Table to the Equations of Opeca hado for a handary type					
No.	Description	Sun Gear A Z _a	Planet Gear B	Internal Gear C	Carrier D
1	Rotate sun gear A once while holding carrier	+1	$-\frac{Z_a}{Z_b}$	$-\frac{Z_a}{Z_c}$	0
2	System is fixed as a whole while rotating $+(z_a/z_c)$	$+\frac{Z_a}{Z_c}$	$+\frac{Z_a}{Z_c}$	$+\frac{Z_a}{Z_c}$	$+\frac{Z_a}{Z_c}$
3	Sum of 1 and 2	$1 + \frac{Z_a}{Z_c}$	$\frac{Z_a}{Z_c} - \frac{Z_a}{Z_b}$	0 (fixed)	$+\frac{Z_a}{Z_c}$

Table 13-1 Equations of Speed Ratio for a Planetary Type

Table 13-2 Equations of Speed Ratio for a Solar Type

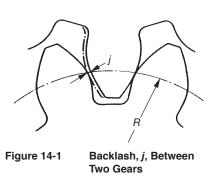
No.	Description	Sun Gear A Z _a	Planet Gear B	Internal Gear C	Carrier D
1	Rotate sun gear A once while holding carrier	+1	$-\frac{Z_a}{Z_b}$	$-\frac{Z_a}{Z_c}$	0
2	System is fixed as a whole while rotating $+(z_a/z_c)$	- 1	- 1	- 1	- 1
3	Sum of 1 and 2	0 (fixed)	$-\frac{Z_a}{Z_b}-1$	$-\frac{Z_a}{Z_c}-1$	- 1

SECTION 14 BACKLASH

Up to this point the discussion has implied that there is no backlash. If the gears are of standard tooth proportion design and operate on standard center distance they would function ideally with neither backlash nor jamming.

Backlash is provided for a variety of reasons and cannot be designated without consideration of machining conditions. The general purpose of backlash is to prevent gears from jamming by making contact on both sides of their teeth simultaneously. A small amount of backlash is also desirable to provide for lubricant space and differential expansion between the gear components and the housing. Any error in machining which tends to increase the possibility of jamming makes it necessary to increase the amount of backlash by at least as much as the possible cumulative errors. Consequently, the smaller the amount of backlash, the more accurate must be the machining of the gears. Runout of both gears, errors in profile, pitch, tooth thickness, helix angle and center distance - all are factors to consider in the specification of the amount of backlash. On the other hand, excessive backlash is objectionable, particularly if the drive is frequently reversing or if there is an overrunning load. The amount of backlash must not be excessive for the requirements of the job, but it should be sufficient so that machining costs are not higher than necessary.

In order to obtain the amount of backlash desired. it is necessary to decrease tooth thickness. See Figure 14-1. This decrease must almost always be greater than the desired backlash because of the errors in manufacturing and assembling. Since the amount of the decrease in tooth thickness depends upon the accuracy of machining, the allowance for a specified backlash will vary according to the manufacturing conditions

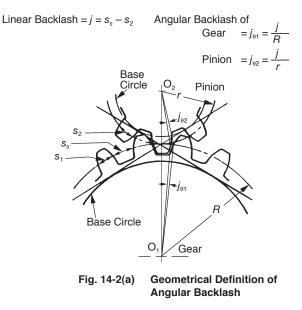


It is customary to make half of the allowance for backlash on the tooth thickness of each gear of a pair, although there are exceptions. For example, on pinions having very low numbers of teeth, it is desirable to provide all of the allowance on the mating gear so as not to weaken the pinion teeth. In spur and helical gearing, backlash allowance is usually obtained by sinking the hob deeper into the blank than the theoretically standard depth. Further, it is true that any increase or decrease in center distance of two gears in any mesh will cause an increase or decrease in backlash. Thus, this is an alternate way of designing backlash into the system.

