 \pm 0.04) \times 10^{-10}$	Mars perturbation	E. M. Standish (personal communication, 2001)
	$(1.00 \pm 0.01) \times 10^{-10}$	Mars perturbation	*Pitjeva* (2001)
4 Vesta	$(1.40 \pm 0.04) \times 10^{-10}$	asteroid perturbation	*Sitarski and Todorovic-Juchniewicz* (1992)
	$(1.69 \pm 0.05) \times 10^{-10}$	asteroid perturbation	*Hilton* (1999)
	$(1.36 \pm 0.05) \times 10^{-10}$	asteroid perturbation	*Michalak* (2000)
	$(1.31 \pm 0.02) \times 10^{-10}$	asteroid perturbation	*Viateau and Rapaport* (2001)
	$(1.34 \pm 0.02) \times 10^{-10}$	Mars perturbation	E. M. Standish (personal communication, 2001)
	$(1.36 \pm 0.01) \times 10^{-10}$	Mars perturbation	*Pitjeva* (2001)
	$(1.38 \pm 0.03) \times 10^{-10}$	spacecraft perturbation*	Konopliv et al. (personal communication, 2002)
10 Hygiea	$(5.6 \pm 0.7) \times 10^{-11}$	asteroid perturbation	*Michalak* (2001)
11 Parthenope	$(2.6 \pm 0.1) \times 10^{-12}$	asteroid perturbation	*Viateau and Rapaport* (1997b)
	$(2.56 \pm 0.07) \times 10^{-12}$	asteroid perturbation	*Viateau and Rapaport* (2001)
15 Eunomia	$(4 \pm 1) \times 10^{-12}$	asteroid perturbation	*Hilton* (1997)
	$(1.2 \pm 0.4) \times 10^{-11}$	asteroid perturbation	*Michalak* (2001)
16 Psyche	$(9 \pm 3) \times 10^{-12}$	asteroid perturbation	*Viateau* (1999)
20 Massalia	$(2.44 \pm 0.4) \times 10^{-12}$	asteroid perturbation	*Bange* (1998)
45 Eugenia	$(3.0 \pm 0.1) \times 10^{-12}$	observation of satellite	*Merline et al.* (1999)
52 Europa	$(2.6 \pm 0.9) \times 10^{-11}$	asteroid perturbation	*Michalak* (2001)
87 Sylvia	$(7.6 \pm 0.6) \times 10^{-12}$	observation of satellite	*Margot et al.* (2001)
88 Thisbe	$(7 \pm 1) \times 10^{-12}$	asteroid perturbation	*Michalak* (2001)
90 Antiope	$(4.14 \pm 0.05) \times 10^{-13}$	observation of satellite	*Merline et al.* (2002)
121 Hermione	$(4.7 \pm 0.8) \times 10^{-12}$	asteroid perturbation	*Viateau* (1999)
243 Ida	$(2.2 \pm 0.3) \times 10^{-14}$	observation of satellite†	*Petit et al.* (1997)
253 Mathilde	$(5.19 \pm 0.02) \times 10^{-14}$	spacecraft perturbation	*Yeomans et al.* (1998)
433 Eros	$(3.6 \pm 0.9) \times 10^{-15}$	spacecraft perturbation	*Yeomans et al.* (2000)
444 Gyptis	$(4 \pm 2) \times 10^{-12}$	asteroid perturbation	*Michalak* (2001)
511 Davida	$(5.6 \pm 0.7) \times 10^{-11}$	asteroid perturbation	*Michalak* (2001)
704 Interamnia	$(4 \pm 2) \times 10^{-11}$	asteroid perturbation	*Landgraff* (1992)
	$(3.5 \pm 0.9) \times 10^{-11}$	asteroid perturbation	*Michalak* (2001)
762 Pulcova	$(1.28 \pm 0.02) \times 10^{-12}$	observation of satellite	*Merline et al.* (2002)
1999 KW_4	$(1.1 \pm 0.2) \times 10^{-18}$	observation of satellite	*Margot et al.* (2001)
2000 DP_{107}	$(2.2 +1.0,-0.3) \times 10^{-19}$	observation of satellite	*Margot et al.* (2001)
2000 UG_{11}	$(5 +1,-2) \times 10^{-21}$	observation of satellite	*Margot et al.* (2001)

* The Konopliv et al. mass of Vesta was determined from its perturbation of Eros and, indirectly, the *NEAR Shoemaker* spacecraft during a close approach (0.416 AU) between Vesta and Eros.

