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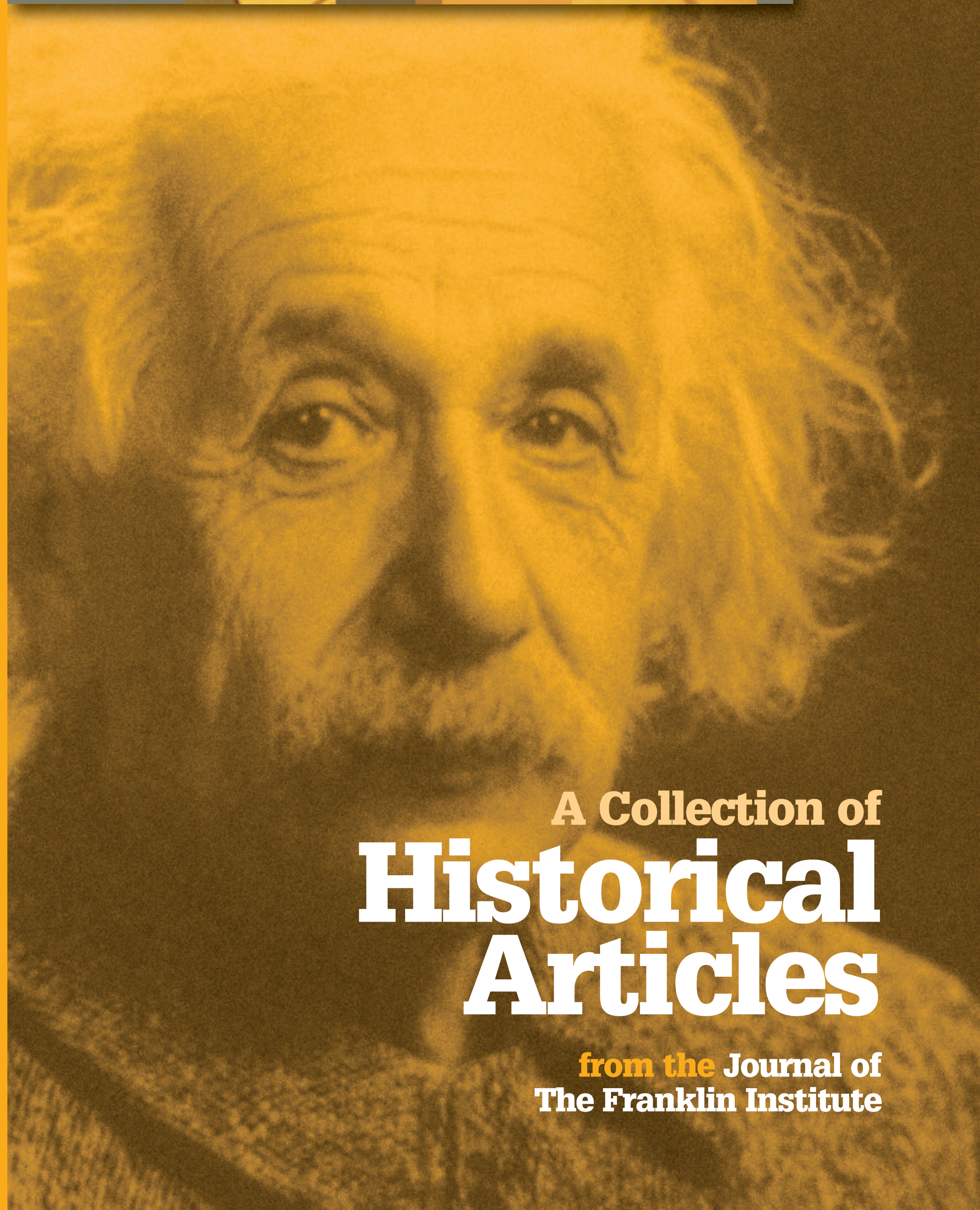

A Collection of Historical Articles

from the Journal of The Franklin Institute

Journal of
The Franklin Institute

Aims and Scope

The Journal of The Franklin Institute has an established reputation for publishing high-quality papers in the field of engineering and applied mathematics. Its current focus is on information and communication systems, signal processing, wavelets, sensor fusion, computer and communication networks, neural networks, control theory, nonlinear dynamics, fractals and chaos theory.

The editors and the editorial board encourage authors to put forth evolving new topics for publication provided they fall within the current scope of the journal as noted above. Special focus issues with a strong theoretical foundation in newly evolving topics are planned for the future.

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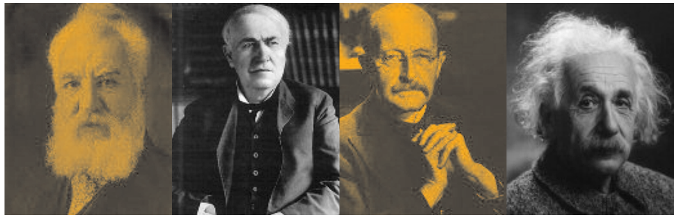

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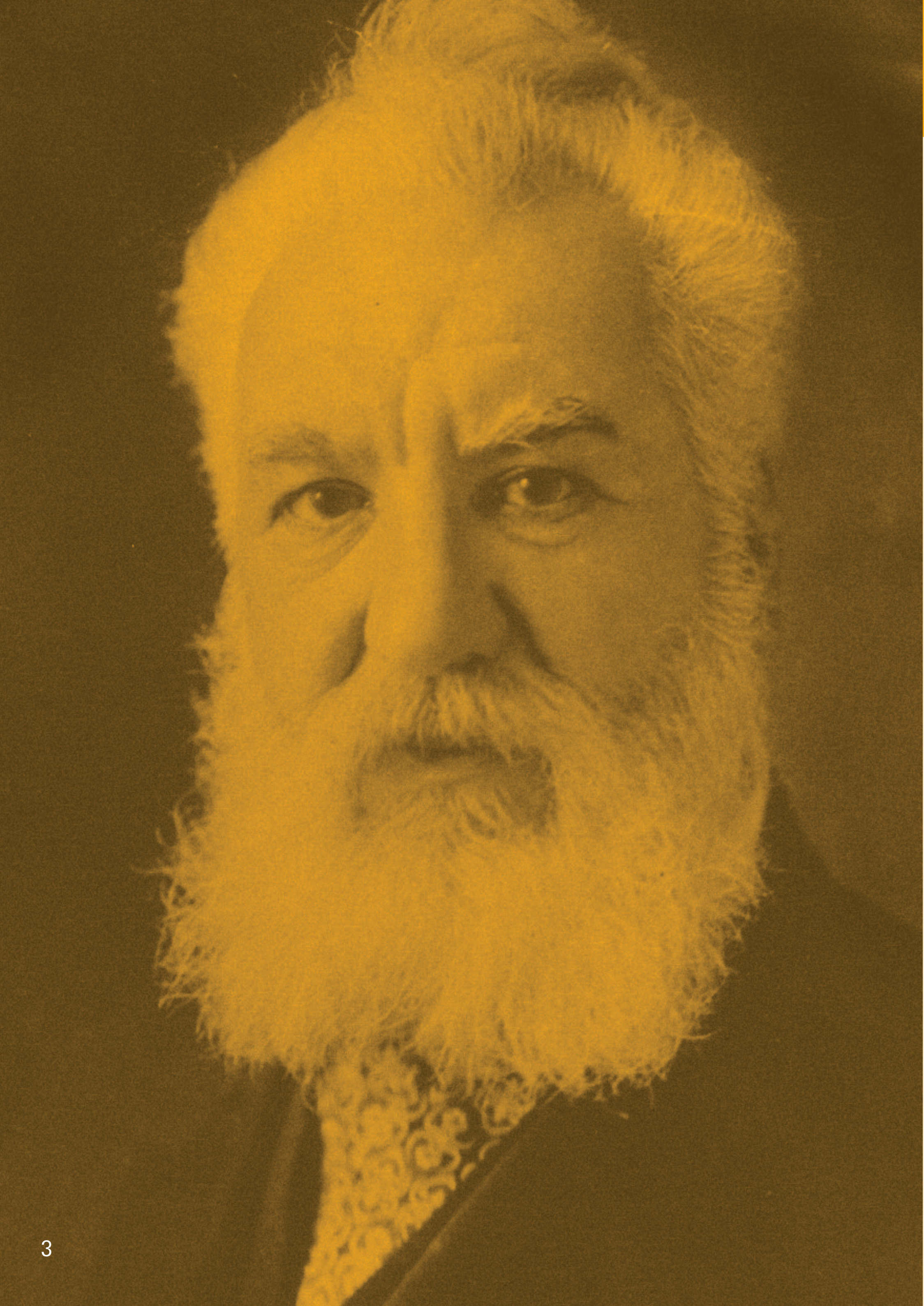

THE PHOTOPHONE.

By Alexander Graham Bell.

Read before the American Association for the Advancement of Science, August, 1880.

In bringing before you some discoveries made by Mr. Sumner Tainter and myself, which have resulted in the construction of apparatus for the production and reproduction of sound by means of light, it is necessary to explain the state of knowledge which formed the starting point of our experiments. I shall first describe the remarkable substance selenium, and the manipulations devised by various experiments; but the final result of our researches has evidenced the the class of substances sensitive to light vibrations until we can propound the fact of such sensitiveness being a general property of all matter. We have found this property in gold, silver, platinum, iron, steel, brass, copper, zinc, lead, antimony, German silver, Jenkins' metal, Babbitt's metal, ivory, celluloid, gutta percha, hard rubber, soft vulcanized rubber, paper, parchment, wood, mica and silvered glass; and the only substances from which we have not obtained results are carbon and thin microscopic glass. We find that when a vibratory beam of light falls upon these substances they emit sounds—the pitch of which depends upon the frequency of the vibratory change in the light. We find farther that, when we control the form or character of the light-vibration on selenium, and probably on the other substances, we control the quality of the sound and obtain all varieties of articulate speech. We can thus, without a conducting wire as in electric telephony, speak from station to station, wherever we can project a beam of light. We have not had opportunity of testing the limit to which this photophonic influence can be extended, but we have spoken to and from points 213 metres apart; and there seems no reason to doubt that the results will be obtained at whatever distance a beam of light can be flashed from one observatory to another. The necessary privacy of our experiments hitherto has alone prevented any attemps at determing the extreme distance at which this new method of vocal communication will be available. I shall now speak of selenium.

In the year 1817 Berzelius and Gottlieb Gahn made an examination of the method of preparing sulphuric acid in use at Gripsholm. During the course of this examination, they observed in the acid a sediment of a partly reddish, partly clear brown color, which, under the action of the blow-pipe, gave out a peculiar odor, like that attributed by Klaproth to tellurium. As tellurium was a substance of extreme rarity, Berzelius attempted its production from this deposit; but he was unable after many experiments, to obtain further indications of its presence. He found plentiful signs of sulphur mixed with mercury, copper, zinc, iron, arsenic and lead, but no trace of tellurium. It was not in the nature of Berzelius to be disheartened by this result. In science every failure advances the boundary of knowledge as well as every success, and Berzelius felt, that if the characteristic odor that had been observed did not proceed from tellurium, it might possibly indicate the presence of some substance then unknown to the chemist. Urged on by this hope he returned with renewed ardor to his work. He collected a great quantity of the material, and submitted the whole mass to various chemical processes. He succeeded in separating successively the sulphur, the mercury, the copper, the tin and the other known substances whose presence had been indicated by his tests—and, after all these had been eliminated, there still remained a residue which proved upon examination to be what he had been in search of—a new elementary substance. The chemical properties of this new element were found to resemble those of tellurium in so remarkable a degree that Berzelius gave to the substance the name of "selenium," from the Greek wore *selene*, the moon—("tellurium," as is well known, being derived from *tellus*, the earth).