In the following, we give the fundamental equations for the determination of backlash in a single gear mesh. For the determination of backlash in gear trains, it is necessary to sum the backlash of each mated gear pair. However, to obtain the total backlash for a series of meshes, it is necessary to take into account the gear ratio of each mesh relative to a chosen reference shaft in the gear train. For details, see Reference 10 at the end of the technical section.

14.1 Definition Of Backlash

Backlash is defined in **Figure 14-2(a)** as the excess thickness of tooth space over the thickness of the mating tooth. There are two basic ways in which backlash arises: tooth thickness is below the zero backlash value; and the operating center distance is greater than the zero backlash value.



If the tooth thickness of either or both mating gears is less than the zero backlash value, the amount of backlash introduced in the mesh is simply this numerical difference:

$$j = S_{std} - S_{act} = \Delta S \tag{14-1}$$

where:

- = linear backlash measured along the pitch circle (Figure 14i 2(b))
- no backlash tooth thickness on the operating pitch circle, which Sete is the standard tooth thickness for ideal gears

= actual tooth thickness

Backlash, Along Line-of-Action = $j_{\alpha} = j \cos \alpha$

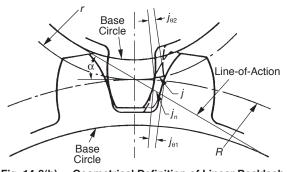
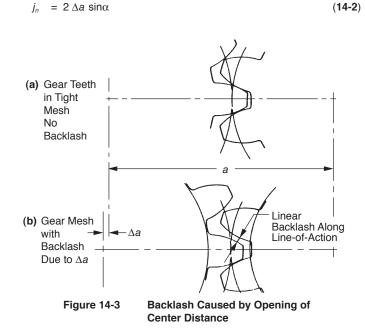


Fig. 14-2(b) **Geometrical Definition of Linear Backlash**

When the center distance is increased by a relatively small amount, Δa , a backlash space develops between mating teeth, as in Figure 14-3. The relationship between center distance increase and linear backlash j_n along the line-of-action is:



This measure along the line-of-action is useful when inserting a feeler gage between teeth to measure backlash. The equivalent linear backlash measured along the pitch circle is given by:

$$i = 2 \Delta a \tan \alpha$$
 (14-3a)

where.

 Δa = change in center distance α = pressure angle

Hence, an approximate relationship between center distance change and change in backlash is:

$$\Delta a = 1.933 \Delta j$$
 for 14.5° pressure angle gears (14-3b)

$$\Delta a = 1.374 \Delta j$$
 for 20° pressure angle gears (14-3c)

Although these are approximate relationships, they are adequate for most uses. Their derivation, limitations, and correction factors are detailed in Reference 10.

Note that backlash due to center distance opening is dependent upon the tangent function of the pressure angle. Thus, 20° gears have 41% more backlash than 14.5° gears, and this constitutes one of the few advantages of the lower pressure angle.

Equations (14-3) are a useful relationship, particularly for converting to angular backlash. Also, for fine pitch gears the use of feeler gages for measurement is impractical, whereas an indicator at the pitch line gives a direct measure. The two linear backlashes are related by:

$$j = \frac{J_n}{\cos\alpha} \tag{14-4}$$

The angular backlash at the gear shaft is usually the critical factor in the gear application. As seen from Figure 14-2(a), this is related to the gear's pitch radius as follows:

$$j_{\theta} = 3440 \frac{J}{R_{\star}}$$
 (arc minutes) (14-5)

Obviously, angular backlash is inversely proportional to gear radius. Also, since the two meshing gears are usually of different pitch diameters, the linear backlash of the measure converts to different angular values for each gear. Thus, an angular backlash must be specified with reference to a particular shaft or gear center.

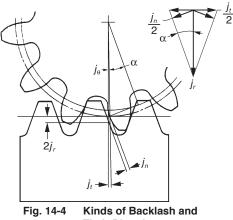
Details of backlash calculations and formulas for various gear types are given in the following sections.