† The mass of 243 Ida was determined based on the constraint that its satellite is in a stable orbit, not from actual observation of the orbit.

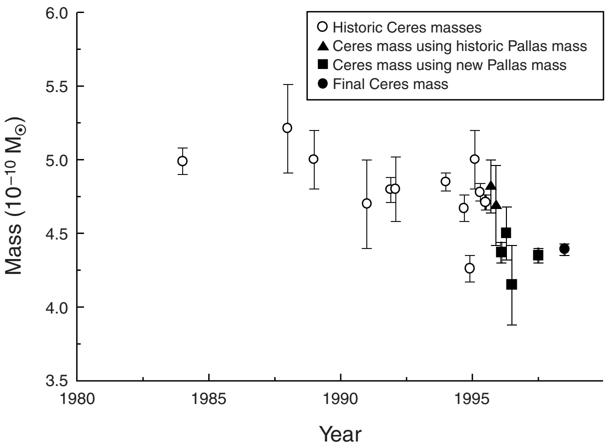

Fig. 2. Historic determinations of the mass of 1 Ceres.

be significant. An example of the effect of unmodeled perturbations can be seen in the ephemeris of Mars. The single largest source of uncertainty in Mars' motion is perturbations by unmodeled asteroids (*Standish and Williams,* 1990). As a result of these perturbations the uncertainty in Mars' position increases by about 0.01 arcsec per century. *Williams* (1984) identifies 36 asteroids with diameter ≥100 km (*Tedesco et al.,* 2002), including Ceres, that are significant perturbers of Mars. Aside from Ceres, the median Mars-asteroid distance at mean opposition for these asteroids' distance is 5.2× greater than the Ceres-asteroid distance at mean opposition. These smaller Ceres-asteroid distances at mean opposition have two consequences. First, the median force on Ceres is 27× greater than that acting on Mars. Second, the encounters take place over a much longer period of time. The median Ceres-asteroid synodic period is 11× greater than the median Mars-asteroid period. A quantitative determination of the increase in the perturbation would require a significant effort, but this qualitative analysis shows that the perturbation of Ceres is easily tens to hundreds of times greater than the perturbation of Mars. Hence, unmodeled perturbations from several medium-sized asteroids can easily dominate over the perturbation of a single large asteroid.

Ceres and Pallas made their closest approach to each other nearly 200 years ago, about the time they were discovered. Thus, determination of their masses from mutual perturbations rely critically on the oldest observations. *Michalak* (2000) made mass determinations for Ceres from its perturbations of Pallas and vice versa. These two determinations produced results similar to *Hilton* (1999). Michalak assumed that there were systematic errors in the early observations and chose not to use these determinations in his final weighted average for the masses of Ceres and Pallas. Hilton examined the oldest observations of Ceres, Pallas, and Vesta, and found no evidence for systematic errors in the observations. It is still possible that perturbations from many long-range encounters over 200 years have made the results unreliable. These observations have a RMS (O–C) ~ 3" in both right ascension and declination, so a small systematic error may not be evident.

Hilton et al. (1996) point out that 348 May should be ideal for determining the mass of Ceres even though it is subject to perturbations from several other asteroids. Although *Davis and Bender* (1977) found an encounter between May and 511 Davida, Hilton et al. failed to find this encounter, and Davis and Bender failed to find the encounter with Ceres. In both searches for encounters the authors made the assumption that perturbations were small enough that an encounter with one asteroid would not change the orbit of the perturbed asteroid enough to hide an encounter with another asteroid. Since Hilton et al. and Davis and Bender used different initial conditions for May and each missed an encounter found by the other, this assumption appears to be false.

