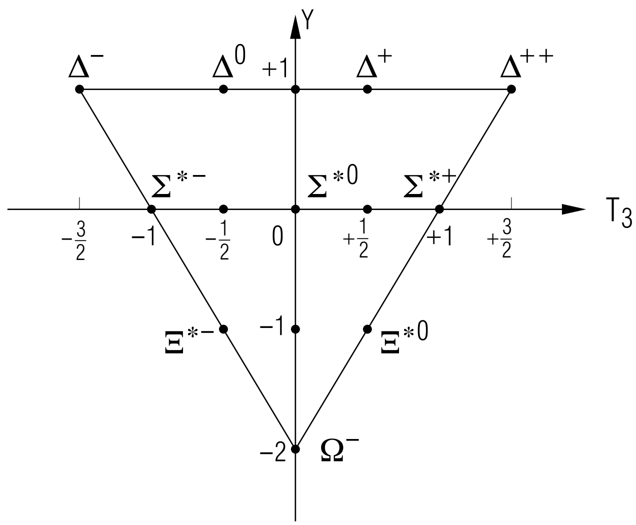
MISN-0-282

SU(3) AND THE QUARK MODEL



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

SU(3) AND THE QUARK MODEL

by

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Input Skills:

1. Explain briefly the isospin concept and calculate the total isotopic spin for a system of particles (MISN-0-278).
2. Explain how hadrons interact via the strong interaction (MISN-0-280).
3. List the basic couplings of the weak interaction and explain the function of the W particle (MISN-0-281).

Output Skills (Knowledge):

K1. Give the three forms of the Y vs. T plots for hadrons, and list the values of Y and T for the particles in each one.

K2. State the properties that all particles in any supermultiplet have in common.

K3. Give the essential quark characteristics for each of the three quarks: baryon number, spin, strangeness and charge.

K4. Write the symbols for the three quarks and the three antiquarks.

K5. State how many quarks and/or antiquarks make up mesons, baryons, and antibaryons.

K6. Discuss the model which accounts for the difference in particle masses within a supermultiplet.

K7. Discuss the basics of particle decays and interactions, in terms of quarks.

Output Skills (Problem Solving):

S1. Given the quark content of a particle, calculate B, Y, T, T_3, Q for that particle, and conversely.

S2. Given a particle decay, draw a quark diagram that represents it.

External Resources (Required):

1. M. J. Longo, *Fund. of Elem. Part. Physics*, McGraw-Hill (1973).
2. G. F. Chew, et al, *Scientific American*, Feb. 1964.

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1. Abstract

We here deal deal with a way of classifying the hadrons: according to SU(3) symmetry operations. This system is known as the quark model or the eight-fold way. The quark model of high energy physics is analogous to the periodic table of the elements in that it provides an ordering of the particles.

2. Readings

Longo, Chapter 8

G. F. Chew, M. Gell-Mann, and A. H. Rosenfeld, "Strongly Interacting Particles," *Scientific American* (Feb. 1964).

3. The Quark Concept

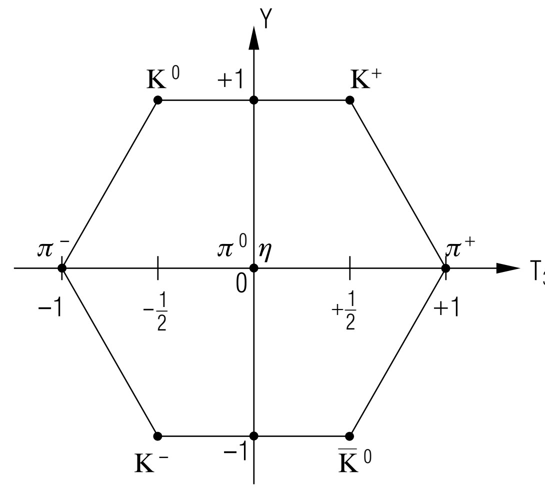
3a. Quarks as Elementary Particles. One of the goals of high energy physics is to explain the properties (mass, spin, charge, isotopic spin, strangeness) of the hadrons in terms of something more fundamental. The idea of the quark model is to invent a small set of particles, imbue them with appropriate properties, and use them to construct the hadrons much as neutrons and protons are used to construct the various atomic nuclei. The hypothetical fundamental building blocks are called quarks. They have never been isolated and have never been observed.

3b. Observed Symmetry Patterns in Hypercharge and Isospin. The quark model deals with the isotopic spin (both magnitude and 3-component), hypercharge, strangeness, and baryon number of the hadrons.

We begin with an observation that, if the particle properties are plotted a certain way, then there are only three forms to the plots. All of the plots have hypercharge on the vertical axis and 3-component of isotopic spin on the horizontal axis. We plot on the same graph only those particles which have the same baryon number, spin, and intrinsic

parity.[1] The particles on any one diagram have nearly the same mass.

