Solid State Lighting

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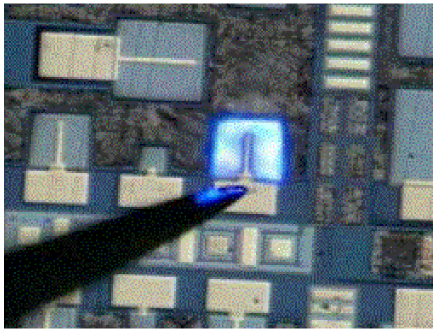
From a Torch to Blue and White LEDs and to Solid State Lamps



Blue LED on Si, Courtesy of SET, Inc.

Research Areas

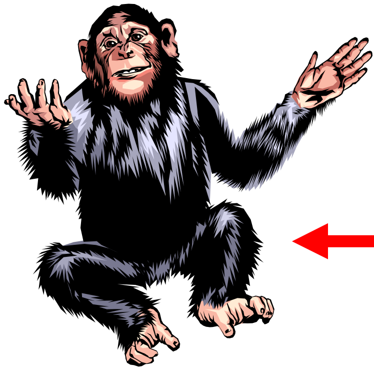
- Plasma wave electronics
  - THz resonant emission and detection
- Wide band gap materials and devices
  - MOSHFET, UV LEDs, SAW, polarization, transport
- Sensitive skin
  - Flexible substrates, nano gauges, electrotextiles, TFTs, OTFTs
- Novel device CAD
  - AIM-Spice, opto/thermo/micro CAD
- Lab-on-the-WEB
  - http://nina.ecse.rpi.edu/shur/remote
- Broadband center

http://nina.ecse.rpi.edu/shur

- **Solid state lighting**

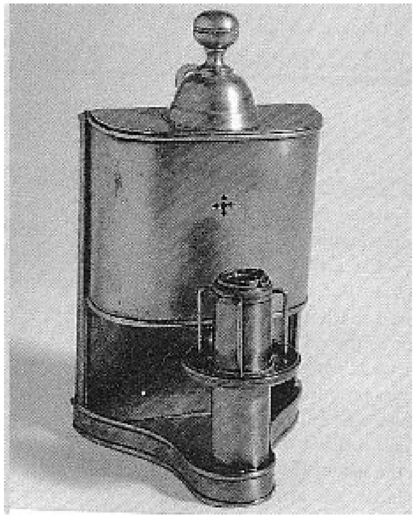
Talk Outline

- History of General and Electric Lighting
- Advantages of Solid State Lighting
- Introduction to Photometry and Colorimetry
- Optimization of Solid State Lamps
- Emerging applications
- Vision

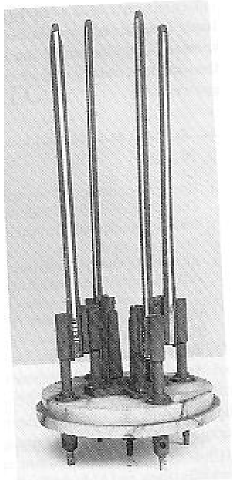


Lighting – prerequisite of human civilization

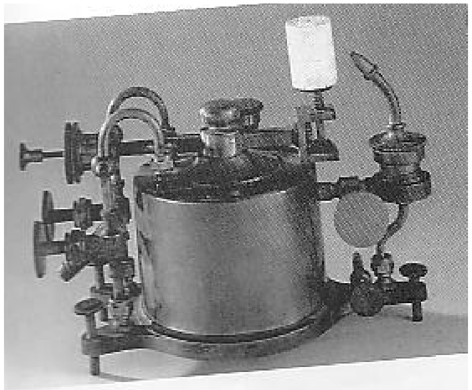
- 500,000 years ago- first torch
- 70,000 years ago – first lamp (wick)
- 1,000 BC – the first candle
- 1772 - gas lighting
- 1784 Agrand lamp - the first lamp relied on research (Lavoisier)
- 1826 -Limelight - solid-state lighting device
- 1876 – Yablochkov candle
- 1879 – Edison bulb



Agrand lamp



Yablochkov candle (1876)

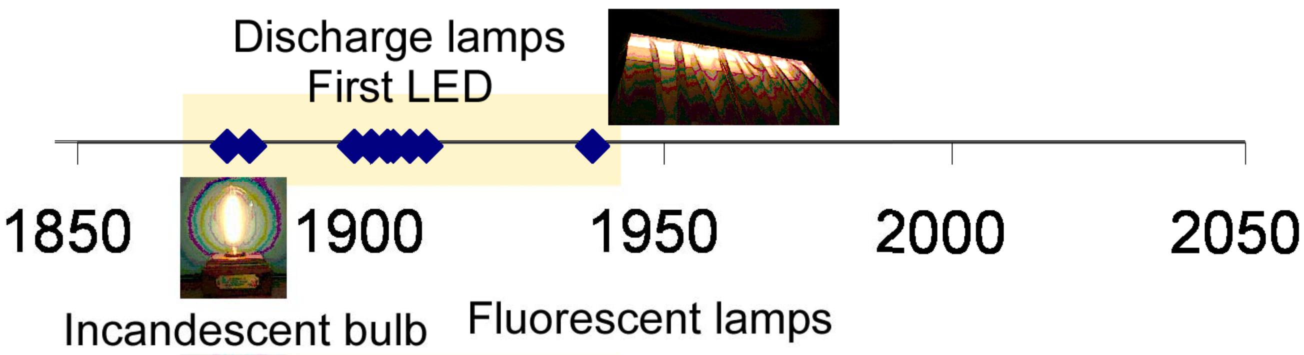


Limelight

Edison bulb (1879)

History of Electric Lighting

- 1876 Pavel Yablochkov. First electric lighting device
- 1879 Thomas Alva Edison. Incandescent lamp
- 1897 Nernst. Filament made of cerium oxide-based solid electrolyte.
- 1900 Peter Cooper Hewitt. Mercury vapor lamp. 1903. A. Just and F. Hanaman. Tungsten filament
- 1904 Moor. Discharge lamps with air
- 1907 H. J. Round. First LED (SiC)
- 1910 P. Claude. Discharge lamps with inert gases
- 1938 GE and Westinghouse Electric Corporation Fluorescent lamps.



A Note on Carborundum.

To the Editors of Electrical World:

SIRS:—During an investigation of the unsymmetrical passage of current through a contact of carborundum and other substances a curious phenomenon was noted. On applying a potential of 10 volts between two points on a crystal of carborundum, the crystal gave out a yellowish light. Only one or two specimens could be found which gave a bright glow on such a low voltage, but with 110 volts a large number could be found to glow. In some crystals only edges gave the light and others gave instead of a yellow light green, orange or blue. In all cases tested the glow appears to come from the negative pole, a bright blue-green spark appearing at the positive pole. In a single crystal, if contact is made near the center with the negative pole, and the positive pole is put in contact at any other place, only one section of the crystal will glow and that the same section wherever the positive pole is placed.

There seems to be some connection between the above effect and the e.m.f. produced by a junction of carborundum and another conductor when heated by a direct or alternating current; but the connection may be only secondary as an obvious explanation of the e.m.f. effect is the thermoelectric one. The writer would be glad of references to any published account of an investigation of this or any allied phenomena.

NEW YORK, N. Y. H. J. ROUND.

First LED (1907)

Lighting in James Joyce Room
Shakespeare Hotel, Bernadinu 8/8 Vilnius, Lithuania
(09/05/02)

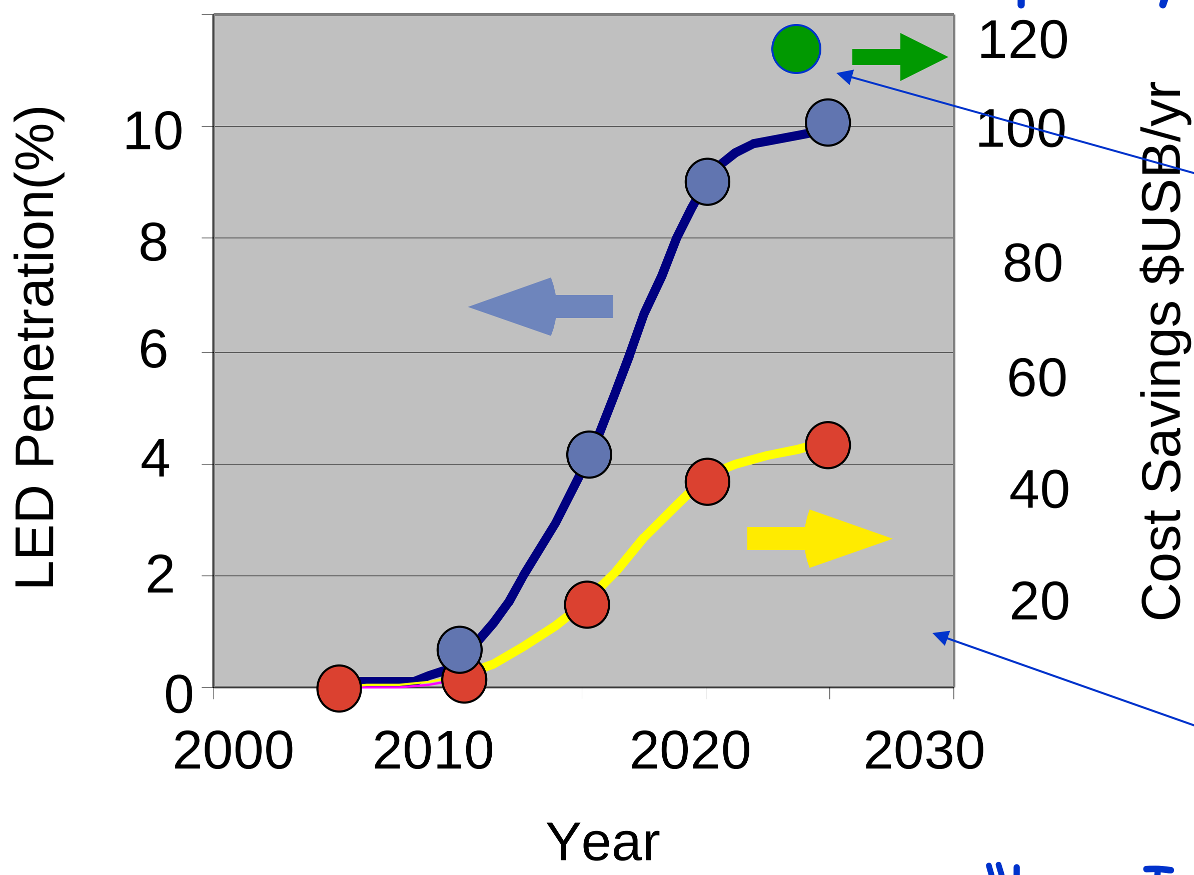
First LED stamp

Incandescent

Fluorescent

Benefits of LED Lighting

An improvement of luminous efficiency by 1% saves 2 billions dollars per year.

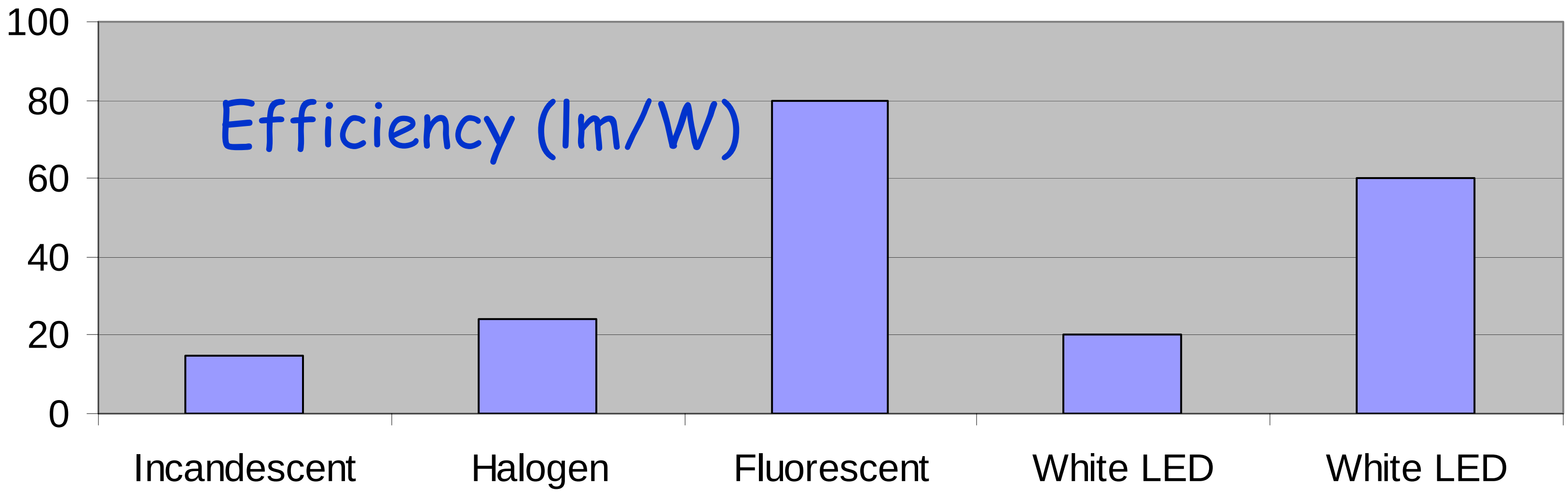


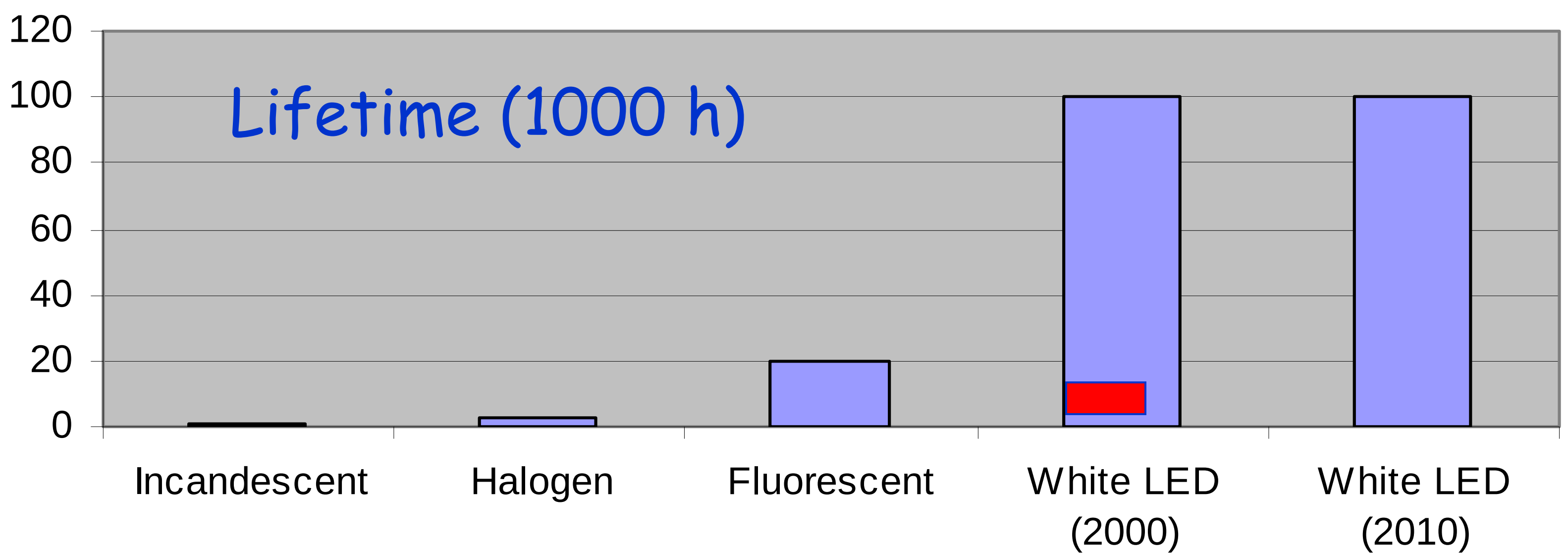
"High Investment Model"

"Low Investment Model"

Data from R. Haitz, F. Kish, J. Tsao, and J. Nelson
Innovation in Semiconductor Illumination: Opportunities for National Impact (2000)

Solid State Lighting





Challenges of Solid State Lighting

- Reduce cost
- Improve efficiency of light generation
- Improve efficiency of light extraction
- Improve quality of light

