Quantum Numbers

In this section we will cover the following topics:

- Electric Charge
- Baryon Number
- Lepton Number
- Spin
- Parity
- Isospin
- Strangeness
- Charge Conjugation

Electric Charge Q

Quantum Numbers are quantised properties of particles that are subject to constraints. They are often related to symmetries.

Electric Charge Q is conserved in all interactions.

✓

	π^-	+	p	$\rightarrow$	K^0	+	Λ^0
Q:	-1		+1		0		0

Strong Interaction

✓

	K^0	$\rightarrow$	π^+	+	π^-
Q:	0		+1		-1

Weak Interaction

Baryon Number B

Baryons have B = +1 Antibaryons have B = -1 Everything else has B = 0	or	Quarks have B = $+\frac{1}{3}$ Antiquarks have B = $-\frac{1}{3}$ Everything else has B = 0

Baryons = $qqq = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ Mesons = $q\bar{q} = \frac{1}{3} + -\frac{1}{3} = 0$

Baryon Number B is conserved in Strong, EM and Weak interactions.

Total (quarks – antiquarks) is constant.

Baryon Number

✓		π^-	+	p	$\rightarrow$	K^0	+	Λ^0
	B:	0		+1		0		+1

Strong Interaction

✓		Λ^0	$\rightarrow$	p	+	π^-
	B:	+1		+1		0

Weak Interaction

Since the proton is the lightest baryon it cannot decay if B is conserved e.g:

✗		p	$\not\rightarrow$	e^+	+	π^0
	B:	+1		0		0

Lepton Number L

Leptons have L = +1
Antileptons have L = -1
Everything else has L = 0

Lepton Number L is conserved in Strong, EM and Weak interactions but is also separately conserved within lepton families:

e^- and ν_e have $L_e = 1$	e^+ and $\bar{\nu}_e$ have $L_e = -1$
μ^- and ν_μ have $L_\mu = 1$	μ^+ and $\bar{\nu}_\mu$ have $L_\mu = -1$
τ^- and ν_τ have $L_\tau = 1$	τ^+ and $\bar{\nu}_\tau$ have $L_\tau = -1$

L_e, L_μ and L_τ are separately conserved.

Lepton Number

Pair Production ✓

	γ	+	N	$\rightarrow$	e^+	+	e^-	+	N
L_e:	0		0		-1		+1		0

Pion Decay ✓

	π^+	$\rightarrow$	μ^+	+	ν_μ
L_μ:	0		-1		+1

Muon Decay ✓

	μ^+	$\rightarrow$	e^+	+	ν_e	+	$\bar{\nu}_\mu$
L_e :	0		-1		+1		0
L_μ :	-1		0		0		-1
L :	-1		-1		+1		-1

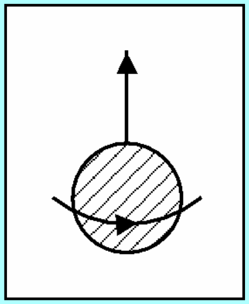
Lepton Number

	μ^+	$\not\to$	e^+	+	γ	
L_e :	0		-1		0	Forbidden
L_μ :	-1		0		0	Forbidden
L :	-1		-1		0	OK

L is conserved but neither L_e or L_μ separately.

The decay $\mu^+ \to e^+ + \gamma$ has not been observed and has a "Branching Ratio" $< 10^{-9}$.