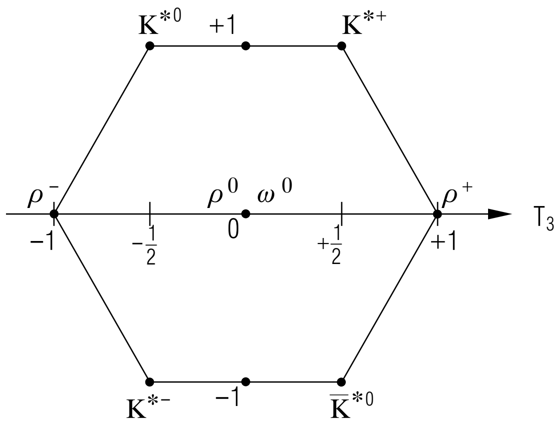
We start with the 0^- mesons (spin 0, odd parity):



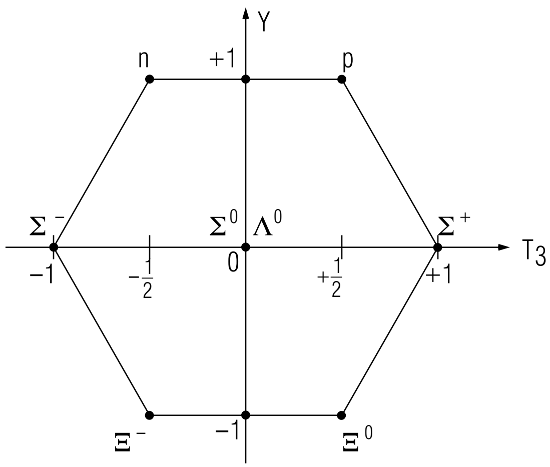
Both the π^0 and the η have $T_3 = 0$. However the π^0 belongs to an isospin triplet and has $T = 1$ while the η is an isospin singlet and has $T = 0$. This plot contains 8 particles and they are jointly called an octet.

There is an octet of 1^- mesons:

[1] A hadron can be assigned an intrinsic parity (+ or −) depending on whether or not the wave function of the particle, when the particle has zero orbital angular momentum, changes sign with operation by the parity operator. Intrinsic parity is denoted by a + or − superscript on the spin. This is a minor technical point for this discussion. You should realize that all particles in a given diagram have the same spin but there may be more than one diagram corresponding to a given spin.



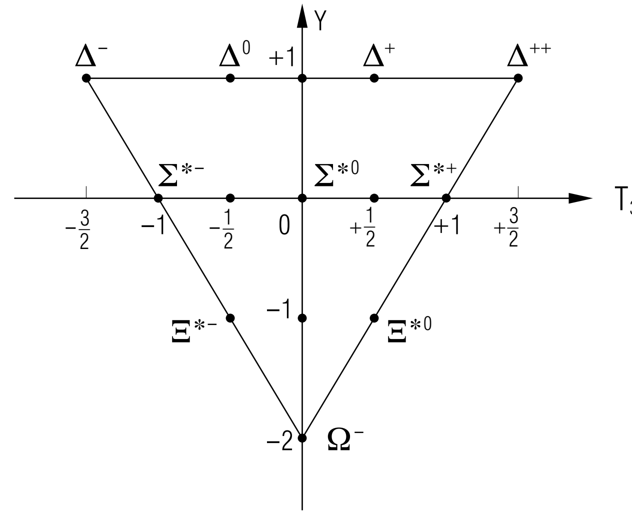
There is an octet of $1/2^+$ baryons:



The octet always consists of an isospin doublet with $Y = +1$, an isospin triplet with $Y = 0$, an isospin singlet with $Y = 0$, and an isospin doublet with $Y = -1$.

The second type pattern to be considered is the decimet, composed of ten particles: an isospin quartet with $Y = +1$, an isospin triplet with $Y = 0$, an isospin doublet with $Y = -1$, and an isospin singlet with $Y = -2$. Only baryons have been found to form decimets and, as we

shall see, the quark model provides a reason why. Here is a decimet of $3/2^+$ baryons:



This is the only complete decimet known but there are undoubtedly others at higher masses (yet to be discovered "resonances").

It is important to note that if a particle belongs to a given multiplet, all of its isospin partners belong to the same multiplet. The patterns shown here combine several sets of isospin partners to form a larger pattern than that provided by isospin alone.

We have now described two of the three forms, the octet and the decimet. The third form is the simplest. It consists of a single particle with $Y = 0$, $T = 0$, and $T_3 = 0$ and is called a singlet. It is easy to confuse one of these particles with the isotopic spin singlet which occurs in the octet of the same spin and parity. For example, the $\phi(1019)$ meson may be a 1^- singlet meson. If differs from the ω^0 only in mass. Which belongs to the octet and which to the singlet? We shall see that the quark model assigns different quark content to the $Y = 0$, $T = 0$ singlet and to the $Y = 0$, $T = 0$ particle in the octet. But quarks are not observable, so this distinction cannot be used. The point is that some assignments of particles to octets or singlets are arbitrary at present and, in fact, the physical particle may be some superposition of the two states.

If all particles (including those undetected at present) are put into singlets, octets, and decimets, the patterns have some predictive power. For example:

a. There are no $T > 3/2$ particles.

b. There can be no $Y > +1$ particles.

c. There can be no $Y < -2$ particles.

d. $Y = 1$ particles have either $T = 3/2$ or $T = 1/2$.

e. $Y = 0$ particles have either $T = 1$ or $T = 0$.

f. $Y = -1$ particles have $T = 1/2$.

g. $Y = -2$ particles have $T = 0$.

3c. Overview of the Quark Model of Elementary Particles. The job of the quark model is to explain these patterns. The basic ideas are:

a. There are 3 quarks and 3 antiquarks. (Recent developments have caused physicists to postulate the existence of a fourth quark but we shall discuss the three quark model now and introduce the fourth quark later.)

b. A meson consists of a quark and an antiquark.

c. A baryon consists of 3 quarks; an antibaryon of 3 antiquarks.

d. The quarks are assigned values of Y, T, and T_3.

e. By combining quarks in all possible ways which are consistent with (b) and (c), it is possible to construct those particles that appear in the singlets, octets, and decimets, and no others. In this regard, two different combinations of quark states may actually have the same quark content but they are combined to give two different values of the magnitude of the total isotopic spin. For example, a $T_3 = 1/2$ and a $T_3 = -1/2$ particle may combine to give either a $T = 1$, $T_3 = 0$ state or a $T = 0$, $T_3 = 0$ state.

f. The quark content of a particle in a certain multiplet is the same as the quark content of the analogous particle in other multiplets of the same type. For example, the quark content of the ρ^- is the same as the quark content of the π^-. The *dynamics* of the quarks are different and this explains the different mass and spin of the ρ^- and π^-.

g. Quarks are also assigned spin and baryon number. The total angular momentum of the quarks is the spin of the composite particle and the net baryon number of the quarks is the baryon number of the composite particle.

