How to do a Roll-down test

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To skip the explanations, go straight to the section "step by step to do a rolldown test" on page 2.

The roll-down test as proposed by Death to All Spammers of the electric vehicle discussion list consists of reaching steady speed on a level, then coasting to a stop and measuring the time t and/or distance d to stop for a given starting speed v. The idea is to do several such tests and determine the coefficients of rolling resistance and air resistance.

The retarding force due to rolling resistance friction is given by

$$F_{rr} = C_{rr} N = C_{rr} mg$$

where Crr is the (unitless) coefficient of rolling resistance, whose values can be: 0.006 to 0.01 for a low rolling resistance car tire on a smooth road, 0.010 to 0.015 for ordinary car tires on concrete.

The retarding force due to aerodynamic drag is given by

$$F_d = \frac{1}{2}\rho C_d A V^2$$

where Cd is the coefficient of drag, which can take values of 1 for a non-recumbent bicycler to 0.5 for a truck to 0.3 for an aerodynamic car to 0.1 for the Dodge Intrepid ESX (see http://en.wikipedia.org/wiki/Drag_coefficient for examples), ρ is the density of air ($\rho = 1.2\frac{kg}{m^3}$ at standard conditions), A the frontal area (projected) of the vehicle, and V the velocity.

When coasting to a stop on a level both these forces will act:

$$F_{total} = F_{rr} + F_d = -C_{rr} mg - \frac{1}{2}\rho C_d A V^2$$

where the minus signs are due to acceleration against the direction of motion.

The acceleration thus experienced is

$$a = \frac{F_{total}}{m} = -C_{rr} g - \frac{1}{2m}\rho C_d A V^2$$

This is in fact a differential equation

$$\frac{dV}{dt} = -C_{rr} g - \frac{1}{2m}\rho C_d A V^2 = -a - bV^2$$

where for convenience I have used $a = C_{rr} g$ which takes values from 0.06 to 0.2 m/s^2, and $b = \frac{\rho C_d A}{2m}$ which takes values $b = \frac{1.2x0.5x4}{2x500} = 0.0024/\text{m}$ for a small car heavily affected by wind (small mass and not aerodynamic) up to $b = \frac{1.2x1x2}{2x50} = 0.024/\text{m}$ for a light and non-aerodynamic bicycle and down to $b = \frac{1.2x0.1x2}{2x1000} = 0.00012\ /\text{m}$ for a heavy car or truck slightly affected by wind (large mass and aerodynamic).

The differential equation can be integrated

$$\int_{V_{start}}^{V_{end}} \frac{dV}{-C_{rr} g - \frac{1}{2m}\rho C_d A V^2} = \int_{t_{start}}^{t_{end}} dt$$

to find

$$= \sqrt{\frac{2m}{\rho C_d A C_{rr} g}} \tan^{-1}\left(V_{start}\sqrt{\frac{\rho C_d A}{2mC_{rr} g}} \right) = \Delta t$$

