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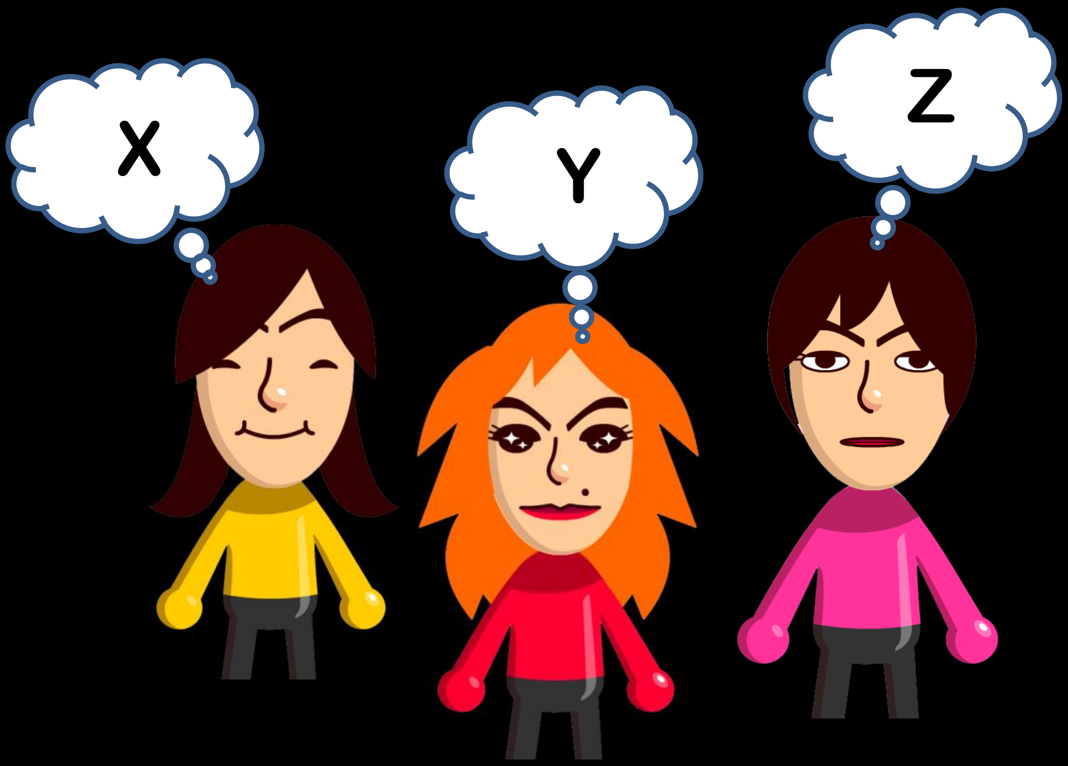
Science of teh Interwebs

Auctions

Lecture 9 (September 30, 2008)

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RESERVE NOTE
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THE UNITED STATES
OF AMERICA
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THIS NOTE IS LEGAL TENDER
FOR ALL DEBTS, PUBLIC AND PRIVATE
Treasurer of the United States.
SERIES
1999
Secretary of the Treasury.
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TEN
HAMILTON
DOLLARS


10
THE UNITED STATES OF AMERICA
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IN GOD WE TRUST
U.S. TREASURY
10
TEN DOLLARS
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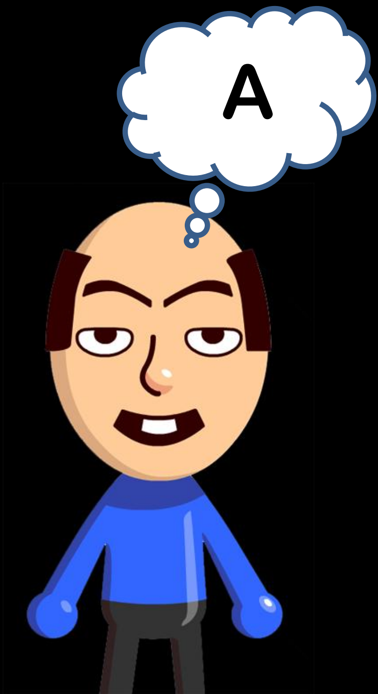
Types of Auctions