Cost of Light

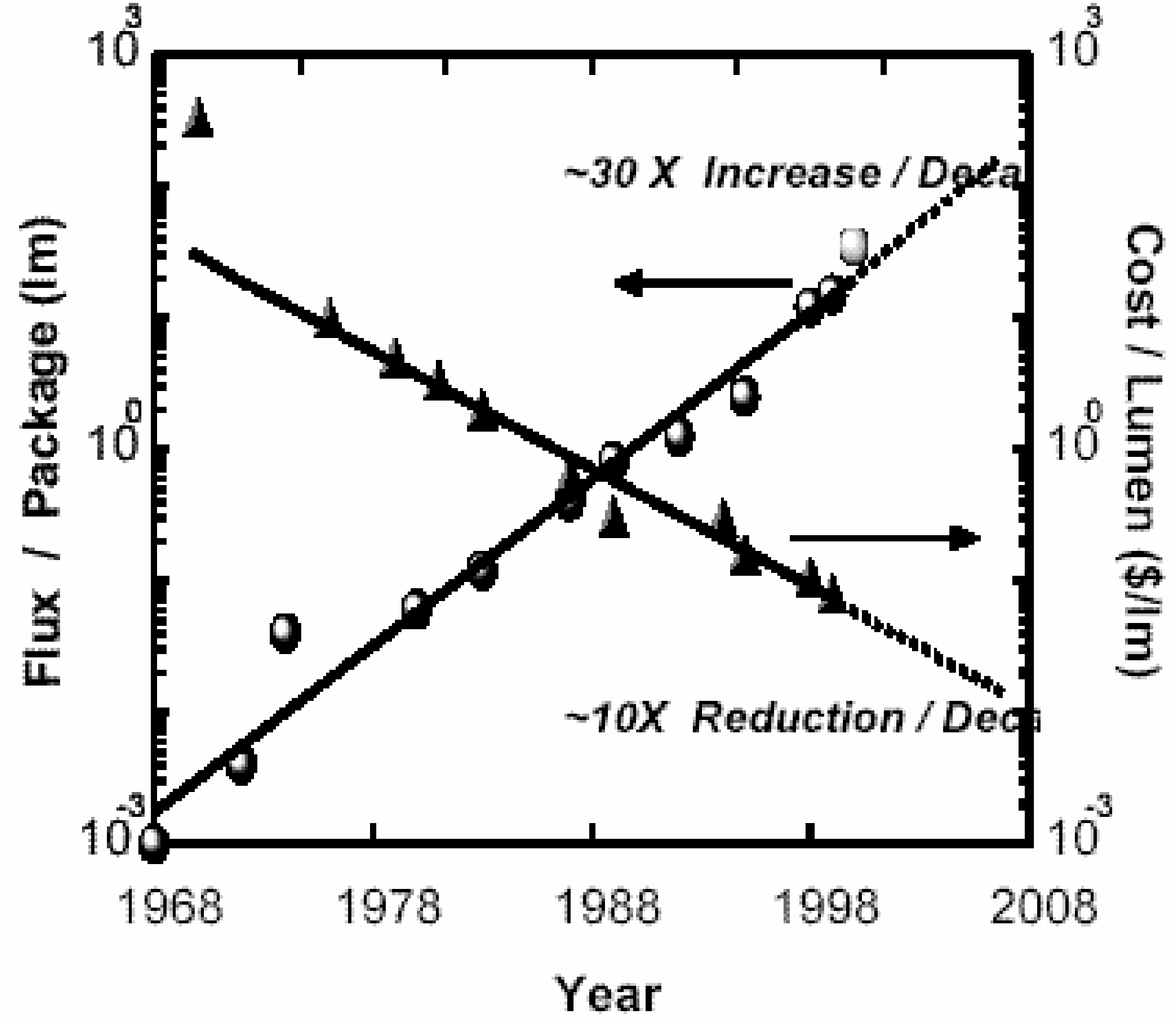
Fluorescent tubes:
dollar per lumen 0.01

•Incandescent bulbs:
Dollar per lumen 1/1100

White LED:
dollar per lumen
0.25 (Lumileds)
0.66 (Nichia)

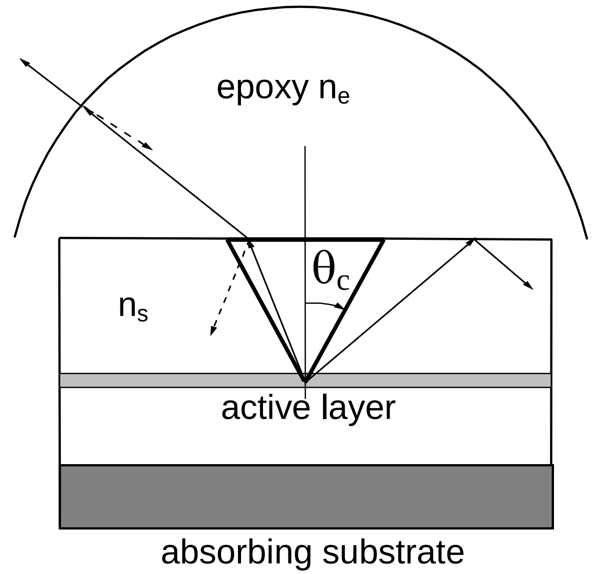
Re LED:
dollar per lumen0.1

Evolution of lm/package and cost/lm for red LEDs



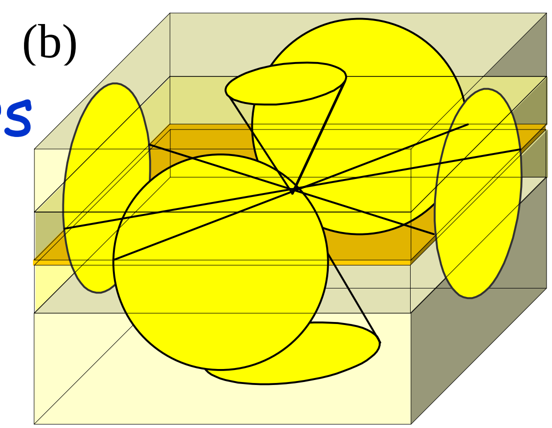
After Roland Haitz Agilent Technologies

Challenges in light extraction



Conventional LED chip grown on an absorbing substrate.

High-brightness LED chip design with thick transparent window layers
Light escapes through 6 cones

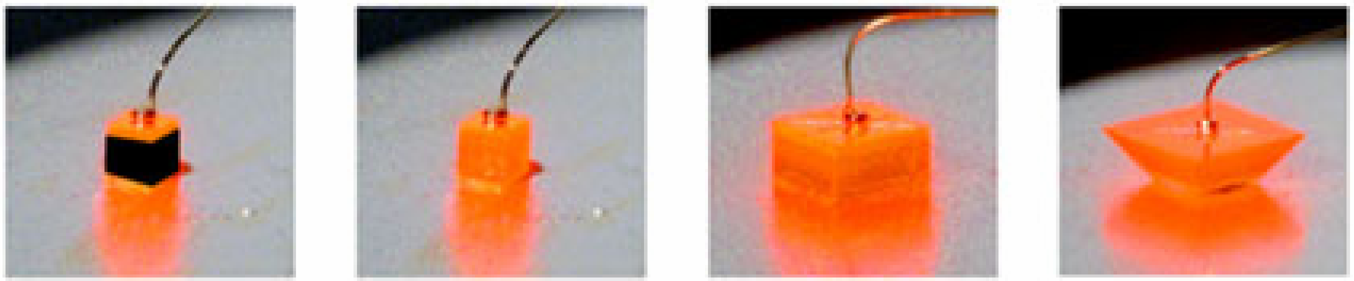


From A. Žukauskas, M. S. Shur, R. Gaska, *MRS Bull.* **26**, 764, 2001.

Progress in AlInGaP LEDs

Lumileds LED semiconductor material
AlInGaP technology for Red, Orange and Amber colors

1991	1994	1998	2001
	3x flux Improvement	15x flux improvement	22x flux improvement

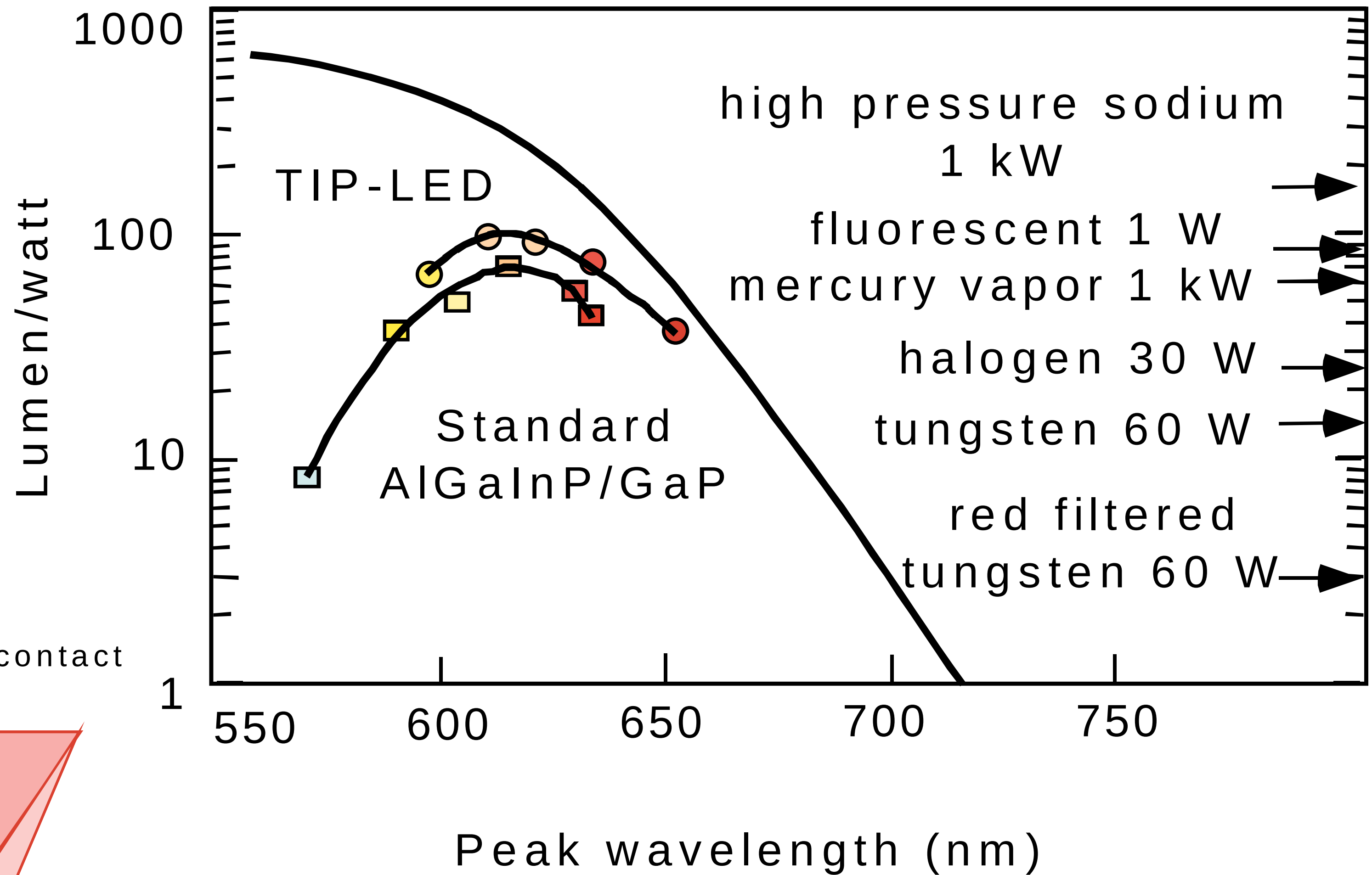


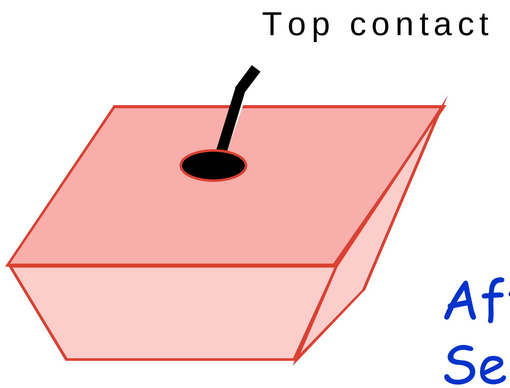
LUXEON LUXEON

Lumileds invests heavily to develop leading technology in LED material. Our AlInGaP technology leads the world in performance for Red, Orange, and Amber light. And we continue to improve performance.

After http://www.lumileds.com/technology/tutorial/slide2.htm

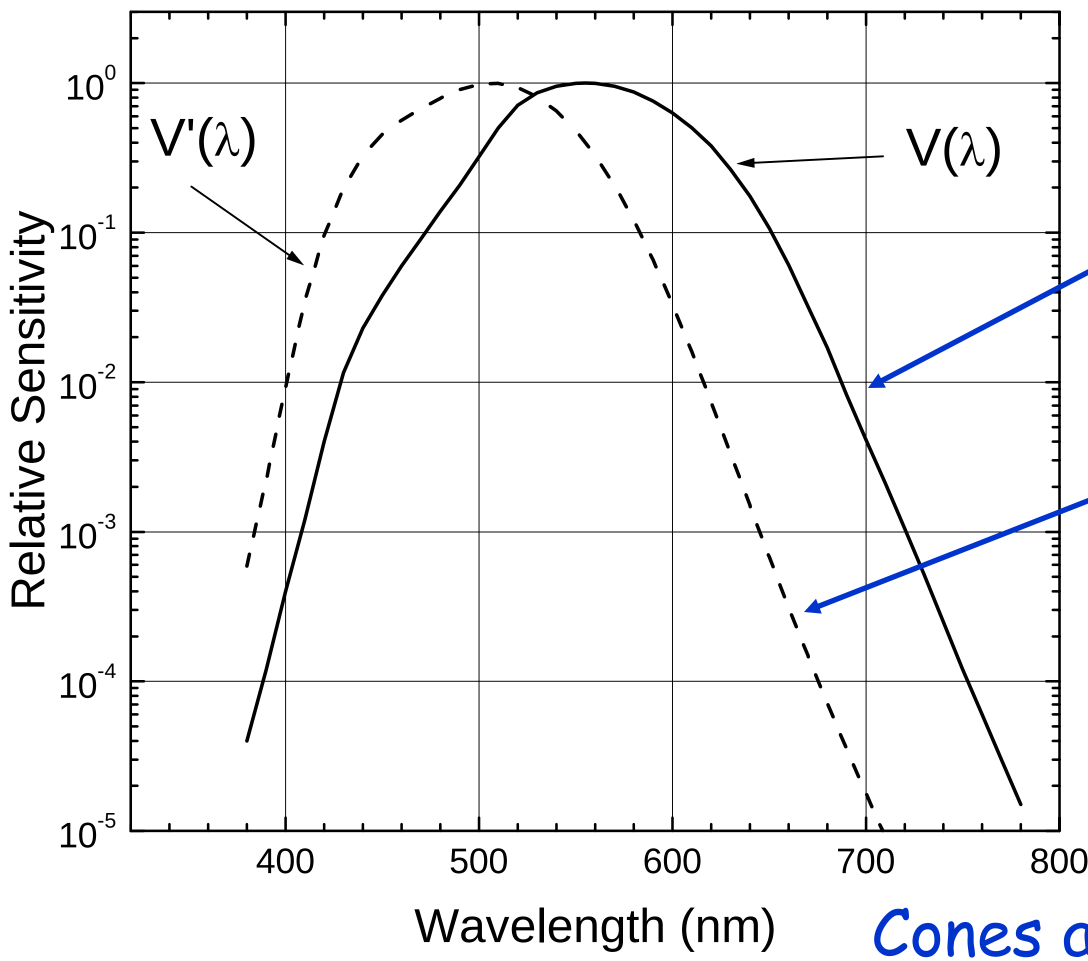
Light Extraction : TIP-LEDs from LumiLeDs





After M.O. HOLCOMB *et al.* (2001), Compound Semiconductor **7**, 59, 2001).

Photometry: Eye sensitivity



Photopic vision (cones) @ High Illumination

Scotopic vision (rods) @ Low illumination

Cones are red, green, and blue

Radiometry and Photometry

Watt

W/nm

Photopic vision eye sensitivity

$$\Phi_\upsilon = 683\,\mathrm{lm/W} \times \int \Phi_e V(\lambda)\, d\lambda$$

Luminous flux

W/nm

Wavelength (nm)

$$I_\upsilon = d\Phi_\upsilon / d\omega = 683\,\mathrm{lm/W} \times \int I_e V(\lambda)\, d\lambda$$

Luminous intensity

(Candela = lm/sr - SI unit)

1/60 of the luminous intensity per square centimeter of a blackbody radiating at the temperature of 2,046 degrees Kelvin

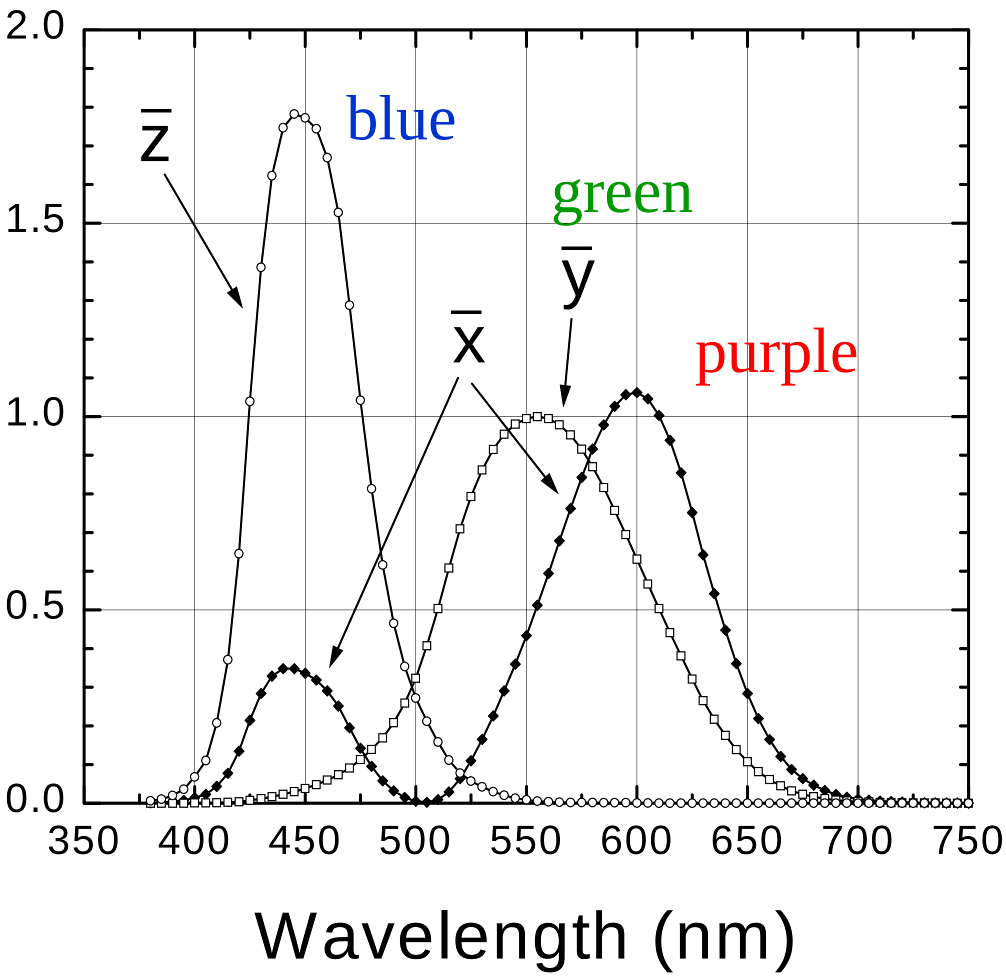
Luminous efficiency: power into actuation of vision (lm/W)

How much light do you need?

