Forces and Fields in Special Relativity*

W. P. Ganley
Bryn Mawr College, Bryn Mawr, Pennsylvania

(Received 14 January 1963)

In most undergraduate courses that include Special Relativity, the force transformation laws are not usually discussed. Since force is an extremely fundamental physical concept, the suggestion is made that the force transformations always be included with the discussion of the transformations for mass, length and time. Two examples of force transformations applied to electromagnetic systems are described in detail, along with an elementary derivation of the force transformation laws that requires no formal calculus.

RELATIVISTIC TRANSFORMATION OF FORCES

MODERN students of physics rarely have a single comprehensive course in special relativity—they meet it *en passant* in introductory physics, optics, atomic physics, etc. As a result, undergraduates, graduate students, and even teachers may be quite familiar with the Lorentz transformations for mass, length, and time, while not really being on very good terms with the force transformation, if they have ever seen it at all! Yet the idea of force is intimately bound to the physicist's concept of science as a whole. It is directly related to the definitions of gravitational and electromagnetic fields, and is indissolubly wedded (through Newton's second law of motion) to the three basic quantities of mechanics, mass, length, and time. When one stops to think, it seems really incredible that most undergraduate textbooks that deal with the fundamentals of special relativity (books specifically on the subject excepted) not only do not derive the force transformation laws but *do not even discuss the possibility* that forces may be transformed relativistically in a way resembling the transformations for mass, length and time![1]

As an example of the confusion that can arise from an inadequate conception of relativistic force transformations, consider a recent note in this Journal entitled "Relative Force between Moving Charges."[2] There, Kolb discusses two electromagnetic systems: (1) two charges moving side-by-side in a direction normal to the line joining them, and (2) two straight parallel current-carrying wires moving parallel to themselves. He states, for case (1), that "if they are moving side-by-side relative to an observer there exists an attracting magnetic force which partially cancels the Coulomb force . . . as the speed of light is approached, the magnetic force becomes nearly equal to the Coulomb force, and the net force between the electrons approaches zero." In treating case (2), he states that, for a moving observer, "the wires still attract, and the field is as strong . . ." for the moving as for the stationary wires, and that "there is no way to make the force vanish."

In case (1) the author is correct in his conclusion that the force exerted by one charge on the other vanishes as their velocity approaches that of light. In case (2), the author is essentially correct insofar as he has implicitly considered, here, velocities low compared with that of light, since he is discussing speeds of the order of electron drift velocities in a wire (though he mentions relativity). There is excellent reason for questioning, however, the process by which he reaches his correct conclusion in case (1), and for questioning the conclusion itself for case (2), for velocities appreciable with respect to the speed of light.

Figure 1 shows a frame S and a frame S_0, coincident at time $t=t_0=0$, moving with a relative velocity $\mathbf{v}$ along the x axis. The general transformation between the force ($\mathbf{f}$) on a particle measured by observer 0 and that measured by observer 0_0 is

$$\gamma(u_0)\mathbf{f}_0=\gamma(u)\left\{\mathbf{f}+\mathbf{v}\left[\frac{\mathbf{f}\cdot\mathbf{v}}{v^2}(\gamma(v)-1)-\frac{\mathbf{f}\cdot\mathbf{u}}{c^2}\gamma(v)\right]\right\},$$

* This work was supported in part by a grant from the National Science Foundation.

[1] Two of the exceptions are Robert B. Leighton, *Principles of Modern Physics* (McGraw-Hill Book Company Inc., New York, 1960); and Edwin F. Taylor, *Introductory Mechanics* (John Wiley & Sons, Inc., New York, 1963).

[2] Kemp Bennett Kolb, Am. J. Phys. **30**, 929 (1962).