Although tellurium and selenium are alike in many respects, they differ in their electrical properties; tellurium being a good conductor of electricity, and selenium, as Berzelius showed, a non-conductor. Knox discovered in 1837, that selenium became a conductor when fused; and Hittorff, in 1852, showed that it conducted, at ordinary temperatures, when in one of its allotropic forms. When selenium is rapidly cooled from a fused condition it is a non-conductor. In this, its vitreous form, it is of a dark-brown color, almost black by reflected light, having an exceedingly brilliant surface. In thin films it is transparent, and appears of a beautiful ruby red by transmitted light. When selenium is cooled from a fused condition with extreme slowness, it presents an entirely different appearance, being of a dull lead color, and having throughout a granulated or crystalline structure, and looking like a metal. In this form it is perfectly opaque to light,

even in very thin films. This variety of selenium has long been known as "granular" or "crystalline" selenium, or, as Regnault called it, "metallic" selenium. It was selenium of this kind that Hittorff found to be a conductor of electricity at ordinary temperatures. He also found that its resistance to the passage of an electrical current diminished continuously by heating up to the point of fusion, and that the resistance suddenly increased in passing from the solid to the liquid condition. It was early discovered that exposure to sunlight hastens the change of selenium from one allotropic form to another; and this observation is significant in the light of recent discoveries.

Although selenium has been known for the last sixty years, it has not yet been utilized to any extent in the arts, and it is still considered simply as a chemical curiosity. It is usually supplied in the form of cylindrical bars. These bars are sometimes found to be in the metallic condition, but more usually they are in the vitreous or non-conducting form. It occurred to Willoughby Smith that on account of the high resistance of crystalline selenium, it might be usefully employed at the short end of a submarine cable in his system of testing and signaling during the process of submersion. Upon experiment the selenium was found to have all the resistance required—some of the bars employed measuring as much as 1,400 megohms—a resistance equivalent to that which would be offered by a telegraph wire long enough to reach from the earth to the sun! But the resistance was found to be extremely variable. Experiments were made to ascertain the cause of this variability. Mr. May, Mr. Willoughby Smith's assistant, discovered that the resistance was less when the selenium was exposed to light than when it was in the dark.

In order to be certain that temperature had nothing to do with the effect, the selenium was placed in a vessel of water, so that the light had to pass through from one to two inches of water in order to reach the selenium. The approach of a lighted candle was found to be sufficient to cause a marked deflection of the needle of the galvanometer connected with the selenium, and the lighting of a piece of magnesium wire caused the selenium to measure less than half the resistance it did the moment before.

These results were naturally at first received by scientific men with some incredulity, but they were verified by Sale, Draper, Moss and others. When selenium is exposed to the action of the solar spectrum the maximum effect is produced, according to Sale, just outside the red end of the spectrum, in a point nearly coincident with the maximum of the heat rays; but, according to Adams, the maximum effect is produced in the greenish-yellow or most luminous part of the spectrum. Lord Rosse exposed selenium to the action of non-luminous radiations from hot bodies, but could produce no effect; whereas a thermopile under similar circumstances gave abundant indications of a current. He also cut off the heat rays from luminous bodies by the interposition of liquid solutions, such as alum, between the selenium and the source of light, without affecting the power of the light to reduce the resistance of the selenium; whereas the interposition of these same substances almost completely neutralized the effect upon the thermopile. Adams found that selenium was sensitive to the cold light of the moon, and Werner Siemens discovered that, in certain extremely sensitive varieties of selenium, heat and light produced opposite effects. In Siemens' experiments special arrangements were made for the purpose of reducing the resistance of the selenium employed. Two fine platinum wires were coiled together in the shape of a double flat spiral in the zigzag shape, and were laid upon a plate of mica so that the disks did not touch one another. A drop of melted selenium was then placed upon the platinum wire arrangement, and a second sheet of mica was pressed upon the selenium, so as to cause it to spread out and fill the spaces between the wires. Each cell was about the size of a silver dime. The selenium cells were then placed in a paraffine bath and exposed for some hours to a temperature of 210°C., after which they were allowed to cool with extreme slowness. The results obtained with these cells were very extraordinary; in some cases the resistance of the cells, when exposed to light, was only one-fifteenth of their resistance in the dark.

Without dwelling further upon the researches of others, I may say that the chief information concerning the effect of light upon the conductivity of selenium will be found under the names of Willoughby Smith, Lieutenant Sale, Draper and Moss, Professor W. G. Adams, Lord Rosse, Day, Sabini, Dr. Werner Siemens and Dr. C. W. Siemens. All observations by these various authors had been made by means of galvanometers; but it occurred to me that the telephone, from its extreme sensitiveness to electrical influences, might be substituted with advantage. Upon consideration of the subject, however, I saw that the experiments could not be conducted in the ordinary way, for the following reason: The law of audibility of the telephone is

precisely analogous to the law of electric induction. No effect is produced during the passage of a continuous and steady current. It is only at the moment of change from a stronger to a weaker state, or *vice versa*, that any audible effect is produced, and the amount of effect is exactly proportional to the amount of variation in the current. It was, therefore, evident that the telephone could only respond to the effect produced in selenium at the moment of change from light to darkness, or *vice versa;* and that it would be advisable to intermit the light with great rapidity, so as to produce a succession of changes in the conductivity of the selenium, corresponding in frequency to musical vibrations within the limits of the sense of hearing. For I had often noticed that currents of electricity, so feeble as to produce scarcely any audible effects from a telephone when the circuit was simply opened or closed, caused very perceptible musical sounds when the circuit was rapidly interrupted, and that the higher the pitch of sound the more audible was the effect. I was much struck by the idea of producing sound by the action of light in this way. Upon further consideration it appeared to me that all the audible effects obtained from varieties of electricity could also be produced by variations of light acting upon selenium. I saw that the effect could be produced at the extreme distance at which selenium would respond to the action of a luminous body, but that this distance could be indefinitely increased by the use of a parallel beam of light, so that we could telephone from one place to another without the necessity of a conducting wire between the transmitter and receiver. It was evidently necessary, in order to reduce this idea to practice, to devise an apparatus to be operated by the voice of a speaker, by which variations could be produced in a parallel beam of light corresponding to the variations in the air produced by the voice.

I proposed to pass light through a large number of small orifices, which might be of any convenient shape, but were preferably in the form of slits. Two similarly perforated plates were to be employed. One was to be fixed and the other attached to the centre of a diaphragm actuated by the voice, so that the vibration of the diaphragm would cause the movable plate to slide to and fro over the surface of the fixed plate, thus alternately enlarging and contracting the free orifices for the passage of light. In this way the voice of a speaker could control the amount of light passed through the perforated plates without completely obstructing its passage. This apparatus was to be placed in the path of a parallel beam of light, and the undulatory beam emerging from the apparatus could be received at some distant place upon a lens, or other apparatus, by means of which it could be condensed upon a sensitive piece of selenium placed in a local circuit with a telephone and galvanic battery. The variations in the light produced by the voice of the speaker should cause corresponding variations in the electrical resistance of the selenium employed; and the telephone in circuit with it should reproduce audibly the tones and articulations of the speaker's voice. I obtained some selenium for the purpose of producing the apparatus shown; but found that its resistance was almost infinitely greater than that of any telephone that had been constructed, and I was unable to obtain any audible effects by the action of light. I believed, however, that the obstacle could be overcome by devising mechanical arrangements for reducing the resistance of the selenium, and by constructing special telephones for the purpose. I felt so much confidence in this that, in a lecture delivered before the Royal Institute of Great Britain, upon the 17th of May, 1878, I announced the possibility of hearing a shadow by interrupting the action of light upon selenium. A few days afterward my ideas upon this subject received a fresh impetus by the announcement made by Mr. Willoughby Smith before the Society of Telegraph Engineers that he had heard the action of a ray of light falling upon a bar of crystalline selenium, by listening to a telephone in circuit with it.

It is not unlikely that the publicity given to the speaking telephone during the last few years may have suggested to many minds in different parts of the world somewhat similar ideas to my own.

Although the idea of producing and reproducing sound by the action of light, as described above, was an entirely original and independent conception of my own, I recognize the fact that the knowledge necessary for its conception has been disseminated throughout the civilized world, and that the idea may, therefore, have occurred to many other minds. *The fundamental idea, on which rests the possibility of producing speech by the action of light, is the conception of what may be termed an undulatory beam of light in contradistinction to a merely intermittent one.* By an undulatory beam of light I mean a beam that shines continuously upon the selenium receiver but the intensity of which upon that receiver is subject to rapid changes, corresponding to the changes in the vibratory movement of a particle of air during the transmission of a sound of definite quality through the atmosphere.

The curve that would graphically represent the changes of light would be similar in shape to that representing the movement of the air. I do not know whether this conception had been clearly realized by "J. F. W.," of Kew, or by Mr. Sargent, of Philadelphia, but to Mr. David Brown, of London, is undoubtedly due the honor of having distinctly and independently formulated the conception, and of having devised apparatus—though of a crude nature—for carrying it into execution. It is greatly due to the genius and perseverance of my friend, Mr. Sumner Tainter, of Watertown, Mass., that the problemn of producing and reproducing sound by the agency of light has at last been successfully solved.

The first point to which we devoted our attention was the reduction of the resistance of crystalline selenium within manageable limits. The resistance of selenium cells employed by former experimenters was measured in millions of ohms, and we do not know of any record of a selenium cell measuring less than 250,000 ohms in the dark. *We have succeeded in producing sensitive selenium cells measuring only* 300 *ohms in the dark, and* 155 *ohms in the light.* All former experimenters seemed to have used platinum for the conducting part of their selenium cells, excepting Werner Siemens, who found that iron and copper might be employed. We have also discovered that brass, although chemically acted upon by selenium, forms an excellent and convenient material; indeed, we are inclined to believe that the chemical action between the brass and selenium has contributed to the low resistance of our cells by forming an intimate bond of union between the selenium and brass. We have observed that melted selenium behaves to the other substances as water to a greasy substance, and we are inclined to think that when selenium is used in connection with metals not chemically acted upon by it, the points of contact between selenium and the metal offer a considerable amount of resistance to the passage of a galvanic current. By using brass we have been enabled to construct a large number of selenium cells of different forms. The mode of applying the selenium is as follows: The cell is heated, and when hot enough a stick of selenium is rubbed over the surface. In order to acquire conductivity and sensitiveness, the selenium must next undergo a process of annealing.