In a followup to the work of *Hilton et al.* (1996) and *Davis and Bender* (1977), I attempted to determine the mass of 511 Davida using the software described in *Hilton* (1999). Using the previously determined mass for Ceres, both Ceres and Davida were included as perturbers of May. A search was then made for additional encounters. If one was found, the mass of the perturbing asteroid was added to the model and the process repeated. This process found encounters between May and seven asteroids (1 Ceres, 16 Psyche, 52 Europa, 87 Sylvia, 451 Patientia, 511 Davida, and 704 Interamnia). At this point, the sparse observational history of May made it impractical to determine the masses of any of these asteroids.

Is there evidence for significant noise being induced into the orbits of perturbed asteroids used in the determination of asteroid masses? *Viateau and Rapaport* (1997b, 2001) include perturbations from eight asteroids in their determination of the mass of 11 Parthenope from the perturbation of 17 Thetis. Aside from the perturbation by Vesta, most of these perturbations are small, but they are cumulatively large enough to affect the mass determined at the 1σ level. *Michalak* (2001) shows how encounters with other large asteroids affect the mass determined for a given large asteroid using a particular test asteroid.

The *Hilton* (1999) ephemeris positions of Juno have large systematic residuals with respect to the early (pre-1825) observed positions. A search for perturbers of Juno was made following the same scheme described above for May. The final model included perturbations from nine asteroids (1 Ceres, 2 Pallas, 4 Vesta, 16 Psyche, 24 Themis, 87 Sylvia, 216 Kleopatra, 511 Davida, and 704 Interamnia). However, including all these perturbing asteroids was still insufficient to remove all the systematic error.

Several authors such as *Sitarski and Todorovic-Juchniewicz* (1995), *Carpino and Knežević* (1996), *Viateau and Rapaport* (1998), *Michalak* (2000), and *Goffin* (2001) have determined the masses of Ceres, Pallas, and Vesta by taking the weighted mean of mass determinations of several perturbed asteroids. The results for each individual determination can vary widely. For example, Goffin's determination of the mass of Pallas used 16 asteroids with individual determinations ranging from $(0.48 \pm 0.56) \times 10^{-10}$ to $(2.8 \pm 1.1) \times 10^{-10}$ $M_\odot$. The unweighted mean of the mass determinations is 1.4×10^{-10} $M_\odot$ with a variance of 0.8×10^{-10} $M_\odot$. *Michalak* (2000) rejected approximately 20% of the mass determinations he made due to large residuals. *Michalak* (2001) made mass determinations of seven other asteroids, and rejected a similar proportion of the results. This rejection rate indicates the level at which the perturbation of test asteroids by other unmodeled asteroids is great enough to be significant in at least 20% of the cases. It is not clear how many of the adopted mass determinations are contaminated by smaller perturbations caused by unmodeled asteroids.

The conclusion is that the classical method of determining asteroid masses is limited by uncertainty in the masses of the large asteroids and perturbations by other, unmodeled asteroids. Examination of those authors who have made mass determinations of asteroids based on the perturbations of multiple test asteroids suggest that the actual uncertainty in the masses is on the order of 10^{-11} $M_\odot$.

The limitation of long-range perturbations can be eliminated by reducing the time around the encounter by the test asteroid to a short enough period that the encounter truly can be treated as a two-body deflection of a ballistic particle. Reducing the observing period requires an increase in the accuracy of the observations of the perturbed body. Past mass determinations made by optical observation of a test asteroid used the detection of changes in the mean longitude of a few arcseconds over periods on the order of 50 yr. If the perturbations of other asteroids are to be ignored, then the period of observation of the test asteroid needs to be reduced to a fraction of an orbital period (i.e., a few months), requiring that the position of the test asteroid be determined to a few milliarcseconds.