Ascending Bid or English Auctions

Descending Bid or Dutch Auctions

First Price Sealed Bid Auctions

Second Price Sealed Bid or Vickrey Auctions



Seller

The Seller does not know the buyers' valuations

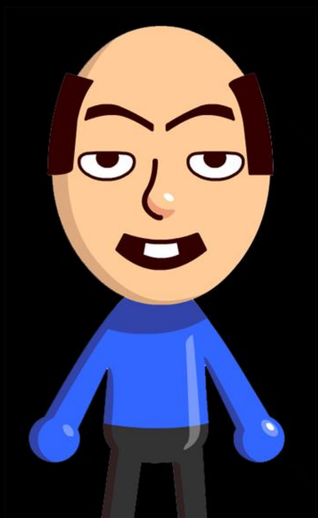
The Buyers don't know each other's valuations

Buyers

Descending bid and first price sealed bid are essentially equivalent from the buyer's perspective

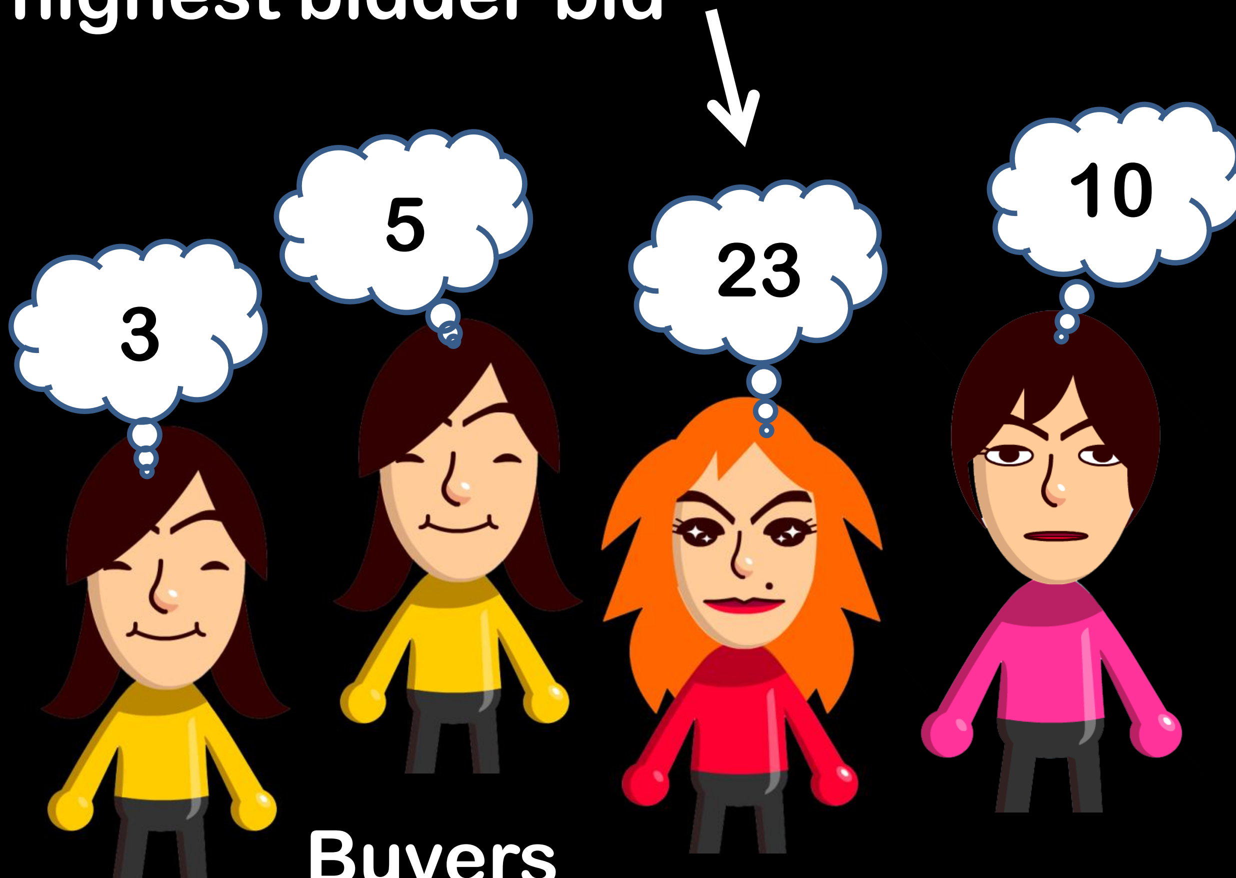
In an ascending bid auction, each buyer will want to stay in the auction until the precise moment when the price reaches his value.