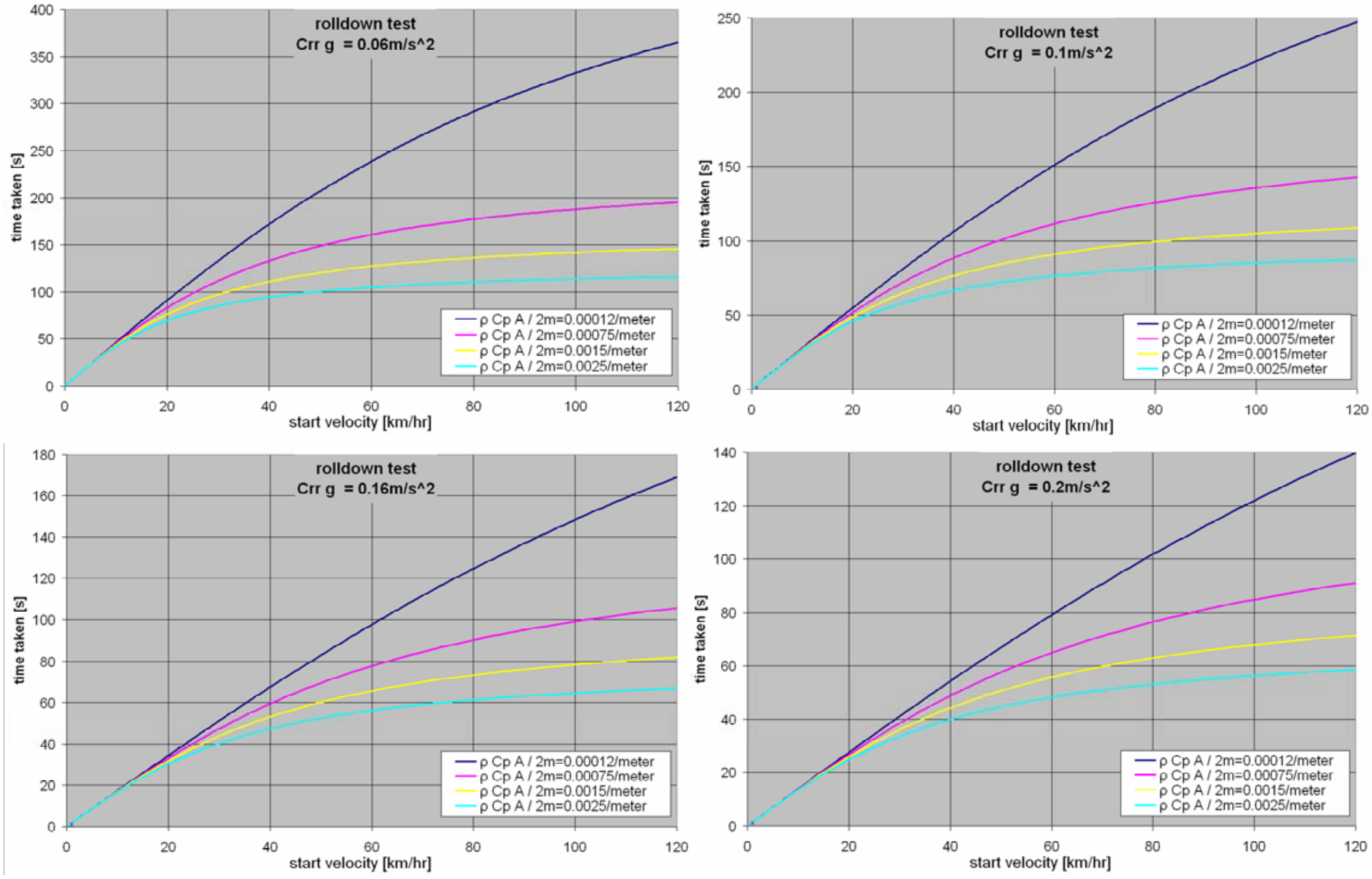
for a roll-to-stop test (where V_{end}=0).
This can be rewritten as

$$V_{start} = \sqrt{\frac{2mC_{rr}g}{\rho C_d A}} \tan\left(\Delta t \sqrt{\frac{\rho C_d A C_{rr} g}{2m}}\right) = \sqrt{\frac{a}{b}} \tan\left(\Delta t \sqrt{ab}\right)$$

We'd like to do two rolldown tests from two different starting speeds, from the two datapoints determine the curve going thru them, and thus determine a and b. But it looks like the equation cannot be solved.
However the problem can be tackled in two stages. First, determine *a* by a low-speed roll-down test, where the rolling resistance dominates. Then do a high-speed test where both factors *a* and *b* enter. Knowing *a*, we can find *b*.

How slow is slow enough that wind resistance doesn't matter?

Plotted below is the function $V_{start} = \sqrt{\frac{a}{b}} \tan\left(\Delta t \sqrt{ab}\right)$ for various values of *a* and *b*. The initial velocity (x-axis) determines the time taken for roll-down (y-axis). These are plots of time required for roll-down to stop, as it depends on initial velocity. The axes will be stretched differently according to the values of the aero and rolling coefficients. Each plot has the value of *a* fixed and four different values of *b*; together the plots span the range of likely values for cars and trucks.



What we are looking for is to operate in the linear parts of these curves, (where the time taken to roll to a stop is a linear function of velocity). In other words, below about 10-15km/hr.

Step-by-step to do a rolldown test:

1. Determine $C_{rr}g$

A. Drive at a steady low speed, and start timing the moment you take your foot off the electrons.
Stop timing as soon as you come to a stop.
v_{start} is your starting speed and *t* is the time it took to roll to a stop.
Do a few tests at different low speeds and take the ratios v_{start}/t. They should be the same if you are starting slow enough. This is the value of $C_{rr}g$. Also try both directions and average (to eliminate any small slope).

B. If you don't trust your speedometer you can measure roll-down distance d and time t, with $C_{rr}g = \frac{2d}{t^2}$ (still at low speeds). I am using mks units on my charts so take care accordingly (velocity v_{start} in m/s).