Type of Activity	Illuminance (lx = lm/m^2)
Orientation and simple visual tasks (public spaces)	30-100
Common visual tasks (commercial, industrial and residential applications)	300-1000
Special visual tasks, including those with very small or very low contrast critical elements	3,000-10,000

Colorimetry: Chromaticity Coordinates

1931 CIE color matching functions: purple, green, and blue



$$X = \int \bar{x}(\lambda) S(\lambda)\, d\lambda$$

$$Y = \int \bar{y}(\lambda) S(\lambda)\, d\lambda$$

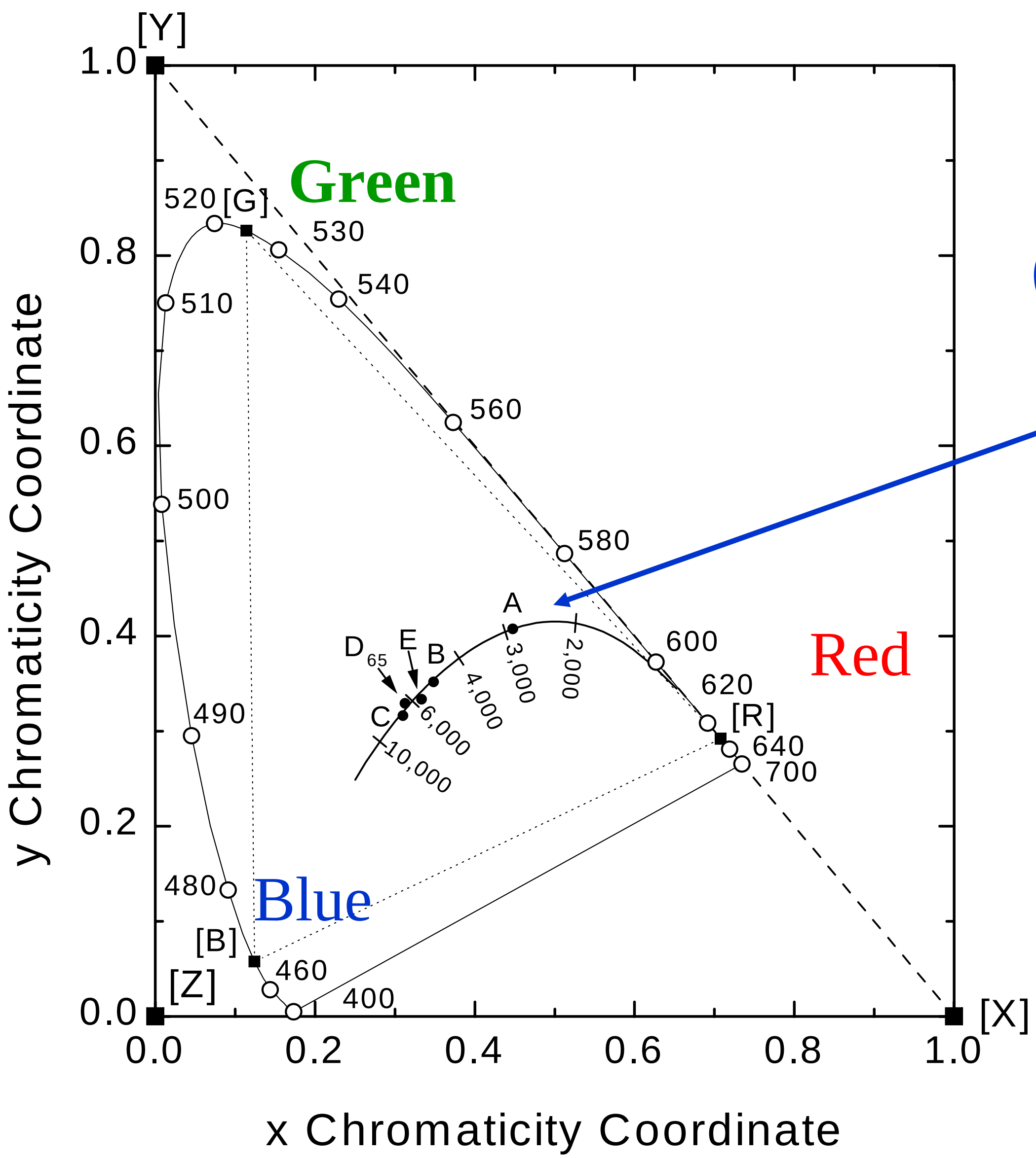
$$Z = \int \bar{z}(\lambda) S(\lambda)\, d\lambda$$

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$x+y+z = 1$$

Color Coordinates

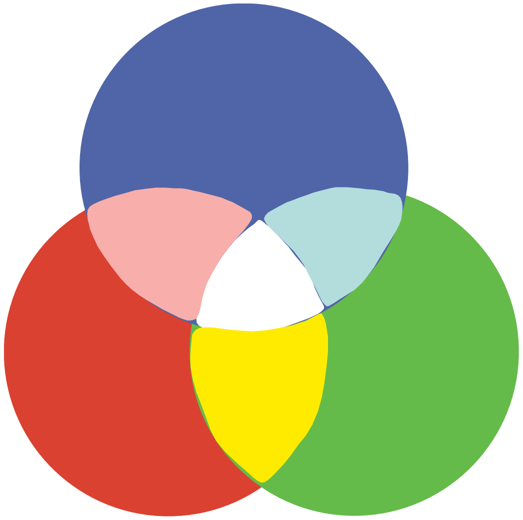


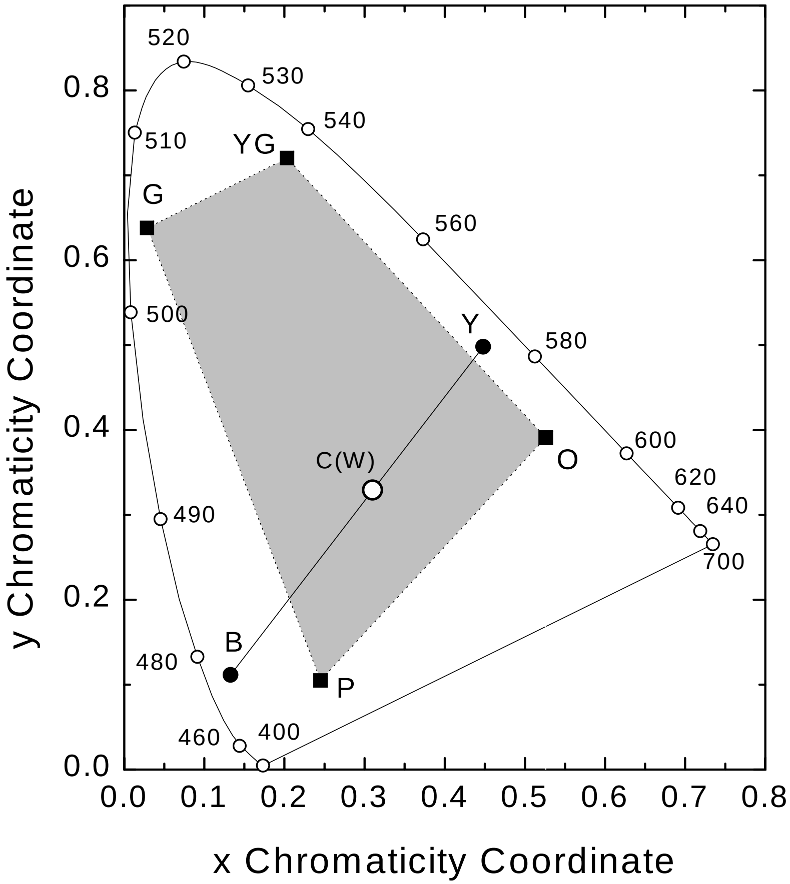
Planckian locus
(black body radiation)

Is it really that bad?
Just remember:
White is Black!

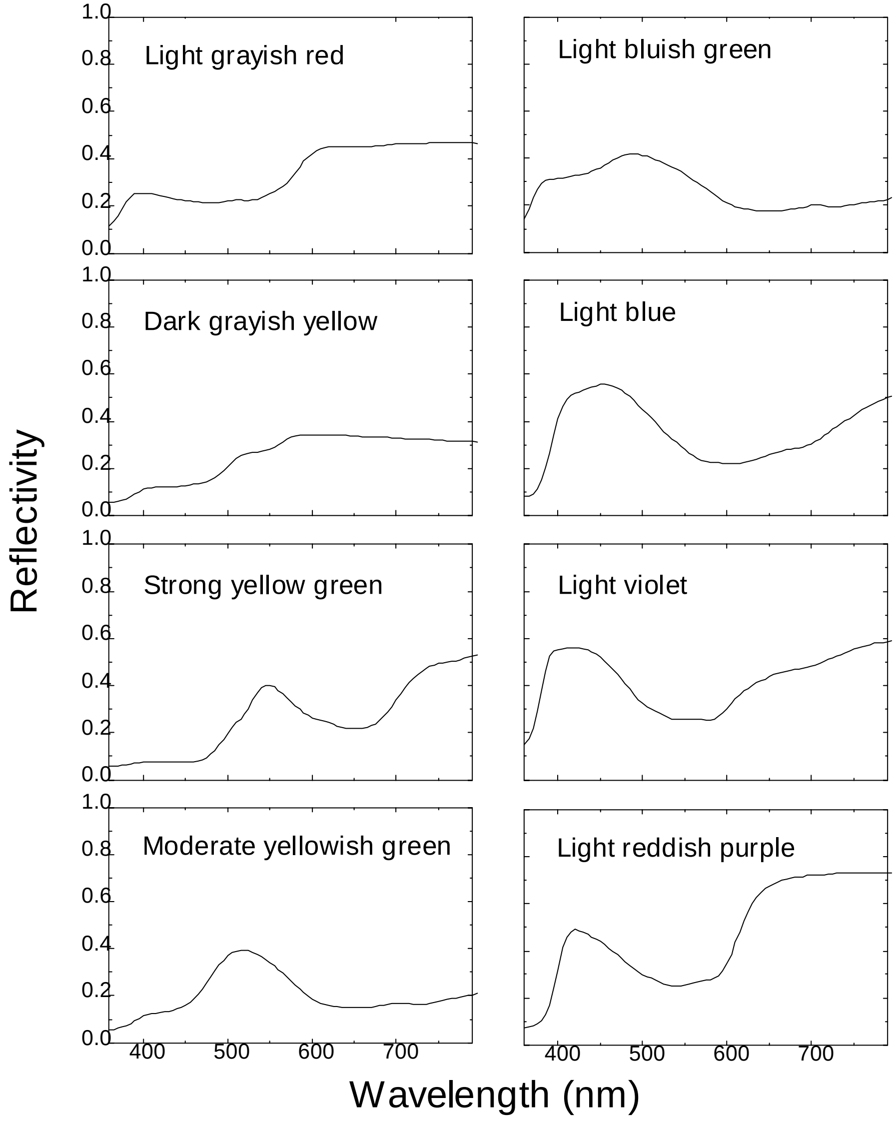
Color Mixing

- Red, green, blue appear as white
- Red and blue appear as magenta
- Green and blue give cyan
- Red and green give yellow





Color Rendering



General Color Rendering Index

R_a (CRI)

integrates the reflectivity data for 8 specified samples

Special color rendering indices,

refer to six additional test samples

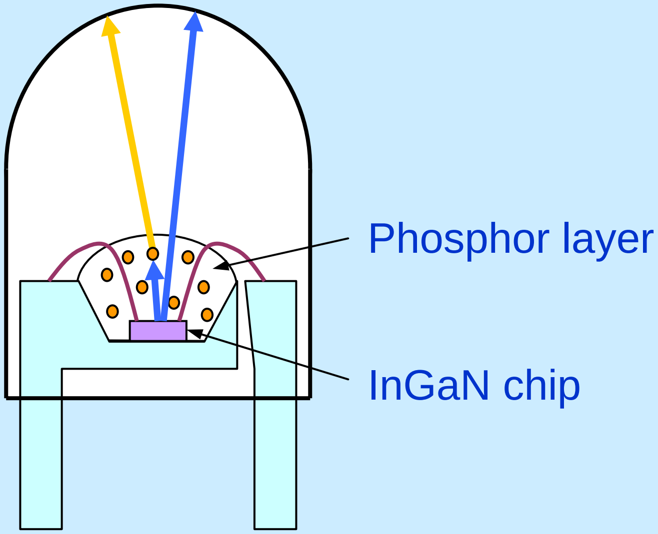
R_a varies up to 100

100 is the best.

R_a might be negative

Generating White Light

Phosphor-conversion white LED



Multichip white LED

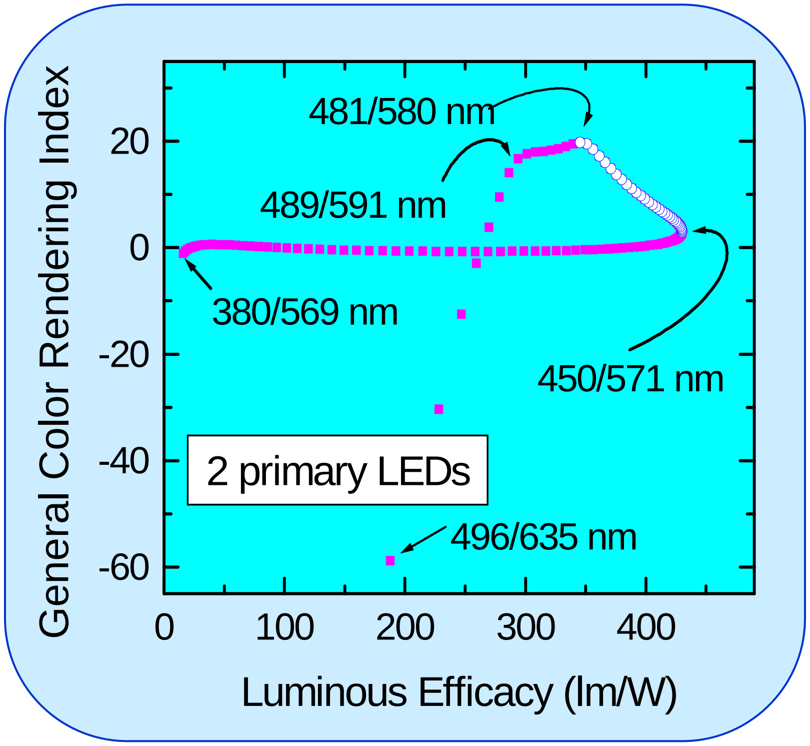
How to Optimize Solid State lamp: maximize the objective function

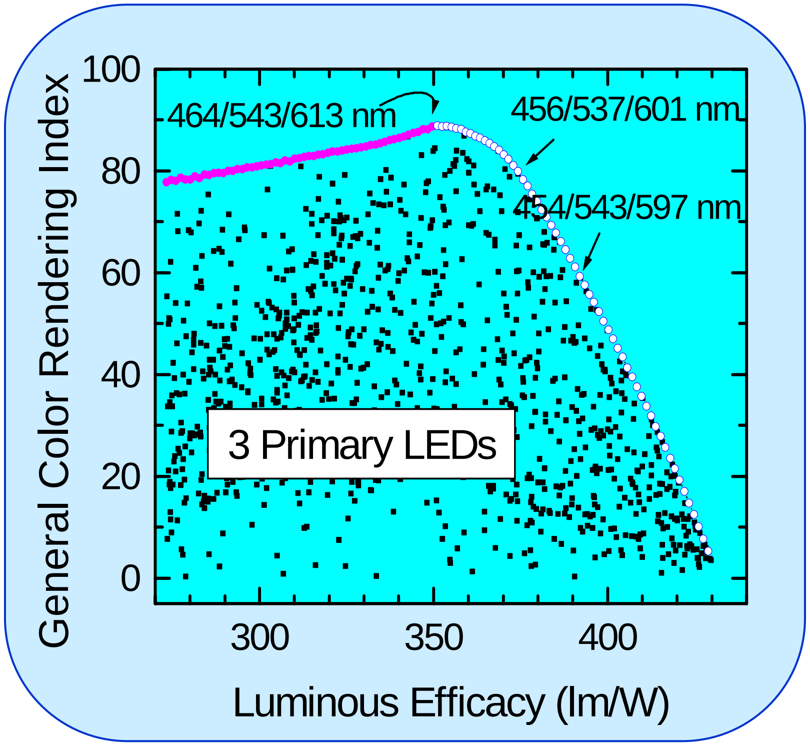
$$F_{\sigma}^{(K)}\left(\lambda_1, \ldots, \lambda_n, I_1, \ldots, I_n\right) = \sigma K + \left(1 - \sigma\right) R_a$$

where σ is the weight that controls the trade-off between the efficacy, K, and color rendering index, Ra
λ_1, λ_2, λ_3,... and I_1, I_2, I_3 are the peak wavelengths and relative powers of the primary LEDs, respectively

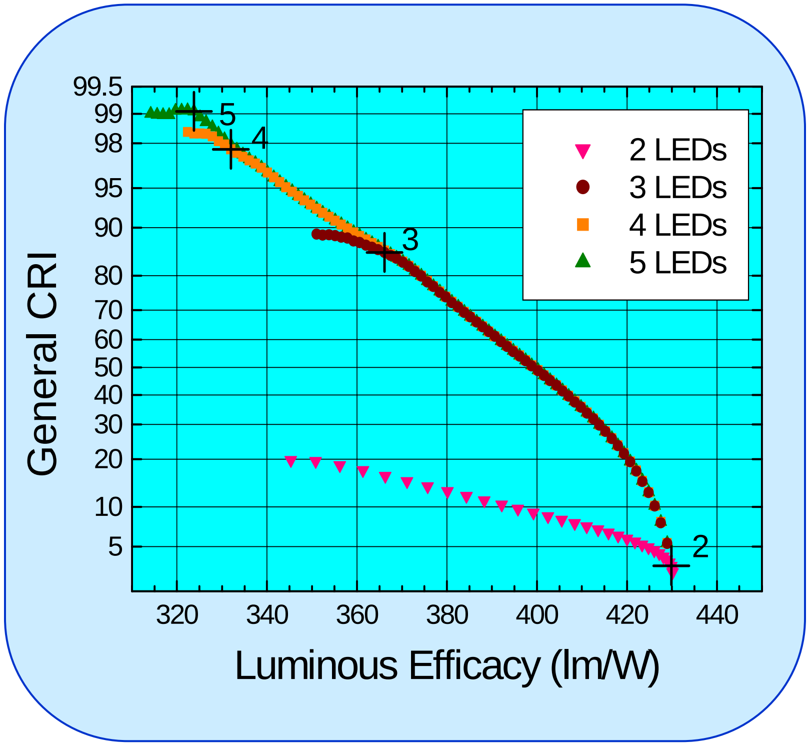
For a dichromatic lamp (two primary LEDs), the optimal solutions can be obtained by simple searching within the wavelength space involving complementary pairs of blue and yellow-green LEDs. .