Second Price Auctions

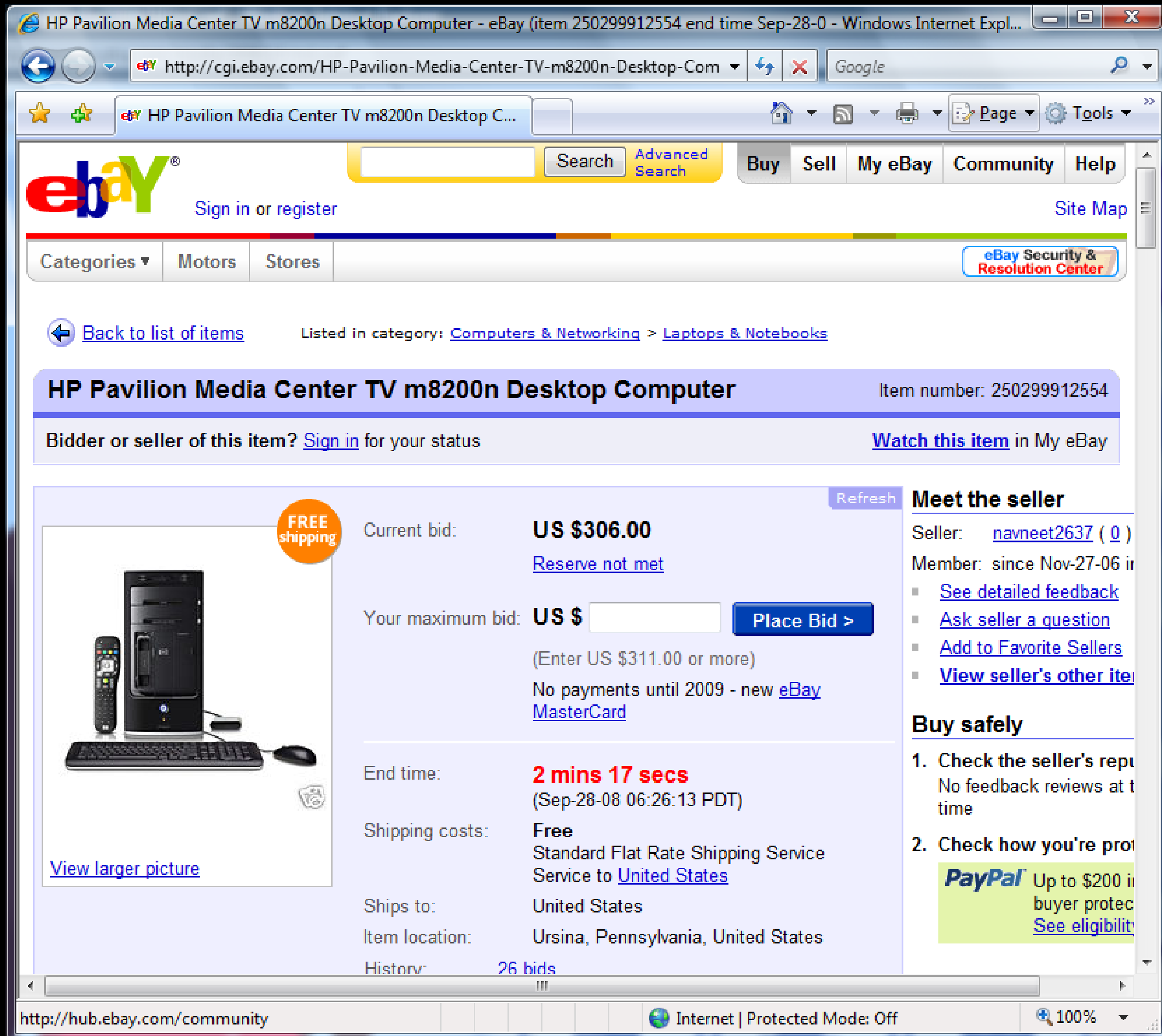


Seller

The highest bidder wins, but pays the what the second highest bidder bid



Buyers



eBay is equivalent to a second price auction

Bidding your true value is a dominant strategy in a second price sealed bid auction

v_i = bidder i's value for the object

b_i = bidder i's bid for the object

A bidder's **strategies** are bids as functions of their values

The payoff to bidder i with value v_i and bid b_i is:

$$\begin{cases} v_i - \max_{j \neq i} b_j & \text{if } b_i > \max_{j \neq i} b_j \\ 0 & \text{otherwise} \end{cases}$$

v_i = bidder i's value for the object

b_i = bidder i's bid for the object