14.2 Backlash Relationships

Expanding upon the previous definition, there are several kinds of

backlash: circular backlash j_t , normal backlash j_n , center backlash j_r and angular backlash j_{θ} (°), see Figure 14-4.

Table 14-1 reveals relationships among circular backlash j_t , normal backlash j_n and center backlash j_r . In this definition, j_r is equivalent to change in center distance, Δa , in Section 14.1.



Their Direction

Table 14-1	The Relationships among the Backlashes
------------	--

No.	Type of Gear Meshes	The Relation between Circular Backlash <i>j</i> _t and Normal Backlash <i>j</i> _n	The Relation between Circular Backlash j_t and Center Backlash j_r	
1	Spur Gear	$j_n = j_t \cos \alpha$	$j_r = \frac{j_t}{2\tan\alpha}$	
2	Helical Gear	$j_{nn} = j_{tt} \cos \alpha_n \cos \beta$	$j_r = \frac{j_{tt}}{2 \tan \alpha_t}$	
3	Straight Bevel Gear	$j_n = j_t \cos \alpha$	$j_r = \frac{j_t}{2\tan\alpha\sin\delta}$	
4	Spiral Bevel Gear	$j_{nn} = j_{tt} \cos \alpha_n \cos \beta_m$	$j_r = \frac{j_{tt}}{2\tan\alpha_t \sin\delta}$	
5	Worm Worm Gear	$j_{nn} = j_{tt1} \cos \alpha_n \cos \gamma$ $j_{nn} = j_{tt2} \cos \alpha_n \cos \gamma$	$j_r = \frac{j_{tt2}}{2\tan\alpha_x}$	

Circular backlash j_t has a relation with angular backlash j_{θ} , as follows:

$$j_{\theta} = j_t \frac{360}{\pi d} \text{ (degrees)} \tag{14-6}$$

have the following relationships:

$$j_n = j_t \cos\alpha$$

$$j_r' = \frac{j_t}{2\tan\alpha}$$
(14-11)

14.2.1 Backlash Of A Spur Gear Mesh

From Figure 14-4 we can derive backlash of spur mesh as:

$$j_r = \frac{j_t \cos\alpha}{2\tan\alpha}$$

$$j_r = \frac{j_t}{2\tan\alpha}$$
(14-7)

14.2.2 Backlash Of Helical Gear Mesh

The helical gear has two kinds of backlash when referring to the tooth space. There is a cross section in the normal direction of the tooth surface n, and a cross section in the radial direction perpendicular to the axis, t.

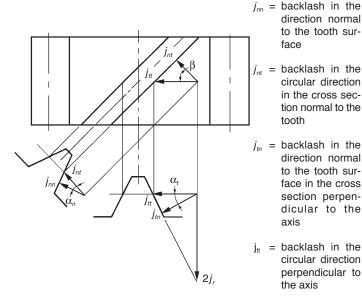


Fig. 14-5 Backlash of Helical Gear Mesh

These backlashes have relations as follows:

In the plane normal to the tooth:

 $j_{nn} = j_{nt} \cos \alpha_n \tag{14-8}$

On the pitch surface:

$$j_{nt} = j_{tt} \cos \beta \tag{14-9}$$

In the plane perpendicular to the axis:

$$\begin{cases} j_{tr} = j_{tt} \cos\alpha_t \\ j_r = \frac{j_{tt}}{2\tan\alpha_t} \end{cases}$$
(14-10)

14.2.3 Backlash Of Straight Bevel Gear Mesh

Figure 14-6 expresses backlash for a straight bevel gear mesh.