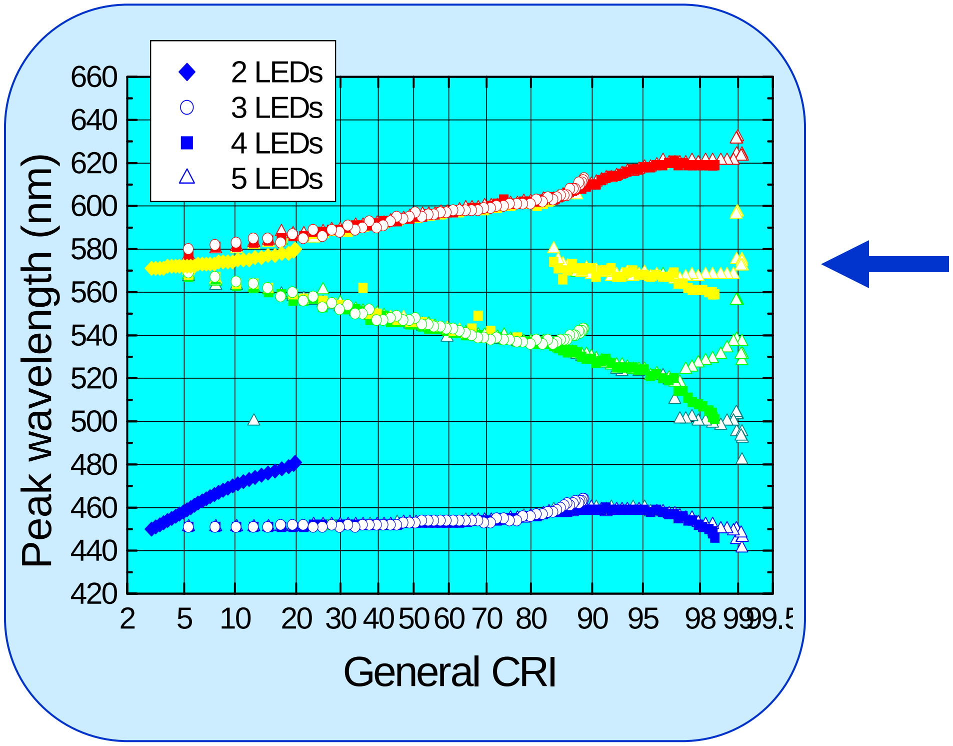
Optimization of Polychromatic Solid-State White Lamps





General CRI, Luminous Efficacy, and LED Wavelengths of Optimized Polychromatic Lamps





The 30-nm line widths (A. Žukauskas, R. Vaicekauskas, F. Ivanauskas, R. Gaska and M. S. Shur, Optimization of White Polychromatic Semiconductor Lamps, Appl. Phys. Lett. 80, 234 (2002)). Crosses mark points that are suggested for highest reasonable CRI for each number of primary sources.

General color rendering index and efficacy for optimized polychromatic solid-state lamps (color temperature T_S = 4870 K)

Type of lamp	R_a	Efficacy (lm/W)
Dichromatic	3	430
Trichromatic	85	366
Quadrichromatic	98	332
Quintichromatic	99	324

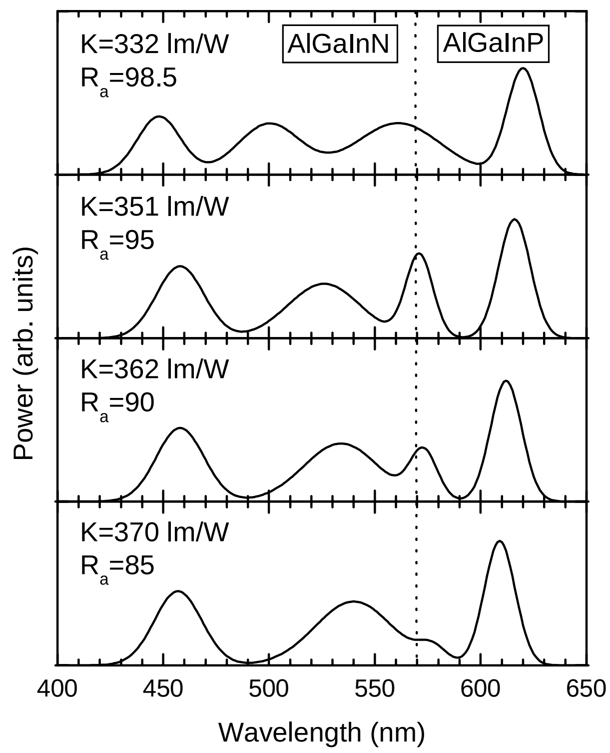
From A. Zukauskas, R. Vaicekauskas, G. Kurilcik, Z. Bliznikas, K. Breive, J. Krupic, A. Rupsys, A. Novickovas, P. Vitta, A. Navickas, V. Raskauskas, M. S. Shur, and R. Gaska, Quadrichromatic white solid-state lamp with digital feedback, SPIE 5187-24

Peak wavelengths of primary LEDs (nm) for optimized polychromatic solid-state lamps (color temperature T_S = 4870 K)

blue	cyan	green	yellow-green	amber to red	
452	-	-	**571**	-	2
453	-	537	-	604	3
454	509		**561**	619	4
448	493	531	**572**	623	5

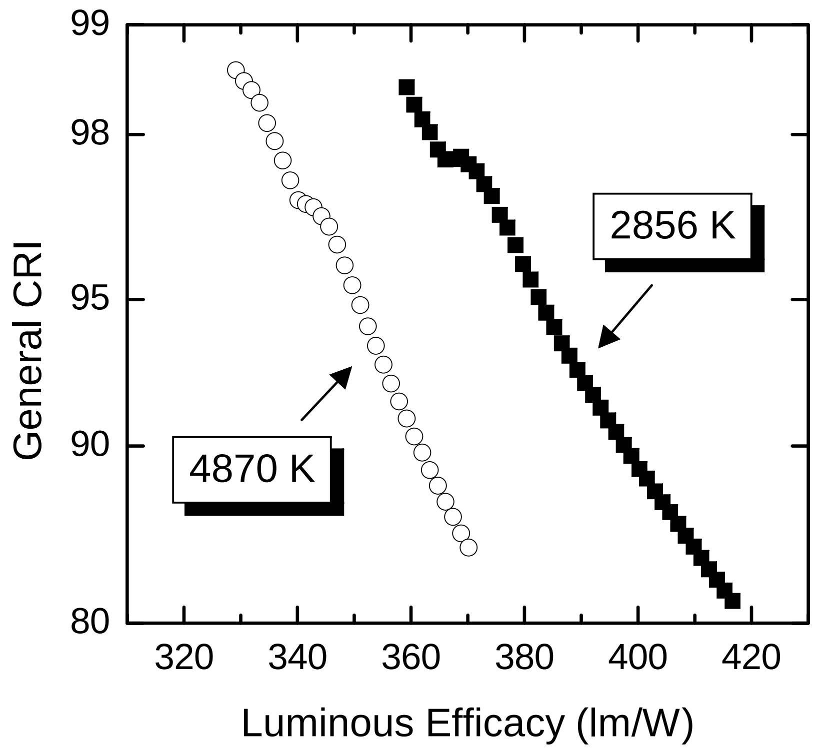
From A. Zukauskas, R. Vaicekauskas, G. Kurilcik, Z. Bliznikas, K. Breive, J. Krupic, A. Rupsys, A.Novickovas, P. Vitta, A. Navickas, V. Raskauskas, M. S. Shur, and R. Gaska, Quadrichromatic white solid-state lamp with digital feedback, SPIE 5187-24

Model spectra of white emission from quadrichromatic lamp (CT = 4870 K) for different points on optimal boundary of the (*K*,*R*a) phase distribution



From R. Gaska, A. Žukauskas, M. S. Shur, and M. A. Khan, "Progress in III-nitride based white light sources", (Invited paper), Volume 4776, pp. 82-96 (2002)

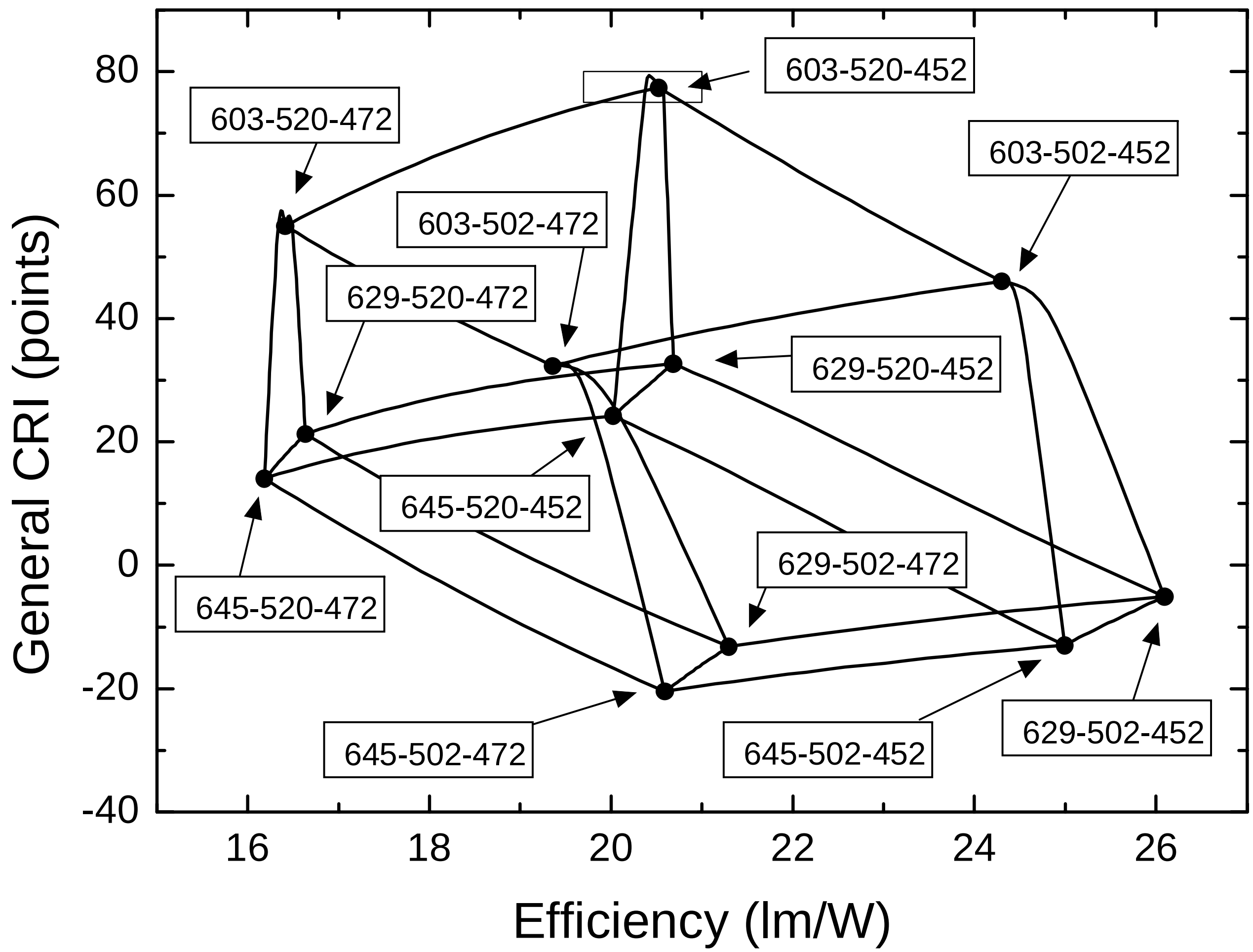
Optimization depends on color temperature



Optimal boundaries of the (*K*,*R*a) phase distribution for quadrichromatic lamps at two color temperatures

From R. Gaska, A. Žukauskas, M. S. Shur, and M. A. Khan, "Progress in III-nitride based white light sources", (Invited paper), Volume 4776, pp. 82-96 (2002)

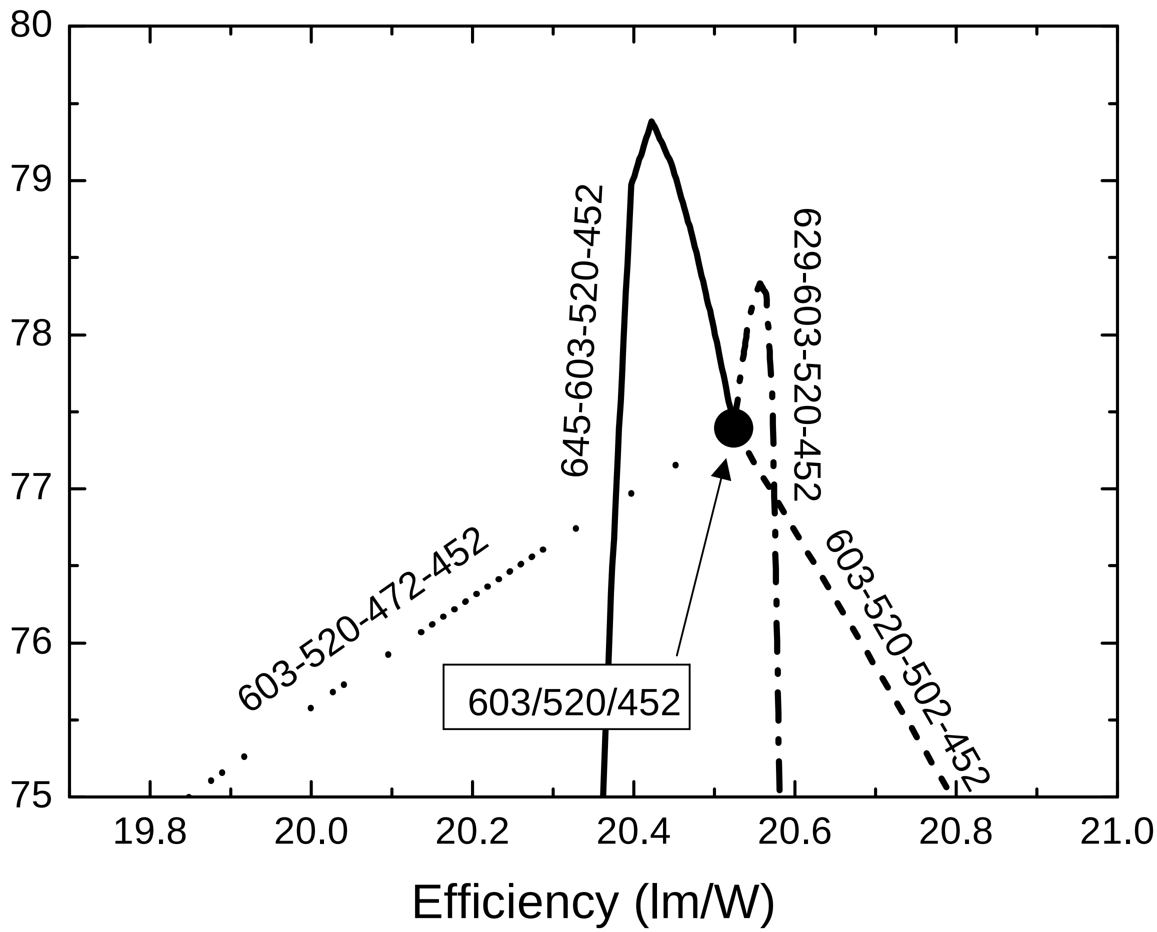
Luxeon^{tm} LED Combination



From theory to practice

From A. Zukauskas, R. Vaicekauskas, G. Kurilcik, Z. Bliznikas, K. Breive, J. Krupic, A. Rupsys, A. Novickovas, P. Vitta, A. Navickas, V. Raskauskas, M. S. Shur, and R. Gaska, Quadrichromatic white solid-state lamp with digital feedback, SPIE 5187-24