$$\text{Payoff} = \begin{cases} v_i - \max\limits_{j \neq i} b_j & \text{if } b_i > \max\limits_{j \neq i} b_j \\ 0 & \text{otherwise} \end{cases}$$

Theorem: Bidding $b_i = v_i$ is a dominant strategy

If $b_i > v_i$ bidder i could get object and pay more than what she values it for (and thus go negative)

If $b_i < v_i$ bidder i could fail to obtain the object; obtaining the object can get her positive payoff

v_i = bidder i's value for the object

b_i = bi… …e object

…max b_j
…$j \neq i$

In a second price auction, your bid does not affect how much you pay; it just affects whether you get the object or not

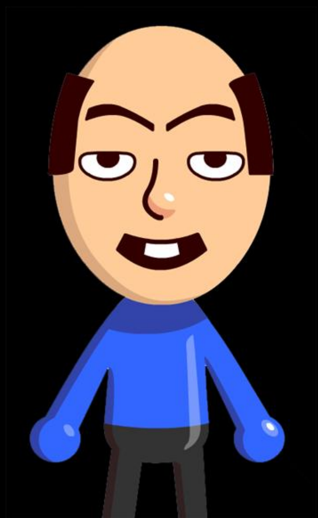
The… …y

If $b_i > v_i$ … …more
than what… …egative)