4. Quark Content

4a. Quark Constituents of Hadrons. It is possible to build all of the hadrons we have been discussing from the following three quarks (and their antiquarks):

Quark	B	T	T_3	σ	S	Y	Q
u	1/3	1/2	1/2	1/2	0	1/3	2/3
d	1/3	1/2	−1/2	1/2	0	1/3	−1/3
s	1/3	0	0	1/2	−1	−2/3	−1/3
$\overline{\text{u}}$	−1/3	1/2	−1/2	1/2	0	−1/3	−2/3
$\overline{\text{d}}$	−1/3	1/2	1/2	1/2	0	−1/3	1/3
$\overline{\text{s}}$	−1/3	0	0	1/2	1	2/3	1/3

Here B = baryon number, T = magnitude of isospin, T_3 = 3-component of isospin, σ = spin in units of $\hbar$, Y = hypercharge, and Q = charge in units of e. The first 5 entries (B, T, T_3, σ, and S) are assigned. Y and Q are calculated from B, S, and T_3: $Y = B + S$, $Q = Y/2 + T_3$.

The s quark is the only quark with non-vanishing strangeness. In fact the number of s quarks in the particle defines its strangeness. Conservation of strangeness then stems from the conservation of the number of s quarks. Conservation of baryon number also stems from conservation of quarks.

4b. Quark Constituents of Mesons. Mesons are constructed by combining a quark with an antiquark. There are 6 different combinations of a quark with a different antiquark: $d\bar{s}$, $u\bar{s}$, $\overline{\text{d}}$s, $\overline{\text{u}}$s, u$\overline{\text{d}}$, and $\overline{\text{u}}$d. Each of these has a definite baryon number (0), strangeness, charge, isotopic spin magnitude, and 3-component of isotopic spin. These combinations can be identified with particles on the basis of the values of these quantities.

	S	Q	T_3	T	Particle
$d\bar{s}$	1	0	$-1/2$	$1/2$	K^0 , K^{*0}
$u\bar{s}$	1	1	$1/2$	$1/2$	K^+ , K^{*+}
$\bar{d}s$	-1	0	$1/2$	$1/2$	$\overline{\mathrm{K}}^0$, $\overline{\mathrm{K}}^{*0}$
$\bar{u}s$	-1	-1	$-1/2$	$1/2$	K^- , K^{*-}
$u\bar{d}$	0	1	1	1	π^+ , ρ^+
$\bar{u}d$	0	-1	-1	1	π^- , ρ^-

Given the values in the table of Sect. 4a, you should be able to generate this table.

It is of interest to check out the isotopic spin quantum numbers. The combination $\mathrm{d\bar{s}}$ has $T_3 = 1/2$. The maximum T can be is $1/2$ since the combination consists of a $T = 1/2$ particle and a $T = 0$ particle.

Hence T must be $1/2$. The combination $\mathrm{u\bar{d}}$ has $T_3 = 1$ and $T_{max} = 1$ so T must be 1. You should check the other combinations.

There are 3 combinations of a quark with its own antiquark: $\mathrm{u\bar{u}}$, $\mathrm{d\bar{d}}$, and $\mathrm{s\bar{s}}$. None of these has a definite value of T. For example, $\mathrm{u\bar{u}}$ has $T_3 = 0$ but can be either a $T = 1$ or a $T = 0$ state. Quantum mechanically, the state $\mathrm{u\bar{u}}$ is a linear superposition of the $T = 1$ and $T = 0$ states. Similarly for the other combinations. It is, however, possible to form combinations of the states $\mathrm{u\bar{u}}$, $\mathrm{d\bar{d}}$ and $\mathrm{s\bar{s}}$which are states of pure T. These are:

a. $(1/\sqrt{2})(\mathrm{u\bar{u}} - \mathrm{d\bar{d}})$: $T = 1$, $T_3 = 0$: π^0 meson

b. $(1/\sqrt{6})(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} - 2\mathrm{s\bar{s}})$: $T = 0$, $T_3 = 0$: ηmeson

c. $(1/\sqrt{3})(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} + \mathrm{s\bar{s}})$: $T = 0$, $T_3 = 0$: η'meson

The meaning of the linear combination in (a), for example, is this: if one could examine the quark content of the π^0, one would find $\mathrm{u\bar{u}}$ half the time and $\mathrm{d\bar{d}}$ half the time. The probability of finding a certain quark-antiquark combination is given by the square of the coefficient in front of that combination. For (b), it is not that there are 2 s and 2 $\bar{\mathrm{s}}$ quarks in the η. Rather, the η has a 2/3 probability of being an $\mathrm{s\bar{s}}$, a 1/6 probability of being a $\mathrm{u\bar{u}}$, and a 1/6 probability of being a $\mathrm{d\bar{d}}$.

Of the three $T_3 = 0$ combinations, one forms part of the isospin triplet of the octet, one forms the isospin singlet of the octet, and the last forms the SU(3) singlet.

4c. Meson Supermultiplets and Quarks. There evidently exist in nature many meson octets and singlets. They differ in spin, intrinsic parity, and mass. The particles of the lowest mass octet are stable under the strong interaction. Others are not: they decay to members of the lowest octet in times on the order of 10^{-24} sec and are the meson resonances.