In the cross section perpendicular to the tooth of a straight bevel gear, circular backlash at pitch line j_i , normal backlash j_n and radial backlash j'_r

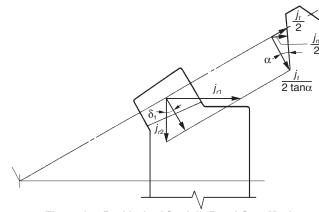


Fig. 14-6 Backlash of Straight Bevel Gear Mesh

The radial backlash in the plane of axes can be broken down into the components in the direction of bevel pinion center axis, j_{r1} , and in the direction of bevel gear center axis, j_{r2} .

$$j_{r1} = \frac{J_t}{2\tan\alpha \sin\delta_1}$$

$$j_{r2} = \frac{j_t}{2\tan\alpha \cos\delta_1}$$
(14-12)

14.2.4 Backlash Of A Spiral Bevel Gear Mesh

Figure 14-7 delineates backlash for a spiral bevel gear mesh.

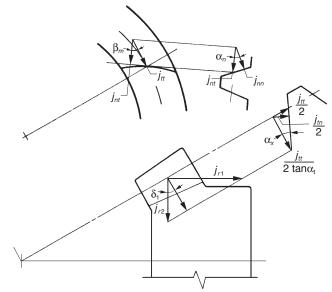


Fig. 14-7 Backlash of Spiral Bevel Gear Mesh

In the tooth space cross section normal to the tooth:

$$j_{nn} = j_{nt} \cos \alpha_n \tag{14-13}$$

On the pitch surface:

$$i_{nt} = j_{tt} \cos\beta_m \tag{14-14}$$

In the plane perpendicular to the generatrix of the pitch cone:

$$\begin{aligned}
j_{tr} &= j_{tt} \cos \alpha_t \\
j_{r}' &= \frac{j_{tt}}{2 \tan \alpha_t}
\end{aligned}$$
(14-15)

The radial backlash in the plane of axes can be broken down into the components in the direction of bevel pinion center axis, j_{r1} , and in the direction of bevel gear center axis, j_{r2} .

$$j_{r1} = \frac{j_{tt}}{2\tan\alpha_{t}\sin\delta_{1}}$$

$$j_{r2} = \frac{j_{tt}}{2\tan\alpha_{t}\cos\delta_{1}}$$
(14-16)

14.2.5 Backlash Of Worm Gear Mesh

Figure 14-8 expresses backlash for a worm gear mesh.

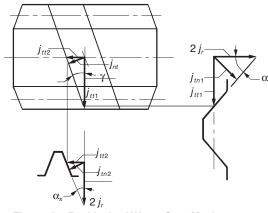


Fig. 14-8 Backlash of Worm Gear Mesh

On the pitch surface of a worm:

 $\begin{aligned}
j_{nt} &= j_{tt1} \sin \gamma \\
j_{nt} &= j_{tt2} \cos \gamma \\
\tan \gamma &= \frac{j_{tt2}}{j_{tt2}}
\end{aligned}$ (14-17)

In the cross section of a worm perpendicular to its axis:

$$\begin{cases}
j_{tr1} = j_{tr1} \cos \alpha_t \\
j_r = \frac{j_{tr1}}{2 \tan \alpha_r}
\end{cases}$$
(14-18)

In the plane perpendicular to the axis of the worm gear:

 $j_{tn2} = j_{tt2} \cos\alpha_x$ $j_r = \frac{j_{tt2}}{2\tan\alpha_x}$ (14-19)

14.3 Tooth Thickness And Backlash

There are two ways to generate backlash. One is to enlarge the center distance. The other is to reduce the tooth thickness. The latter is much more popular than the former. We are going to discuss more about the way of reducing the tooth thickness. In **SECTION 10**, we have discussed the standard tooth thickness s. In the meshing of a pair of gears, if the tooth thickness of pinion and gear were reduced by Δs_1 and Δs_2 , they would generate a backlash of $\Delta s_1 + \Delta s_2$ in the direction of the pitch circle.

Let the magnitude of Δs_1 , Δs_2 be 0.1. We know that $\alpha = 20^\circ$, then:

 $j_t = \Delta s_1 + \Delta s_2 = 0.1 + 0.1 = 0.2$

We can convert it into the backlash on normal direction:

 $j_n = j_t \cos \alpha = 0.2 \cos 20^\circ = 0.1879$

Let the backlash on the center distance direction be j_r , then:

$$\dot{t}_r = \frac{J_t}{2\tan\alpha} = \frac{0.2}{2\tan 20^\circ} = 0.2747$$

These express the relationship among several kinds of backlashes. In application, one should consult the JIS standard.