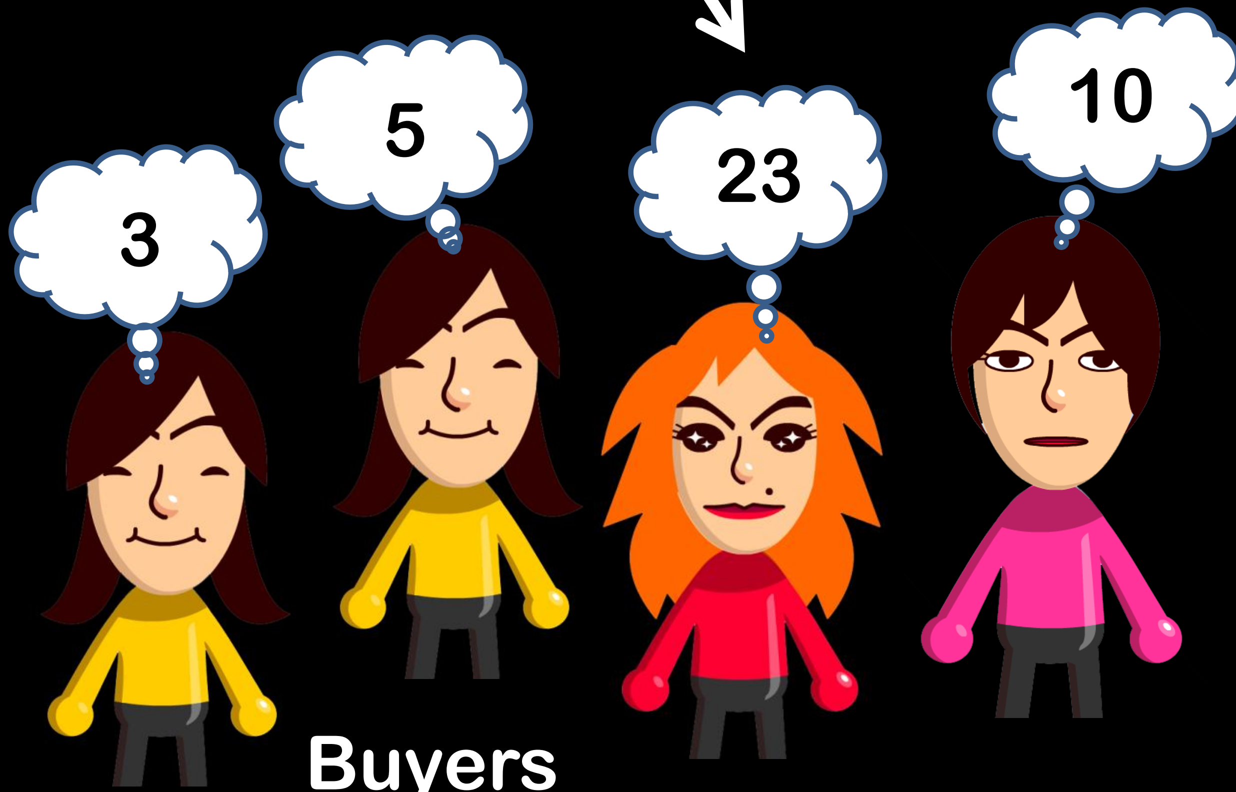
If $b_i < v_i$ bidder i … …n the object;
obtaining the object can get him positive payoff

First Price Auctions

The highest bidder wins, and pays her bid



Seller



Buyers

Bidding your true value is NOT a dominant strategy in a first price sealed bid auction

Since your bid also affects what you pay, you will tend to underbid

First Price Auctions

Suppose all bids v_i are uniformly distributed between [0,1] and there are N other bidders

Suppose all bidders bid $s(v_i)$ where s( ) is a strictly increasing function

What is the probability that bidder who bids $s(v_i)=b_i$ will win? $(v_i)^{N-1}$

What is the net payoff for value v_i and bid $s(v_i)=b_i$ if they win? (v_i-b_i)

Expected payoff to bidder i with value v_i and bid $s(v_i)=b_i$: $(v_i)^{N-1}(v_i-s(v_i))$

Expected payoff to bidder i with value v_i and bid $s(v_i)=b_i$: $(v_i)^{N-1}(v_i-s(v_i))$

Bidders could pretend their value is some u_i instead of v_i

For s( ) to be an equilibrium, this deviation must make i worse off:

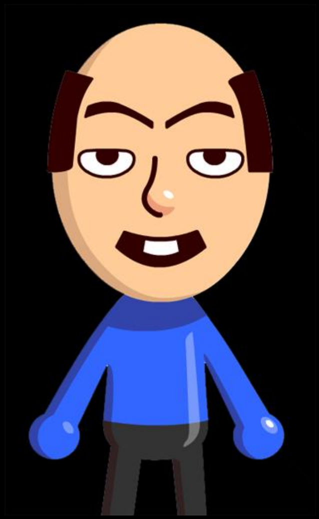
$$(v_i)^{N-1}(v_i-s(v_i)) \geq (u_i)^{N-1}(v_i-s(u_i))$$

It can be shown that $s(v) = v(N-1)/N$ is an equilibrium strategy

E.g., For N=2:

$$v_i(v_i-v_i/2) \geq u_i(v_i-u_i/2) \text{ is true for all } v_i, u_i$$

Seller Revenue



Which is better for the Seller: first or second price auctions?

In first price auctions the bidders pay less than their true valuation

In second price auctions they only pay the second largest valuation

Assume N valuations uniformly distributed in [0,1]

In expectation, the highest value is N/(N+1) and the second highest value is (N-1)/(N+1)

So, seller revenue is the same in both cases

g2g
ttyl