The *GAIA* mission (*GAIA,* 2000) will produce single-observation accuracies that are good enough to make asteroid mass determinations by observing a perturbed body over the period of a few months. Another high accuracy source of data is radar time delay-Doppler observations. Since radar observations can determine the position of the center of mass of the perturbed asteroid with an accuracy of 1 km or better (*Ostro,* 1993), the perturbed motion of an asteroid could be detected over time periods as short as a few days. However, since it relies on a signal sent out from a radar station, this technique has a r^{-4} falloff in the return signal. The perturbed asteroids are usually only tens of kilometers in diameter, so observations require the use of the Arecibo radio telescope, which is restricted in its declination and hour angle range.

5. EARLY SIZE DETERMINATIONS

Determination of the mean diameters, and hence the volumes and bulk densities, of even the largest asteroids has taken well over a century to produce reliable results. The first attempt to determine the asteroid mean diameters was made by *Herschel* (1802). Using a projection system to measure their angular diameters, he determined 161.6 miles (260.0 km) for the diameter of Ceres and 147 miles (237 km) for Pallas. These results are nearly a factor of 4 too small for Ceres and more than a factor of 2 too small for Pallas. A similar determination by *Schröter* (1811) produced diameters of 2613 km for Ceres, 3380 km for Pallas, and 2290 km for Juno. Clearly, direct observation of the disks of the asteroids using early nineteenth century equipment and techniques produced inaccurate results. As late as 1979 the *Schubart and Matson* (1979) radius for Ceres was uncertain by 75 km, leading to an uncertainty in its bulk density of 50%.

Bruhns (1856) made the first indirect determination of the size of the asteroids using their brightnesses, a technique that requires knowledge of the asteroid's albedo. Bruhns chose an average of the albedos of Saturn, Uranus, Neptune, and the Galilean satellites, all of which have albedos much higher than that of most asteroids. Thus, the diameters he determined for 39 asteroids were too small. In particular, he found the diameter of Ceres to be 365 km; Pallas, 277 km; Juno, 180 km; and Vesta, 367 km.

Barnard (1895), using impersonal filar micrometer observations, determined the diameters of Ceres (780 ± 80 km), Pallas (490 ± 100 km), Juno (190 ± 20 km), and Vesta (390 ± 40 km). These diameters were the definitive values for the first half of the twentieth century. Micrometer determinations of the asteroid diameters tended to produce values that are significantly smaller than the current diameters. For example, *Dollfus* (1971) determined the diameters of the first four asteroids: Ceres, 770 km; Pallas, 490 km; Juno, 195 km; Vesta, 390 km. Like Barnard's diameters, they are all systematically too small. A discussion of the sources of systematic errors in micrometer measurements can be found in *de Vaucouleurs* (1964).

Hamy (1899) made the first attempt to determine the diameter of Vesta using an interferometer. The result, 390 ± 50 km, similar to that of direct measurements, is too small.

The first twentieth century determination of asteroid sizes was made by *Windorn* (1967), who estimated the albedos of 1 Ceres, 2 Pallas, 4 Vesta, and 7 Iris from their polarimetric properties. These albedos then allowed him to estimate effective diameters from their photometry.

Allen (1971) pioneered the radiometric method of determining asteroid diameters. This method compares the amount of reflected visible radiation with the amount of radiated infrared radiation. Since these two quantities depend complimentarily on the albedo, the diameter and albedo are determined simultaneously.

Early polarimetric and radiometric diameters have uncertainties of about 100 km. Thus, the density of Ceres had an uncertainty of about 15% and the densities of Vesta and Pallas had uncertainties of about 20%. The uncertainties in the diameters and masses of the asteroids contributed approximately equally to the uncertainty in the densities. Subsequent improvement in the models, and a combination of both polarimetry and radiometry techniques for the albedo estimates, has reduced the uncertainty for the diameters of the largest asteroids to about 5% (*Tedesco et al.,* 2002). However, the use of the radiometric method to determine the volume of an asteroid becomes severely limited since the shapes of the smaller asteroids depart from a spheroid. Even the largest of the S-class asteroids, 15 Eunomia, departs significantly from a spheroidal shape (see Fig. 3).