C. If you suspect some nonlinear effect near stopping, you can measure the acceleration at slow speeds but away from 0. For example if you measure the time to go from 10km/hr to 5km/hr, then $C_{rr}g = \frac{\Delta v}{t} = \frac{10km/hr - 5km/hr}{t}$. Remember to use consistent units. If you also don't have a reliable speedometer at low speeds you can use measurement of time and distance only. Put three marks on the ground at some distance you can coast at low speeds, eg at 0m, 10m, and 15m. Measure the time t_1 to coast from 1st to 2nd mark and the total time t_2 to coast (on the same run – use the 'lap' feature of your stopwatch) from 1st (not 2nd) to 3rd mark, and the corresponding distances x_1 and x_2.from 1st to 2nd and from 1st to 3rd marks. The acceleration experienced is $C_{rr}g = \frac{2}{t_2 - t_1}\left(\frac{x_2}{t_2} - \frac{x_1}{t_1}\right)$.

2. Determine *ρC*

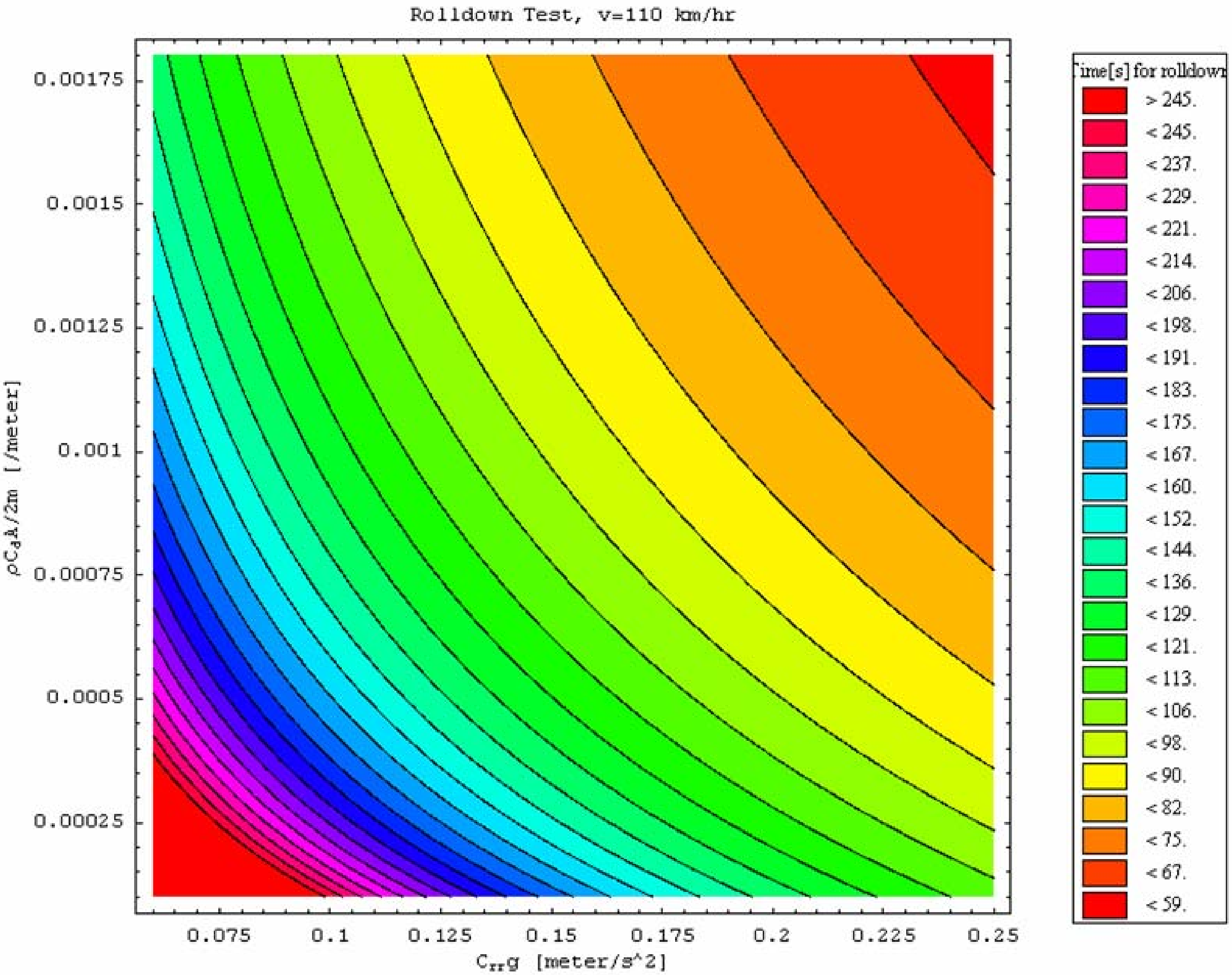
Now do a test with high starting speed (where wind resistance is important, e.g. above 70km/hr).