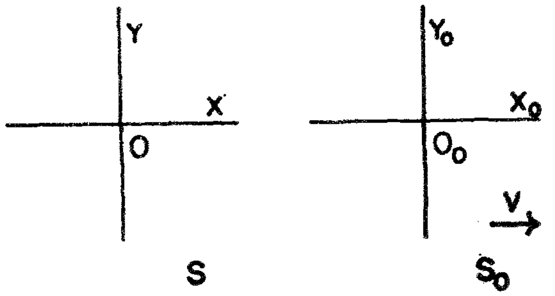

FIG. 1. The two reference frames, S and S_0, moving with relative velocity v along the x axis (the z axis is not shown).

where $\gamma(v)=[1-v^2/c^2]^{-\frac{1}{2}}$, and where the particle has a velocity $\mathbf{u}$ in frame S, and $\mathbf{u}_0$ in S_0.[3] If the particle is at rest in system S_0, $\mathbf{u}_0=\mathbf{0}$ and $\mathbf{u}=\mathbf{v}$. This gives a somewhat simpler relation

$$f_{0x}=f_x, \quad f_{0y}=f_y\gamma, \quad f_{0z}=f_z\gamma.$$

These equations hold for *any force*, not necessarily only for electromagnetic forces, and may therefore be applied to both problems under consideration here. Note that the force measured by one observer is *different* from that measured by the other, except for the special case where the direction of the force coincides with the direction of motion. *This* is the basic fact which underlies the problems discussed here, and *this* is the reason some students who meet these problems for the first time experience great perplexity. If the student is not as familiar with the force transformations as he is with those for mass, length, and time, electrodynamics problems in which they are extremely important appear paradoxical: and the relativity may well be obscured by the electrodynamics, or vice versa.

TWO ELECTRODYNAMICS PROBLEMS

Case (1). Figure 2 shows two point charges (q_1,q_2) instantaneously at rest in the S_0 system on the y_0 axis at positions $y_0=\pm d/2$. The only nonvanishing force component acting on q_1 is given by Coulomb's law

$$f_{0y}=(1/4\pi\epsilon_0)(q_1q_2/d^2).$$

The relativistic force transformation therefore gives, for the force measured by observer 0 in system S where both charges move with velocity $\mathbf{v}$

$$f_y=(1/4\pi\epsilon_0)[q_1q_2/d^2][1-v^2/c^2]^{\frac{1}{2}},$$

and the force does indeed approach 0 as $v\rightarrow c$.

In terms of electric and magnetic fields, the problem is not so simple. It may be solved in at least two somewhat different ways: by using the relativistic transformation for arbitrary electric and magnetic fields, or by finding the fields due to a moving charge from the Liénard–Wiechert (retarded) potentials and then, having the fields, using the well-known expression for the Lorentz force. In the S frame, the fields at the location of q_1 due to the charge q_2 moving with a velocity $\mathbf{v}$ are

$$\mathbf{E}=(1/4\pi\epsilon_0)(q_2\gamma/d^2)\mathbf{j},$$
$$\mathbf{B}=(1/c^2)\mathbf{v}\times\mathbf{E},$$

where the conditions of the present problem have been inserted to simplify the expressions.[4] The force on a charge q_1 moving with a velocity $\mathbf{v}$ is given by the "Lorentz Force."

$$\mathbf{f}=q_1(\mathbf{E}+\mathbf{v}\times\mathbf{B}).$$

Substituting for $\mathbf{B}$ and $\mathbf{E}$, one obtains the following expression, which is identical with that obtained by direct relativistic force transformation

$$f_y=(1/4\pi\epsilon_0)(q_1q_2/d^2)\gamma[1-v^2/c^2].$$

Since the first term contains a factor γ, there is

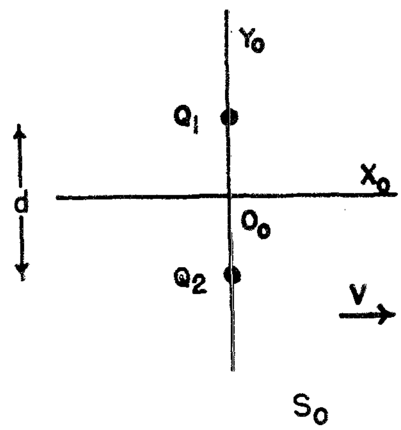

FIG. 2. Charges q_1 and q_2 located on the y_0 axis at $\pm d/2$. The coordinate system has velocity v to the right.