There are two JIS standards for backlash – one is JIS B 1703-76 for spur gears and helical gears, and the other is JIS B 1705-73 for bevel gears. All these standards regulate the standard backlashes in the direction of the pitch circle j_t or j_{tt} . These standards can be applied directly, but the backlash beyond the standards may also be used for special purposes. When writing tooth thicknesses on a drawing, it is necessary to specify, in addition, the tolerances on the thicknesses as well as the backlash. For example:

Circular tooth thickness 3.141 -0.050

Backlash 0.100 ... 0.200

14.4 Gear Train And Backlash

The discussions so far involved a single pair of gears. Now, we are going to discuss two stage gear trains and their backlash. In a two stage gear train, as **Figure 14-9** shows, j_1 and j_4 represent the backlashes of first stage gear train and second stage gear train respectively.

If number one gear were fixed, then the accumulated backlash on number four gear j_{i74} would be as follows:

$$j_{t74} = j_1 \frac{d_3}{d_2} + j_4 \tag{14-20}$$

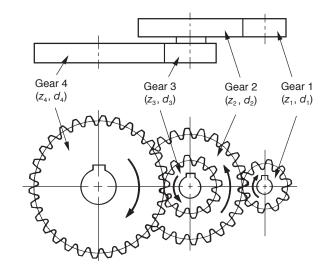


Fig. 14-9 Overall Accumulated Backlash of Two Stage Gear Train

This accumulated backlash can be converted into rotation in degrees:

$$j_{\theta} = j_{t74} - \frac{360}{\pi d_4}$$
 (degrees) (14-21)

The reverse case is to fix number four gear and to examine the accumulated backlash on number one gear $j_{\rm tT1}.$

$$j_{tT1} = j_4 \frac{d_2}{d_3} + j_1 \tag{14-22}$$

This accumulated backlash can be converted into rotation in degrees:

$$j_{\theta} = j_{tT1} \frac{360}{\pi d_1}$$
 (degrees) (14-23)

14.5 Methods Of Controlling Backlash

In order to meet special needs, precision gears are used more frequently than ever before. Reducing backlash becomes an important issue. There are two methods of reducing or eliminating backlash – one a static, and the other a dynamic method.

The static method concerns means of assembling gears and then making proper adjustments to achieve the desired low backlash. The dynamic method introduces an external force which continually eliminates all backlash regardless of rotational position.

14.5.1 Static Method

This involves adjustment of either the gear's effective tooth thickness or the mesh center distance. These two independent adjustments can be used to produce four possible combinations as shown in **Table 14-2**.

Table 14-2				
		Center Distance		
	Fixed	Adjustable		
Fixed	I	III		
Adjustable	II	IV		
	Fixed	Center Fixed I		

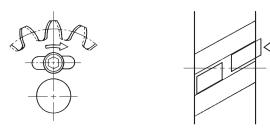
Case I

By design, center distance and tooth thickness are such that they yield the proper amount of desired minimum backlash. Center distance and tooth thickness size are fixed at correct values and require precision manufacturing.

Case II

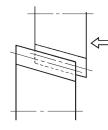
With gears mounted on fixed centers, adjustment is made to the effective tooth thickness by axial movement or other means. Three main methods are:

- Two identical gears are mounted so that one can be rotated relative to the other and fixed. See Figure 14-10a. In this way, the effective tooth thickness can be adjusted to yield the desired low backlash.
- A gear with a helix angle such as a helical gear is made in two half thicknesses. One is shifted axially such that each makes contact with the mating gear on the opposite sides of the tooth. See Figure 14-10b.
- The backlash of cone shaped gears, such as bevel and tapered tooth spur gears, can be adjusted with axial positioning. A duplex lead worm can be adjusted similarly. See Figure 14-10c.