We have to solve $V_{start}\sqrt{\frac{b}{a}} = \tan\left(\Delta t\sqrt{ab}\right)$ for the unknown b(my abbreviation for $\rho C_d A/2m$), possible in several ways:

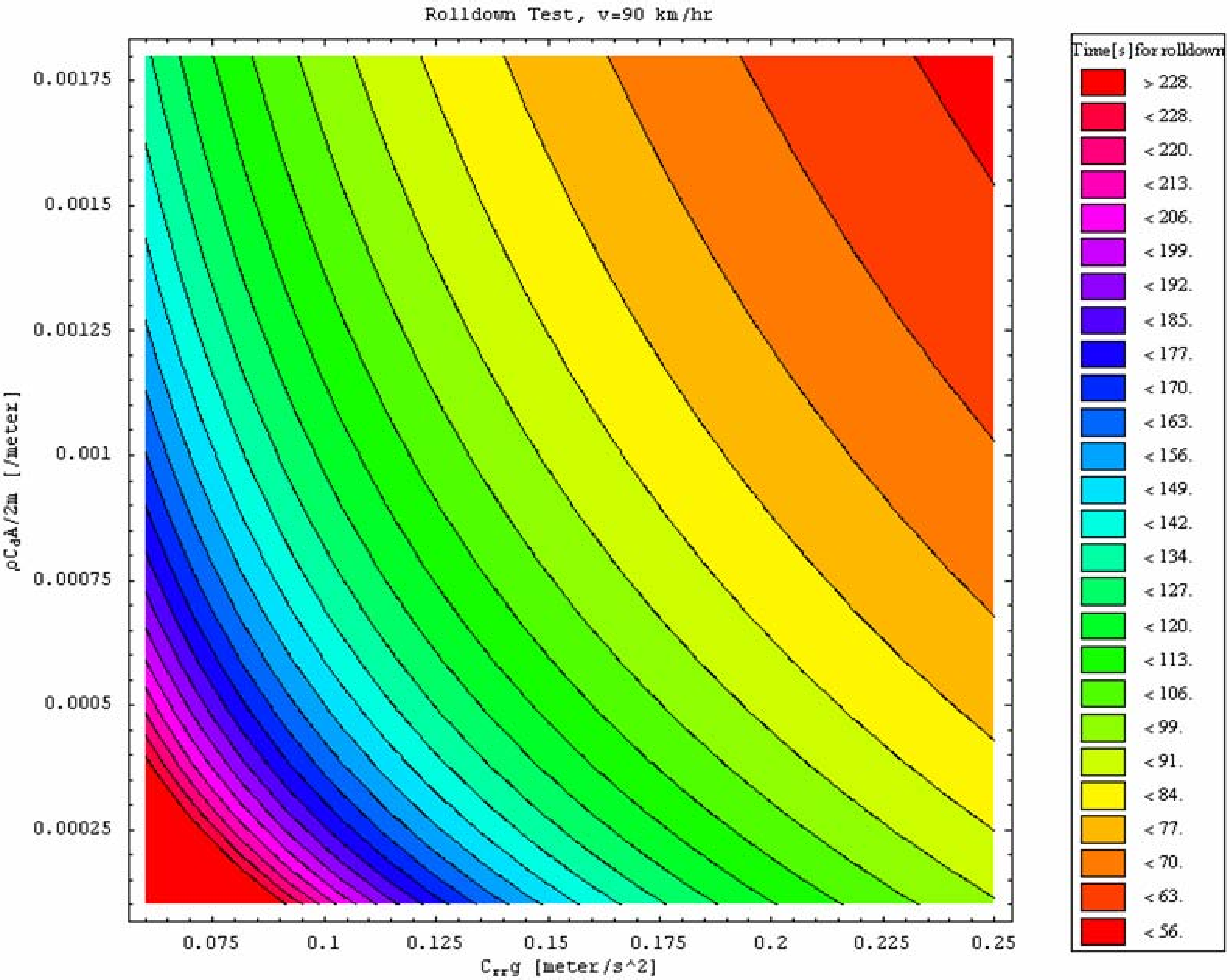
A. Use the contour plots below. Each plot is for a different starting velocity (labeled at top, e.g. the first is for a v_{start} of 110km/hr). Find your value of $C_{rr}g$ on the x-axis, go up till you hit the color for your measured time, and read off the value of $\rho C_d A/2m$ on the y-axis Remember these are in mks units; meters/s^2 for $C_{rr}g$ and 1/meters for $\rho C_d A/2m$. You might do this at several different high speeds to get a better value. Or,

