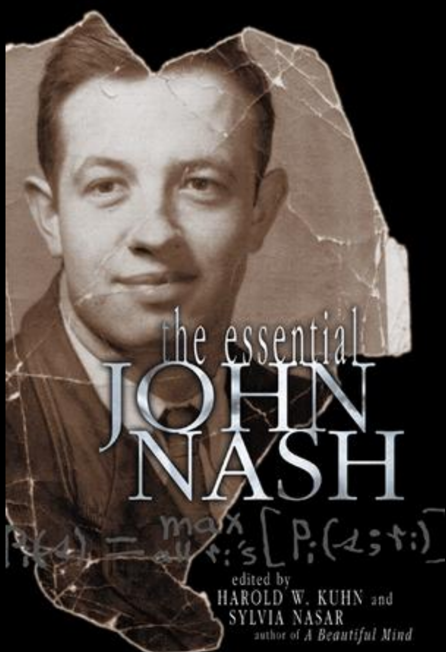
15-396

Science of teh Interwebs

Preliminaries of Game Theory

Lecture 8 (September 25, 2008)



Players

Roommate 2

Roommate 1

	WiFi	Wired
WiFi	10,10	15,5
Wired	5,15	5,5

Payoffs

Strategies for Player 1: S_1 = {WiFi, Wired}

Strategies for Player 2: S_2 = {WiFi, Wired}

S = { (WiFi, Wired), (WiFi, WiFi), (Wired, WiFi), (Wired, Wired) }

We assume that everything a player cares about is summarized in the player’s payoff

We also assume that each player knows everything about the game

Prisoner's Dilemma

Suspect 1 \ Suspect 2	Conf	No Conf
Conf	-4,-4	0,-10
No Conf	-10,0	-1,-1

Best Responses

A strategy s_1* is a **best response** by player 1 to a strategy s_2 for player 2 if

$$\pi_1(s_1^*, s_2) \geq \pi_1(s_1, s_2)$$

for all strategies $s_1 \in S_1$.

		Suspect 2	
		Conf	No Conf
Suspect 1	Conf	-4,-4	0,-10
	No Conf	-10,0	-1,-1

If Suspect 2 does not confess, then confessing is a best response for Suspect 1

Dominant Strategy

A strategy s_1* is a **Dominant Strategy** for player 1 if s_1* is a Best Response to every possible strategy for player 2.

Suspect 2

Suspect 1	Conf	No Conf
Conf	-4,-4	0,-10
No Conf	-10,0	-1,-1

Confessing is a dominant strategy for both Suspects!

		Player 2: I	Player 2: II
Player 1	I	3,3	1,1
Player 1	II	1,1	0,0

I is a dominant strategy for both players

Optimal Pricing

Firm 1 \ Firm 2	H	L
H	2,2	0,3
L	3,2	5,1

(L,H) will be played

Player 2

Player 1

	L	M	R
t	3,3	2,2	2,1
m	2,2	1,2	3,1
b	1,2	3,1	2,3

Neither player has a dominant strategy

Nash Equilibrium

A pair of strategies (s_1*,s_2*) is in Nash Equilibrium if s_1* is a Best Response by player 1 to s_2*, and s_2* is a Best Response by player 2 to s_1*.

Player 2

Player 1		L	M	R
	t	3,3	2,2	2,1
	m	2,2	1,2	3,1
	b	1,2	3,1	2,3

Coordination Game

		Player 2	
		L	R
Player 1	L	1,1	0,0
	R	0,0	1,1

Nash Equilibria: (L,L), (R,R)

Dove-Hawk

		Animal 2	
		D	H
Animal 1	D	3,3	1,5
	H	5,1	0,0

Nash Equilibria: (D,H), (H,D)

Matching Pennies

		Player 2: H	Player 2: T
Player 1	H	-1,+1	+1,-1
	T	+1,-1	-1,+1

No pure Nash Equilibria Exist!

Randomized Strategies

		Player 2: H	Player 2: T
Player 1	H	-1,+1	+1,-1
	T	+1,-1	-1,+1

Player 1 picks H with probability p and Player 2 picks H with probability q

Player 2

Player 1	H	T
H	-1,+1	+1,-1
T	+1,-1	-1,+1

(p=1/2,q=1/2) is an equilibrium!

E[Payoff for P1 doing H] = $(-1)q + (+1)(1-q) = 1-2q$

E[Payoff for P1 doing T] = $(+1)q + (-1)(1-q) = 2q-1$

Player 1 will choose H if $1-2q > 2q-1$. i.e., if $q < 1/2$

Player 1 will choose T if $1-2q < 2q-1$. i.e., if $q > 1/2$

We say that (p*,q*) is a mixed strategy Nash Equilibrium if p* is a best response by player 1 to q* and q* is a best response by player 2 to p*

Player 2

Player 1		L	R
	U	1,1	4,0
	D	2,1	1,3

(p=2/3,q=3/4) is an equilibrium!

Player 1 is only willing to randomize if the expected payoffs of U and D are equal:

$$q+4(1-q)=2q+(1-q), \text{ so } q=3/4$$

Goalie

Kicker		*l*	*r*
	L	.58,.42	.95,.05
	R	.93,.07	.70,.30

Player 1 \ Player 2	L	R
U	1,1	3,0
D	0,3	2,2

The only Nash Equilibrium is (U,L)

But (U,L) gives each player a payoff of 1, whereas (D,R) gives them 2.

Nash Equilibrium not always socially optimal

g2g
ttyl