Three meson octets and three meson singlets have been experimentally completed. They are given by the Particle Data Group following the meson table and the quark content is given in the following chart. You should consult the meson table of the Particle Data Group and observe that there are many other mesons which have been discovered and which do not form completed octets as yet. It is a good exercise to see where these particles might fit into as yet incomplete groups and then watch for discoveries of particles that fit the missing places.

Meson Supermultiplets with Approximate Masses

Note: The last particle listed in the octet and the particle listed as the corresponding singlet may be different mixtures of the same quark states.

Quark Content	Spin 0	Spin 1	Spin 2
Octet:			
$\mathrm{u\bar{d}}$	$\pi^+(140)$	$\rho^+(770)$	$\mathrm{A_2}^+(1310)$
$(1/2)(\mathrm{u\bar{u}} - \mathrm{d\bar{d}})$	$\pi^0(135)$	$\rho^0(770)$	$\mathrm{A_2}^0(1310)$
$\mathrm{\bar{u}d}$	$\pi^-(140)$	$\rho^-(770)$	$\mathrm{A_2}^-(1310)$
$\mathrm{u\bar{s}}$	$\mathrm{K}^+(494)$	$\mathrm{K}^{*+}(892)$	$\mathrm{K}_N{}^+(1420)$
$\mathrm{d\bar{s}}$	$\mathrm{K}^0(498)$	$\mathrm{K}^{*0}(892)$	$\mathrm{K}_N{}^0(1420)$
$\mathrm{\bar{d}s}$	$\overline{\mathrm{K}}^0(498)$	$\overline{\mathrm{K}}^{*0}(892)$	$\overline{\mathrm{K}}_N{}^0(1420)$
$\mathrm{\bar{u}s}$	$\mathrm{K}^-(494)$	$\mathrm{K}^{*-}(892)$	$\overline{\mathrm{K}}_N{}^-(1420)$
$(1/6)(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} - 2\mathrm{s\bar{s}})$	$\eta(549)$	$\phi(1019)$	$\mathrm{f}'(1514)$
Singlet:			
$(1/3)(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} + \mathrm{s\bar{s}})$	$\eta'(958)$	$\omega(784)$	$\mathrm{f}(1270)$

4d. Spin and Quark Components. Presumably the energy (and hence the mass) increases rather drastically with orbital angular momentum. Spin 0 states can be constructed if the quark and antiquark have antiparallel spins and zero orbital angular momentum. This is the lowest mass set of particles. Spin 0 particles also result if the quark spins are parallel and the quarks have 1 unit of orbital angular momentum directed opposite to the spins. These particles evidently have masses on the order of 800 MeV larger than the first set of particles.

Spin 1 mesons result from any one of the following combinations:

a. Quark spins parallel and zero orbital angular momentum. This is evidently the lowest mass set of spin 1 mesons.

b. Quark spins antiparallel and 1 unit of orbital angular momentum.

c. Quark spins parallel and 2 units of orbital angular momentum directed opposite to the spins.

4e. Large Masses as Excited States. Quantum mechanically, one can consider the mesons of a large mass set (the spin 1 mesons, for example) to be excited states of mesons in the lowest mass set. All the K^-'s, for example, have the same properties except spin and mass and can be thought of as having the same quark content.

4f. Baryon Supermultiplets and Quarks. The baryons are constructed from 3 quarks, antibaryons from 3 antiquarks. Note that this prescription automatically satisfies the rules for assignment of baryon number and automatically makes the baryons fermions: they must have spin of half a positive odd integer. There are 10 ways to combine 3 quarks 3 at a time. They are:

	Y	T_3	T
uuu	1	3/2	3/2
uud	1	1/2	3/2 or 1/2
udd	1	−1/2	3/2 or 1/2
ddd	1	−3/2	3/2
uus	0	1	1
uds	0	0	1 or 0
dds	0	−1	1
uss	−1	1/2	1/2
dss	−1	−1/2	1/2
sss	−2	0	0

These form into groups of eight spin 1/2 baryons and ten spin 3/2 baryons.

The baryon octet:

quarks:	T	particle:
uud	1/2	p
udd	1/2	n
uds	0	Λ^0
uus	1	Σ^+
uds	1	Σ^0
dds	1	Σ^-
uss	1/2	Ξ^0
dss	1/2	Ξ^-

The baryon decimet:

quarks:	T	particle:
uuu	3/2	Δ^{++}
uud	3/2	Δ^+
udd	3/2	Δ^0
ddd	3/2	Δ^-
uus	1	Σ^{*+}
uds	1	Σ^{*0}
dds	1	Σ^{*-}
uss	1/2	Ξ^{*0}
dss	1/2	Ξ^{*-}
sss	0	Ω^-

Note that uud and udd can each form two different states, one with $T = 3/2$ and one with $T = 1/2$. The $T = 1/2$ states (uud with $T_3 = 1/2$ and udd with $T_3 = -1/2$) occur in the spin 1/2 octet while the $T = 3/2$ states (uud with $T_3 = 1/2$ and udd with $T_3 = -1/2$) are augmented with the other two states in the isospin multiplet (uuu with $T = 3/2$ and ddd with $T = 3/2$ and $T_3 = -3/2$) and occur in the spin 3/2 decimet.

Similarly the combination uds can form two different states, one with $T = 1$, $T_3 = 0$ and one with $T = 0$, $T_3 = 0$. The $T = 1$ state occurs in both groups while the $T = 0$ state occurs only in the octet.