We simply heat the selenium over a gas stove and observe its appearance. When the selenium attains a certain temperature, the beautifully reflecting surface becomes dimmed. A cloudiness gradually extends over it, somewhat like the film of moisture produced by breathing upon a mirror. This appearance gradually increases, and the whole surface is soon seen to be in the metallic granular or crystalline condition. The cell may then be taken off the stove, and cooled in any suitable way. When the heating process is carried too far, the crystalline selenium is seen to melt. Our best results have been obtained by heating the selenium until it crystallizes, and continuing the heating until signs of melting appear, when the gas is immediately put out. The portions that had melted instantly recrystallize, and the selenium is found upon cooling to be a conductor, and to be sensitive to light. The whole operation occupies only a few minutes. This method has not only the advantage of being expeditious, but it proves that many of the accepted theories on this subject are fallacious. Our new method shows that fusion is unnecessary; that conductivity and sensitiveness can be produced without long heating and slow cooling: and that crystallization takes place during the heating process. We have found that on removing the source of heat immediately on the appearance of the cloudiness, distinct and separate crystals can be observed under the microscope, which appear like leaden snow-flakes on a ground of ruby-red. Upon removing the heat, when crystallization is further advanced, we perceive under the microscope masses of these crystals arranged like basaltic columns standing detached from one another; and at a still higher point of heating, the distinct columns are no longer traceable, but the whole mass resembles metallic pudding stone, with here and there a separate snow-flake, like a fossil, on the surface. Selenium crystals formed during slow cooling after fusion present an entirely different appearance, showing distinct facets.

We have devised about fifty forms of apparatus for varying a beam of light in the manner required, but only a few typical varieties need be shown. The source of light may be controlled or a steady beam may be modified at any point in its path. The beam may be controlled in many ways. For instance, it may be polarized and then affected by electrical or magnetic influences in the manner discovered by Faraday and Dr. Ker. The beam of polarized light, instead of being passed through a liquid, may be reflected from the polished pole of an electro-magnet. Another method of affecting a beam of light is to pass it through a lens of variable focus. I observe that a lens of this kind has been invented in France by Dr. Cusco, and is fully described in a recent paper in *La Nature*, but Mr. Tainter and I have used such

a lens in our experiments for months past. The best and simplest form of apparatus for producing the effect remains to be described. This consists of a plane mirror of flexible material—such as silvered mica or microscope glass. Against the back of this mirror the speaker's voice is directed. The light reflected from this mirror is thus thrown into vibrations corresponding to those of the diaphragm itself.

In arranging the apparatus for the purpose of reproducing sound at a distance, any powerful source of light may be used, but we have experimented chiefly with sunlight. For this purpose a large beam is concentrated by means of a lens upon the diaphragm mirror and after reflection is again rendered parallel by means of another lens. The beam is received at a distant station upon a parabolic reflector, in the focus of which is placed a sensitive selenium cell, connected in a local circuit with a battery and telephone. A large number of trials of this apparatus have been made with the transmitting and receiving instruments so far apart that sounds could not be heard directly through the air. In illustration I shall describe one of the most recent of these experiments. Mr. Tainter operated the transmitting instrument, which was placed on the top of the Franklin school-house in Washington, and the sensitive receiver was arranged in one of the windows of my laboratory, 1325 L street, at a distance of 213 metres. Upon placing the telephone to my ear I heard distinctly from the illuminated receiver the words "Mr. Bell, if you hear what I say, come to the window and wave your hat." In laboratory experiments the transmitting and receiving instruments are necessarily within earshot of one another, and we have, therefore, been accustomed to pooling the electric circuit connected with the selenium receiver, so as to place the telephones in another room. By such experiments we have found that articulate speech can be reproduced by the oxyhydrogen light, and even by the light of a kerosene lamp. The loudest effects obtained from light are produced by rapidly interrupting the beam by the perforated disk. The great advantage of this form of apparatus for experimental work is the noiselessness of its rotation, admitting the close approach of the receiver without interfering with the audibility of the effect heard from the latter; for it will be understood that musical tones are emitted from the receiver when no sound is made at the transmitter. A silent motion thus produces a sound. In this way musical tones have been heard even from the light of a candle. When distant effects are sought, another apparatus is used. By placing an opaque screen near the rotating disk, the beam can be entirely cut off by a slight motion of the hand, and musical signals like the dots and dashes of the Morse telegraph code can thus be produced at the distant receiving station.

We have made experiments with the object of ascertaining the nature of the rays that affect selenium. For this purpose we have placed in the path of an intermittent beam various absorbing substances. Professor Cross has been kind enough to give me his assistance in conducting these experiments. When a solution of alum or bisulphide of carbon is employed, the loudness of the sound produced by the intermittent beam is very slightly diminished; but a solution of iodine in bisulphide of carbon cuts off most, but not all, of the audible effect. Even an apparently opaque sheet of hard rubber does not entirely do this. When the sheet of hard rubber was held near the disk interrupter, the rotation of the disk interrupted what was then an invisible beam, which passed over a space of about twelve feet before it reached the lens, which finally concentrated it upon the selenium cell. A faint but perfectly perceptible musical tone was heard from the telephone connected with the selenium. This could be interrupted at will by placing the hand in the path of the invisible beam. It would be premature, without further experiments, to speculate too much concerning the nature of these invisible rays, but it is difficult to believe that they can be bent rays, as the effect is produced through two sheets of hard rubber containing between them a saturated solution of alum. Although effects are produced as above shown by forms of radiant energy which are invisible, we have named the apparatus for the production and reproduction of sound in this way "The Photophone," because an ordinary beam of light contains the rays which are operative.

It is a well known fact that the molecular disturbance produced in a mass of iron by the magnetizing influence of an intermittent electrical current can be observed as sound by placing the ear in close contact with the iron. It occurred to us that the molecular disturbance produced in crystalline selenium by the action of an intermittent beam of light should be audible in a similar manner without the aid of a telephone or battery. Many experiments were made to verify this theory without definite results. The anomalous behavior of the hard rubber screen suggested the thought of listening to it also. This experiment was tried with extraordinary success. I held the sheet in close contact with my ear, while a beam of intermittent light was

focused upon it by a lens. A distinct musical note was immediately heard. We found the effect intensified by arranging the sheet of hard rubber as a diaphragm, and listening through a hearing tube. We then tried crystalline selenium in the form of a thin disk, and obtained a similar but less intense effect. The other substances which I enumerated at the beginning of my address were now successively tried in the form of thin disks, and sounds were obtained from all but carbon and thin glass. We found hard rubber to produce a louder sound than any other substance we tried, excepting antimony, and paper and mica to produce the weakest sounds. *On the whole, we feel warranted in announcing as our conclusion that sounds can be produced by the action of a variable light from substances of all kinds, when in the form of thin diaphragms.* We have heard from interrupted sunlight very perceptible musical tones through tubes of ordinary vulcanized rubber, of brass, and of wood. These were all the materials at hand in tubular form, and we have had no opportunity since of extending the observations to other substances.

I am extremely glad that I have the opportunity of making the first publication of these researches before a scientific society, for it is from scientific men that my work of the last six years has received its earliest and kindest recognition. I gratefully remember the encouragement which I received from the late Professor Henry at a time when the speaking telephone existed only in theory. Indeed, it is greatly due to the stimulus of his appreciation that the telephone became an accomplished fact. I cannot state too highly also the advantage I received in preliminary experiments on sound vibrations in this building from Professor Cross, and near here from my valued friend Dr. Clarence J. Blake. When the public were incredulous of the possibility of electrical speech, the American Academy of Arts and Sciences, the Philosophical Society of Washington, and the Essex Institute of Salem recognized the reality of the results and honored me by their congratulations. The public interest, I think, was first awakened by the judgment of the very eminent scientific men before whom the telephone was exhibited in Philadelphia, and by the address of Sir William Thomson before the British Association for the Advancement of Science.

At a later period, when even practical telegraphers considered the telephone as a mere scientific toy, Professor John Pierce, Professor Eli W. Blake, Dr. Channing, Mr. Clark and Mr. Jones, of Providence, R. I., devoted themselves to a series of experiments for the purpose of assisting me in making the telephone of practical utility; and they communicated to me, from time to time, the result of their experiments with a kindness and generosity I can never forget. It is not only pleasant to remember these things and to speak of them, but it is a duty to repeat them, as they give a practical reputation to the often repeated stories of the blindness of scientific men to unaccredited novelties, and of their jealousy of unknown inventors who dare to enter the charmed circle of science. I trust that the scientific favor which was so readily accorded to the telephone may be extended by you to this new claimant, the photophone.

Solidification under Pressure.—Walther Spring has investigated the phenomena of universal regelation, or, in other words, the property which the particles of solid bodies possess of uniting by cohesive action under pressure. The pressure in some of his experiments exceeded 20,000 atmospheres, but in the majority of cases from 2,000 to 3,000 atmospheres were sufficient to secure his results. He concludes that all solid bodies possess the property of uniting when they are in close contact. This property is more or less pronounced in different bodies, and it appears to be a function of the hardness. Soft bodies are easily soldered; hard bodies, with difficulty. There is, however, another element which influences the union, to which he gives the name of waxiness, and which is illustrated by Tresca's experiments upon the flow of solids. In every case the bodies which were submitted to pressure were changed into a denser variety; prismatic sulphur, for example, which has a specific weight of 1·96, was changed into octahedric sulphur, with a specific weight of 2·05. We may infer from this fact that the state which matter assumes depends somewhat upon the volume which it is obliged to occupy, through the action of external forces. These experiments seem calculated to throw new light upon some important geological questions; the layers of primary rocks, for example, may have resulted from the union of the grains of sand or mud which were borne along by the waters; the great pressure would have ground and thrust and broken and soldered them anew so as to give them the form which they now possess; faults and cleavages of every description would be comparable to the crevices of glaciers.—*Bull de l'Acad. Roy. de Belg.* C.

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DESCRIPTION OF THE EDISON STEAM DYNAMO.

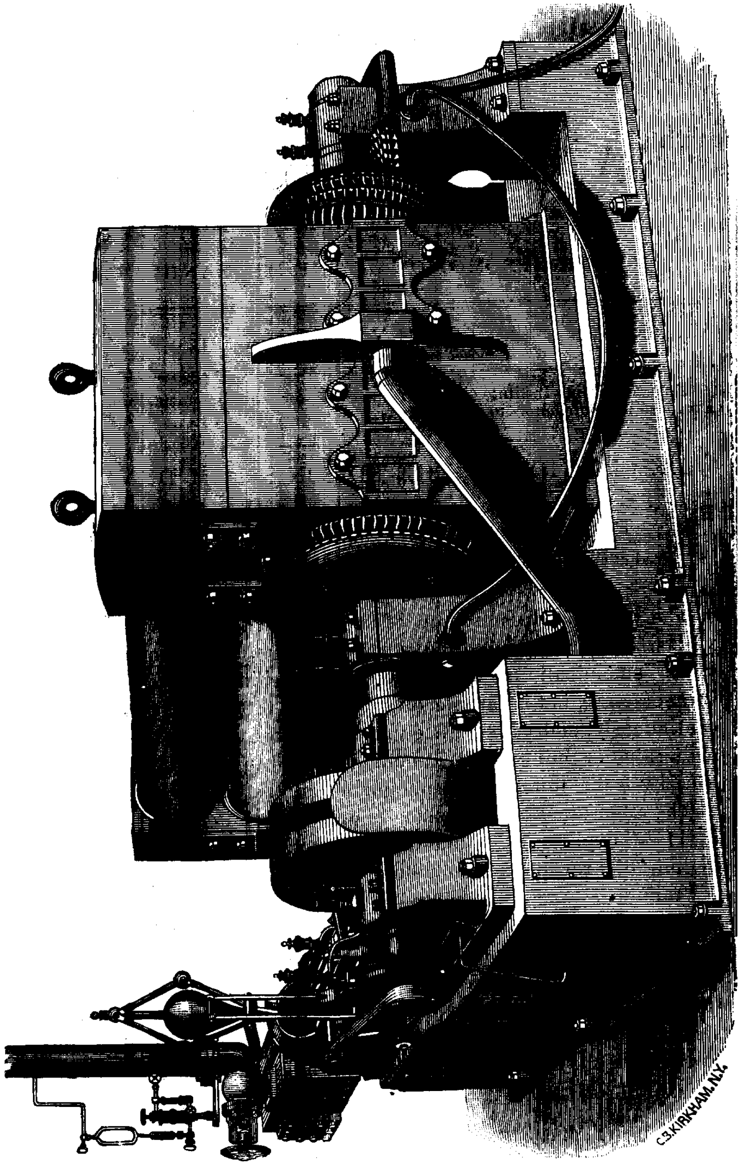
By T. A. EDISON, Ph.D., and CHARLES T. PORTER.