[3] W. Rindler, *Special Relativity* (Oliver & Boyd, Edinburgh, Scotland and London, 1960), pp. 83, 43.

[4] W. K. H. Panofsky and M. Phillips, *Classical Electricity and Magnetism* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962), 2nd ed., pp. 317, 330, 333.

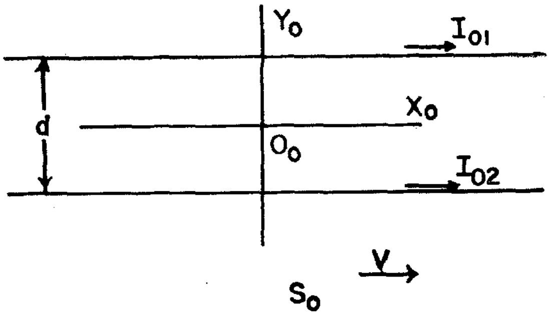

FIG. 3. Two very long current-carrying wires, a distance d apart, parallel to each other and to the x axis. The coordinate system has velocity v to the right.

nothing that can truly be identified with the ordinary Coulomb force unless the law of charge conservation is discarded; it, therefore, is incorrect to say that a magnetic force "partially cancels the *Coulomb* force." One can, indeed, say that there is an *electrostatic force* and an *electromagnetic force*, and that these cancel one another as the velocity of the charges (or the observer) nears the velocity of light. Note, however, that, in the limiting case, each of these terms is infinitely great, despite the fact that their difference vanishes.

The "moral" to be drawn from the tale of the two charges is that an electromagnetic system that consists only of static charges in one preferred frame consists both of charges and currents in inertial frames moving with respect to the first one and that, in general, the *retarded* potentials are required to describe the interaction.

Case (*2*). Figure 3 shows two very long straight wires of length L_0 in system S_0, carrying currents I_{01} and I_{02} in the direction of positive x_0. The wires pass through points $y=\pm d/2$, and are parallel to the x and x_0 axes. The force exerted on a length L_0 of the wire carrying the current I_{02} as seen by observer 0_0 is

$$f_{0y}=(\mu_0/2\pi)(I_{01}I_{02}L_0/d).$$

The relativistic transformation at once gives the force seen in the S frame, where the wires move with velocity $\mathbf{v}$:

$$f_y=(\mu_0/2\pi)(I_{01}I_{02}L_0/d)[1-v^2/c^2]^{\frac{1}{2}}.$$

This force vanishes in the limit of $v\rightarrow c$. Note that if each observer measures the *force per unit length* on the wire in his own system, *these* indeed are equal, since the two observers cannot agree on the meaning of a unit length, the discrepancy being the factor $[1-v^2/c^2]^{\frac{1}{2}}$ that appears in the well-known Lorentz contraction formula. But, if we wish to discuss the force on an object, we must decide what the object is! Let observer 0_0, then, paint two spots on his wire to determine a length L_0 in his system and a length $L=L_0\times[1-v^2/c^2]^{\frac{1}{2}}$ in system S. The force on *this* length (the only meaningful quantity in this case, since infinitely long circuits are difficult to come by) is given by the equations above, and differs for the two observers.

It might very well be expected that one could do this problem by a method more meaningful to the student of electrodynamics. Consider, therefore, the relativistic transformations for charge and current[4,5]:

$$i_x=\gamma i_{0x}$$
$$\rho=-\gamma[i_{0x}(v/c^2)],$$

where i_x is the current density, and is assumed to be merely I/A, A being the constant cross-sectional area of each of the wires (invariant in the relativistic transformation).[6] The physical significance of these transformations is that the *uncharged*, current-carrying wire, when viewed by a moving observer, becomes a *charged*, current-carrying wire, the charge and current being determined by the equations above.