Spin S



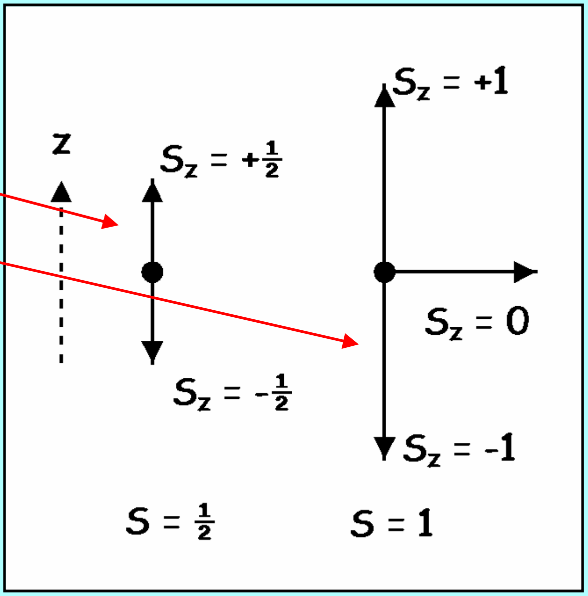
Spin is an intrinsic property of all particles:

$0\hbar, 1\hbar, 2\hbar, 3\hbar, \ldots$ Bosons

$\frac{1}{2}\hbar, \frac{3}{2}\hbar, \frac{5}{2}\hbar, \frac{7}{2}\hbar, \ldots$ Fermions

Spin is like angular momentum but a Quantum Mechanical effect.

For spin S there are $2S+1$ states of different S_z (like $2J+1$ in Angular Momentum) e.g.



For Spin $S = \frac{1}{2}$, S_z can be $+\frac{1}{2}$ or $-\frac{1}{2}$ (2 states).

For Spin $S = 1$, S_z can be $+1, 0, -1$ (3 states).

For a process $a + b \rightarrow c + d$ the cross section

$$\sigma \sim (2S_c + 1)(2S_d + 1) \times \text{Other Factors}$$

This can be used to determine the spin of unknown particles.

Parity P

Parity is a Quantum Mechanical concept.

For a wavefunction $\Psi(r)$ and Parity operator P, the Parity Operator reverses the coordinates r to $-r$.

$$P\,\Psi(r) \rightarrow \Psi(-r) = \lambda\,\Psi(r)$$

But $P^2\,\Psi(r) \rightarrow \Psi(r) = \lambda^2\,\Psi(r)$

i.e. $\lambda^2 = 1$ so that $\lambda = \pm 1$

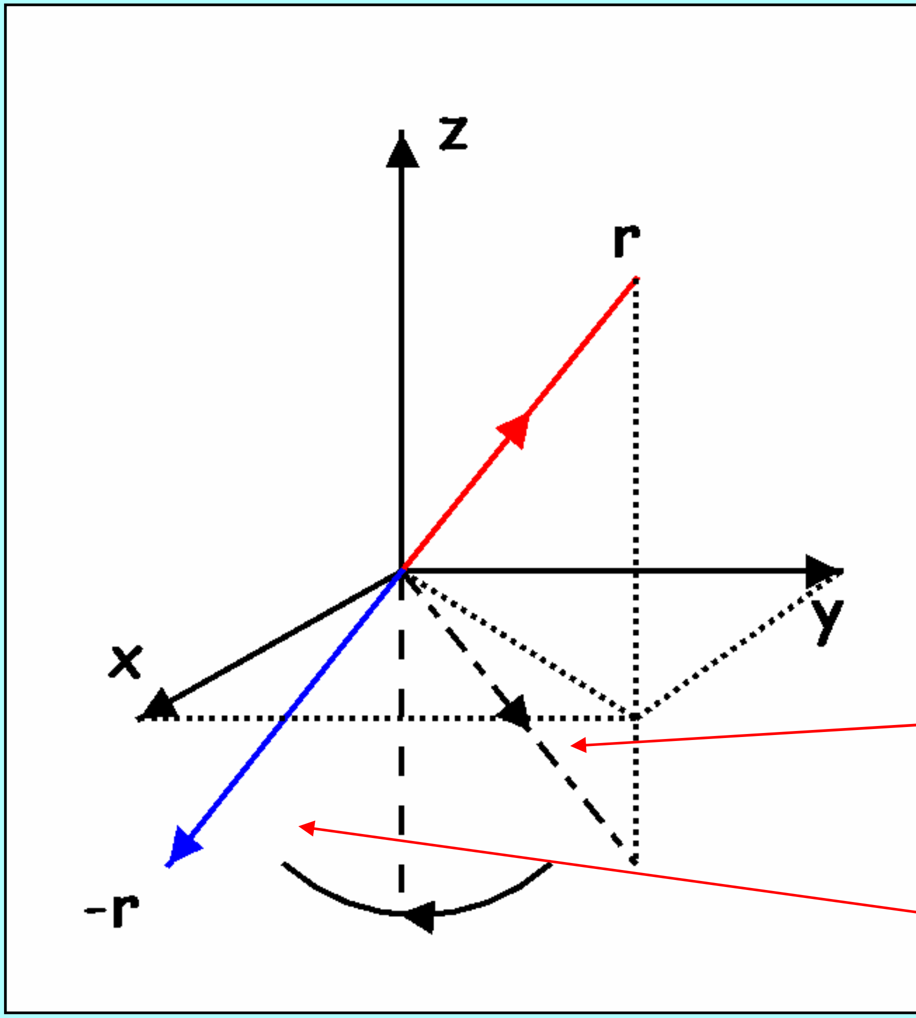
If an operator O acts on a wavefunction Ψ such that Ψ is unchanged