Four LED Segment



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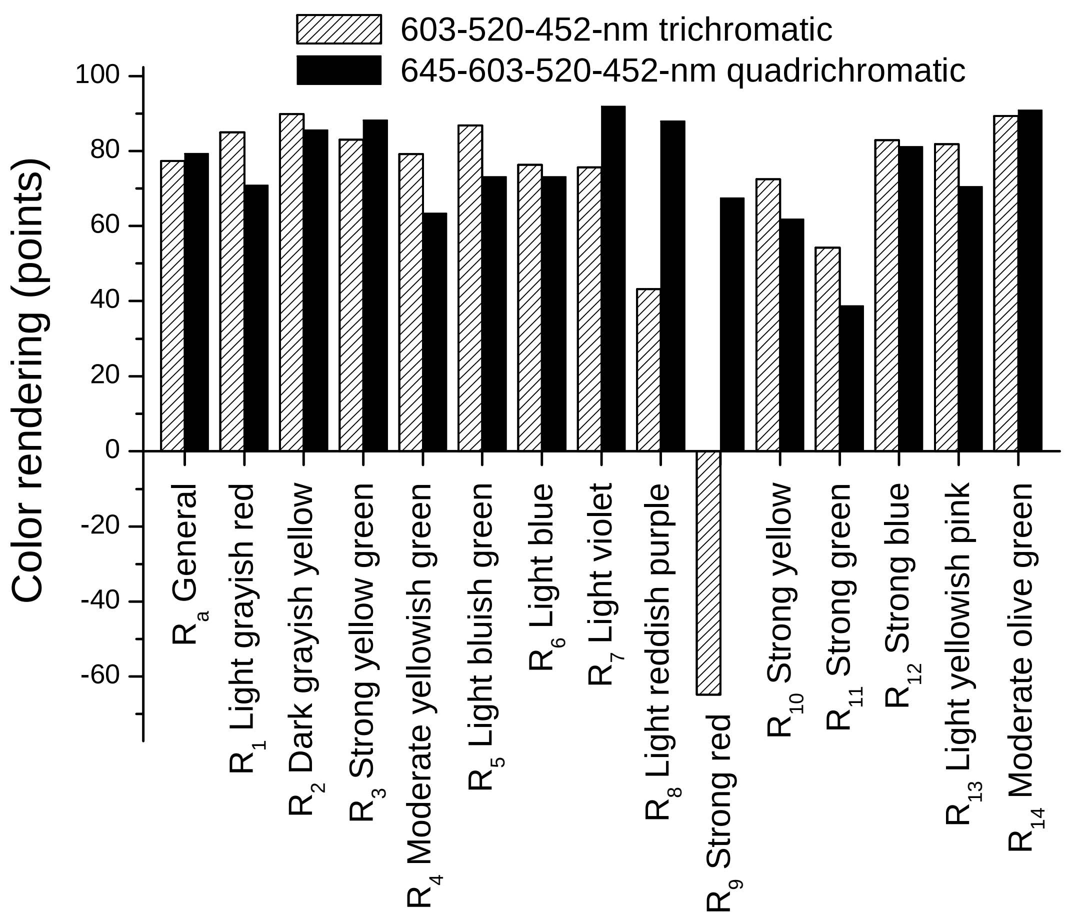
Why do we need quadrichromatic lamp?



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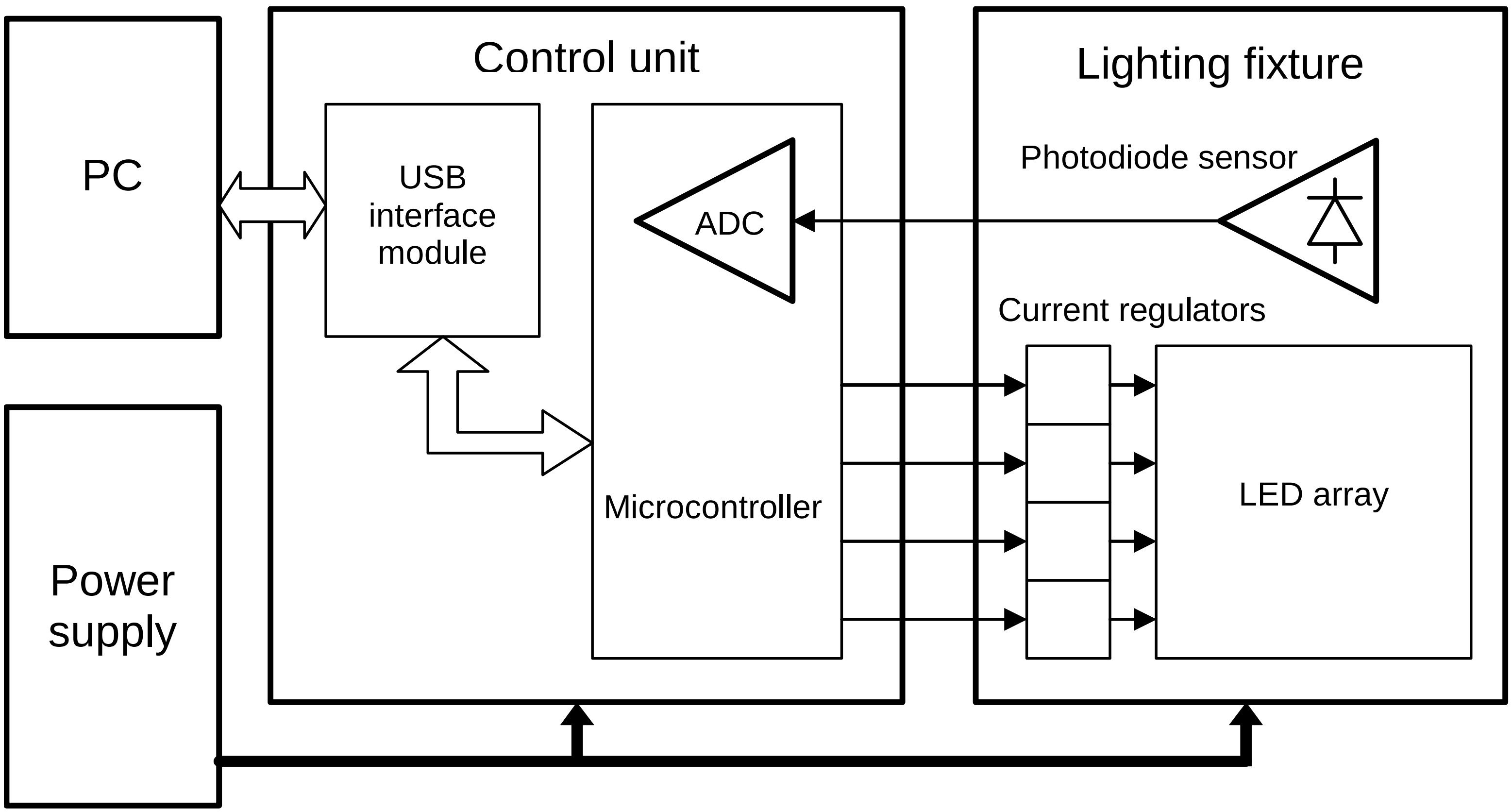
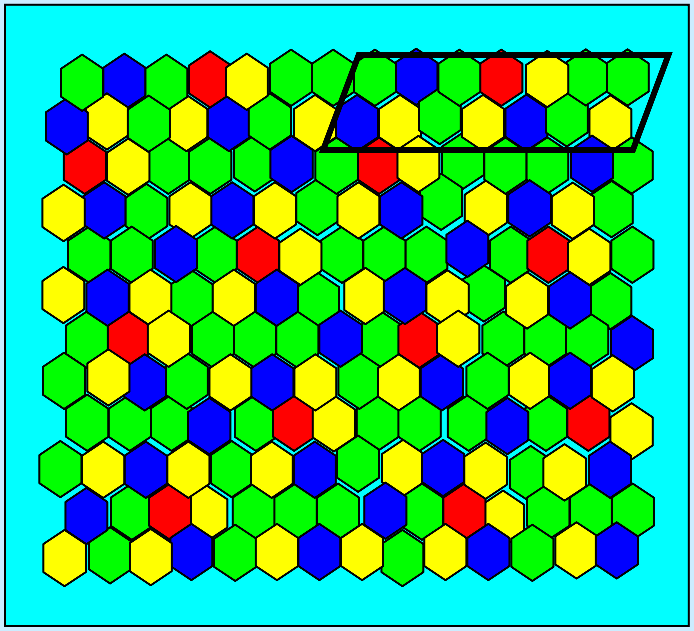
U of V/SET/RPI Versatile Solid State Lamp

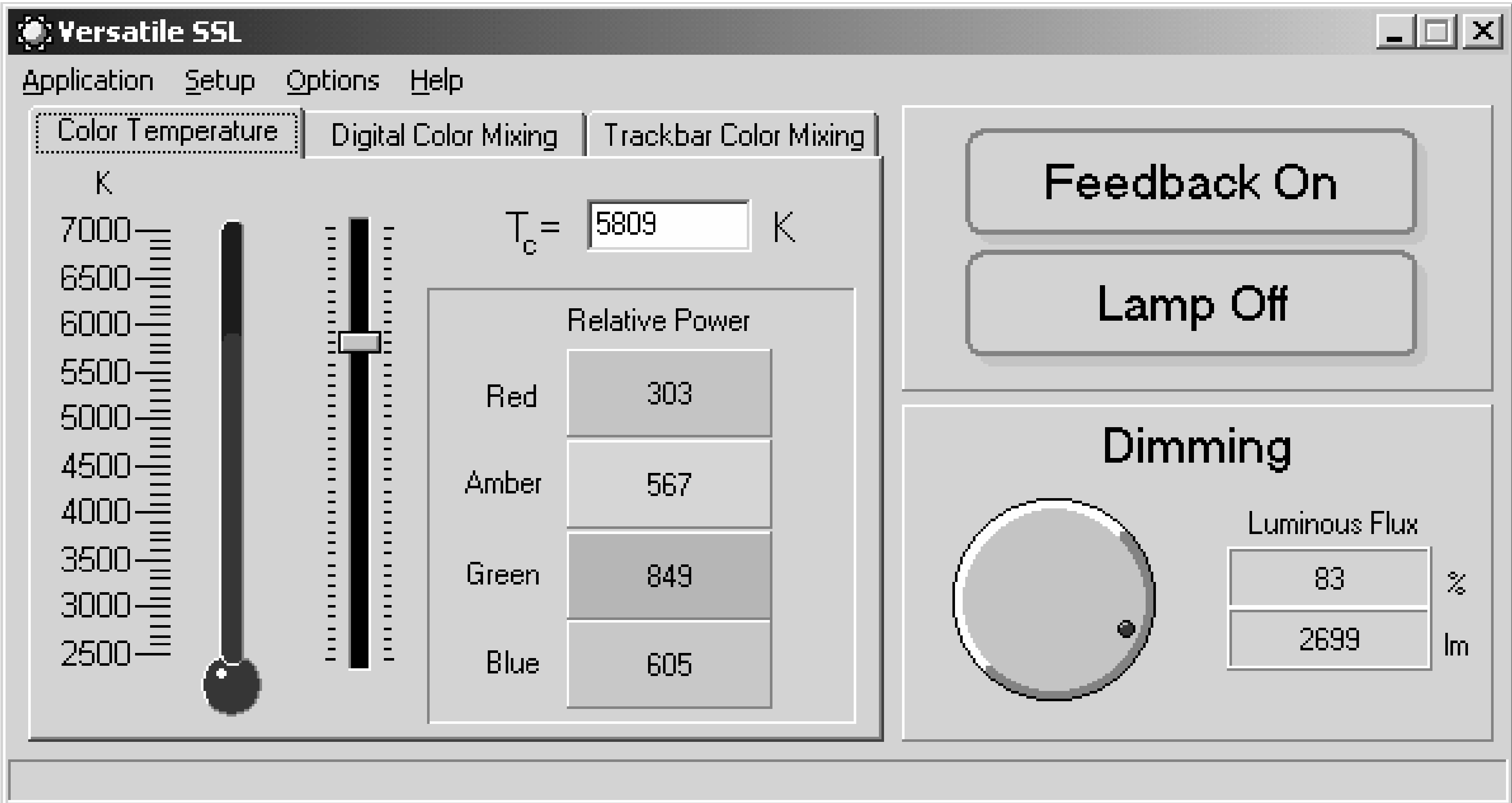


From A. Zukauskas, R. Vaicekauskas, G. Kurilcik, Z. Bliznikas, K. Breive, J. Krupic, A. Rupsys, A. Novickovas, P. Vitta, A. Navickas, V. Raskauskas, M. S. Shur, and R. Gaska, Quadrichromatic white solid-state lamp with digital feedback, SPIE 5187-24

High-power LED based Versatile White Multichip Lamp

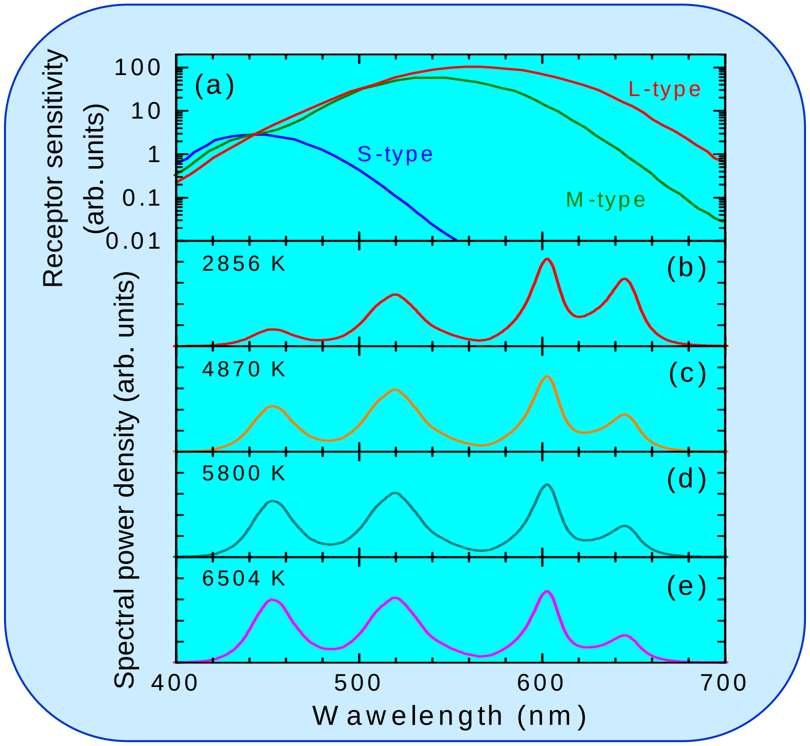


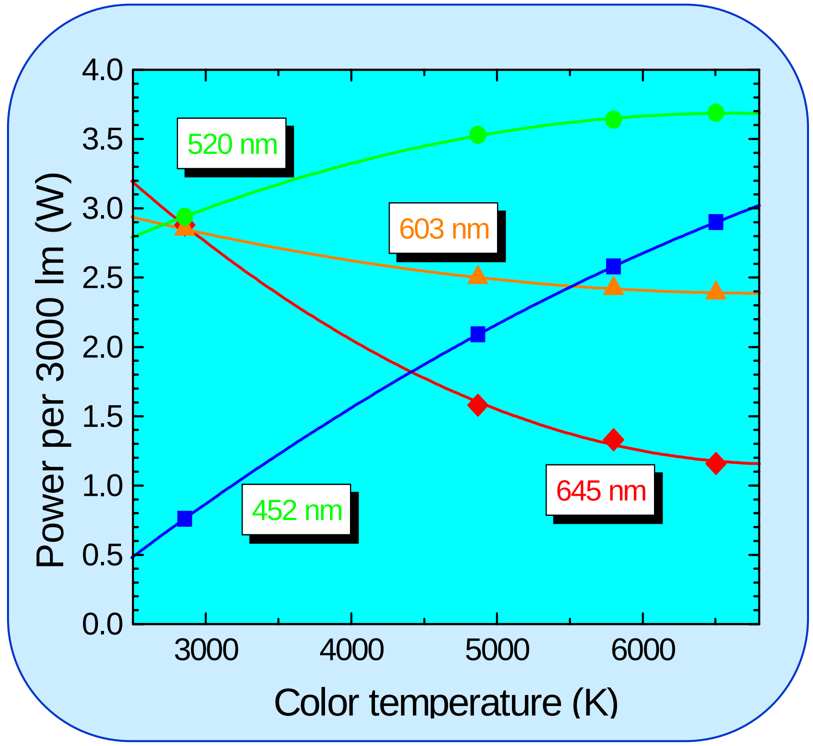
VSSL User Interface



From A. Zukauskas, R. Vaicekauskas, G. Kurilcik, Z. Bliznikas, K. Breive, J. Krupic, A. Rupsys, A. Novickovas, P. Vitta, A. Navickas, V. Raskauskas, M. S. Shur, and R. Gaska, Quadrichromatic white solid-state lamp with digital feedback, SPIE 5187-24

Color Temperature Control in a Quadrichromatic Source of White Light





From A. Zukauskas, R. Vaicekauskas, G. Kurilcik, Z. Bliznikas, K. Breive, J. Krupic, A. Rupsys, A. Novickovas, P. Vitta, A. Navickas, V. Raskauskas, M. S. Shur, and R. Gaska, Quadrichromatic white solid-state lamp with digital feedback, SPIE 5187-24