(a) Rotary Adjustment

(b) Parallel Adjustment



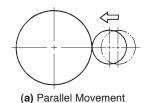
(c) Axial Adjustment

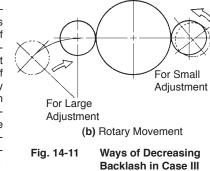
Fig. 14-10 Ways of Reducing Backlash in Case II

Case III

Center distance adjustment of backlash can be accomplished in two wavs:

- Linear Movement Figure 14-11a shows adjustment along the line-of-centers in a straight or parallel axes manner. After setting to the desired value of backlash, the centers are locked in place.
- Rotary Movement Figure 14-11b shows an alternate way of achieving center distance adjustment by rotation of one of the gear centers by means of a swing arm on an eccentric bushing. Again, once the desired backlash setting is found, the positioning arm is locked.





Case IV

Adjustment of both center distance and tooth thickness is theoretically valid, but is not the usual practice. This would call for needless fabrication expense.

14.5.2 Dynamic Methods

Dynamic methods relate to the static techniques. However, they involve a forced adjustment of either the effective tooth thickness or the center distance.

1. Backlash Removal by Forced Tooth Contact

This is derived from static Case II. Referring to **Figure 14-10a**, a forcing spring rotates the two gear halves apart. This results in an effective tooth thickness that continually fills the entire tooth space in all mesh positions.

2. Backlash Removal by Forced Center Distance Closing

This is derived from static Case III. A spring force is applied to close the center distance; in one case as a linear force along the line-of-centers, and in the other case as a torque applied to the swing arm.

In all of these dynamic methods, the applied external force should be known and properly specified. The theoretical relationship of the forces involved is as follows:

 $F > F_1 + F_2$ (14-24)

where:

 F_1 = Transmission Load on Tooth Surface

 F_2 = Friction Force on Tooth Surface

If $F < F_1 + F_2$, then it would be impossible to remove backlash. But if *F* is excessively greater than a proper level, the tooth surfaces would be needlessly loaded and could lead to premature wear and shortened life. Thus, in designing such gears, consideration must be given to not only the needed transmission load, but also the forces acting upon the tooth surfaces caused by the spring load. It is important to appreciate that the spring loading must be set to accommodate the largest expected transmission force, F_1 , and this maximum spring force is applied to the tooth surfaces continually and irrespective of the load being driven.

3. Duplex Lead Worm

A duplex lead worm mesh is a special design in which backlash can be adjusted by shifting the worm axially. It is useful for worm drives in high precision turntables and hobbing machines. Figure 14-12 presents the basic concept of a duplex lead worm.

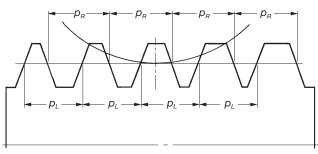


Fig. 14-12 **Basic Concepts of Duplex Lead Worm**

The lead or pitch, p_L and p_R , on the two sides of the worm thread are not identical. The example in **Figure 14-12** shows the case when $p_B > p_I$. To produce such a worm requires a special dual lead hob.

The intent of Figure 14-12 is to indicate that the worm tooth thickness is progressively bigger towards the right end. Thus, it is convenient to adjust backlash by simply moving the duplex worm in the axial direction.

SECTION 15 GEAR ACCURACY

Gears are one of the basic elements used to transmit power and position. As designers, we desire them to meet various demands:

- 1. Minimum size.
- 2. Maximum power capability.
- 3. Minimum noise (silent operation).
- 4. Accurate rotation/position.

To meet various levels of these demands requires appropriate degrees of gear accuracy. This involves several gear features.

15.1 Accuracy Of Spur And Helical Gears

This discussion of spur and helical gear accuracy is based upon JIS B 1702 standard. This specification describes 9 grades of gear accuracy - grouped from 0 through 8 - and four types of pitch errors:

Single pitch error.