For observer 0, moving with respect to the wires, the problem of finding the force exerted on a length L of wire number 2 resolves itself into two parts: finding an electromagnetic force and an electrostatic force. Retarded potentials need not be employed in this particular example since the wires are very long and, therefore, the charge and current densities are not changing with time. The electromagnetic force is

$$f_y^{(M)}=+\frac{\mu_0}{2\pi}\frac{I_1I_2L}{d}=+\frac{\mu_0}{2\pi}\frac{I_{01}I_{02}}{d}\gamma^2\left(\frac{L_0}{\gamma}\right).$$

The electrostatic force per unit length between two very long wires having static charge densities, respectively, of ρ_1 and ρ_2 (and cross sec-

[5] David L. Webster, Am. J. Phys. 29, 841 (1961).
[6] Terms referring to static charges in S_0 are omitted from the transformations since there are none in the present case.

tions A) is

$$F^{(E)}/L = -(1/2\pi\epsilon_0)(\rho_1\rho_2A^2/d),$$

and therefore, the electrostatic force on length L of wire 2 is

$$f_y^{(E)} = -\frac{1}{2\pi\epsilon_0}\frac{I_{01}I_{02}\gamma^2}{d}\left(\frac{L_0}{\gamma}\right)\frac{v^2}{c^4}.$$

The sum of these forces is identical with the result obtained earlier from the relativistic transformation equation for the transverse force.

As in case (1), the "moral" to be drawn here is that a system involving currents in one reference frame involves both currents and charges in any inertial frame not at rest with respect to the first.[7] To treat a more general problem, of course, the *retarded* potentials would be necessary.

RELATIVISTIC TRANSFORMATION OF FIELDS

In the above example, the S_0 system contains only magnetic fields and magnetic forces, while the S system contains both magnetic fields and forces and electrostatic fields and forces. The reader may well inquire about the general possibility of turning purely magnetic fields and forces into purely electrostatic fields and forces. In this connection, the reader may refer to a recent article by Webster, in which a calculation similar to the foregoing one is made.[5] It may be instructive to review this calculation briefly:

Considering the two long current-carrying wires, Webster calculates the forces that are exerted not on a wire but on the charge carriers in one of the wires (ignoring the electrostatic forces on charge carriers due to the slight transverse polarizations of the individual wires that are necessary to keep these charge carriers from fleeing the vicinity of the wires under the influence of the external forces).

In the system at rest with respect to the wires (but moving with respect to the charge carriers), the force is purely magnetic and results from a field that is purely magnetic. In order to compute the force on the charge carriers in a system at rest with respect to them (but moving with respect to the lattice ions of the wire), Webster notes that the moving observer sees an apparent charge on the wires, a charge given by the same relativistic transformation used earlier in this paper. He, therefore, computes an electrostatic force on the charge carriers equal to the value $f_y^{(E)}$ obtained in this paper. Note that this is not the total force on the *wire*. As Kolb points out,[2] the observer moving with the charge carriers (assumed here to be conduction electrons), observes an apparent motion of the lattice ions. This constitutes a current in each wire, and results in an additional force $f_y^{(M)}$ on the *wire*. However, the conduction electrons are at rest in this system, and do not interact with these currents.

Webster has, therefore, demonstrated that a *magnetic force* may indeed be changed into a purely *electrostatic force!* The casual reader may assume that he has also shown that a pure *magnetic field* may be transformed into a pure *electrostatic field*, since the relativistic transformation of current (and the resultant magnetic field) is not discussed in the article in question. Nevertheless, although there are no forces on the electron due to uniform magnetic fields in the system where the electron is at rest, a magnetic field is nonetheless present in this case. The *observer* (not being an electron himself) could detect it.[8]

Can *magnetic fields* ever be changed into *electrostatic fields*, as is the case with the forces? The answer is negative: this fact may be shown by examining the transformation equations for the electric and magnetic fields from S_0 to S. These transformations may be obtained by applying the force transformation to the expression for the Lorentz force; they are given by Panofsky and Phillips as[4]

$$E_x = E_{0x},$$
$$B_x = B_{0x},$$
$$\mathbf{E}_t = \gamma(\mathbf{E}_{0t} + \mathbf{v}\times\mathbf{B}_{0t}),$$
$$\mathbf{B}_t = \gamma(\mathbf{B}_{0t} - \mathbf{v}\times\mathbf{E}_{0t}/c^2),$$

(where x indicates the longitudinal direction and t indicates the transverse direction).

