Cylindric algebras edit

Abbreviation: $\mathbf{CA}_\alpha$

Definition 1. A *cylindric algebra* of dimension α is a Boolean algebra with operators $\mathbf{A} = \langle A, \vee, 0, \wedge, 1, -, c_i, d_{ij} : i, j < \alpha\rangle$ such that for all $i, j < \alpha$

the c_i are increasing: $x \leq c_i x$

the c_i semi-distribute over $\wedge$: $c_i(x \wedge c_i y) = c_i x \wedge c_i y$

the c_i commute: $c_i c_j x = c_j c_i x$

the diagonals d_{ii} equal the top element: $d_{ii} = 1$

$d_{ij} = c_k(d_{ik} \wedge d_{kj})$ for $k \neq i, j$

$c_i(d_{ij} \wedge x) \wedge c_i(d_{ij} \wedge -x) = 0$ for $i \neq j$

Remark: This is a template. Click on the 'Edit text of this page' link at the bottom to add some information to this page.

It is not unusual to give several (equivalent) definitions. Ideally, one of the definitions would give an irredundant axiomatization that does not refer to other classes.

Morphisms. Let $\mathbf{A}$ and $\mathbf{B}$ be ... . A morphism from $\mathbf{A}$ to $\mathbf{B}$ is a function $h : A \to B$ that is a homomorphism: $h(x...y) = h(x)...h(y)$

Definition 2. An ... is a structure $\mathbf{A} = \langle A, ...\rangle$ of type $\langle ...\rangle$ such that

... is ...: *axiom*

... is ...: *axiom*

Basic Results.

Examples.

1.

Finite Members. $f(n) =$ number of members of size n.

$f(1) =$ 1	$f(6) =$
$f(2) =$	$f(7) =$
$f(3) =$	$f(8) =$
$f(4) =$	$f(9) =$
$f(5) =$	$f(10) =$

Subclasses.

Representable cylindric algebras subvariety

Superclasses.

Diagonal free cylindric algebras subreduct

Two-dimensional cylindric algebras subreduct

Properties. (description)
Feel free to add or delete properties from this list. The list below may contain properties that are not relevant to the class that is being described.

Classtype	variety
Equational theory	undecidable for $\alpha \geq 3$, decidable otherwise
Quasiequational theory	
First-order theory	
Locally finite	no
Residual size	unbounded
Congruence distributive	yes
Congruence modular	yes
Congruence n-permutable	yes, $n = 2$
Congruence regular	yes
Congruence uniform	yes
Congruence extension property	yes
Definable principal congruences	
Equationally def. pr. cong.	
Amalgamation property	
Strong amalgamation property	
Epimorphisms are surjective	

References

[1] Roger Maddux, *Introductory course on relation algebras, finite-dimensional cylindric algebras, and their interconnections*, Algebraic Logic (Proc. Conf. Budapest 1988) ed. by H. Andreka, J. D. Monk, and I. Nemeti, Colloq. Math. Soc. J. Bolyai 54 North-Holland Amsterdam, 1991, 361–392 `http://www.math.iastate.edu/maddux/papers/raca.ps`