[Read at the Philad'a meeting of the Amer. Soc. Mech. Engs., April, 1882.]

The central Edison station of the first district in New York City will, when fully equipped, be supplied with twelve dynamos, each of which is nominally rated as a 1200 light machine, at 16 candle-power incandescence, but is capable of supplying 1400 lights of this power, continuously, and with high economy, without heating the armature, or burning or injuring the commutator or brushes. This increased capacity is due to improvements in the lamp itself.

The armature of each dynamo is driven by a Porter-Allen engine, of $11\frac{3}{16}''$ diameter of cylinder by 16″ stroke, directly connected, and making 350 revolutions per minute, giving a piston travel of 933 feet per minute.

The steam is supplied by eight Babcock & Wilcox boilers, of 2000 aggregate horse-powers, and which will work under a pressure of about 120 pounds. These occupy the basement of the building. Over them, the first and second floors being removed, an iron superstructure is erected entirely separated from the walls of the building, and on this the combined dynamos and engines are placed.



One-half of this equipment is now nearly ready for service, and the remainder is expected to be completed during the coming season.

The armature of the dynamo is of the form commonly known as the Siemens armature, but in its construction and "connecting up" it differs radically from all others.

The foundation of the armature, or the iron core which is built upon the shaft, is made up of sheet-iron disks, separated from each other by sheets of tissue paper, and bolted together. This has all the advantages of a solid iron core in strengthening the magnetic field, while it completely prevents the great loss of power by local currents, which wonld circulate in the iron if it were solid. In the place of insulated wires, the cylindrical face of the armature is made up of heavy copper bars, trapezoidal in section, each bar being insulated, and also separated frcm its neighbors and from the iron core underneath by an air space.

The connection between the bars on opposite sides of the armature, to form the electrical circuit, is made by copper disks, of the same diameter as the core. At each end of the core are one-half as many of these copper disks as there are bars, each disk being insulated from its neighbors, and the whole being bolted together in such a manner as to form, with the disks of sheet-iron constituting the core, one solid mass. Each disk is formed with projecting lugs on its opposite sides, to which the two bars are connected.

The connections between the opposite surfaces of an armature are of no benefit in generating an electric current, but are a necessary evil, introducing useless resistance into the circuit. By using for this connection copper disks in the manner described, a great weight of copper is disposed in a limited space, and so this useless resistance, and consequent loss of energy, is reduced to a minimum.

This method, moreover, reduces the work to a simple machine construction, in which all the parts are duplicates, and the operations can be much cheapened and facilitated by the use of special tools.

The spaces between the armature bars admit of a free circulation of air, thereby preventing the accumulation of heat, and increasing to an enormous degree the capacity of the machine. The armature is at intervals wound with piano wire over the bars to resist the centrifugal force developed by their revolution.

The commutator and brushes of an electrical machine are the parts subject to the greatest depreciation. In this machine all parts of the end of the armature are so constructed as to be easy of access, and they can be quickly and cheaply repaired, or removed and replaced by new parts, when necessary. Any accident would require but a short stoppage for repairs.

Provision is made for keeping a continuous and rapid circulation of air over the entire face of the armature.

This armature is 27·8″ in diameter by 61″ long. The commutator adds 18″ to this length, and is itself 12$\frac{3}{4}$″ in diameter. The armature shaft is of steel, 7$\frac{3}{4}$″ in diameter, having a total length of 10′3″. The journals are 6$\frac{1}{2}$″ in diameter by 15″ long, and run in Babbit metal bearings in pillow blocks of the box form, giving the greatest stiffness with minimum of weight.

Provision is made for continuous water circulation underneath the boxes, and for continuous lubrication, with traps to prevent the creeping of the oil along the shaft and reaching the commutator, and drains to receive it as it runs through the bearings and convey it to a drip pan.

The magnet is made up of two immense cast iron "pole pieces," between the semi-cylindrical faces of which the armature revolves, twelve cylindrical soft iron cores attached to these pole pieces, and made magnetic by an electrical current circulated in the wire wound around them, and four soft iron keepers connecting the back ends of the cores. Eight of the cores are attached to the upper pole piece, and four to the lower one.

The width of these "poles" is 49″, and their height 61$\frac{1}{2}$″. The length of the twelve soft iron cores is 57″, the diameter of the eight upper ones is 8″, and of the four lower ones 9″.

The four soft iron keepers are each 11″ wide by 9″ in thickness, and the total length of the magnet is 94″.

The magnet is insulated by cast zinc bases 3″ in thickness.

The weight of the dynamo is as follows:

Armature and shaft, . .	9,800 lbs.
Two pillow blocks, . .	1,340 "
Magnet, complete, . .	33,000 "
Zinc bases, . . .	680 "
Total, . . .	44,820 lbs.

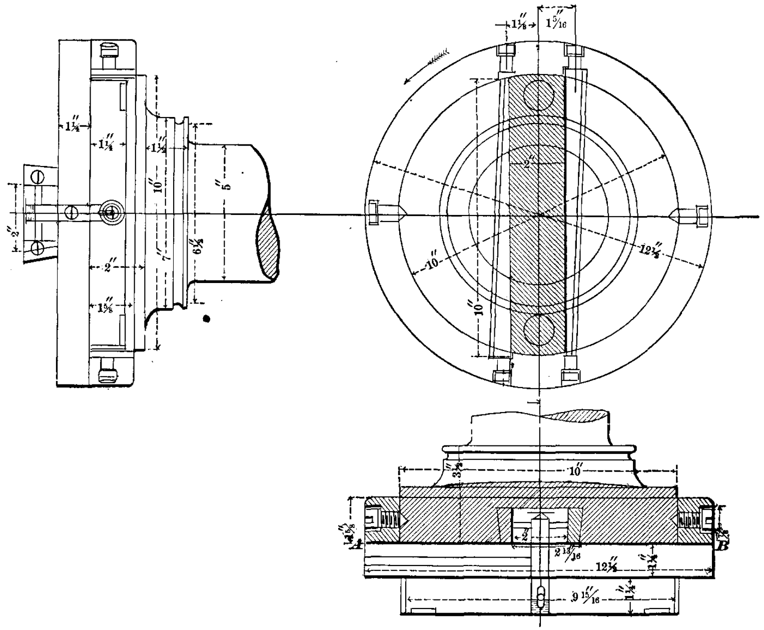
Description of the Edison steam dynamo T. A. Edison and C. T. Porter

The copper is distributed as follows:

In the armature bars, . .	590 lbs.
" " disks, . .	1,350 "
" magnet wire, . .	1,500 "
Total, . . .	3,440 lbs.

Fig. 1. SELF-ADJUSTING COUPLING.

Side Elevation. Front Elevation. Tongue in section on the line A B



Plan. Flange on shaft, ring and wedges in section.

Mr. Edison was early impressed with the conviction that to give steady and reliable motion to these armatures it would be necessary to connect an engine to each one of them directly. This combination has been termed by him the STEAM DYNAMO.

In adapting the Porter-Allen engine to this service, a special construction in some respects was found to be called for. These special features will be briefly described.

It seemed important to avoid a rigid connection between the engine and the armature shafts, which would require the entire series of bearings to be maintained absolutely in line. In place of this, therefore, a self-adjusting coupling has been introduced, of the form shown in Fig. 1, and illustrated by a working model, which will permit of considerable errors of alignment without any abnormal friction being produced in the bearings.

The point of difficulty was the backlash, the engine having no fly-wheel, except the heavy armatnre itself, which was to be driven through the coupling. Provision was made for taking this up by steel keys of a somewhat peculiar form, between which the tongues of the coupling move freely, while they themselves are immovable. These keys are held between set-screws threaded in wrought iron rings covering the flanges on the ends of the shaft. All the faces liable to move upon each other are oiled from a central reservoir. This coupling is a very compact affair, without a projection anywhere above its surface, and gives every promise of completely answering its purpose.

The engine is made with a forked bed and two shaft bearings and a double crank, and so is completely self-contained. It is shown in plan and elevation in Plate 1.

The shaft having no support beyond these bearings on either side, unusual stiffness was required in the crank-pin to prevent deflection under the great strains to which it is subjected.

A novel form of pin (see Fig 2) was proposed by Mr. Richards, which is found to possess all the rigidity required. It is provided with flanges which are let into each crank, and held each by four screws, as shown, while the shanks of the pin are also forced firmly into the cranks.

Special appliances enabled the work of putting the cranks together in this manner to be done with extreme and uniform accuracy.

The engine is so arranged as to have the valve gear on the side furthest from the dynamo. The engineer has not to go between the engine and dynamo, when running, for any purpose.

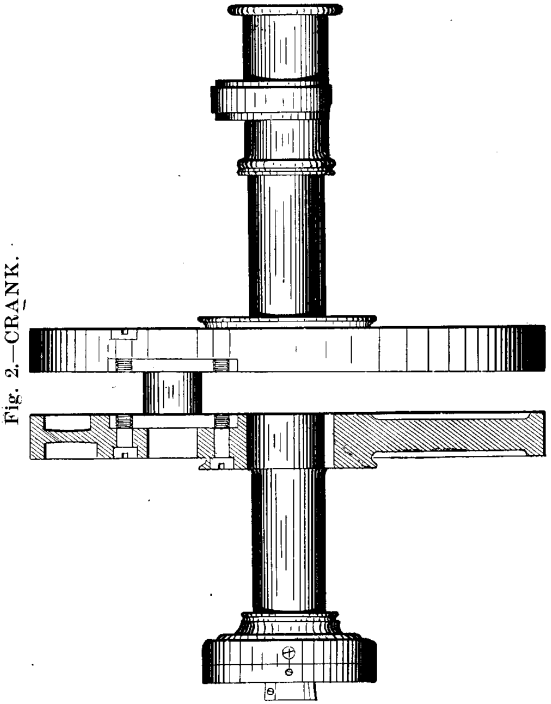
The connecting-rod (Fig. 3), is of steel, and the crank-pin boxes are formed directly in the end of it.

This end is finished from a solid forging, and chambered out for Babbit metal. The bolts are then fitted, after which it is parted and holes are drilled for holding the Babbit securely.

In the connecting-rods for single crank engines of this type per-

manent length of rod is secured by forming the crank pin end solid, and taking up the wear by a wedge closing up the inside box. In these double crank engines this construction is impracticable, but the same object is attained by forming the crosshead end in the manner shown, in which the strap is made permanent and the inside box is closed up by a key bearing against a steel plate.

Fig. 2.—CRANK.