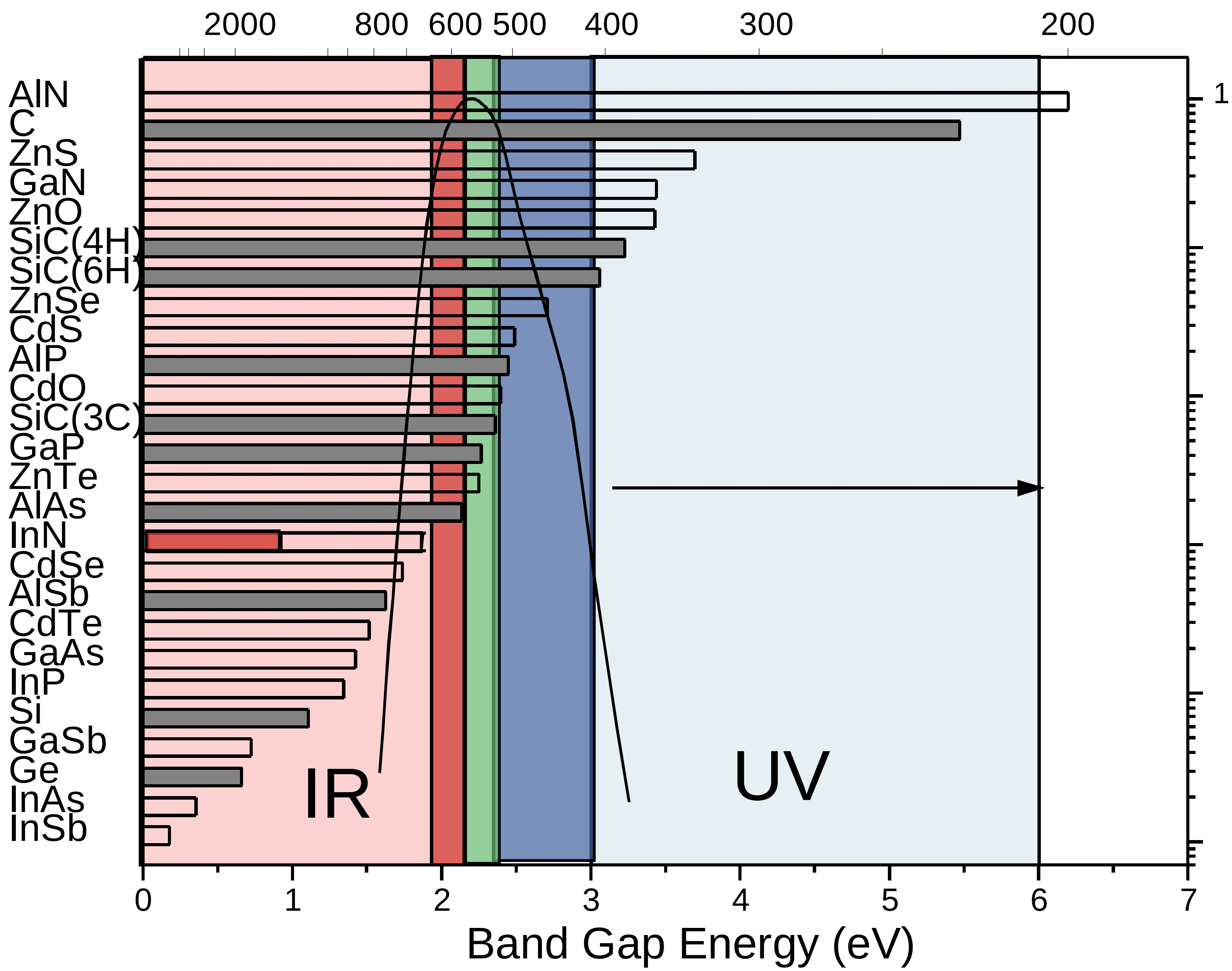
Applications of U of V/RPI/SET quadrichromatic Versatile Solid-State Lamp: Phototherapy of seasonal affective disorder at Psychiatric Clinic of Vilnius University

- Seasonal affective disorder (SAD) strikes some people in the northern latitudes
- Bright white light is known to counteract SAD
- The exact mechanism of treatment is not known

Preliminary results for SAD Treatment

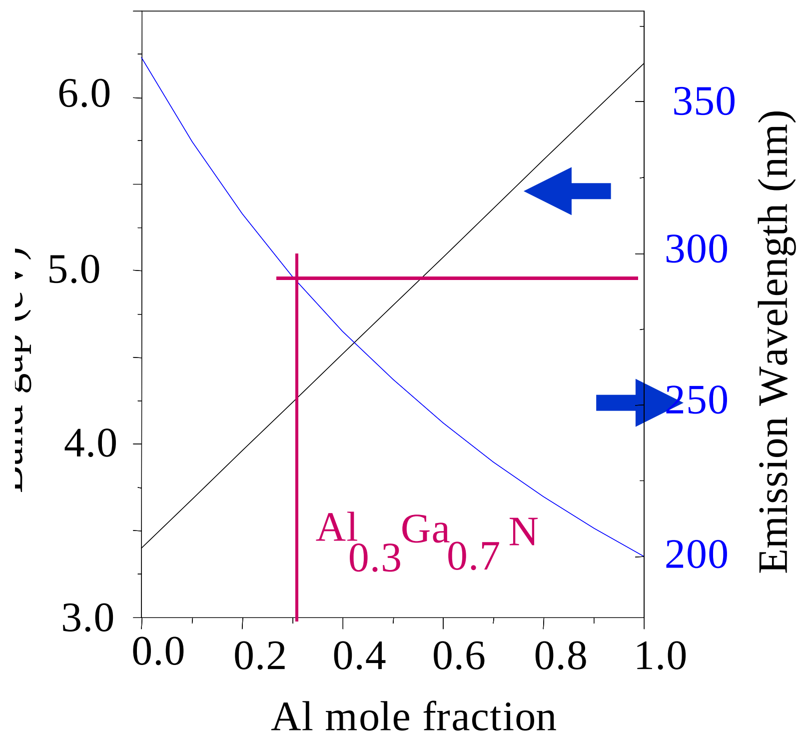
A pilot investigation of color temperature selection was carried out on healthy subjects. Preliminary results showed that subjects with the emotional scale exhibiting anxiety required light with a higher color temperature.

SEMICONDUCTOR MATERIALS SYSTEMS FOR HIGH BRIGHTNESS LEDs



http://nina.ecse.rpi.edu/shur/

III-Nitride $Al_xGa_{1-x}N$ UV Emitters require high Al molar fraction (X)



AlGaN heterostructures involve strain and polarization:

Strain Energy Band Engineering

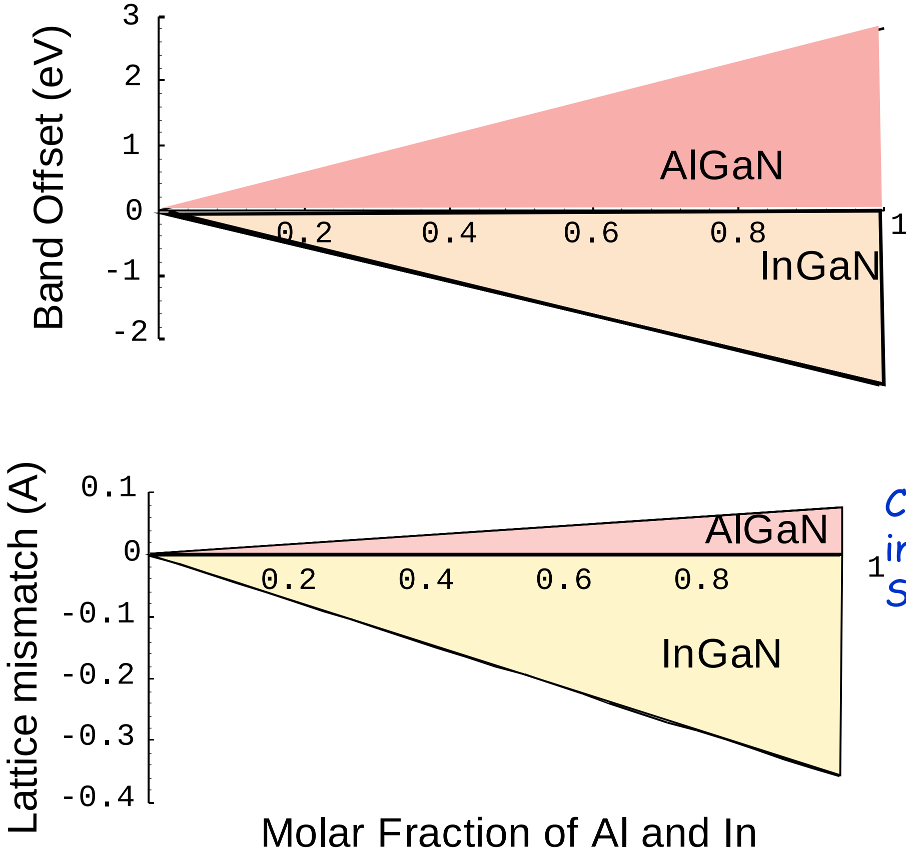
- Quaternary
- Strain relief via superlattice buffers
- MEMOCVD
- Non-polar substrates
- Homoepitaxy

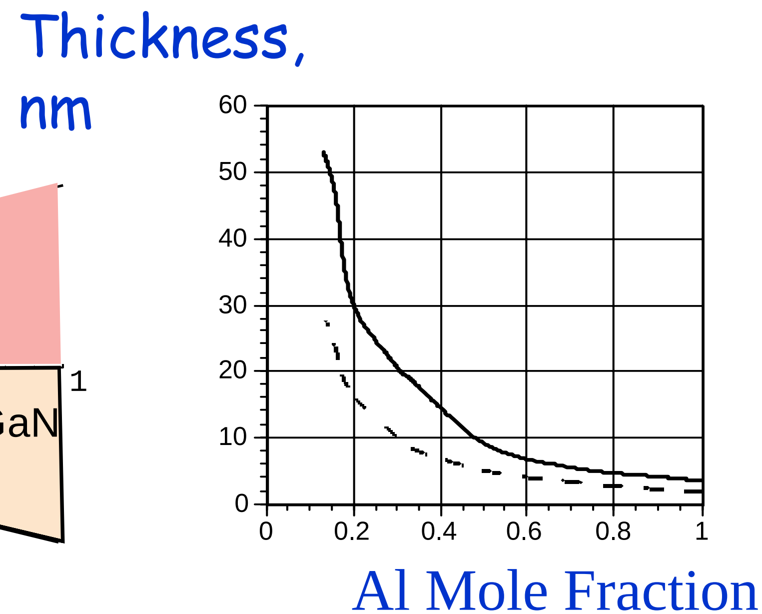
Thermal and current management

Strain Energy Band Engineering

Use AlGaInN - quaternary

Use strain controlling superlattices

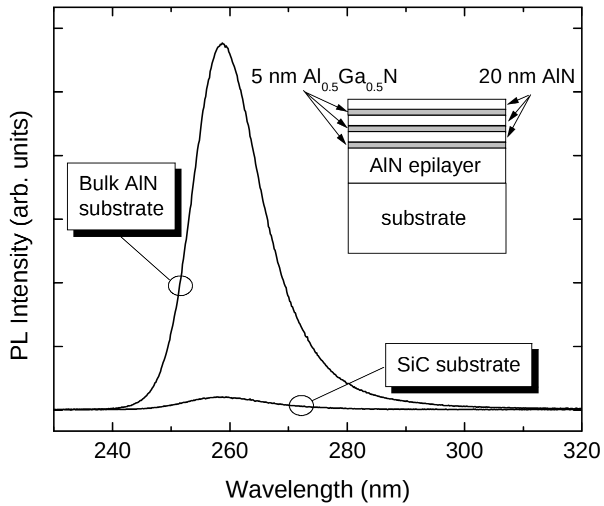


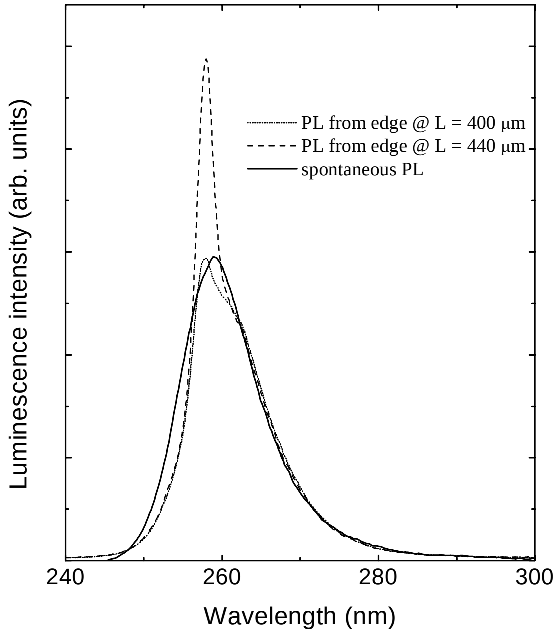


Critical thickness as a function of Al mole fraction in AlxGa1-xN/GaN: superlattice (solid line), SIS structure (dashed line).

From A. D. Bykhovski, B. L. Gelmont, and M. S. Shur, J. Appl. Phys. 81 (9), 6332-6338 (1997)

AlN/AlGaN Room Temperature Photoluminescence





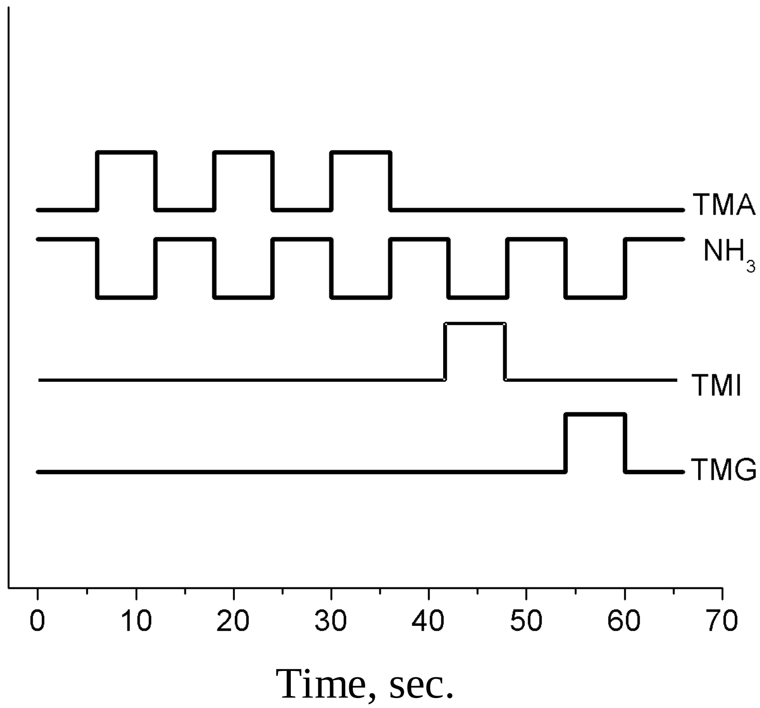
PL signal from MQWs on bulk AlN is approximately 28 times stronger compared to the structure grown over SiC.

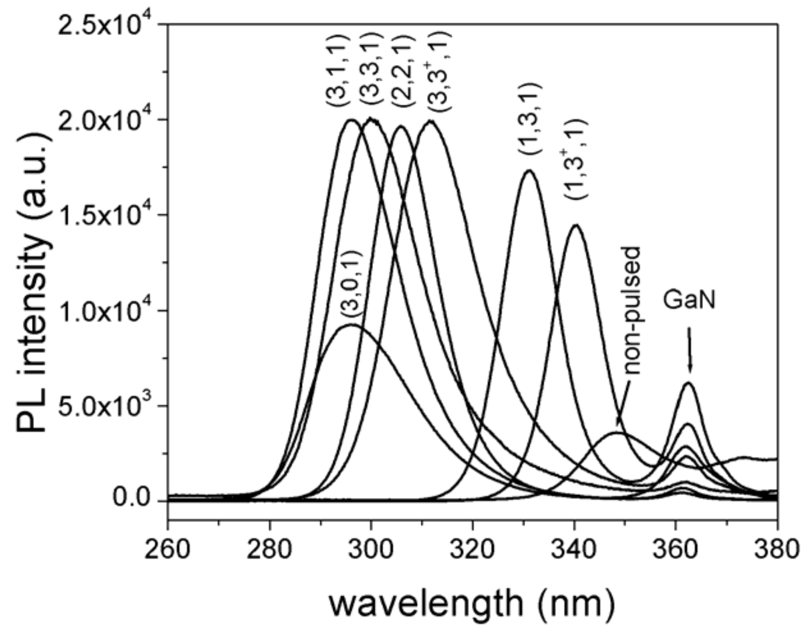
Stimulated emission at 258 nm.