$$O\,\Psi = \lambda\,\Psi$$

Ψ is an Eigenfunction of O and λ is the Eigenvalue.

Hence the eigenvalues of Parity are $+1$ (even) and -1 (odd).

Parity



The Parity Operator reverses the coordinates r to -r.

Equivalent to a reflection in the x-y plane followed by a rotation about the z axis

Reflection in x-y plane

Rotation about z axis

Parity

Parity is a multiplicative quantum number. The parity of a composite system is equal to the product of the parities of the parts:

$$\Psi = \Phi_A \Phi_B \ldots \qquad P_\Psi = P_A \times P_B$$

One can show that a state with angular momentum ℓ has parity

$$P = (-1)^\ell$$

For a system of particles:

$$P\,(\text{overall}) = P\,(\text{relative motion}) \times P\,(\text{intrinsic})$$

For Fermions $P\,(\text{antiparticle}) = (-1) \times P\,(\text{particle})$

For Bosons $P\,(\text{antiparticle}) = P\,(\text{particle})$

Arbitrarily assign $n, p \rightarrow P = +1 \qquad \bar{p}, \bar{n} \rightarrow P = -1$

Others determined from experiment (angular distributions)

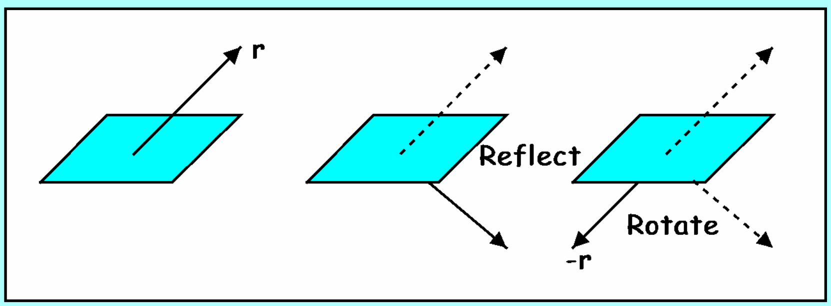
Parity of $\pi^+, \pi^-, \pi^0 \rightarrow P = -1$