Worden and Stein (1979) determined diameters from speckle interferometry observations of Pallas and Vesta. The diameter for Vesta, 550 ± 23 km, is in good agreement with modern determinations, but the diameter for Pallas, 673 ± 55 km, is approximately 26% greater than the currently accepted value. The authors point out that the nonlinearity and nonuniformity of the photographic film they used could cause systematic errors; furthermore, these errors were more likely to affect the diameter of Pallas. Until recently, detector nonlinearity has remained the greatest obstacle to using speckle interferometry to determine asteroid sizes and shapes.

Observing stellar occultations by asteroids is another method of inferring asteroid shapes. This method has the advantage of making accurate (σ ~ a few kilometers) determinations of the length of the chord observed. Individual chord lengths are determined by timing the length of the occultation at a known place within the path on Earth's surface. The rate of motion of the asteroid from its ephemeris allows the timing data to be converted to a length. The shape is then formed by observing multiple chords at several different places across the path. Thus, the asteroid's projected shape at a specific rotational phase and relative position can be determined from multiple chords observed during a single occultation.

Occultation observations have a severe restriction. Since the track of an asteroid occultation on the surface of Earth is only a couple hundred kilometers wide, the ephemeris for the asteroid needs an accuracy of about 1 arcsec to assure that most of the observers are within the path of the shadow. There is also a significant chance that the shadow will pass over a large body of water or be unobservable at one or more locations due to local weather conditions.

TABLE 2. Occultations of asteroids with mass determinations.

Asteroid	No. Occult. Obs.	Max. No. Chords
1 Ceres	2	13
2 Pallas	9	141
4 Vesta	2	21
10 Hygiea	5	2
11 Parthenope	1	1
15 Eunomia	3	2
45 Eugenia	1	1
121 Hermione	2	3
216 Kleopatra	3	10
433 Eros	1	10
704 Interamnia	4	11

As of February 27, 2002, the International Occultation Timing Association (IOTA) had collected data from 331 occultations of 213 asteroids (*Dunham,* 2002). These occultations cover the period from February 19, 1958, through February 26, 2002, and include several observations of asteroid occultations for which mass determinations have been made (see Table 2).

Since many chords need to be observed to determine the shape of an asteroid, a large number of observers stationed along the expected path of the asteroid's shadow are required. For example, 22 teams of observers obtained just seven chords during an occultation of SAO 85009 by Pallas (*Wasserman et al.,* 1979). Of the 331 occultations for which

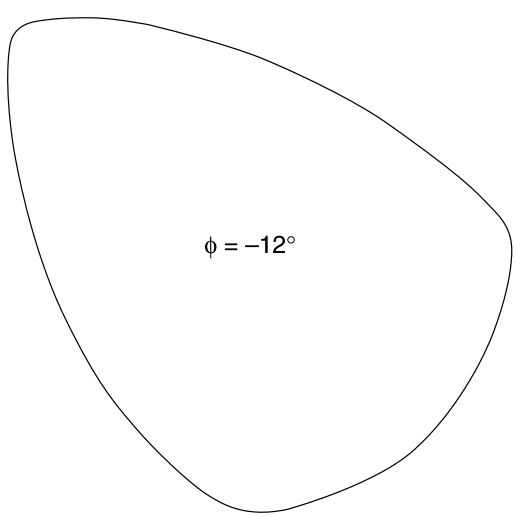

Fig. 3. The shape of 15 Eunomia at a rotation phase of –12°. From *Ostro and Connelly* (1984).