From R. Gaska, C. Chen, J. Yang, E. Kuokstis, A. Khan, G. Tamulaitis, I. Yilmaz, M. S. Shur, J. C. Rojo, L. Schowalter, . Deep-ultraviolet emission of AlGaN/AlN quantum wells on bulk AlN, accepted at APL

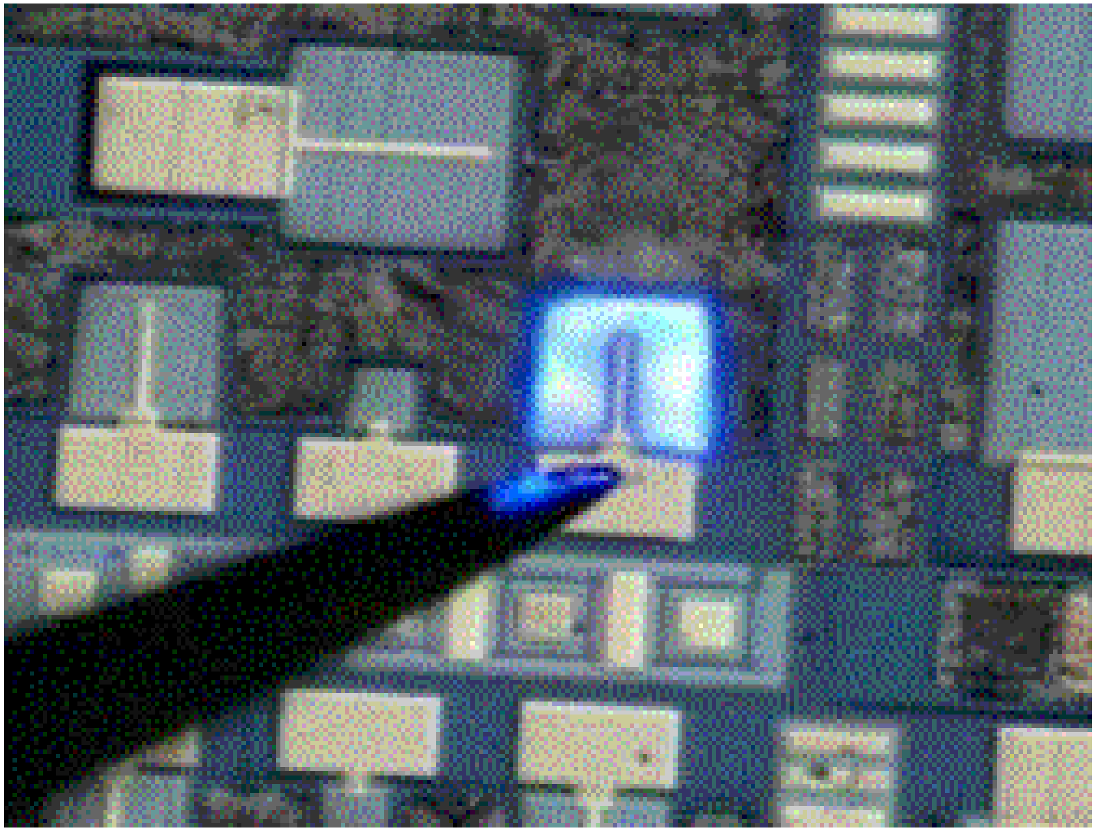
From R. Gaska, C. Chen, J. Yang, E. Kuokstis, A. Khan, G. Tamulaitis, I. Yilmaz, M. S. Shur, J. C. Rojo, L. Schowalter, . Deep-ultraviolet emission of AlGaN/AlN quantum wells on bulk AlN, accepted at APL

Pulsed Atomic Epitaxy: A representative growth unit cell of PALE.
New Development – MEMO-CVD



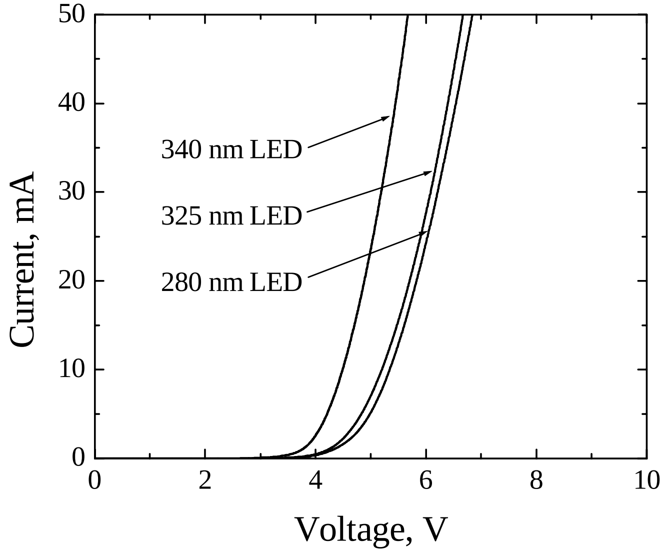


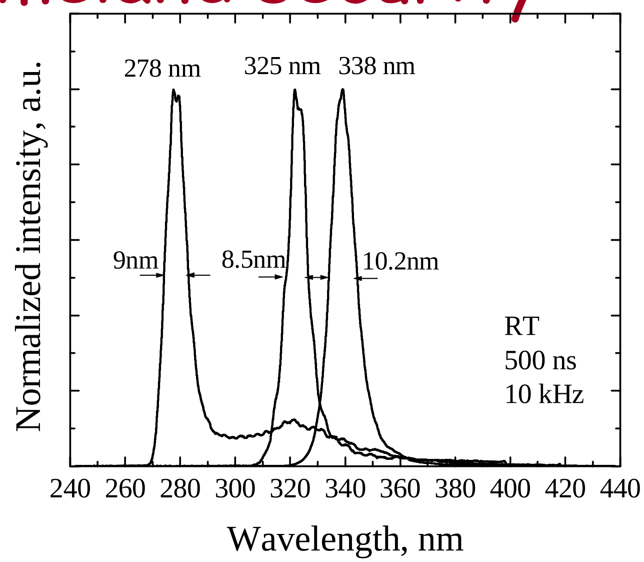
Ultimate heteroepitaxy: blue LED on Si



Courtesy of SET, Inc.

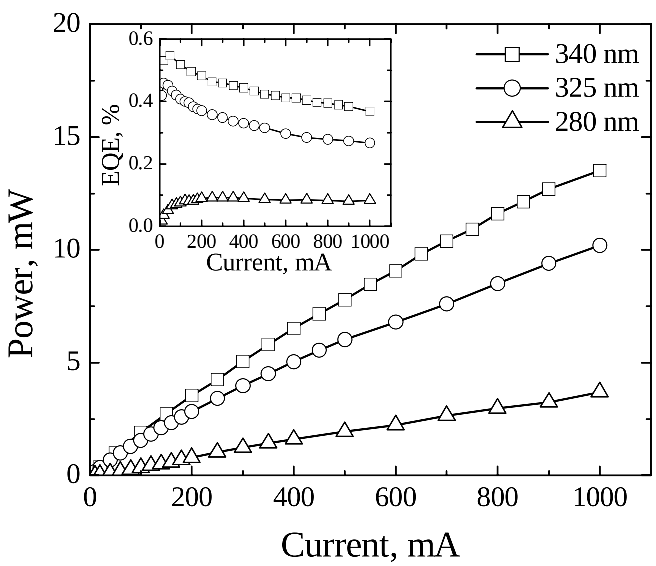
USC/SET UV LED for solid state lighting and homeland security

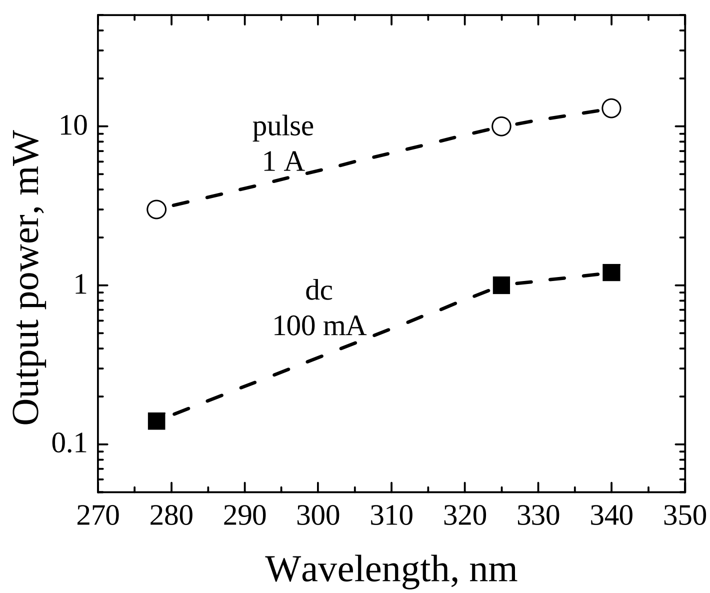




From:

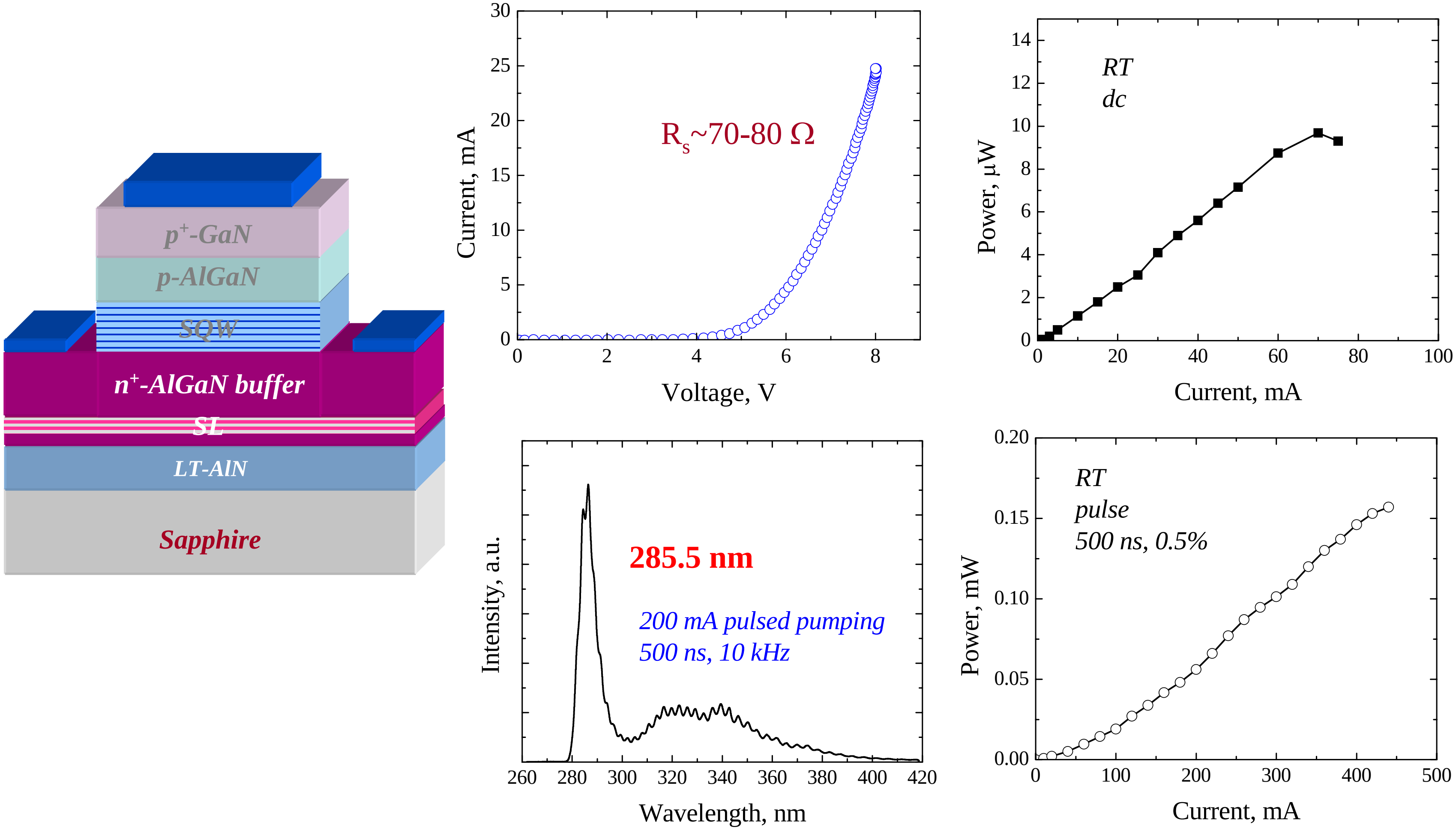
A. Chitnis, V. Adivarahan, J. Zhang, M. Shatalov, S. Wu, J. Yang, G. Simin, M. Asif Khan, X. Hu, Q. Fareed, R. Gaska, and M. S. Shur, Milliwatt power AlGaN quantum well deep ultraviolet Light emitting diodes, physics status solidi (2003) To be published





http://nina.ecse.rpi.edu/shur/

Sub-milliwatt power 285 nm Emission UV LED on Sapphire.



V. Adivarahan, S. Wu, A. Chitnis, R. Pachipulusu, V. Mandavilli, M. Shatalov, J. P. Zhang, M. Asif Khan, G. Tamulaitis, A Sereika, I. Yilmaz, M. S. Shur, and R. Gaska, AlGaN Single Quantum Well Light Emitting Diodes with Emission at 285 nm, Appl. Phys. Lett., Vol. 81, Issue 19, pp. 3666-3668 (2002)

Solid State Lighting Funding

Nothing comes from nothing (Fresco at Vilnius University)

US DOE Mission Statement

Develop viable methodologies to conserve 50% of electric lighting load by the year 2010

From Status Report:
Edward D. Petrow
DOE's Solid State Lighting Initiative For General Illumination
Photonics West
January 24, 2001
San Jose, CA

San Francisco, California, June 24, 2002

U.S. Secretary of Energy Spencer Abraham told the United States Energy Association that his department is exploring the use of solid-state lighting utilizing Light Emitting Diodes (LEDs) as part of its ongoing campaign to reduce energy usage in the U.S. He displayed a Luxeon LED light source from Lumileds Lighting and called high-power LEDs "a revolutionary technological innovation that promises to change the way we light our homes and businesses."

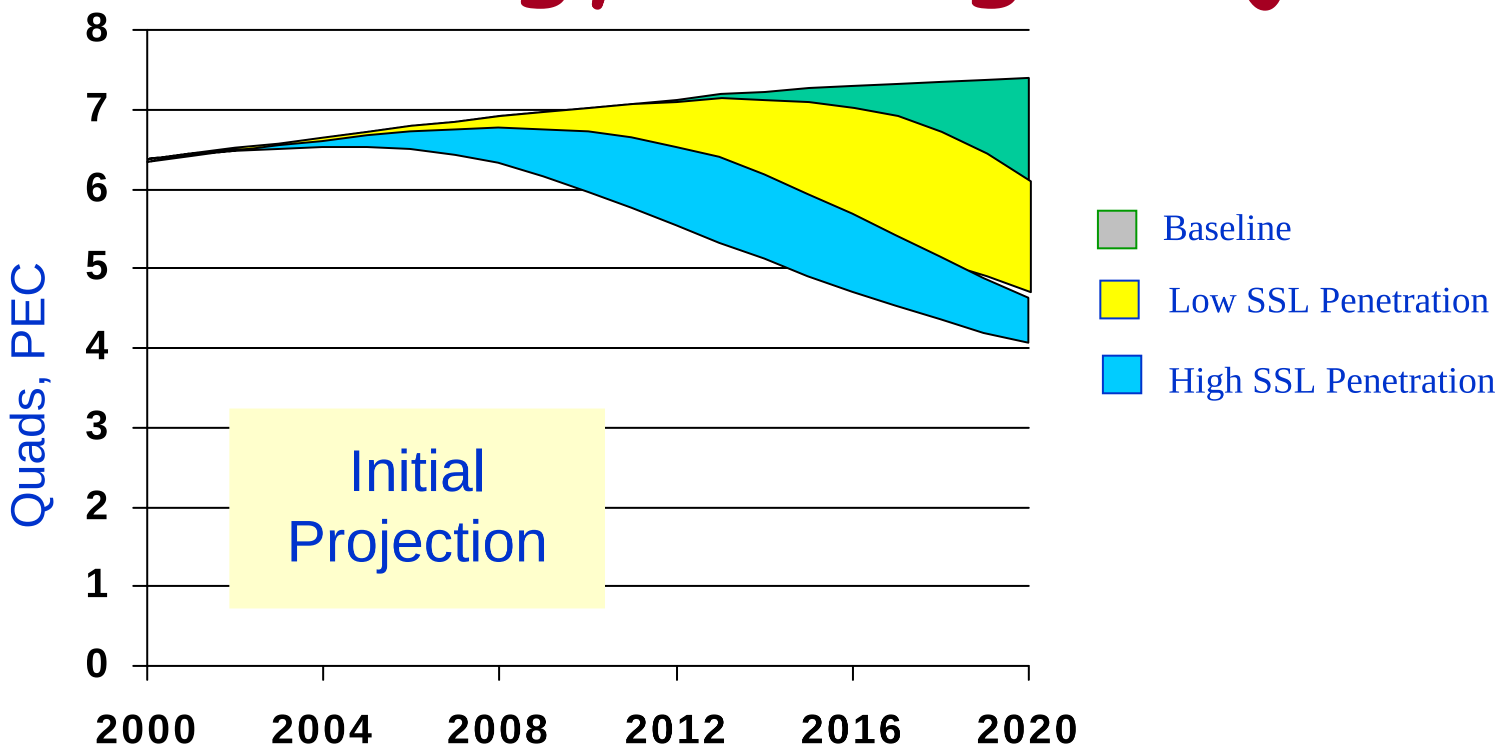
White 5 W 120 lm
5500 K color temperature Lumileds Luxeon

How energy saving will be achieved

R&D Topic Area	% of Goal
New Light Sources	25%
Solid State Lighting	45%
Advanced Electronics & Integrated Controls	15%
Improved Fixtures & Market Penetration	10%
Human Factors	5%

From Status Report: Edward D. Petrow
DOE's Solid State Lighting Initiative For General Illumination, *Photonics West January 24, 2001 San Jose, CA*

DOE Energy Saving Projection



Note: The projection assumes a constant primary energy consumption conversion ratio of 3.22 from end-use electricity to PEC. (source: BTS Core Databook, 2000)

From Status Report: Edward D. Petrow
DOE's Solid State Lighting Initiative For General Illumination,
Photonics West January 24, 2001 San Jose, CA