If $\mathbf{E}_0$ is 0 in frame S_0 (pure magnetic field),

[7] One might object to the preceding treatment on the basis that charge has apparently been created in the moving system. However, for real, finite closed circuits, the currents return on the opposite sides of the loops, giving charges of like value but opposite sign. Thus, for any closed circuit, charge remains conserved. See Ref. 5.

[8] Actually, the magnetic field *does* interact with a real electron, since the latter has a magnetic moment.

we find for the fields in the S system

$$E_x=0,$$
$$B_x=B_{0x},$$
$$\mathbf{E}_t=\gamma\mathbf{v}\times\mathbf{B}_{0t},$$
$$\mathbf{B}_t=\gamma\mathbf{B}_{0t}.$$

Is there a reference frame S in which the fields are purely electrostatic (i.e., $\mathbf{B}=0$)? If so, then $B_{0x}=0$ and $\mathbf{B}_{0t}=0$, giving both $\mathbf{E}_0$ and $\mathbf{B}_0=0$, a trivial case. Similarly, if $\mathbf{B}_0$ is 0 in frame S_0 (pure electrostatic field), the fields in S are

$$E_x=E_{0x},$$
$$B_x=0,$$
$$\mathbf{E}_t=\gamma\mathbf{E}_{0t},$$
$$\mathbf{B}_t=-\gamma\mathbf{v}\times\mathbf{E}_{0t}/c^2.$$

Now to get a pure magnetic field in S, E_{0x}, and $\mathbf{E}_{0t}$ must be 0, resulting in another triviality. In fact, if $|\mathbf{E}|>|c\mathbf{B}|$ in any one inertial frame, $|\mathbf{E}|>|c\mathbf{B}|$ in any other inertial frame (see Ref. 4, Chap. 18, Problem 2).

NEWTONIAN RELATIVITY

The comments by Kolb become more understandable when one considers the problems of the two charges and the two currents from a nonrelativistic point of view.[2] The Galilean velocity addition formulas obviously substantiate his conclusions for case (2) (the currents). His result, that the force between two parallel current-carrying wires is identical in all inertial systems, is the natural result of employing Newtonian laws: *of course*, all nonaccelerating observers measure the same forces, in the case of nonrelativistic physics!

Case (1) (the charges) is a little more interesting. In a "Newtonian world," retarded potentials become unnecessary, since one considers either that the velocities of all objects are small compared with the velocity of light, or one assumes the velocity of light to be infinite. Therefore, for the two charges, one uses Coulomb's law to find the electrostatic force and Ampere's law to find the magnetic force (where the current elements $I_1\Delta l$ and $I_2\Delta l$ are interpreted as q_1v and q_2v). The electrostatic and magnetic forces become

$$f_y^{(E)}=(1/4\pi\epsilon_0)(q_1q_2/d^2),$$

and

$$f_y^{(M)}=-\frac{\mu_0}{4\pi}\frac{I_1I_2(\Delta l)^2}{d^2}=-\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{d^2}\left(\frac{v^2}{c^2}\right),$$

where μ_0 has been written as $1/\epsilon_0c^2$.

If $\mathbf{v}$ is assumed very small, the magnetic force is negligible; the electrostatic force is not more exact than terms of order v^2/c^2 and, therefore, the total force is given unambiguously by the electrostatic force alone. Alternatively, if c is assumed to be infinite, then the second term is identically equal to zero and the first term is exactly correct. Either way, the Newtonian law of force transformation—that forces are measured to be the same value in all inertial systems—is valid.

If, however, these nonrelativistic equations are now *incorrectly* applied to the case $v\rightarrow c$, the sum of $f_y^{(E)}$ and $f_y^{(M)}$ nevertheless still approaches zero, because the magnetic force *does* become ". . . nearly equal to the Coulomb force."