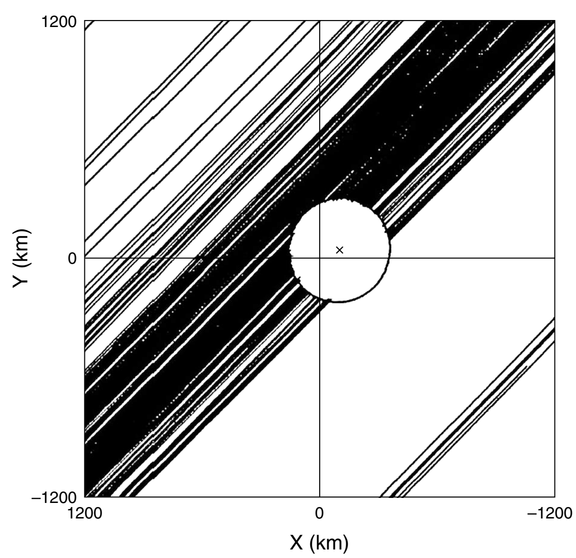

Fig. 4. The shape of 2 Pallas determined by 141 chords observed in its occultation of 1 Vulpeculae by *Dunham et al.* (1990).

IOTA has collected observations, only 56 occultations yielded five or more chords and only four occultations yielded 20 or more chords. Figure 4 shows the result from a total of 141 chords from the occultation of 1 Vulpeculae by Pallas in 1983 (*Dunham et al.,* 1990).

A single occultation gives only a two-dimensional projection of the shape of an asteroid. Thus, a single occultation cannot give enough information to reliably determine the volume for a nonspheroidal asteroid.

6. MODERN SIZE DETERMINATIONS

The determinations of the shapes of asteroids no longer have to rely on the indirect methods developed and used during most of the previous 30 years. Instead, using both spacebased observing platforms and groundbased adaptive optics, direct imaging dominates with much more accurate angular resolution.

The most direct method has been through encounters of asteroids by spacecraft. *Galileo* imaged both 951 Gaspra (*Belton,* 1994) and 243 Ida (*Belton et al.,* 1996), while *NEAR Shoemaker* imaged 253 Mathilde (*Yeomans et al.,* 1998) and 433 Eros (*Yeomans et al.,* 2000). These encounters have allowed highly accurate determinations of the size and shape of these four irregular asteroids. Integrating over the shape of the asteroids (e.g., *Yeomans et al.,* 2000) allows an accurate determination of their volumes to be derived. Combined with mass information, the densities of three of these asteroids, Ida, Mathilde, and Eros, have been determined. In the case of Eros, additional gravity field data show that its composition is probably homogeneous. Although spacecraft imaging *in situ* is highly accurate, it is also extremely expensive. Thus, this technique cannot be counted on for providing volumes for most asteroids.

Direct imaging using optical telescopes has been greatly improved recently by using instruments either in space (*Thomas et al.,* 1997; *Storrs et al.,* 1999) or using adaptive optics at groundbased observatories (*Drummond et al.,* 1998; *Merline,* 2002).

Thomas et al. (1997) have determined the size and shape of Vesta to an uncertainty of only 5 km using the Hubble Space Telescope. This gives its volume with an accuracy of 4%, which is comparable to that attainable from occultation studies. *Storrs et al.* (1999) have provided similar results for several smaller main-belt asteroids. Their results also show that even relatively large (mean diameter ~300 km) asteroids may have nonspheroidal shapes.

Drummond et al. (1999) have used adaptive optics techniques to produce groundbased determinations of the sizes and shapes of Ceres and Vesta with absolute accuracies similar to those produced from Hubble observations. *Drummond and Christou* (1994) indicates that these more recent groundbased observations do not suffer from the systematic errors of earlier ones.

An even more accurate method of asteroid size and shape determination exists in the form of radar time delay-Doppler observations. This method was developed theoretically by *Ostro et al*. (1988) and has been refined to provide

TABLE 3. Current best estimates of asteroid bulk densities.