Parity

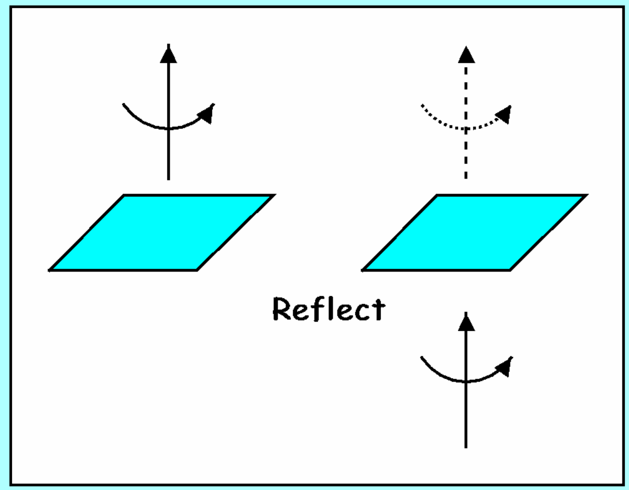
We label mesons by J^p – Spin$^{\text{Parity}}$ corresponding to how their wavefunctions behave:

Examples of things that have these properties

$J^p = 0^-$	Pseudoscalar	(Pressure...)
0^+	Scalar	(Mass, time, wavelength...)
1^-	Vector	(Momentum, position...)
1^+	Axial Vector	(Spin, angular momentum...)
2^+	Tensor	



Vector $r \rightarrow -r \therefore P = -1$



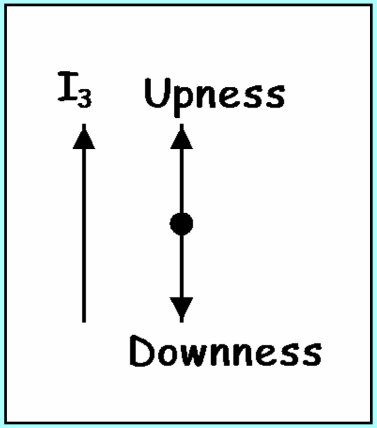
Axial Vector $r \rightarrow r \therefore P = +1$

Parity is conserved Strong and EM Interactions but NOT Weak.

Isospin I

Used mostly in Nuclear Physics from charge independence of nuclear force $p \leftrightarrow p = n \leftrightarrow n = p \leftrightarrow n$ sometimes called Isobaric Spin/Isotopic Spin T (or t!).

Isospin is represented by a 'spin' vector I with component I_3 along some axis.



$I = \frac{1}{2}$: p has $I_3 = +\frac{1}{2}$ ($\uparrow$), n has $I_3 = -\frac{1}{2}$ ($\downarrow$)

$I = 1$: π^- has $I_3 = -1$, π^0 has $I_3 = 0$, π^+ has $I_3 = +1$

I_3 really only counts the number of u and d quarks

u has $I_3 = +\frac{1}{2}$	$\bar{u}$ has $I_3 = -\frac{1}{2}$
d has $I_3 = -\frac{1}{2}$	$\bar{d}$ has $I_3 = +\frac{1}{2}$

$p = uud = \frac{1}{2} + \frac{1}{2} + -\frac{1}{2} = \frac{1}{2}$ $\quad \pi^- = \bar{u}d = -\frac{1}{2} + -\frac{1}{2} = -1$

Isospin

I_3 can be related to charge Q and baryon number B:

$$\boxed{Q = B/2 + I_3}$$

For a proton $B = 1$, $I_3 = +\frac{1}{2}$ and hence $Q = 1$

Since the Strong Interaction doesn't distinguish p from n or u from d, I and I_3 are conserved in Strong Interactions.