- Pitch variation error.
- Accumulated pitch error.
- Normal pitch error.

Single pitch error, pitch variation and accumulated pitch errors are closely related with each other.

15.1.1 Pitch Errors of Gear Teeth

- 1. Single Pitch Error (f_{pt})
 - The deviation between actual measured pitch value between any adjacent tooth surface and theoretical circular pitch.
- 2. Pitch Variation Error (f_{pu}) Actual pitch variation between any two adjacent teeth.
- In the ideal case, the pitch variation error will be zero. 3. Accumulated Pitch Error (F_{p})
 - Difference between theoretical summation over any number of teeth interval, and summation of actual pitch measurement over the same interval.
- 4. Normal Pitch Error (f_{pb})

It is the difference between theoretical normal pitch and its actual measured value.

The major element to influence the pitch errors is the runout of gear flank groove.

Table 15-1 contains the ranges of allowable pitch errors of spur gears and helical gears for each precision grade, as specified in JIS B 1702-1976.

Table 15-1 The Allowable Single Pitch Error, Accumulated Pitch Error and Normal Pitch Error, µm

Grade	Single Pitch Error	Accumulated Pitch Error	Normal Pitch Error	
Grade	f _{pt}	F_{p}	$f_{ ho b}$	
JIS 0	0.5W + 1.4	2.0W + 5.6	0.9W' + 1.4	
1	0.71 <i>W</i> + 2.0	2.8W + 8.0	1.25W' + 2.0	
2	1.0W + 2.8	4.0W + 11.2	1.8W' + 2.8	
3	1.4W + 4.0	5.6W + 16.0	2.5W' + 4.0	
4	2.0W + 5.6	8.0W + 22.4	4.0W' + 6.3	
5	2.8W + 8.0	11.2W + 31.5	6.3W' + 10.0	
6	4.0W + 11.2	16.0W + 45.0	10.0W' + 16.0	
7	8.0W + 22.4	32.0W + 90.0	20.0W' + 32.0	
8	16.0W + 45.0	64.0W + 180.0	40.0W' + 64.0	

In the above table, W and W' are the tolerance units defined as:

$$W = \sqrt[3]{d} + 0.65m \,(\mu m)$$
 (15-1)

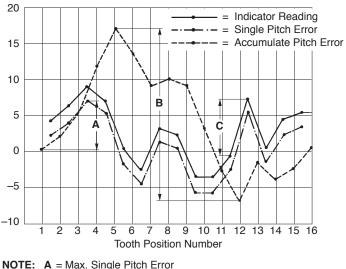
$$W' = 0.56 W + 0.25m (\mu m)$$
 (15-2)

The value of allowable pitch variation error is k times the single pitch error. Table 15-2 expresses the formula of the allowable pitch variation error.

Table 15-2 The Allowable Pitch Variation Error, µm

Single Pitch Error, f _{pt}		Pitch Variation Error, f _{pu}
less than 5		1.00 <i>f</i> _{pt}
5 or more, but less than	10	1.06 <i>f</i> _{pt}
10 or more, but less than	20	1.12f _{pt}
20 or more, but less than	30	1.18 <i>f</i> _{pt}
30 or more, but less than	50	1.25 <i>f</i> _{pt}
50 or more, but less than	70	1.32 <i>f</i> _{pt}
70 or more, but less than	100	1.40 <i>f</i> _{pt}
100 or more, but less than	150	1.50f _{pt}
more than 150		1.60 <i>f</i> _{pt}

Figure 15-1 is an example of pitch errors derived from data measurements made with a dial indicator on a 15 tooth gear. Pitch differences were measured between adjacent teeth and are plotted in the figure. From that plot, single pitch, pitch variation and accumulated pitch errors are extracted and plotted.



B = Max. Accumulated Error

C = Max. Pitch Variation Error

Fig. 15-1 Examples of Pitch Errors for a 15 Tooth Gear