5. Su(3) Operators

5a. The Quark-Quark Interaction and SU(3). There is more to the quark model than just the construction of particles from quarks. The chief idea behind the model is that the interaction which gives rise to the particles has a high degree of symmetry. The symmetry we are talking about is very much analogous to the ideas of invariance under parity or time reversal. That is, there is a group of operators (eight in number) which do not change the interaction when they operate on it (similar in nature to the fact that the electromagnetic interaction does not change when operated on by the parity operator i.e. when ($\vec{r}$) is replaced by (-$\vec{r}$) and ($\vec{p}$) by (-$\vec{p}$). The eight operators are collectively known as the SU(3) indicates that the basis of the group consists of 3 independent states (the 3 quarks).

5b. Hadron-Hadron Interaction: SU(3) from QQ Interaction. The abstract properties of the operators can be discussed and some general properties of the strong interaction derived. We shall not do this since the mathematics required (group theory) is generally not part of an undergraduate education and, although the ideas are not difficult, it would take too much time. Suffice it to say that the construction of hadrons from quarks, with properties as given, forces SU(3) symmetry on the strong interaction.

5c. Operations on Quarks: Quark Model Basic Postulates. Since we have already postulated the quarks, we can list the operators of SU(3) in terms of how they transform the quarks. The operators are denoted by λ_i:

	operator		causes:
$\lambda_1 \mathrm{u} = 0$	$\lambda_1 \mathrm{d} = \mathrm{u}$	$\lambda_1 \mathrm{s} = 0$	$\mathrm{d} \to \mathrm{u}$
$\lambda_2 \mathrm{u} = \mathrm{d}$	$\lambda_2 \mathrm{d} = 0$	$\lambda_2 \mathrm{s} = 0$	$\mathrm{u} \to \mathrm{d}$
$\lambda_3 \mathrm{u} = \mathrm{u}$	$\lambda_3 \mathrm{d} = \mathrm{d}$	$\lambda_3 \mathrm{s} = 0$	
$\lambda_4 \mathrm{u} = 0$	$\lambda_4 \mathrm{d} = 0$	$\lambda_4 \mathrm{s} = \mathrm{u}$	$\mathrm{s} \to \mathrm{u}$
$\lambda_5 \mathrm{u} = \mathrm{s}$	$\lambda_5 \mathrm{d} = 0$	$\lambda_5 \mathrm{s} = 0$	$\mathrm{u} \to \mathrm{s}$
$\lambda_6 \mathrm{u} = 0$	$\lambda_6 \mathrm{d} = 0$	$\lambda_6 \mathrm{s} = \mathrm{d}$	$\mathrm{s} \to \mathrm{d}$
$\lambda_7 \mathrm{u} = 0$	$\lambda_7 \mathrm{d} = \mathrm{s}$	$\lambda_7 \mathrm{s} = 0$	$\mathrm{d} \to \mathrm{s}$
$\lambda_8 \mathrm{u} = \mathrm{u}/3$	$\lambda_8 \mathrm{d} = \mathrm{d}/3$	$\lambda_8 \mathrm{s} = -\mathrm{s}/3$	

One of the operators turns a u into a d quark, another turns a u into an s, etc., so that each quark is turned into each of the others by one of the operators. In addition, λ_3 and λ_8 are special in that they do not change the character of the quark. λ_3 produces the same quark state but multiplied by twice its isotopic spin. λ_8 produces the same quark state but multiplied by its hypercharge.

In the language of quantum mechanics the quark states are chosen to be eigenstates of the operators corresponding to the 3-component of isotopic spin and hypercharge. Another way of saying the same thing is that each quark has a definite value for T_3 and Y, and these values are constants of its motion; a u quark, for example, always has $T_3 = 1/2$ and $Y = 1/2$. The basic postulates of the quark model are:

a. the fundamental interaction which produces the hadrons is invariant under the SU(3) operators,

b. a meson is composed of a quark and an antiquark,

c. a baryon is composed of three quarks.

These postulates give rise to the grouping of hadrons into singlets, octets, and decimets. Operating on one of the particles of an octet, for example, turns it into one or more of the other particles in the octet. SU(3) provides the rationale for grouping the hadrons in to supermultiplets, as the octets, decimets and singlets are collectively called.

5d. SU(3) Operators, States, Insufficiency. The eight operators are not arbitrarily chosen. Every operator which operates in a space which is specified by 3 basis states can be written as a linear combination of these 8, augmented by the identity operator ($\lambda_0 \mathrm{u} = \mathrm{u}$, $\lambda_0 \mathrm{d} = \mathrm{d}$, $\lambda_0 \mathrm{s} = \mathrm{s}$). Several conclusions can be drawn from this statement. First, the 8 operators ($\lambda_1, \ldots, \lambda_8$) are not unique. One can form many other sets of 8 independent operators but these will always be linear combinations of those we have written down. Different authors, in fact, use different sets but all sets will lead to the same physical conclusions. Second, if strong interaction physics can be described in terms of what happens to 3 independent states (i.e. 3 quarks) then the theory can be written in terms of the 8 SU(3) operators and the identity operator. There is nothing that can be done to a quark which is not describable by some combination of these operators. There is however evidence that things do happen in nature which are not describable by the SU(3) operators and physicists no are forced to postulate a fourth quark and deal with the operators of su(4). You should also realize that the SU(3) operators deal with isotopic spin and hypercharge. There are two other important quantities, namely spin and baryon number, which are used to describe quarks and which are outside the domain of SU(3).

6. Su(3) and Interactions

6a. Interaction Invariance under Selected SU(3) Operations. The postulates and mathematical reasoning behind the quark model and SU(3) symmetry seem to be invalid physically. If the strong interaction is invariant under the SU(3) operators, then all the hadrons of a given octet or decimet should have the same mass. This follows because the operators change one quark into another, or what is the same thing, change one member particle of a supermultiplet (as the singlets, octets, and decimets are called) into another member of the same supermultiplet without changing any of the interactions. Since the interactions are presumably responsible for the masses, all the particles of a supermultiplet must have the same mass.