The weight of the reciprocating parts of this engine is as follows:

Piston, with rod,	83 lbs.
Crosshead,	42 "
Connecting-rod,	109 "
Total,	234 lbs.

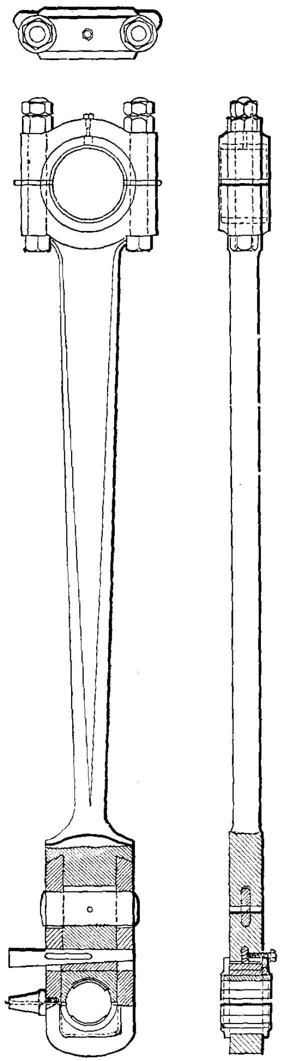


Fig. 3—CONNECTING ROD.

The initial acceleration of this mass, or the force required, on the dead centres, to give it the motion necessary to relieve the crank from strain, is as follows:

$350^2 \times \cdot 66 \times \cdot 000341 = 27 \cdot 57$,

or 27·57 times the weight of the mass, which gives

$234 \times 27 \cdot 57 = 6451$ lbs.

The formula is $R^2\, l\, c$, when

$R =$ the revolutions per minute;

$l =$ the length of the crank in decimals of a foot; and

$c =$ the coefficient of centrifugal force.

The connecting-rod is 48″, or 6 cranks, in length. This affects the initial acceleration, making this to be on the dead centre farthest from the crank 7526 lbs., and on the dead centre nearest to the crank 5376 lbs., a difference of 40 per cent.

The area of the cylinder is 98·2 square inches.

The area of the piston rod, 1¾ inches diameter, is 2·4 square inches, leaving area of cylinder at crank end 95·8 square inches.

The initial accelerating forces are therefore as follows, viz.: at the end of the cylinder farthest from the crank 77 lbs., and at the end of the cylinder nearest to the crank 56 lbs., on the square inch of piston area.

The counterweight was after some trials fixed at 135 lbs. This leaves 99 lbs. of the reciprocating parts running unbalanced. It is found that this is not sufficient to disturb the stability of the engine, while on the other hand the counterweight

is not so great as to exert an objectionable strain in the vertical direction.

The total weight of the engine is 6445 lbs.

The engine and dynamo are mounted on a cast-iron base plate, made for convenience in two parts, and bolted together.

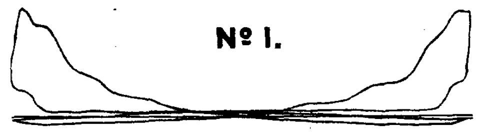
The dimensions of this base plate are as follows: length 14 feet, width 8 feet 9 inches; and its weight is 10,300 lbs. The entire weight is therefore as follows:

Base plate, . . .	10,300 lbs.
Dynamo, ...	44,800 "
Engine, . . .	6,450 "
Total, . . .	61,550 lbs.

Fig. 4 is a perspective view of the Dynamo and Engine combined.

The last and most careful test of one of these dynamos gives the following results, as shown by the indicator diagrams, which are here reproduced full size; scale 80 lbs. to the inch.*

The lamps used in all the trials were of the older construction, of which 8½ lamps, at 16 candle power incandescence, require one horse-power of electrical energy.



Since these were placed for experimental uses, improvements in the lamp have increased their economy, so that one horse-power is sufficient to maintain fully 10 of the present lamps at 16 candle power incandescence.

Diagram No. 1 shows the friction of engine and dynamo at 350 revolutions per minute, requiring . . 13·63 HP

Diagram No. 2 shows the resistance with the magnet circuit on = ... 19·17 HP

Field 5·78 ohms, 103 volts.

*As many persons might doubt about these diagrams having been really taken from any engine and by any indicator at this speed, we have examined the originals taken by a Tabor indicator, and can vouch for their accuracy.—Ed. J. F. I.

The increased resistance due to the magnets was . 5·54 H.P.
Of this, the calculated energy developed in the magnets

was $\dfrac{103^2 \times 44{\cdot}3}{5{\cdot}78 \times 33{,}000} =$ . . . 2·46 H.P.

Leaving energy to be accounted for by local currents in iron core of armature, and in armature bars, . 3·08 H.P.

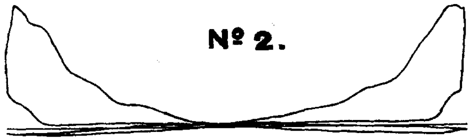
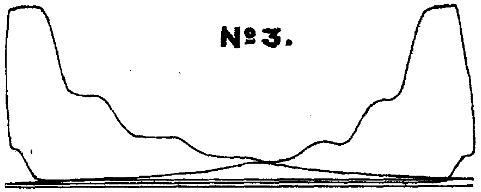


Diagram No. 3 shows the work done in maintaining 300 lamps.



These, in the ratio of $8\frac{1}{2}$ to 10, were equal to 353 lamps of the present construction. The pressure was maintained also at 102 volts, representing 25 candle power, in place of 98 volts, representing 16 candle power incandescence, which requires the number of lamps to be increased in the ratio of 102^2 to 98^2, or to 382 lamps.

The pressure at the armature was 104 volts, showing a loss in the conductor of 2 volts, which would increase the number of lamps as 104 : 102.*

The total correction is therefore as follows:

$$300 \times \frac{10}{8{\cdot}5} \times \frac{102^2}{98^2} \times \frac{104}{102} = 389 \text{ lamps.}$$

The power exerted was . . . 60·6 H.P.
which gives to the indicated horse-power

$$389 \div 60{\cdot}6 = 6{\cdot}42 \text{ lamps.}$$

*The conductors were insufficient, occasioning a loss, that increased with the increase in the number of lamps.

Description of the Edison steam dynamo T. A. Edison and C. T. Porter

The magnet circuit had now a resistance of 5·28 ohms with 104 volts pressure, representing

$$\frac{104^2 \times 44{\cdot}3}{5{\cdot}28 \times 33{,}000} = \quad 2{\cdot}75 \text{ H.P.}$$

Substituting this in place of 2·46 H.P. in the first trial, we have 19·46 H.P., which, deducted from 60·6 H.P., leaves net 41·14 H.P.

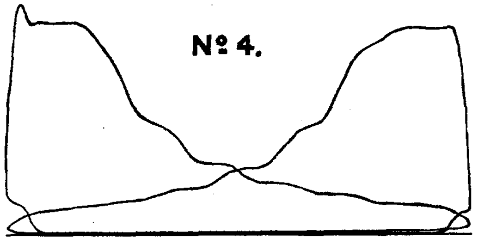
This gives 389 ÷ 41·14 = 9·45 lamps per H.P.

Diagram No. 4 shows the work done in maintaining 700 lamps.

The pressure at the lamps was maintained, as in the preceding trial, at 102 volts, which required at the armature a pressure of 105 volts.

The total correction in this case is therefore

$$700 \times \frac{10}{8{\cdot}5} \times \frac{102^2}{98^2} \times \frac{105}{102} = 919 \text{ lamps.}$$



The power exerted was . . . 115·83 H.P.
giving to the indicated horse-power,

$$919 \div 115{\cdot}83 = 7{\cdot}93 \text{ lamps.}$$

The resistance of the magnet circuit was now 4·78 ohms, with 105 volts pressure, representing, $\dfrac{105^2 \times 44{\cdot}3}{4{\cdot}78 \times 33{,}000} = 3{\cdot}1$ H.P.

Substituting this in place of 2·46 H.P. in the first trial, we have 19·81, which, deducted from 115·83 H.P., leaves net 96·02 H.P.

This gives 919 ÷ 96·02 = 9·57 lamps per H.P.

Diagram No. 5 shows the work done in maintaining 1050 lamps.

The pressure at the lamps was maintained in this trial at only 99 volts, but this required at the armature a pressure of 108 volts, showing a loss of 9 volts in conduction.

The total correction in this case is thus

$$1050 \times \frac{10}{8{\cdot}5} \times \frac{99^2}{98^2} \times \frac{108}{99} = 1375 \text{ lamps.}$$

The power was . . . . 168·4 HP
giving to the indicated horse-power

$$1375 \div 168{\cdot}4 = 8{\cdot}16 \text{ lamps.}$$

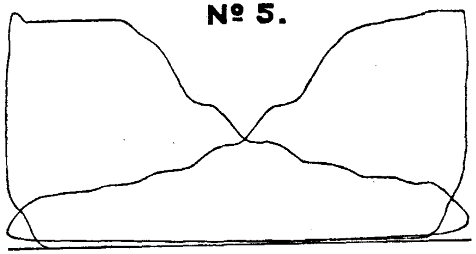
The resistance of the magnet circuit was now 3·28 ohms, with 108 volts pressure, representing

$$\frac{108^2 \times 44{\cdot}3}{3{\cdot}28 \times 33{,}000} = 4{\cdot}77 \text{ HP.}$$

Substituting this in place of 2·45 HP in the first trial, we have 21·48 HP, which, deducted from 168·4 HP, leaves net 146·92 HP.

This gives 1375 ÷ 146·92 = 9·36 lamps per HP.

Nº 5.



It will be seen that the losses of efficiency due to undiscovered resistances are only

In the first case, .	10 — 9·45 = ·55 HP per lamp,
In the second case, . .	10 — 9·57 = ·43 HP per lamp, and
In the third place, .	10 — 9·36 = ·64 HP per lamp,

Averaging 5·4 per cent.

The friction in the journals of the armature, when driven in this manner, does not increase with the resistance, and, on account of the action of the reciprocating parts of the engine, that in its bearings is also nearly a constant quantity, whatever the load may be.

The above figures show this very clearly, the subtraction of the friction diagram in each case exhibiting substantially the same net power per lamp.

THE PHYSICAL REALITY OF LIGHT-QUANTA.*

BY

MAX PLANCK, Ph.D.

University of Berlin;
Franklin Medallist; Member of the Institute.

To MY keen regret I am prevented by pressing duties from receiving in person the high honor which The Franklin Institute has conferred upon me. Accordingly I ask permission to lay before you as a token of my profound gratitude a short communication upon a scientific question which at the present time occupies the foreground of interest.

You all know that theoretical physics, which developed and progressed consistently through two hundred years, and only a generation ago seemed near to its final conclusion, has now entered a critical period, fraught with serious consequences. Not that its fundamental principles have been questioned! For its most general and at the same time its simplest laws, such as the Principle of the Conservation of Energy, the Laws of Thermodynamics, and the Fundamental Equations of the Electromagnetic Field, are just the ones which have so far withstood successfully the most severe trials, and serve now as ever before as the guides for wider exploration. It would then be entirely incorrect to speak of a breakdown of the science. It is our mental pictures of the occurrences, in which these principles find their application, that in recent times have been thrown into confusion. Many concepts which could be counted among the simplest and the most evident in the world, have turned out to be obscure, doubtful, even contradictory, and it is certain that in many respects we must straightaway build up from the very beginning, if we are not to lose sight of the most important assumption of physical research, the compatibility of the various laws with one another.