ELEMENTARY DERIVATION OF THE FORCE TRANSFORMATION LAWS

The derivation of the force transformation laws for special relativity is simple in principle, since the transformation laws for mass, length, and time (the more familiar ones) may be assumed known. If relativity is taught in an introductory college (or even high school) course in which the calculus cannot be employed,[9] the following elementary approach is suggested. It is first desirable to show that transverse forces and longitudinal forces yield accelerations in these directions according to the relations

$$\mathbf{a}_t=\mathbf{F}_t/m_0\gamma,$$
$$a_x=F_x/m_0\gamma^3,$$

where the subscripts t and x represent the transverse and longitudinal directions, as previously. $m_0\gamma$ is sometimes called the "transverse mass," and $m_0\gamma^3$ is sometimes called the "longitudinal mass." (One should not hesitate to emphasize to students that the longitudinal mass is of very little physical significance, and that Newton's law in the form $\mathbf{F}=\Delta(m\mathbf{v})/\Delta t$, where $m=m_0\gamma$, is the really basic law.)

[9] One such course with which the author is familiar is the one given to freshmen at Bryn Mawr College—see W. C. Michels and A. L. Patterson, *Elements of Modern Physics* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1951).

It is assumed that persons knowing no calculus are nevertheless familiar with the "delta" notation used above, having seen the acceleration defined as $\mathbf{a}=\Delta\mathbf{v}/\Delta t$, and with the manipulation of "small quantities."

The transverse equation above may be obtained intuitively, but for skeptical students, one may start from the general form of Newton's law. $\Delta(m\mathbf{v})/\Delta t$ is just shorthand for

$$[(m+\Delta m)(\mathbf{v}+\Delta\mathbf{v})-m\mathbf{v}]/\Delta t,$$

which is equal to $[m\Delta\mathbf{v}+\mathbf{v}\Delta m]/\Delta t$, omitting second-order terms.[10] Now they can see that Δm is 0 for transverse forces, since the term $\Delta m\mathbf{v}$ is directed longitudinally, and one cannot have a transverse vector equal to a vector with a longitudinal component. Therefore, $\mathbf{F}_t=m\Delta\mathbf{v}/\Delta t$, where $m=m_0\gamma$, or $\mathbf{F}_t=m_0\gamma\mathbf{a}_t$.

Since the final step of differentiating m cannot be taken here, the longitudinal equation is obtained in another manner. Consider a particle moving with velocity $\mathbf{v}$. Let a force $\mathbf{F}_x$ act in the longitudinal direction for a short time Δt, imparting a velocity increment Δv. The work done by the force is very nearly

$$F_x(v+\tfrac{1}{2}\Delta v)\Delta t,$$

or, neglecting the second-order term

$$F_x v\Delta t.$$

The work is equal to the increase in kinetic energy. Using the Einstein relation, this increase in energy is just Δmc^2, where Δm is the increase in mass. Originally the mass was

$$m=m_0[1-(v^2/c^2)]^{-\frac{1}{2}}.$$

After the time Δt, the mass is

$$m+\Delta m=m_0[1-(v+\Delta v)^2/c^2]^{-\frac{1}{2}}.$$

One may rearrange this quantity (neglecting second-order terms) as follows[11]:

$$m+\Delta m=m_0\left[1-\frac{v^2}{c^2}\right]^{-\frac{1}{2}}\left[1-\frac{2v\Delta v}{c^2(1-v^2/c^2)}\right]^{-\frac{1}{2}}$$

$$=m_0\left[1-\frac{v^2}{c^2}\right]^{-\frac{1}{2}}\left[1+\frac{v\Delta v}{c^2(1-v^2/c^2)}\right]$$

$$=m_0\gamma\left[1+\frac{v\Delta v}{c^2}\gamma^2\right].$$

Therefore, one sees that the increase in mass is

$$\Delta m=m_0\gamma^3(v\Delta v/c^2),$$

giving the increase of kinetic energy

$$\Delta E=m_0\gamma^3 v\Delta v.$$

Equating work and kinetic energy, one obtains

$$F_x=m_0\gamma^3(\Delta v/\Delta t),$$

or, using the definition of acceleration

$$F_x=m_0\gamma^3 a_x.$$

Now the following problem may be done: suppose a force F acts on a mass m for a *short* time Δt. Using the acceleration, one may compute the small distance traveled as a result of this acceleration. The problem is of course trivial in a system (S_0) at rest with respect to the mass, and we can now also do it, using the same formalism, in the moving system.