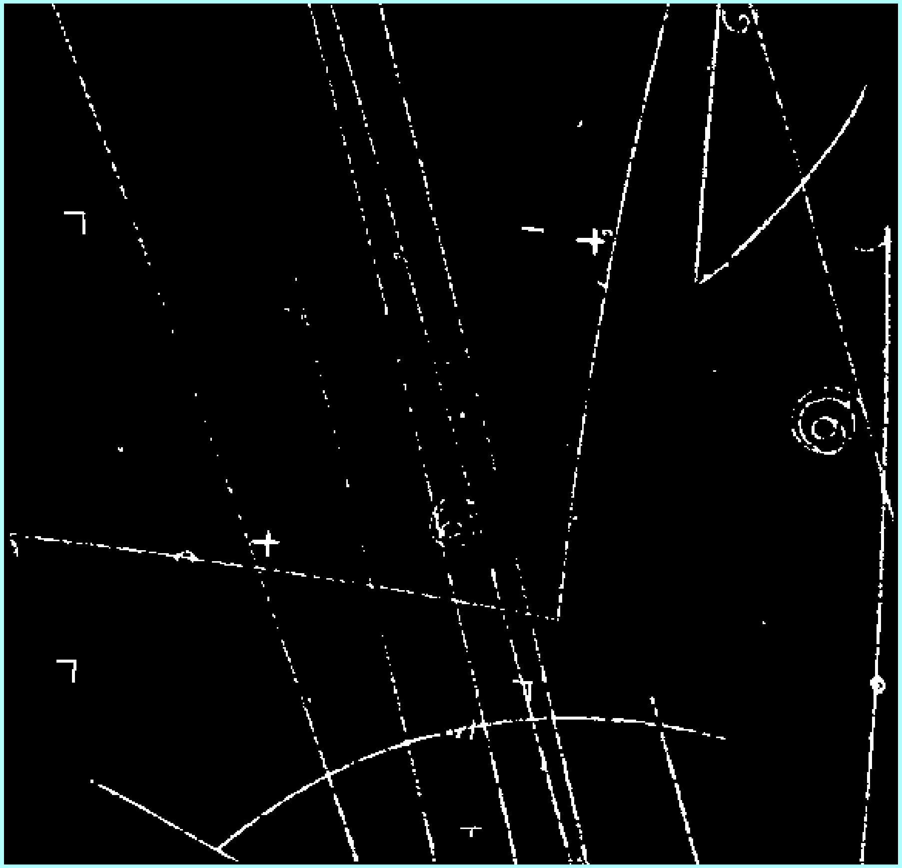
This is equivalent to saying that the number of
$(u - \bar{u}) - (d - \bar{d}) = \text{constant}$

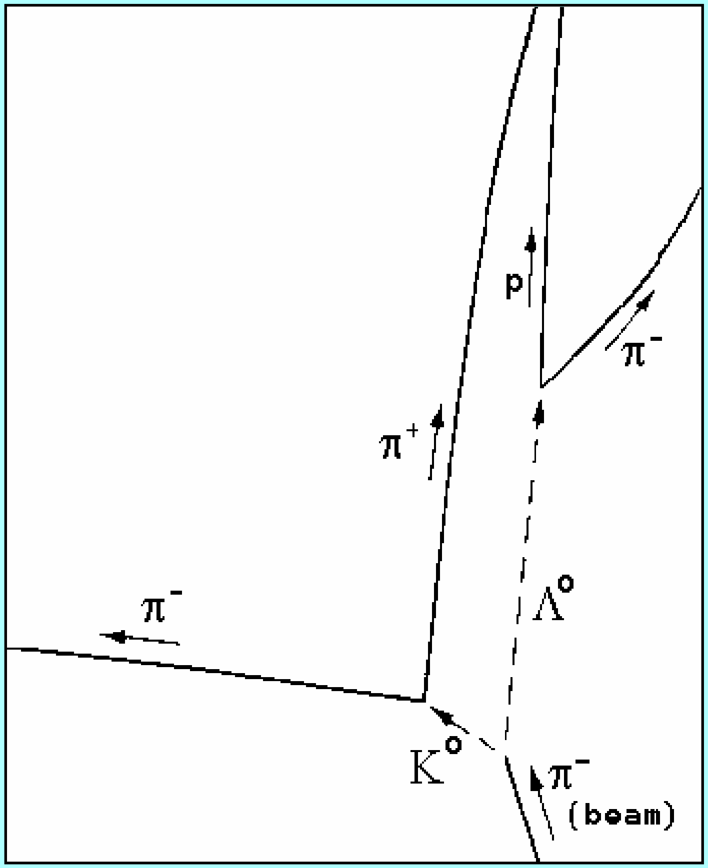
In Weak Interactions where $u \rightleftharpoons d$, I and I_3 are NOT conserved.