For example, no occurrence in nature appears more simple and more obvious, alike to the layman and to the investigator, than the motion of a material body, a sphere for instance, and to this simplicity of ideas corresponds completely the simplicity of the

* Presented at the Stated Meeting of the Institute held Wednesday, May 18, 1927. Translated from the German by Dr. T. D. Cope, Department of Physics, University of Pennsylvania. The translator wishes to thank Profs. H. C. Richards and H. C. Barker for valuable criticisms and suggestions.

laws which so far have been recognized as governing the motion. It is no wonder that man has sought for ages, even from the time of the Greek philosophers, to reduce all physical occurrences to the motions of material bodies, and that he was later on strongly confirmed in this endeavor by the success of the brilliant discoveries of Galileo, Kepler, and Newton.

To-day we know with certainty that the laws of mechanics possess only an approximate validity. I am not referring now to the emendation brought about by the Theory of Relativity. For, although this theory has affected our views fundamentally, nevertheless in the last analysis it means not a complication, but on the contrary a simplification and a refinement of the classical mechanics. What we are now considering has to do with something entirely different, something far more revolutionary. That is to say, experience has obliged us to recognize the inevitable conclusion, that not only are the laws of mechanics not fundamental, but that even the basal concept of mechanics, the material particle, under the circumstances of sharply curved motion, immediately loses its meaning. When a particle, an electron, for example, moves in this way, it means nothing to speak of a definite position which the electron occupies at a definite time. The more sharply the path curves, the more blurred becomes the position of the electron. It becomes indefinite and, so to speak, spreads out into surrounding space, much as a beam of light, when striking the edge of a screen, instead of continuing forward as a unit, bends and scatters in all directions. If the path of the electron is a periodic or a quasi-periodic one, and if it extends over a very small amount of space, as in Bohr's model of the atom, then at every instant the electron spreads over its entire path, and its motion resembles rather the vibrations of a standing wave in a continuous medium than those of an oscillating particle.

Thus corpuscular mechanics resolves itself into an undulatory mechanics, the principles of which in all their details have not by any means been fully investigated. Still, thanks to the ideas introduced into science by L. de Broglie and E. Schroedinger, these principles have already established a solid foundation. We may place confidence in them all the more because their consequences agree completely with the mathematical postulates which had been introduced earlier into quantum mechanics by W. Heisenberg, solely upon the basis of facts of experience.

If thus at least a prospect is offered of gradually attaining a deeper insight into the true nature of mechanical energy, still the way toward an understanding of the ~~nature~~ of the energy of electromagnetic radiation seems at the present time to be completely closed. Here we are experiencing in a certain sense a development just the opposite of that in corpuscular motions. While the corpuscular quanta, as we have seen, in sharply curved motions spread out in space and dissolve into wave-forms, it appears on the contrary that radiant energy travelling in absolute vacuum with the speed of light, at high frequencies shrinks together and concentrates itself at separate points which move like corpuscles, and for that reason are called light-quanta.

At the first glance this latter state of affairs must appear far more incomprehensible than the former. For in corpuscular motions we have to do with matter, or with electric charges, and these surely still conceal within themselves so much of the mysterious that therewith may be associated many a riddle, whose solution will only be found when we shall have succeeded in rending the veil of the mystery. But the laws of propagation of radiant energy in absolute vacuum, we had the right to regard as known in all their details since the brilliant success of Maxwell's theory. Absolute vacuum surely conceals within itself no mystery, no matter, no electric charge. It serves only as the carrier of the electromagnetic field. And the laws of this field are represented with a completeness and an exactness, which hold their own against the finest interference measurements, by equations to which a concentration of energy into quanta is entirely foreign. For the elementary quantum of action plays no part in Maxwell's equations. From dimensional considerations it would be entirely impossible to introduce this quantity into Maxwell's equations unless additional constants should appear therein.

Just at the point, then, where conditions seem to be the simplest, and where, according to all previous experience, we had the right to feel ourselves nearest to a final comprehension of Nature, we are baffled by an entirely unexpected mystery. Again and again the question arises, must we really ascribe to the light-quanta a physical reality, or is there after all a way of taking account of them, which preserves the validity of Maxwell's classical electrodynamics?

Many have been the endeavors to answer this question, and still the argument surges to and fro. Let us now give the question a brief consideration.

To begin with, it is clear that a decision can only be reached by the closest attention to the facts of experience. Since in the last analysis we can never measure the electromagnetic field itself, but only its effects upon matter, that is upon measuring instruments, it might at the first glance appear promising to limit the significance of light-quanta solely to the interaction between radiation and matter, that is to the processes of emission and absorption, and on the other hand to deny their existence in the propagation in absolute vacuum. Then all the classical laws of the radiation of energy in vacuum could be retained.

But a closer consideration will show us that this way out leads nowhere if we are to hold fast to the ever trustworthy fundamental principles of physics. First of all, it cannot be doubted that a physical reality must be ascribed to radiant energy in vacuum, as such. This follows from the First Law of Thermodynamics, the Energy Principle, in its application to the emission and absorption of radiant energy. But not only does a beam of radiation possess a definite energy according to the First Law, it has also a definite entropy according to the Second Law, the Principle of the Increase of Entropy. For if the entropy were not present it could not increase. We are thus obliged to ascribe to entropy, just as to energy, an independent existence, without reference to any matter whatever. This conclusion is in no wise affected by the fact that, in order to determine the quantity of entropy, we must measure the temperature of a material body which stands in equilibrium with the radiant energy.

If we now retain also the relationship between entropy and probability introduced by L. Boltzmann, and without it an understanding of the content of the Second Law seems impossible, there follow laws of the fluctuation in space and time of the energy in a beam of radiation of definite temperature.

If we now compare the law of energy-fluctuation deduced from the measured entropy of radiation with the law of fluctuation called for by the classical theory, we find that in addition to the fluctuations yielded by the latter law, there appears a new and entirely different kind of fluctuation, the statistics of which can only be explained by the presence of discrete atoms of energy of

the magnitude of light-quanta. For the fluctuations are much too great at low temperatures to be accounted for by the classical theory.

In this entire study the interaction between radiation and matter plays no part whatever. We then cannot avoid ascribing to light-quanta in absolute vacuum a real physical existence. This was pointed out as early as 1909 by A. Einstein. On the other hand, however, we may not look upon light-quanta as independent of one another. For then we would obtain only the second kind of energy-fluctuation, and not those fluctuations which predominate at high temperatures, and are yielded by the classical theory. These, too, are called for by the law of radiation based upon measurements.

Here opens then that break which in my opinion penetrates to the depths of the structure of the quantum theory. None of the more recent advances has filled it up. Beyond doubt the statistical interdependence of light-quanta is related to the interference phenomena of light rays from the same source. One might perhaps think of it in this way, that each light-quantum carries about with it, as it were, a mark of its origin, and that two light-quanta of like origin can interfere with each other when they meet. But the difficulty would not be avoided by thinking in this way. For at feeble intensities the chances of an encounter are much too small to account for those interference phenomena which actually occur. Rather do Maxwell's equations of the field, which do not involve light-quanta at all, appear to represent the interference phenomena completely and exactly, even down to the feeblest intensities of light.

For these reasons we shall find it impossible to think of the energy of light-quanta as concentrated at separate points in space. Rather there goes out from each light-quantum a kind of action-at-a-distance, and indeed not only at a distant place but also at a distant time, for according to the Theory of Relativity we cannot distinguish in this connection between space and time. In fact, the form of many very general principles of general mechanics and atomic physics brings out the thought that the course of an event is dependent as much upon the final state as upon the initial one. In this way is introduced a certain direct interaction between the two states separated from one another in time. The Principle of Causality would be influenced thereby only in its form, not in its substance. For all that, such trains of thought mean a difficult undertaking for our present-day powers of conception. To carry them out would bring about far-reaching changes in all our views of the physical world.

What is there to be gained which makes such a great sacrifice seem worth while? It would be rash to be willing to express a judgment at the present time. Still I would like to make at least an attempt to point out the direction in which the end sought for may lie. Probably we might catch a glimpse of it in the complete consolidation of the two great regions of physics which still are separated by an impassable chasm, corpuscular physics and physics of the continuum, or wave-physics. If the goal shall some day be attained, these two regions will no longer appear fundamentally different from one another but will represent only the opposite ends of a single region which includes them both.

The classical theory recognizes and treats only the two extreme cases; on the one side, corpuscular motions, on whose outermost border lies the uniform motion of a particle in a straight line; on the other side, wave-motions, on whose outer limit lies the static, homogeneous field. Looked upon from the newly established point of view, there is neither pure corpuscular motion, nor any pure wave-motion. Rather, every corpuscular motion includes something of wave-motion, and every wave-motion something of corpuscular motion. The difference is only gradual and quantitative. In the motion of a particle, as soon as the ratio of the impulse to the curvature of the path, which in motion in a straight line has an infinite value, sinks to the order of magnitude of the universal constant of action, the laws of wave-motion begin to play an appreciable part. And *vice versa*, in monochromatic light, as soon as the ratio of its energy to its frequency, which is infinite in a static field, sinks to the order of magnitude just mentioned, the corpuscular laws begin to be appreciable. In what relation, however, the corpuscular laws stand to the laws of wave-motion in the general case, remains the great problem, to which at the present time a whole generation of investigators is devoting its best efforts. We can entertain no doubt that finally a satisfactory solution will be found, and that then theoretical physics will have made another significant advance toward the attainment of its ultimate goal, the building up of a unified system embracing all physical phenomena.

ON GRAVITATIONAL WAVES.

BY

A. EINSTEIN and N. ROSEN.

ABSTRACT.

The rigorous solution for cylindrical gravitational waves is given. For the convenience of the reader the theory of gravitational waves and their production, already known in principle, is given in the first part of this paper. After encountering relationships which cast doubt on the existence of *rigorous* solutions for undulatory gravitational fields, we investigate rigorously the case of cylindrical gravitational waves. It turns out that rigorous solutions exist and that the problem reduces to the usual cylindrical waves in euclidean space.

I. APPROXIMATE SOLUTION OF THE PROBLEM OF PLANE WAVES AND THE PRODUCTION OF GRAVITATIONAL WAVES.

It is well known that the approximate method of integration of the gravitational equations of the general relativity theory leads to the existence of gravitational waves. The method used is as follows: We start with the equations

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = - T_{\mu\nu}. \qquad (1)$$

We consider that the $g_{\mu\nu}$ are replaced by the expressions

$$g_{\mu\nu} = \delta_{\mu\nu} + \gamma_{\mu\nu}, \qquad (2)$$

where

$$\begin{aligned}\delta_{\mu\nu} &= 1 \quad \text{if} \quad \mu = \nu,\\ &= 0 \quad \text{if} \quad \mu \neq \nu,\end{aligned}$$

provided we take the time coördinate imaginary, as was done by Minkowski. It is assumed that the $\gamma_{\mu\nu}$ are small, i.e. that the gravitational field is weak. In the equations the $\gamma_{\mu\nu}$ and their derivatives will occur in various powers. If the $\gamma_{\mu\nu}$ are everywhere sufficiently small compared to unity one obtains a first-approximation solution of the equations by neglecting in (1) the higher powers of the $\gamma_{\mu\nu}$ (and their derivatives) compared with the lower ones. If one introduces further the $\bar{\gamma}_{\mu\nu}$ instead of the $\gamma_{\mu\nu}$ by the relations

$$\bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \tfrac{1}{2}\delta_{\mu\nu}\gamma_{\alpha\alpha},$$

then (1) assumes the form

$$\bar{\gamma}_{\mu\nu,\,\alpha\alpha} - \bar{\gamma}_{\mu\nu,\,\alpha\nu} - \bar{\gamma}_{\nu\alpha,\,\alpha\mu} + \bar{\gamma}_{\alpha\alpha,\,\mu\nu} = -2T_{\mu\nu}. \tag{3}$$

The specialization contained in (2) is conserved if one performs an infinitesimal transformation on the coördinates:

$$x_\mu{}' = x_\mu + \xi^\mu, \tag{4}$$

where the ξ^μ are infinitely small but otherwise arbitrary functions. One can therefore prescribe four of the $\bar{\gamma}_{\mu\nu}$ or four conditions which the $\bar{\gamma}_{\mu\nu}$ must satisfy besides the equations (3); this amounts to a specialization of the coördinate system chosen to describe the field. We choose the coordinate system in the usual way by demanding that

$$\bar{\gamma}_{\mu\alpha,\,\alpha} = 0. \tag{5}$$

It is readily verified that these four conditions are compatible with the approximate gravitational equations provided the divergence $T_{\mu\alpha,\,\alpha}$ of $T_{\mu\nu}$ vanishes, which must be assumed according to the special theory of relativity.