LED Applications

LED Applications
Signals and Displays

- POWER SIGNALS
  - Traffic Lights
  - Automotive Signage
  - Miscellaneous Signage
- DISPLAYS
  - Alphanumeric Displays
  - Full Color Video Displays

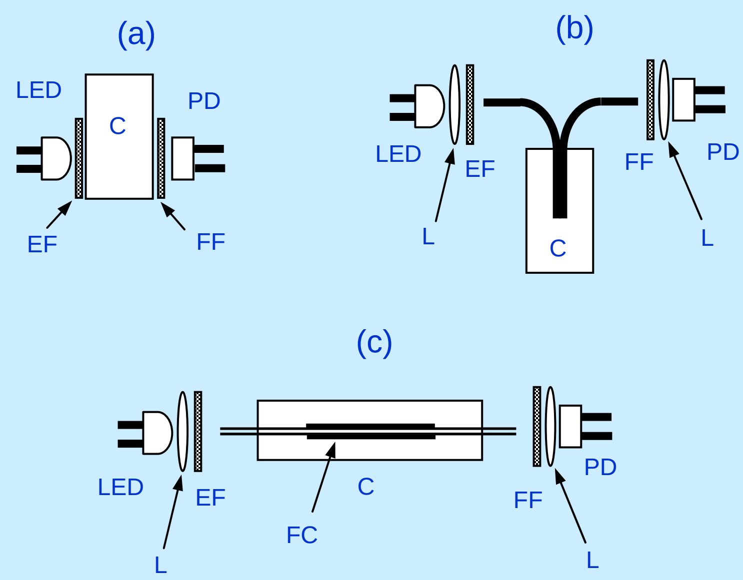
LED Applications (Biomedical)

- MEDICAL APPLICATIONS
  - Phototherapy of Neonatal Jaundice
  - Photodynamic Therapy
  - Photopolymerization of Dental Composites
  - <u>Phototherapy of Seasonal Affective Disorder</u>

- PHOTOSYNTHESIS
  - Plant Growing
  - Photobioreactors

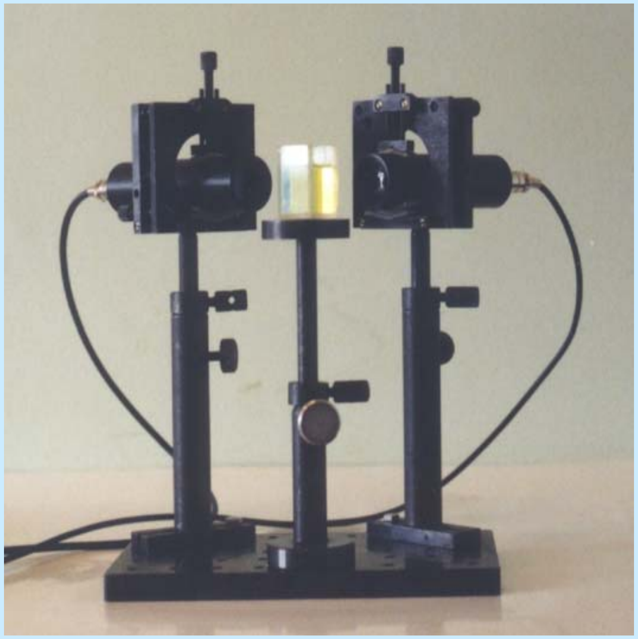
- OPTICAL MEASUREMENTS
  - Fluorescent Sensors
  - Time-Domain and Frequency-Domain Spectroscopy
  - Other Optical Applications

LED-Based Fluorimetry



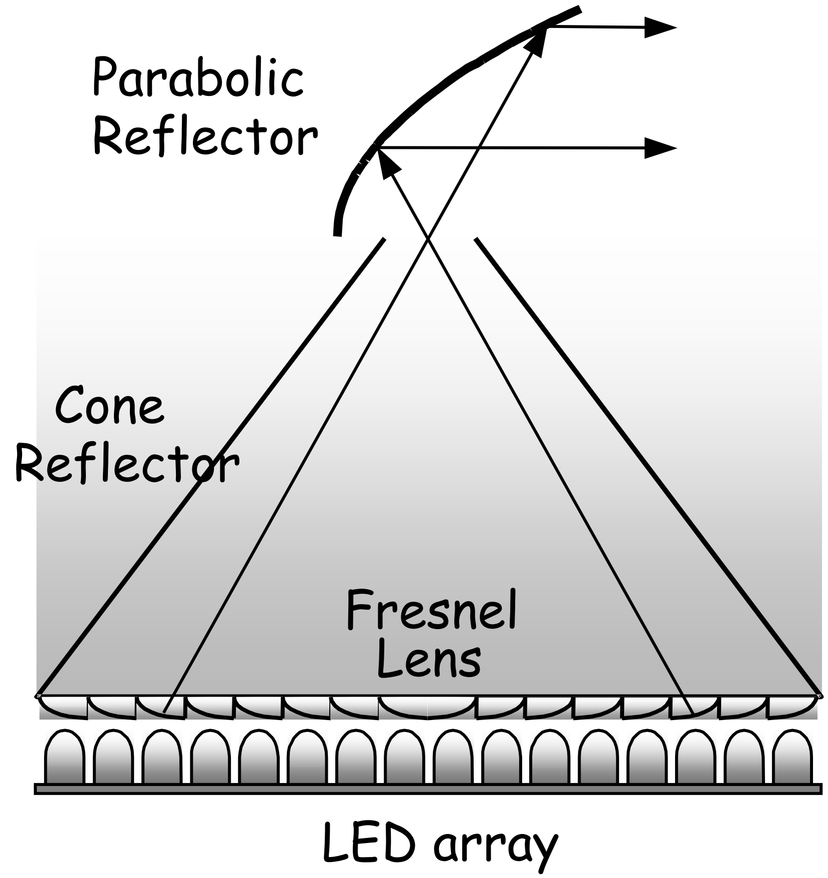
- Design versatility
- Low-noise response
- Electronic modulation
- Low heat production
- Small dimensions
- Longevity
- Durability
- Low cost

UV-LED Based Fluorimeter with Integrated Lock-in Amplifier



LED Applications - Lighting

- ILLUMINATION
  - Local Illumination
  - General Lighting

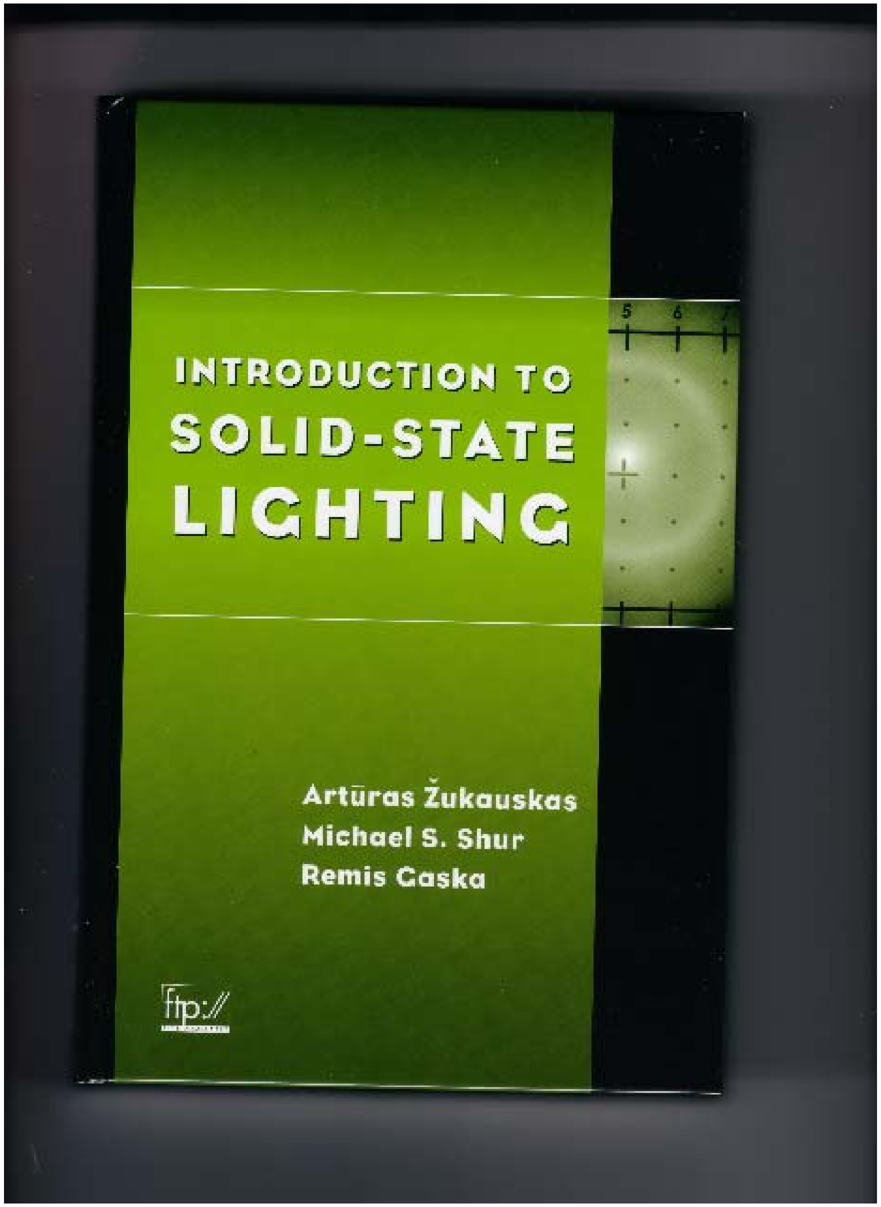


LED Floodlight (after A.García-Botella *et al.*, *J. IES* **29**, 135, 2000)

Grand Challenge:

General Lighting

Solid State Lighting - To probe further:



Book

A. Žukauskas, M. S. Shur, and R. Gaska, Introduction to Solid State Lighting, John Wiley and Sons, 2002, ISBN: 0471215740

Reviews/chapters:

A. Žukauskas, R. Vaicekauskas, F. Ivanauskas, M. S. Shur and R. Gaska, Optimization of white all-semiconductor lamp for solid-state lighting applications, in in Frontiers in Electronics: Future Chips Proceedings of the 2002 Workshop on Frontiers in Electronics (Wofe-02) St. Croix, Virgin Islands, World Scientific Pub Co; (January 15, 2003),

R. Gaska, A. Žukauskas, M. S. Shur, and M. A. Khan, Progress in III-nitride based white light sources, in SPIE Proceedings, to be published (Invited paper)

A. Žukauskas, M. S. Shur, and R. Gaska, Solid State Lighting, in: *Future Trends in Microelectronics: The Nano Millennium*, New York: Wiley, 2002, S. Luryi, J. M. Xu, and A. Zaslavsky, eds.

A. Žukauskas, M. S. Shur, and R. Gaska, Light-emitting diodes: progress in solid-state lighting, MRS Bulletin, pp. 764-769, October (2001)

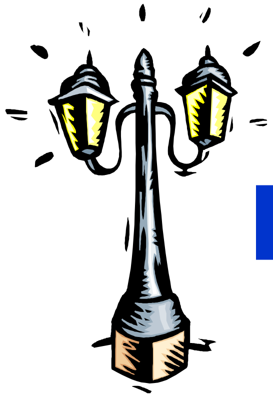
A. Žukauskas, M. S. Shur, R. Gaska, Solid-State Lamps, McGraw Hill Technical Encyclopedia

M. Asif Khan, J. Yang, G. Simin, R. Gaska, and M. S. Shur, Strain energy band engineering approach to AlN/GaN/InN heterojunction devices, pp. 195-214, in Frontiers in Electronics: Future Chips Proceedings of the 2002 Workshop on Frontiers in Electronics (Wofe-02) St. Croix, Virgin Islands, World Scientific Pub Co; (January 15, 2003),

Conclusion

"... it is vital to know that the LED is an ultimate form of lamp, in principle and in practice, and that its development indeed can and will continue until all power levels and colors are realized."

HOLONYAK, N., JR. (2000), "Is the light emitting diode (LED) an ultimate lamp?" *Am. J. Phys.* **68** (9), pp. 864-866.



Appendix: Math of Color Rendering

Reference source

$$S_r(\lambda) \rightarrow S_r(\lambda)\,\rho_i(\lambda), \; i = 1, \ldots, 8$$

Test source

$$S_k(\lambda) \rightarrow S_k(\lambda)\,\rho_i(\lambda), \; i = 1, \ldots, 8$$

USC chromaticity coordinates

$$u = 4x/(-2x + 12y + 3) \;, \; v = 6y/(-2x + 12y + 3)$$

General color rendering index (CRI)

$$R_a = \frac{1}{8}\sum_{i=1}^{8} R_i$$

where R_i is

$$= 100 - 4.60\left\{\left[W_{ki} - W_{ri}\right]^2 + 13^2\left[W_{ki}\left(u'_{ki} - u_r\right) - W_{ri}\left(u_{ri} - u_r\right)\right]^2 + 13^2\left[W_{ki}\left(v'_{ki} - v_r\right) - W_{ri}\left(v_{ri} - v_r\right)\right]^2\right\}^{1/2}.$$

$$W = 25Y^{1/3} - 17$$

$$c = \left(4 - u - 10v\right)/v$$

$$d = \left(1.708v + 0.404 - 1.481u\right)/v$$

$$u'_{ki} = \frac{10.872 + 0.404\,c_r c_{ki}/c_k - 4\,d_r d_{ki}/d_k}{16.518 + 1.481 c_r c_{ki}/c_k - d_r d_{ki}/d_k}$$

$$v'_{ki} = \frac{5.520}{16.518 + 1.481 c_r c_{ki}/c_k - d_r d_{ki}/d_k}$$

Primary Energy Consumption in Buildings by End Use (US, 1998)

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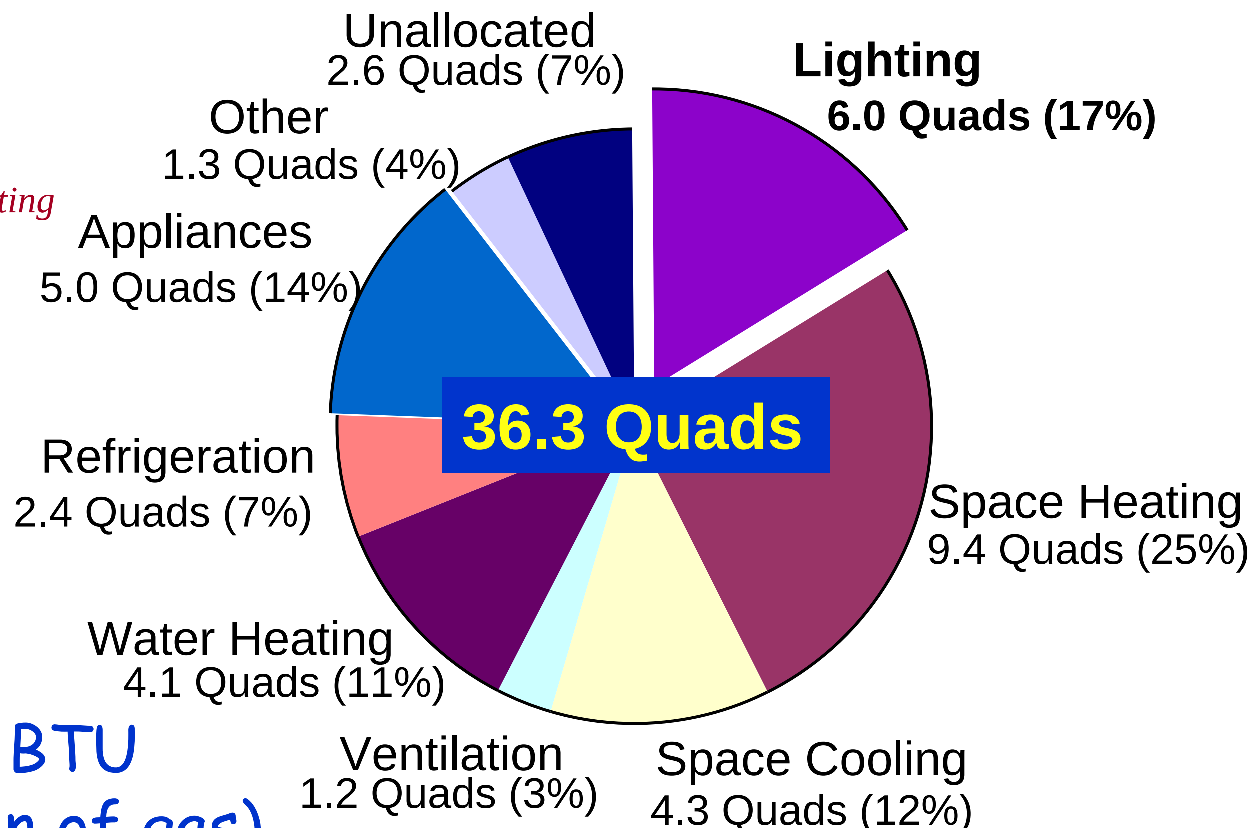
San Jose, CA



1 Quad = 10^{15} BTU
(~8× 10^{9} gallon of gas)

Source: BTS Core Databook, August 2000, Table 1.1.7 and Industrial Building Estimate from Table 1.3.11

C'_L

Cost of light

- Estimated from the cost of the lamp and the electric power consumed divided by the amount of lumens produced over the lifetime. For 1 Mlm·h, this yields a cost

$$C_{1Mlmh} \approx 10^6 \frac{C'_L}{P_L \tau_L \eta'_L} + 10^3 \frac{C_{1kWh}}{\eta'_L}$$

C'_L cost of the bulb

C_{1kWh} price of 1 kW·h power

η'_L luminous efficiency

P_L wattage

τ_L lifetime of the lamp