If the quark model is to be valid, it must be that the interaction responsible for the particles of the supermultiplet is not the strong interaction but some other interaction which is invariant under the operations of SU(3).

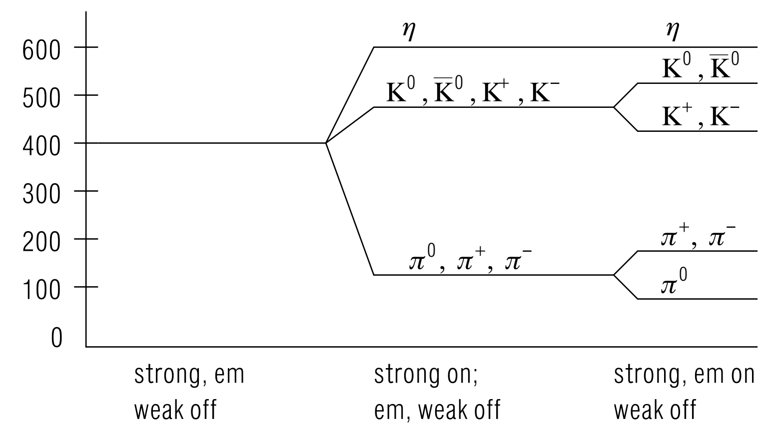
It is presumed that if all interactions would be turned off except this interaction, all particles in a supermultiplet would be identical. Particles in different supermultiplets would still be different (have different spins and different masses) because of internal quark dynamics. For example, the K^0, $\overline{K}^0$, K^+, K^-, π^+, π^-, π^0 and mesons would be experimentally indistinguishable from each other as would the K^{*0}, K^{-*0}, K^{*+}, K^{*-}, ρ^+, ρ^-, ρ^0 and ω mesons but the particles of the second octet would have mass and spin which would be different from the mass and spin respectively of the first octet.

The complete strong interaction is not invariant under all the SU(3) operators. Since the complete strong interaction conserves isotopic spin and hypercharge, it must be invariant under λ_1, λ_2, λ_3, and λ_8, the operators associated with isotopic spin and hypercharge. It is not invariant under λ_4, λ_5, λ_6, or λ_7, may now have different mass.

Looking at the chart of the operators (Sect. 5c), we see that particles which have the same quark content, except for the interchange of a u and d quark, will have the same mass and still be indistinguishable when the strong interaction is turned on. Particles which differ by more than this interchange will generally have different masses and this mass difference is associated with the strong interaction. In more detail, the strong interaction is not invariant under the operators λ_4, λ_5, λ_6, λ_7 of SU(3). These operators interchange u and s quarks or d and s quarks. We conclude that particles which differ in quark content by the substitutions $u \to s$, $d \to s$, $s \to u$, or $s \to d$ differ in mass by virtue of the energy associated with the strong interaction. Particles which differ by the substitutions $u \to d$ or $d \to u$ do not differ in mass by virtue of the strong interaction. For example, K^0, $\overline{K}^0$, K^+, K^- all have the same mass and π^+, π^-, π^0 all have the same mass but these two masses are different and differ from that of the η.

When the electromagnetic interaction is turned on, invariance with respect to isotopic spin is violated and a mass difference between the K^+ and K^0 is generated. Similarly, a mass difference between the π^+ and π^0 appears. The π^+ and π^- have the same mass because one is formed from the other by charge conjugation, and both the electromagnetic and strong interactions are invariant under this operation.

Presumably, the weak interaction produces a mass difference between particle and antiparticle, if charge conjugation invariance is violated. Mass effects of the weak interaction are too small to be observed at this time. The influence on mass of the various parts of the total interaction can be diagrammed as follows (for the lowest mass meson octet):



The mass of the original particle, plotted here at 400 MeV, is of course unknown since the interactions can not be turned off in practice.

6b. Comparison to Atomic Magnetic Splittings. The situation here is very similar to the magnetic states of an electron in an atom. For an electron with a specified principal quantum number corresponding to the values of m_ℓ are degenerate; they all have the same energy. When a magnetic field is turned on, the degeneracy is lifted and states with different m_ℓ have different energy. The splitting is given by $\Delta E = (e/mc)Bm_\ell$. For the spin 0 mesons, the original particle (with all interactions turned off) can be considered a quantum mechanical energy level that is 8-fold degenerate. The strong interaction splits the degeneracy to form 3 states, two of which are still degenerate (one 4-fold and one 3-fold) and the electromagnetic interaction further splits the degeneracy.

6c. Strong Interactions; Decays. Strong decays and interactions preserve strangeness and isotopic spin. In terms of quarks, the net number of u quarks is preserved (counting +1 for u and −1 for $\bar{\mathrm{u}}$); similarly for s and d quarks. Some examples of strong decays are:

a.

$$\begin{array}{lccccc} & \rho^+ & \rightarrow & \pi^+ & + & \pi^0 \\ \text{or:} & \mathrm{u\bar{d}} & \rightarrow & \mathrm{u\bar{d}} & + & (1/\sqrt{2})(\mathrm{u\bar{u}} - \mathrm{d\bar{d}}). \end{array}$$

The spin of one quark flipped to change the particle from spin 1 to spin 0 and the energy was used to form the $\mathrm{u\bar{u}} - \mathrm{d\bar{d}}$ combination. The $\pi^+ - \pi^0$ system has one unit of orbital angular momentum.

b.