It turns out however that these conditions do not completely fix the coördinate system. If $\gamma_{\mu\nu}$ are solutions of (2) and (5), then the $\gamma_{\mu\nu}{}'$ after a transformation of the type (4)

$$\gamma_{\mu\nu}{}' = \gamma_{\mu\nu} + \xi^\mu{}_{,\nu} + \xi^\nu{}_{,\mu} \tag{6}$$

are also solutions, provided the ξ^μ satisfy the conditions

$$[\xi^\mu{}_{,\nu} + \xi^\nu{}_{,\mu} - \tfrac{1}{2}\delta_{\mu\nu}(\xi^\alpha{}_{.\alpha} + \xi^\alpha{}_{,\alpha})]_{,\nu} = 0,$$

or

$$\xi^\mu{}_{,\alpha\alpha} = 0. \tag{7}$$

If a γ-field can be made to vanish by the addition of terms like those in (6), i.e., by means of an infinitesimal transformation, then the gravitational field being described is only an apparent field.

With reference to (2), the gravitational equations for empty space can be written in the form

$$\left.\begin{aligned} \bar{\gamma}_{\mu\nu,\,\alpha\alpha} &= 0. \\ \gamma_{\mu\alpha,\,\alpha} &= 0. \end{aligned}\right\} \tag{8}$$

One obtains plane gravitational waves which move in the

direction of the positive x_1-axis by taking the $\bar{\gamma}_{\mu\nu}$ of the form $\varphi(x_1 + ix_4)(= \varphi(x_1 - t))$, where these $\bar{\gamma}_{\mu\nu}$ must further satisfy the conditions

$$\left.\begin{aligned} \bar{\gamma}_{11} + i\bar{\gamma}_{14} &= 0, \\ \bar{\gamma}_{41} + i\bar{\gamma}_{44} &= 0, \\ \bar{\gamma}_{21} + i\bar{\gamma}_{24} &= 0, \\ \bar{\gamma}_{31} + i\bar{\gamma}_{34} &= 0. \end{aligned}\right\} \tag{9}$$

One can accordingly subdivide the most general (progressing) plane gravitational waves into three types:

(*a*) pure longitudinal waves,

only $\bar{\gamma}_{11}$, $\bar{\gamma}_{14}$, $\bar{\gamma}_{44}$ different from zero,

(*b*) half longitudinal, half transverse waves,

only $\bar{\gamma}_{21}$ and $\bar{\gamma}_{24}$, or only $\bar{\gamma}_{31}$ and $\bar{\gamma}_{34}$ different from zero,

(*c*) pure transverse waves,

only $\bar{\gamma}_{22}$, $\bar{\gamma}_{23}$, $\bar{\gamma}_{33}$ are different from zero.

On the basis of the previous remarks it can next be shown that every wave of type (*a*) or of type (*b*) is an apparent field, that is, it can be obtained by an infinitesimal transformation from the euclidean field ($\bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} = 0$).

We carry out the proof in the example of a wave of type (*a*). According to (9) one must set, if φ is a suitable function of the argument $x_1 + ix_4$,

$$\bar{\gamma}_{11} = \varphi, \qquad \bar{\gamma}_{14} = i\varphi, \qquad \bar{\gamma}_{44} = -\varphi,$$

hence also

$$\gamma_{11} = \varphi, \qquad \gamma_{14} = i\varphi, \qquad \gamma_{44} = -\varphi.$$

If one now chooses ξ' and ξ^4 (with $\xi^2 = \xi^3 = 0$) so that

$$\xi^1 = \chi(x_1 + ix_4), \qquad \xi^4 = i\chi(x_1 + ix_4),$$

then one has

$$\xi^1{}_{,1} + \xi'{}_{,1} = 2\chi', \qquad \xi^1{}_{,4} + \xi^4{}_{,1} = 2i\chi', \qquad \xi^4{}_{,4} + \xi^4{}_{,4} = -2\chi'.$$

These agree with the values given above for γ_{11}, γ_{14}, γ_{44} if one chooses $\chi' = \frac{1}{2}\varphi$. Hence it is shown that these waves are apparent. An analogous proof can be carried out for the waves of type (*b*).

Furthermore we wish to show that also type (*c*) contains apparent fields, namely, those in which $\bar{\gamma}_{22} = \bar{\gamma}_{33} \neq 0$, $\bar{\gamma}_{23} = 0$. The corresponding $\gamma_{\mu\nu}$ are $\gamma_{11} = \gamma_{44} \neq 0$, all others vanishing. Such a wave can be obtained by taking $\xi' = \chi$, $\xi^4 = -i\chi$, i.e. by an infinitesimal transformation from the euclidean space. Accordingly there remain as real waves only the two pure transverse types, the non-vanishing components of which are

$$\gamma_{22} = -\gamma_{33}, \qquad (c_1)$$

or

$$\gamma_{23}. \qquad (c_2)$$

It follows however from the transformation law for tensors that these two types can be transformed into each other by a spatial rotation of the coördinate system about the x_1-axis through the angle $\pi/4$. They represent merely the decomposition into components of the pure transverse wave (the only one which has a real significance). Type c_1 is characterized by the fact that its components do not change under the transformations

$$x_2' = -x_2, \quad x_1' = x_1, \quad x_3' = x_3, \quad x_4' = x_4,$$

or

$$x_3' = -x_3, \quad x_1' = x_1, \quad x_2' = x_2, \quad x_4' = x_4,$$

in contrast to c_2, i.e. c_1 is symmetrical with respect to the x_1–x_2-plane and the x_1–x_3-plane.

We now investigate the generation of waves, as it follows from the approximate (linearized) gravitational equations. The system of the equations to be integrated is

$$\left.\begin{aligned} \bar{\gamma}_{\mu\nu,\,\alpha\alpha} &= -2T_{\mu\nu}, \\ \bar{\gamma}_{\mu\alpha,\,\alpha} &= 0. \end{aligned}\right\} \qquad (10)$$

Let us suppose that a physical system described by $T_{\mu\nu}$ is found in the neighborhood of the origin of coördinates. The γ-field is then determined mathematically in a similar way to that in which an electromagnetic field is determined through an electrical current system. The usual solution is the one

given by retarded potentials

$$\bar{\gamma}_{\mu\nu} = \frac{1}{2\pi}\int \frac{[T_{\mu\nu}]_{(t-r)}}{r}\,dv. \qquad (11)$$

Here r signifies the spatial distance of the point in question from a volume-element, $t = x_4/i$, the time in question.

If one considers the material system as being in a volume having dimensions small compared to r_0, the distance of our point from the origin, and also small compared to the wave-lengths of the radiation produced, then r can be replaced by r_0, and one obtains

$$\bar{\gamma}_{\mu\nu} = \frac{1}{2\pi r_0}\int [T_{\mu\nu}]_{(t-r_0)}\,dv,$$

or

$$\bar{\gamma}_{\mu\nu} = \frac{1}{2\pi r_0}\left[\int T_{\mu\nu}\,dv\right]_{(t-r_0)}. \qquad (12)$$

The $\bar{\gamma}_{\mu\nu}$ are more and more closely approximated by a plane wave the greater one takes r_0. If one chooses the point in question in the neighborhood of the x_1-axis, the wave normal is parallel to the x_1 direction and only the components $\bar{\gamma}_{22}$, $\bar{\gamma}_{23}$, $\bar{\gamma}_{33}$ correspond to an actual gravitational wave according to the preceding. The corresponding integrals (12) for a system producing the wave and consisting of masses in motion relative to one another have directly no simple significance. We notice however that T_{44} denotes the (negatively taken) energy density which in the case of slow motion is practically equal to the mass density in the sense of ordinary mechanics. As will be shown, the above integrals can be expressed through this quantity. This can be done because of the existence of the energy-momentum equations of the physical system:

$$T_{\mu\alpha,\,\alpha} = 0. \qquad (13)$$

If one multiplies the second of these with x_2 and the fourth with $\frac{1}{2}x_2^2$ and integrates over the whole system, one obtains two integral relations, which on being combined yield

$$\int T_{22}\,dv = \frac{1}{2}\frac{\partial^2}{\partial x_4^2}\int x_2^2 T_{44}\,dv. \qquad (13a)$$

Analogously one obtains

$$\int T_{33}dv = \frac{1}{2}\frac{\partial^2}{\partial x_4^2}\int x_3^2 T_{44}dv,$$

$$\int T_{23}dv = \frac{1}{2}\frac{\partial^2}{\partial x_4^2}\int x_2 x_3 T_{44}dv.$$

One sees from this that the time-derivatives of the moments of inertia determine the emission of the gravitational waves, provided the whole method of application of the approximation-equations is really justified. In particular one also sees that the case of waves symmetrical with respect to the x_1–x_2 and x_1–x_3 planes could be realized by means of elastic oscillations of a material system which has the same symmetry properties. For example, one might have two equal masses which are joined by an elastic spring and oscillate toward each other in a direction parallel to the x_3-axis.

From consideration of energy relationship it has been concluded that such a system, in sending out gravitational waves, must send out energy which reacts by damping the motion. Nevertheless, one can think of the case of vibration free from damping if one imagines that, besides the waves emitted by the system, there is present a second concentric wave-field which is propagated inward and brings to the system as much energy as the outgoing waves remove. This leads to an undamped mechanical process which is imbedded in a system of standing waves.

Mathematically this is connected with the following considerations, clearly pointed out in past years by Ritz and Tetrode. The integration of the wave-equation

$$\Box\varphi = -4\pi\rho$$

by the *retarded* potential

$$\varphi = \int \frac{[\rho]_{(t-r)}}{r}dv$$

is mathematically not the only possibility. One can also do it with

$$\varphi = \int \frac{[\rho]_{(t+r)}}{r}dv,$$

i.e. by means of the "advanced" potential, or by a mixture of the two, for example,

$$\varphi = \frac{1}{2}\int \frac{[\rho]_{(t+r)} + [\rho]_{(t-r)}}{r}dv.$$

The last possibility corresponds to the case without damping, in which a standing wave is present.