Asteroid		Density (g cm^{-3})	Reference
1 Ceres*	*Michalak* (2000)	2.03 ± 0.05	*Parker et al.* (2002)
	E. M. Standish (personal communication, 2001)	2.06 ± 0.05	
	Pitjeva (2001)	2.08 ± 0.05	
2 Pallas*	*Goffin* (2001)	3.1 ± 0.3	*Drummond and Cocke* (1989)
	E. M. Standish (personal communication, 2001)	2.9 ± 0.3	
	Pitjeva (2001)	2.6 ± 0.2	
4 Vesta*	E. M. Standish (personal communication, 2001)	3.4 ± 0.2	*Thomas et al.* (1997)
	Pitjeva (2001)	3.5 ± 0.2	
	Konopliv et al. (personal communication, 2002)	3.5 ± 0.3	
16 Psyche		1.8 ± 0.6	*Viateau* (1999)
20 Massalia		2.7 ± 1.1	*Bange* (1998)
45 Eugenia		1.2 (+0.6,–0.3)	*Merline et al.* (1999)
87 Sylvia		1.6 ± 0.3	*Tedesco et al.* (2002)
90 Antiope		1.8 ± 0.8	*Merline et al*. (2002)
121 Hermione		1.8 ± 0.4	*Viateau* (1999)
243 Ida		2.7 ± 0.4	*Petit et al.* (1997)
253 Mathilde		1.3 ± 0.2	*Veverka et al.* (1997)
433 Eros		2.67 ± 0.03	*Yeomans et al.* (2000)
762 Pulcova		1.5 ± 0.4	*Merline et al*. (2002)
1999 KW_4		2.4 ± 0.9	*Margot et al*. (2001)
2000 DP_{107}		1.6 (+1.2,–0.9)	*Margot et al*. (2001)
2000 UG_{11}		1.5 (+0.6,–1.3)	*Margot et al*. (2001)

* Densities using each of the three most recent mass determinations are given for Ceres, Pallas, and Vesta. All three densities are computed using the same volume for comparison.

detailed shapes of asteroids such as 4769 Castalia (*Hudson and Ostro,* 1994) and 216 Kleopatra (*Ostro et al.,* 2000). This technique is limited by a r^{-4} falloff in the return signal.

Ostro and Connelly (1984) pioneered the determination of asteroid shapes from inversion of their light curves. Nowadays, higher-quality data, more powerful computers, and more sophisticated models, such as those used by *Kaasalainen et al*. (2001), could provide very good estimates of asteroid sizes and shapes.

Another future source for asteroid volumes will be GAIA. With a basic resolution of 20 milliarcsec (*GAIA,* 2000), a 50-km-diameter asteroid at 1.5 AU would subtend approximately 2 pixels. Thus, several observations over a rotation period will allow shapes to be determined with an uncertainty of about 40–50%. Although other techniques already available are more accurate, *GAIA* will be able to produce the first large-scale determination of the volumes of large- to medium-sized asteroids.

The bulk density of an asteroid is its mass per unit volume. If the mass and volume of an asteroid are known, its density can easily be derived. There are currently 16 asteroids with reliable masses and volumes for which bulk densities can be derived. The current best estimates for densities of individual asteroids is summarized in Table 3.

7. SUMMARY

The last 10 years have seen great progress in the determination of the masses of asteroids. The classical method of asteroid mass determination by observing the perturbation of a test asteroid over 30–50 years is seriously limited by asteroid interactions not included in the model used to determine the mass. Fortunately, new methods such as spacecraft astrometry, observation of asteroid satellites, and high-accuracy observations (~1 milliarcsec) of the perturbed asteroid over a few months are being developed. These methods will allow better determination of the masses of individual asteroids. Another method that has great promise is radar observations of perturbed asteroids. These observations are of such high accuracy that the perturbation of the massive asteroid can be observed over a few days or months. The greatest drawback to this technique is the r^{-4} falloff in the return signal.

Better determination of asteroid shapes and volumes have also seen major improvements over the last decade. Groundbased adaptive optics and spacecraft-based observations have become major contributors to asteroid shape determinations. Other methods such as deconvolution of asteroid lightcurves and radar image synthesis also have much to contribute as well. Knowledge of both the mass and volume of an asteroid lead directly to the bulk density.

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