In EM Interactions u and d are not changed but because of the different charges u and d can be distinguished. Hence I_3 is conserved but I is NOT conserved.

Strangeness S

Associated production (via SI) of 'strange' particles $\pi^- + p \rightarrow K^0 + \Lambda^0$

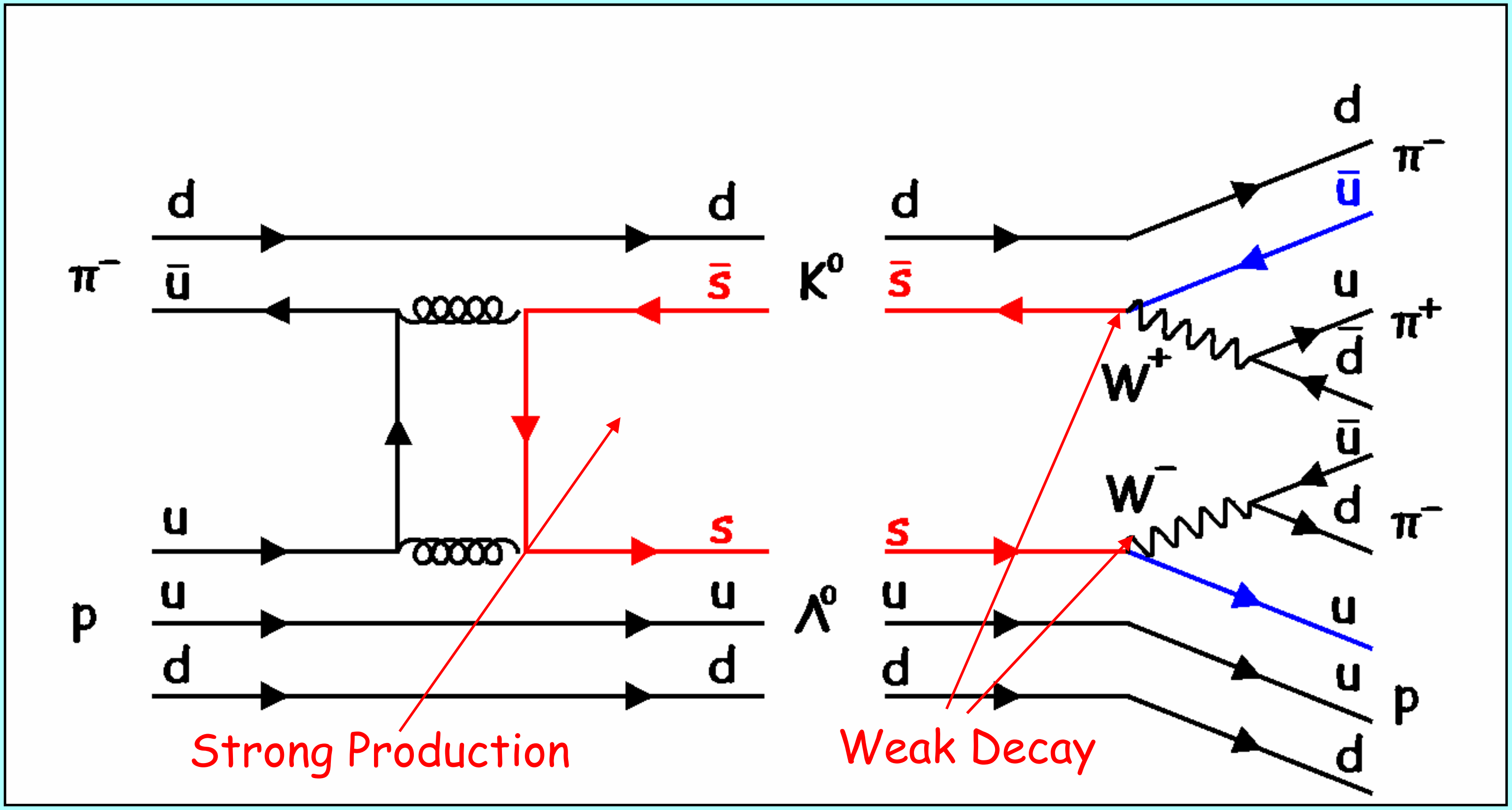




K^0 and Λ^0 'Strange' – decay weakly not strongly.

Strangeness

Assume pair production of new quark s and antiquark $\bar{s}$ by Strong Interaction but once produced s and $\bar{s}$ can only decay weakly.



EPP Elementary Particle Physics

Strangeness

Λ^0 is uds	$\bar{\Lambda}^0$ is $\bar{u}\bar{d}\bar{s}$
K^0 is $d\bar{s}$	$\bar{K}^0$ is $\bar{d}s$
K^+ is $u\bar{s}$	K^- is $\bar{u}s$

The K^0 has an antiparticle the $\bar{K}^0$ although it is neutral, unlike the π^0 which is its own antiparticle.

Strangeness can be combined with Isospin if

$$Q = I_3 + (B + S)/2$$ Gell Mann – Nishijima relation

The s quark has strangeness $S = -1$

Strangeness is conserved in Strong and EM Interactions but NOT in Weak Interactions.

Likewise charm, bottom, top quantum numbers.

Strong and EM Interactions do not change quark flavours. Number of $(u - \bar{u})$, $(d - \bar{d})$, $(s - \bar{s})$, $(c - \bar{c})$, $(b - \bar{b})$, $(t - \bar{t})$ constant.

Weak Interaction changes one quark type to another.

Charge Conjugation C