In either direction, in the rest system (S_0), the solution is

$$\Delta x_0=\tfrac{1}{2}(F_{x0}/m_0)(\Delta t_0)^2,\quad \Delta y_0=(F_{y0}/2m_0)(\Delta t_0)^2.$$

In the transverse direction, in system S

$$\Delta y=\tfrac{1}{2}(F_y/m_0\gamma)(\Delta t)^2.$$

In the longitudinal direction, in system S

$$\Delta x=\tfrac{1}{2}(F_x/m_0\gamma^3)(\Delta t)^2.$$

Since $\Delta x=(1/\gamma)\Delta x_0$ and $\Delta y=\Delta y_0$, we obtain

$$\tfrac{1}{2}(F_x/m_0\gamma^3)(\Delta t)^2=(1/\gamma)\tfrac{1}{2}(F_{0x}/m_0)(\Delta t_0)^2,$$

and

$$\tfrac{1}{2}(F_y/m_0\gamma)(\Delta t)^2=\tfrac{1}{2}(F_{0y}/m_0)(\Delta t_0)^2.$$

Using the Lorentz transformation $\Delta t=\Delta t_0\gamma$, one obtains the desired relations

$$F_x=F_{0x},$$

and

$$F_y=F_{0y}/\gamma.$$

[10] Students knowing no calculus (even nonscience majors) can readily comprehend statements such as this, as long as they are not aware that such statements constitute the basis of differentiation—the word "calculus" seems to frighten students for whom its concepts hold no difficulty.

[11] The expansion of square roots is a perfectly legitimate operation for persons who know no calculus. In fact, the expansion of $1/(1+x)$ (by multiplying numerator and denominator by $(1-x)$ and neglecting terms of second order) and $[1+x]^{\frac{1}{2}}$ (by adding within the bracket the negligible term $x^2/4$) can serve as excellent exercises to introduce the beginning student to the manipulation of small quantities.

To recapitulate, let us inspect and compare the relativistic transformations known as the *time dilation*

$$\Delta t = \Delta t_0 \gamma,$$

the *length contraction*

$$\Delta x = \Delta x_0 / \gamma,$$

$$\Delta y = \Delta y_0,$$

$$\Delta z = \Delta z_0,$$

the relativistic *mass increase*

$$m = m_0 \gamma,$$

and (to coin a phrase) the *force diminution*

$$f_x = f_{0x},$$

$$f_y = f_{0y} / \gamma,$$

$$f_z = f_{0z} / \gamma.$$

It is perhaps worth noting that the relativistic contraction is not only a contraction, but also a rotation of the length vector, $\Delta \mathbf{L} = \mathbf{i}\Delta x + \mathbf{j}\Delta y + \mathbf{k}\Delta z$, in the plane formed by the x component and the sum of the y and z components, *away* from the x axis; similarly, the relativistic force diminution involves a rotation of the force vector in the plane formed by its x component and the sum of its y and z components, *toward* the x axis.

Although little described in this paper is novel, there seems to be a need for the discussion of such fundamental problems on a level below that of most available textbooks on graduate and even on advanced undergraduate electrodynamics and relativity. No attempt has been made here to present the electrodynamics on a level suitable for a secondary school course or a college introductory course without the calculus. That should be the job of the individual teacher—in fact, it is part of the fun of teaching. What has been attempted is the following: (1) to delineate the areas of confusion in the elementary calculation of forces in electromagnetic theory, (2) to attack each area separately, pointing out some of the basic physics that lurks behind the relativistic transformation formulas, and (3) to suggest strongly that teachers of introductory and intermediate courses dealing with relativity at an elementary level give serious consideration to treating the force transformations along with those for mass, length, and time, since all are of fundamental importance.

ACKNOWLEDGMENTS

The author wishes to thank his colleagues in the Bryn Mawr College Physics Department, Walter C. Michels, Rosalie C. Hoyt, and John R. Pruett, for their suggestions and criticisms.