It is to be remarked that one can think of waves generated as described above which approximate plane waves as closely as desired. One can obtain them, for example, through a limit-process by considering the wave-source to be removed further and further from the point in question and at the same time the oscillating moment of inertia of the former increased in proportion.

II. RIGOROUS SOLUTION FOR CYLINDRICAL WAVES.

We choose the coördinates x_1, x_2 in the meridian plane in such a way that $x_1 = 0$ is the axis of rotation and x_2 runs from 0 to infinity. Let x_3 be an angle coördinate specifying the position of the meridian plane. Also, let the field be symmetrical about every plane $x_2 = $ const. and about every meridian plane. The required symmetry leads to the vanishing of all components $g_{\mu\nu}$ which contain one and only one index 2; the same holds for the index 3. In such a gravitational field only

$$g_{11},\quad g_{22},\quad g_{33},\quad g_{44},\quad g_{14}$$

can be different from zero. For convenience we now take all the coördinates real. One can further transform the coördinates x_1, x_4 so that two conditions are satisfied. As such we take

$$\left.\begin{aligned} g_{14} &= 0, \\ g_{11} &= -g_{44}. \end{aligned}\right\} \tag{14}$$

It can be easily shown that this can be done without introducing any singularities.

We now write

$$\left.\begin{aligned} -g_{11} = g_{44} &= A, \\ -g_{22} &= B, \\ -g_{33} &= C, \end{aligned}\right\} \tag{15}$$

where $A, B, C > 0$. In terms of these quantities one calculates that

$$
\left.\begin{aligned}
2\left(R_{11}-\frac{1}{2}g_{11}R\right) &= \frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{1}{2}\left[\frac{B_4^2}{B^2}+\frac{C_4^2}{C^2}\right.\\
&\quad -\frac{B_4C_4}{BC}+\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)\\
&\quad \left.+\frac{B_1C_1}{BC}+\frac{A_1}{A}\left(\frac{B_1}{B}+\frac{C_1}{C}\right)\right],\\
\frac{2A}{B}\left(R_{22}-\frac{1}{2}g_{22}R\right) &= \frac{A_{44}}{A}+\frac{C_{44}}{C}-\frac{A_{11}}{A}-\frac{C_{11}}{C}\\
&\quad +\frac{1}{2}\left[\frac{C_1^2}{C^2}-\frac{C_4^2}{C^2}\right.\\
&\quad \left.+\frac{2A_1^2}{A^2}-\frac{2A_4^2}{A^2}\right],\\
\frac{2A}{C}\left(R_{33}-\frac{1}{2}g_{33}R\right) &= \frac{A_{44}}{A}+\frac{B_{44}}{B}-\frac{A_{11}}{A}-\frac{B_{11}}{B}\\
&\quad +\frac{1}{2}\left[\frac{2A_1^2}{A^2}-\frac{2A_4^2}{A^2}\right.\\
&\quad \left.+\frac{B_1^2}{B^2}-\frac{B_4^2}{B^2}\right],\\
2\left(R_{44}-\frac{1}{2}g_{44}R\right) &= \frac{B_{11}}{B}+\frac{C_{11}}{C}-\frac{1}{2}\left[\frac{B_1^2}{B^2}+\frac{C_1^2}{C^2}\right.\\
&\quad -\frac{B_1C_1}{BC}+\frac{A_1}{A}\left(\frac{B_1}{B}+\frac{C_1}{C}\right)\\
&\quad \left.+\frac{B_4C_4}{BC}+\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)\right],\\
2R_{14} &= \frac{B_{14}}{B}+\frac{C_{14}}{C}-\frac{1}{2}\left[\frac{B_1B_4}{B^2}+\frac{C_1C_4}{C^2}\right.\\
&\quad +\frac{A_4}{A}\left(\frac{B_1}{B}+\frac{C_1}{C}\right)\\
&\quad \left.+\frac{A_1}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)\right],
\end{aligned}\right\} \quad (16)
$$

where subscripts in the right-hand members denote differ-

entiation. If we take as field equations these expressions set equal to zero, replace the second and third by their sum and difference, and introduce as new variables

$$
\left.\begin{aligned}
\alpha &= \log A,\\
\beta &= \tfrac{1}{2}\log (B/C),\\
\gamma &= \tfrac{1}{2}\log (BC),
\end{aligned}\right\} \quad (15a)
$$

we get

$$2\gamma_{44}+\tfrac{1}{2}[\beta_4^2+3\gamma_4^2+\beta_1^2-\gamma_1^2-2\alpha_1\gamma_1-2\alpha_4\gamma_4]=0, \quad (17)$$

$$2(\alpha_{11}-\alpha_{44})+2\gamma_{11}-2\gamma_{44}+[\beta_1^2+\gamma_1^2-\beta_4^2-\gamma_4^2]=0, \quad (18)$$

$$\beta_{11}-\beta_{44}+[\beta_1\gamma_1-\beta_4\gamma_4]=0, \quad (19)$$

$$2\gamma_{11}+\tfrac{1}{2}[\beta_1^2+3\gamma_1^2+\beta_4^2-\gamma_4^2-2\alpha_1\gamma_1-2\alpha_4\gamma_4]=0, \quad (20)$$

$$2\gamma_{14}+[\beta_1\beta_4+\gamma_1\gamma_4-2\alpha_1\gamma_4-2\alpha_4\gamma_1]=0. \quad (21)$$

The first and fourth equations of this group give

$$\gamma_{11}-\gamma_{44}+(\gamma_1^2-\gamma_4^2)=0. \quad (22)$$

The substitution

$$\gamma=\log\sigma, \qquad \sigma=(BC)^{\frac{1}{2}}, \quad (23)$$

leads to the wave equation

$$\sigma_{11}-\sigma_{44}=0, \quad (24)$$

which has the solution

$$\sigma=f(x_1+x_4)+g(x_1-x_4), \quad (25)$$

where f and g are arbitrary functions. Eq. (18) reduces to

$$\alpha_{11}-\alpha_{44}+\tfrac{1}{2}(\beta_1^2-\beta_4^2+\gamma_4^2-\gamma_1^2)=0. \quad (18a)$$

Equation (17) then shows that γ cannot vanish everywhere.

We must now see whether there exist undulatory processes for which γ does not vanish. We note that such an undulatory process is represented, in the first approximation, by an undulatory β, that is by a β-function which, so far as its dependence on x_1 and also its dependence on x_4 is concerned, possesses maxima and minima; we must expect this also for a rigorous solution. We know about γ that $e^\gamma=\sigma$ satisfies the wave equation (24) and therefore takes the form (25). From this, however, the undulatory nature of this quantity does not necessarily follow. We shall in fact show that γ can have no minima.

Such a minimum would imply that the functions f and g in (25) have minima. At a point (x_1, x_4) where this were the case we should have $\gamma_1 = \gamma_4 = 0$, $\gamma_{11} \geqq 0$, $\gamma_{44} \geqq 0$. But by (17) and (20) this is impossible. Therefore γ has no minima, that is it is not undulatory but behaves, at least in a region of space arbitrarily extended in one direction, monotonically. We shall now consider such a region of space.

It is useful to see what sort of transformations of x_1 and x_4 leave our system of equations (14) invariant. For this invariance it is necessary and sufficient that the transformation satisfy the equations

$$\left.\begin{aligned} \frac{\partial \bar{x}_1}{\partial x_1} &= \frac{\partial \bar{x}_4}{\partial x_4}, \\ \frac{\partial \bar{x}_1}{\partial x_4} &= \frac{\partial \bar{x}_4}{\partial x_1}, \end{aligned}\right\} \tag{26}$$

Thus we may arbitrarily choose $\bar{x}_1(x_1, x_4)$ to satisfy the equation

$$\frac{\partial^2 \bar{x}_1}{\partial x_1^2} - \frac{\partial^2 \bar{x}_1}{\partial x_4^2} = 0 \tag{26a}$$

and then (26) will determine the corresponding $\bar{x}_4$. Since e^γ is invariant under this transformation and also satisfies the wave equation, there exists a transformation where $\bar{x}_1$ is respectively equal or proportional to e^γ. In the *new* coördinate system we have

$$e^\gamma = a x_1$$

or

$$\gamma = \log a + \log x_1. \tag{27}$$

If we insert this expression for γ in (17)–(27) the equations reduce to the equivalent system

$$\beta_{11} - \beta_{44} + \frac{1}{x_1}\beta_1 = 0, \tag{28}$$

$$\alpha_1 = \tfrac{1}{2} x_1(\beta_1^2 + \beta_4^2) - \frac{1}{2x_1}, \tag{29}$$

and

$$\alpha_4 = x_1 \beta_1 \beta_4. \tag{30}$$

Equation (28) is the equation for cylindrical waves in a three-dimensional space, if x_1 denotes the distance from the axis of rotation. The equations (29) and (30) determine, for given β, the function α up to an (arbitrary) additive constant, while, by (27), γ is already determined.

In order that the waves may be regarded as waves in a euclidean space these equations must be satisfied by the euclidean space when the field is independent of x_4. This field is represented by

$$A = 1; \quad B = 1; \quad C = x_1^2,$$

if we denote the angle about the axis of rotation by x_3. These relations correspond to

$$\alpha = 0, \quad \beta = -\log x_1, \quad \gamma = \log x_1,$$

and from this we see that the equations (27)–(30) are in fact satisfied.

We have still to investigate whether *stationary* waves exist, that is waves which are purely periodic in the time.

For β it is at once clear that such solutions exist. Although it is not essential, we shall now consider the case where the variation of β with time is sinusoidal. Here β has the form

$$\beta = X_0 + X_1 \sin \omega x_4 + X_2 \cos \omega x_4,$$

where X_0, X_1, X_2 are functions of x_1 alone. From (30) it then follows that α is periodic if and only if the integral

$$\int \beta_1 \beta_4 dx_4$$

taken over a whole number of periods vanishes.

In the case of a stationary oscillation, which is represented by

$$\beta = X_0 + X_1 \sin \omega x_4,$$

this condition is actually fulfilled since

$$\int \beta_1 \beta_4 dx_4 = \int (X_0' + X_1' \sin \omega x_4) \omega X_1 \cos \omega x_4 dx_4 = 0.$$

On the other hand, in the general case, which includes the case of progressive waves, we obtain for this integral the value

$$\tfrac{1}{2}(X_1 X_2' - X_2 X_1')\omega T,$$

where T is the interval of time over which the integral is taken. This does not vanish, in general. At distances x_1 from $x_1 = 0$ great compared with the wave-lengths, a progressive wave can be represented with good approximation in a domain containing many waves by

$$\beta = X_0 + a \sin \omega(x_4 - x_1),$$

where a is a constant (which, to be sure, is a substitute for a function depending weakly on x_1). In this case $X_1 = a \cos \omega x_1$, $X_2 = - a \sin \omega x_1$, so that the integral can be (approximately) represented by $- \frac{1}{2} a\omega^2 T$, and thus cannot vanish and always has the same sign. Progressive waves therefore produce a secular change in the metric.

This is related to the fact that the waves transport energy, which is bound up with a systematic change in time of a gravitating mass localized in the axis $x = 0$.

Note.—The second part of this paper was considerably altered by me after the departure of Mr. Rosen for Russia since we had originally interpreted our formula results erroneously. I wish to thank my colleague Professor Robertson for his friendly assistance in the clarification of the original error. I thank also Mr. Hoffmann for kind assistance in translation.

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