The Charge Conjugation operator reverses the sign of electric charge and magnetic moment (μ).

This implies particle $\rightleftharpoons$ antiparticle.

proton	$\underset{C}{\rightleftharpoons}$	antiproton
$Q = +e$		$Q = -e$
$B = +1$		$B = -1$
μ		$-\mu$

> $|X\rangle$ *is (Dirac) bra/ket notation for* Ψ_X *i.e.* $|\pi^+\rangle \equiv \Psi_{\pi^+}$

$C\,|\pi^+\rangle \rightarrow |\pi^-\rangle$ Hence $\pi^\pm$ not eigenstates of C.

C only has definite eigenvalues for neutral systems such as the π^0.

$$C\,|\pi^0\rangle = \lambda|\pi^0\rangle \qquad C^2\,|\pi^0\rangle = \lambda^2|\pi^0\rangle = |\pi^0\rangle$$

$$\therefore \lambda = \pm 1$$

Charge Conjugation

EM fields come from moving charges which change sign under Charge Conjugation $\therefore C_\gamma = -1$.

$\therefore$ n photons have $C = (-1)^n$

Since $\pi^0 \to \gamma\gamma$ this implies $C_{\pi^0} = +1$ (assuming C invariance in EM decays).

Note $\pi^0 \to \gamma\gamma\gamma$ is then forbidden.

The η (eta) meson (mass 550 MeV/c^2)

$$\eta \to \gamma\gamma$$

$$\eta \not\to \gamma\gamma\gamma$$

i.e. $C_\eta = +1$

C is conserved Strong and EM Interactions but NOT Weak.

Summary

Conserved Quantum Numbers

Quantity		Strong	EM	Weak
Charge	Q	✓	✓	✓
Baryon Number	B	✓	✓	✓
Lepton Number	L	✓	✓	✓
Isospin	I	✓	✗	✗
	I_3	✓	✓	✗
Strangeness	S	✓	✓	✗
Parity	P	✓	✓	✗
Charge Conjugation	C	✓	✓	✗