$$\begin{array}{lccccc} & \mathrm{K}^{*+} & \rightarrow & \mathrm{K}^0 & + & \pi^+ \\ \text{or:} & \mathrm{u\bar{s}} & \rightarrow & \mathrm{d\bar{s}} & + & \mathrm{u\bar{d}} \end{array}$$

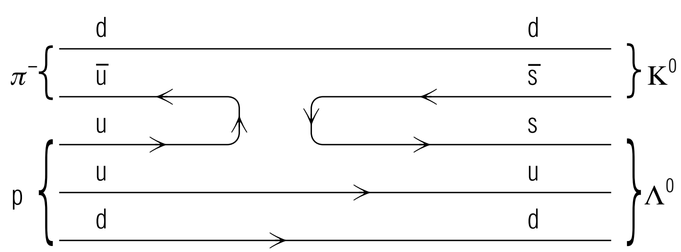
Energy from the $\mathrm{u\bar{s}}$ bond is used to create a $\mathrm{d\bar{d}}$ pair. The d is then coupled to the $\bar{\mathrm{s}}$ to form the K^0 and the $\bar{\mathrm{d}}$ is coupled to the u to form the π^+.

c.

$$\begin{array}{lccccccc} & \pi^- & + & \mathrm{p} & \rightarrow & \mathrm{K}^0 & + & \Lambda^0 \\ \text{or:} & \mathrm{\bar{u}d} & + & \mathrm{uud} & \rightarrow & \mathrm{d\bar{s}} & + & \mathrm{uds} \end{array}$$

The $\bar{\mathrm{u}}$ and one of the u's annihilate and energy is used to create an $\mathrm{s\bar{s}}$ pair. The $\bar{\mathrm{s}}$ couples to the d to form K^0 and the s couples to the ud to form Λ.

This process can be represented schematically by:



6d. Electromagnetic Interactions; Decays. Several particles, stable via the strong interaction, decay electromagnetically. The electromagnetic decay (or interaction) need not conserve the magnitude of isospin although it does conserve the 3-component. Again net quark content cannot change. Quark-antiquark pairs may annihilate to produce a photon, and quark-antiquark pairs may be produced, but these are the only changes allowed. Examples are:

a.

$$\begin{array}{lccccc} & \Sigma^0 & \rightarrow & \Lambda^0 & + & \gamma \\ \text{or:} & \mathrm{uds} & \rightarrow & \mathrm{uds} & + & \gamma \end{array}$$

In the Σ^0 the isotopic spin vectors of the u and the d are aligned and $T = 1$. In the Λ^0 the isotopic spin vectors of the u and the d are antiparallel and $T = 0$. In both cases $T_3 = 0$. This is an example of a change in isospin without change in quark content.

b.

$$\begin{array}{lccccc} & \lambda & \rightarrow & \gamma & + & \gamma \\ \text{or:} & 1/\sqrt{6}(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} - 2\mathrm{s\bar{s}}) & \rightarrow & \gamma & + & \gamma \end{array}$$

This is an example of the annihilation of a quark antiquark pair to produce a pair of γ's.

c.

$$\begin{array}{lccccc} & \pi^0 & \rightarrow & \gamma & + & \gamma \\ \text{or:} & (1/2)(\mathrm{u\bar{u}} - \mathrm{d\bar{d}}) & \rightarrow & \gamma & + & \gamma \end{array}$$

This is also an example of quark-antiquark annihilation.

d.

$$\begin{array}{lccccc} & \gamma & + & \mathrm{p} & \rightarrow & \pi^0 + \mathrm{p} \\ \text{or:} & \gamma & + & \mathrm{duu} & \rightarrow & (1/2)(\mathrm{u\bar{u}} - \mathrm{d\bar{d}}) + \mathrm{duu} \end{array}$$

The energy of the γ is used to create a quark-antiquark pair. Actually, this interaction may occur via a virtual nucleon:

$$\gamma + \mathrm{duu} \rightarrow \mathrm{duu} \rightarrow (1/2)(\mathrm{u\bar{u}} - \mathrm{d\bar{d}}) + \mathrm{duu}$$

where the intermediate proton violates conservation of energy at both its inception and its decay.

6e. Weak Interactions; Decays. The weak interaction changes one type quark into another type quark. Hadrons enter the 6 basic couplings of the weak interaction in 3 ways:

$$\begin{array}{rcl} \mathrm{p} + \bar{\mathrm{n}} & \leftrightarrow & \text{leptons}, \\ \mathrm{p} + \bar{\Lambda} & \leftrightarrow & \text{leptons}, \\ \mathrm{p} + \Lambda & \leftrightarrow & \mathrm{p} + \mathrm{n}. \end{array}$$

In terms of quark content, the first can be written $\mathrm{dud} + \overline{\mathrm{d}}\overline{\mathrm{d}}\overline{\mathrm{u}} \rightarrow$ leptons. If the weak interaction changes a u quark to a d quark, all quarks can be paired with their antiquarks and total annihilation of all quarks can occur. Since leptons presumably do not contain quarks, either this or the other alternative ($\bar{\mathrm{d}}$ to $\bar{\mathrm{u}}$) must occur.

The second interaction requires the weak interaction to change u to s or $\bar{\mathrm{s}}$ to $\bar{\mathrm{u}}$ while the third requires the change $\bar{\mathrm{d}}$ to $\bar{\mathrm{s}}$.

Again, it is seen that the effect of the weak interaction is to change one type quark into another type quark.

