Constructing a covering set for numbers $2^k + p$

While optimizing a program for calculating small members of Sloane's A094076, I noticed that the order of 2 mod $2^1+1=3$, $2^2+1=5$, $2^4+1=17$, $2^8+1=257$, $2^{16}+1=65537$, and 641 were, respectively, 2, 4, 8, 16, 32, and 64. By the Chinese remainder theorem, there are n such that 2^k+n is divisible by one of these primes for all but one residue class mod 64. If a prime q could be found modulo which 2 had order 64, all 2^k+q would be divisible by one of $\{3, 5, 17, 257, 641, 65537, q\}$. In fact there would be 64 clases mod $3 \cdot 5 \cdot 17 \cdot 257 \cdot 641 \cdot 65537 \cdot q$ depending on the choice of which residue mod 641 (or q). By Dirichlet's Theorem, each of these classes would have an infinite number of primes.

A simple Pari search reveals that modulo 6700417, the order of 2 was 64, as required: `forprime(p=3,1e8,if(znorder(Mod(2,p))==64,print(p)))`

Enumerating the powers of 2 mod 6700417, associating the appropriate powers of the other primes, taking the negatives of all powers (since the result in each congruence class should be 0 mod the prime), and applying the Chinese remainder theorem gave 64 classes mod 18446744073709551615. The smallest positive residue was 201446503145165177.

Searching each class, I determined that 3367034409844073483 was the smallest prime of the 64 classes. This 19-digit prime, then, is a counterexample to Dr. Zumkeller's conjecture that an integer k exists such that 2^k+p is prime for each prime p, even though the conjecture holds with probability 1 in the Cramér model.

A067760(100723251572582588) = -1.
A094076(80869739673507329) = -1, where $p_{80869739673507329} = 3367034409844073483$.

(Concerned with sequences A094076 and A067760)

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