If weak processes occur via the W particle, it must be that W's can decay into quark-antiquark pairs where the quark and antiquark are not necessarily of the same type. The W's can undergo the following two types of processes:

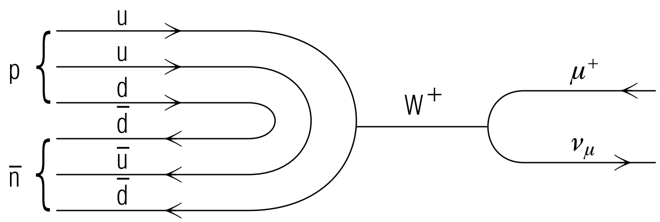
$$\mathrm{W} \leftrightarrow \text{ lepton } + \text{ antilepton}$$

and:

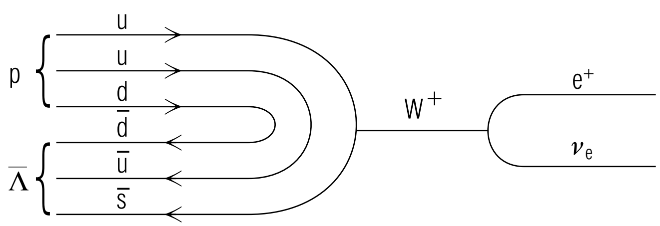
$$\mathrm{W} \leftrightarrow \text{ quark } + \text{ antiquark}\,.$$

Here are examples of each of the three types of weak couplings involving hadrons:

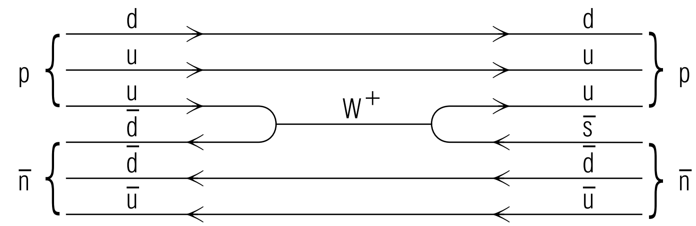
a. $\mathrm{p} + \overline{\mathrm{n}} \rightarrow$ leptons



b. $\mathrm{p} + \bar{\Lambda} \rightarrow$ leptons



c. $\mathrm{p} + \overline{\mathrm{n}} \rightarrow \mathrm{p} + \bar{\Lambda}$



7. Problems with the Quark Model

The quark model leads to some questions, as yet unresolved. The quarks are charged and interact electromagnetically; some other force is needed to bind the quarks in a particle. For example the $\bar{\Xi}$ is composed of 3 negatively charged quarks which electromagnetically repel each other.

A new set of particles, called gluons, has been proposed. The gluons act as exchange particles between quarks and produce the binding required to hold particles together. It may be that the gluons are indeed responsible for the basic strong interaction.

Another problem that arises has to do with the quark content itself. Why do 3 quarks bind together and not 4? Why do quarks and antiquarks form a bound state and not 2 quarks? If ought to be possible to construct a baryon of the form $\mathrm{qqqq\bar{q}}$ and a meson of the form $\mathrm{q\bar{q}q\bar{q}}$.

Beyond this, physicists want to know why there are no particles with fractional baryon number or fractional charges. That is, what prevents a particle of the form qqqq?

Some of these questions are partially answered by a model which is discussed elsewhere.[2]

Acknowledgments

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[2]See "Color and Charm" (MISN-0-283).

PROBLEM SUPPLEMENT

1. Draw quark diagrams, similar to the ones at the end of Sect. 4b or 4d, for these decays:

   a. $\Delta^+ \to \mathrm{n} + \pi^+$

   b. $\Lambda^0 \to \mathrm{p} + \pi^-$

   c. $\pi^+ \to \mu^+ + \nu_\mu$

2. Give the quark content of a particle with baryon number +1, charge $+e$, and strangeness +1. This particle is called an "exotic" particle and is denoted by Z^*. Its quark content cannot be of the form qqq and it is being sought experimentally as a test of the rules for constructing baryons.

3. Find the appropriate characteristics of the following quark combinations and show that they match the characteristics of one or more of the particles. Identify the particle or particles.

   a. uud

   b. $\mathrm{u\bar{s}}$

   c. $\mathrm{\bar{d}\bar{d}\bar{s}}$

   d. $\mathrm{u\bar{d}}$

   e. uds

   f. $\mathrm{\bar{u}d}$

MODEL EXAM

Quark	B	T	T_3	σ	S	Y	Q
u	1/3	1/2	1/2	1/2	0	1/3	2/3
d	1/3	1/2	−1/2	1/2	0	1/3	−1/3
s	1/3	0	0	1/2	−1	−2/3	−1/3
$\bar{u}$	−1/3	1/2	−1/2	1/2	0	−1/3	−2/3
$\bar{d}$	−1/3	1/2	1/2	1/2	0	−1/3	1/3
$\bar{s}$	−1/3	0	0	1/2	1	2/3	1/3

1. See Output Skills K1-K7 in this module's *ID Sheet*. The actual exam may have one or more of these skills, or none.

2. Draw quark diagrams, similar to the ones at the end of Sect. 4b or 4d, for these decays:
   (a) $\Delta^+ \to \mathrm{n} + \pi^+$; (b) $\Lambda^0 \to \mathrm{p} + \pi^-$; (c) $\pi^+ \to \mu^+ + \nu_\mu$.

3. Give the quark content of a particle with baryon number +1, charge +e, and strangeness +1. This particle is called an "exotic" particle and is denoted by Z^*. Its quark content cannot be of the form qqq and it is being sought experimentally as a test of the rules for constructing baryons.

4. Find the appropriate characteristics of the following quark combinations and show that they match the characteristics of one or more of the particles. Identify the particle or particles: (a) uud; (b) $\mathrm{u\bar{s}}$; (c) $\mathrm{\bar{d}\bar{d}\bar{s}}$; (d) $\mathrm{u\bar{d}}$; (e) uds; (f) $\mathrm{\bar{u}d}$.