B. With a calculator, guess and check values of b, or

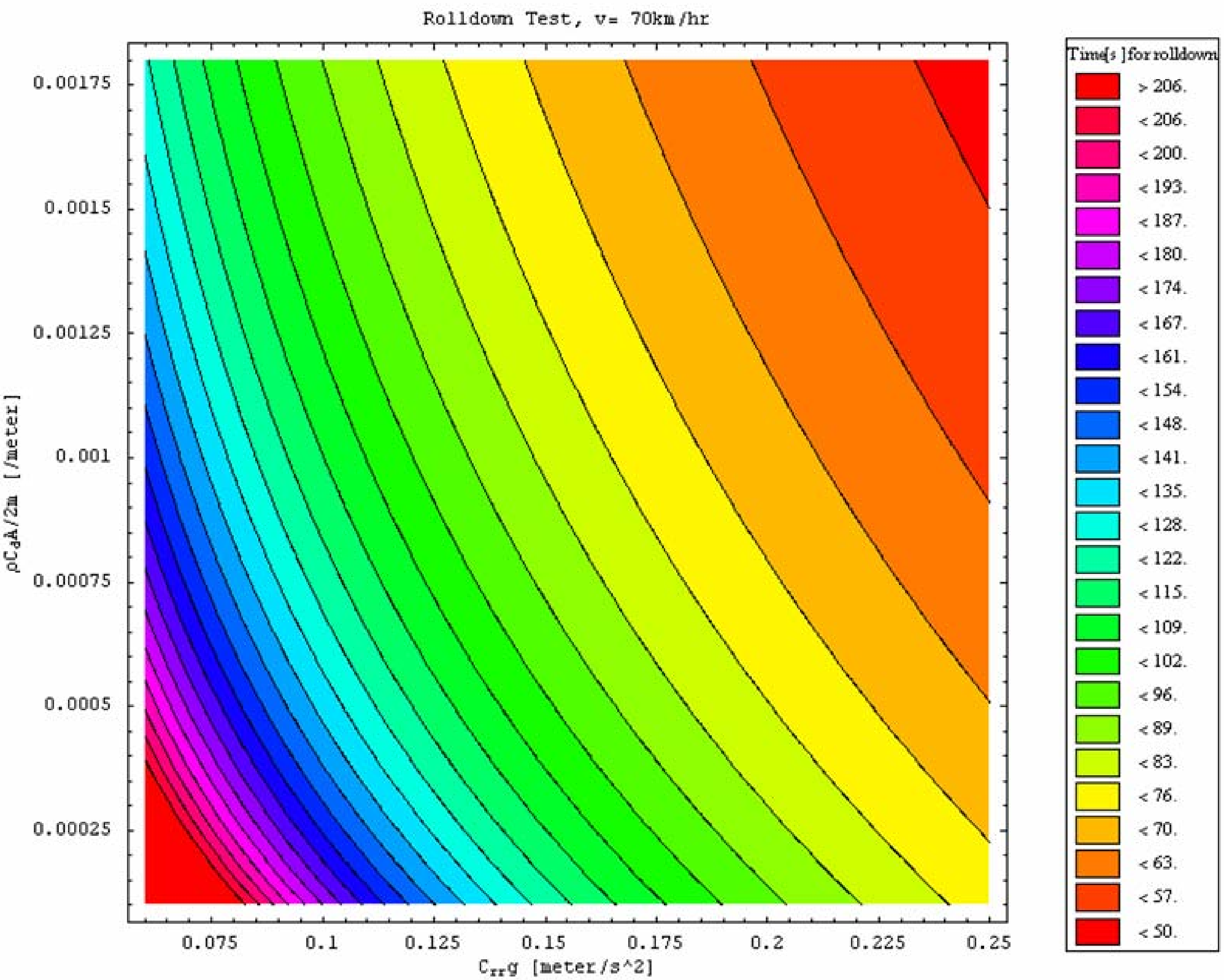
C. Use the tables at the end (for 70km/hr starting speed only).



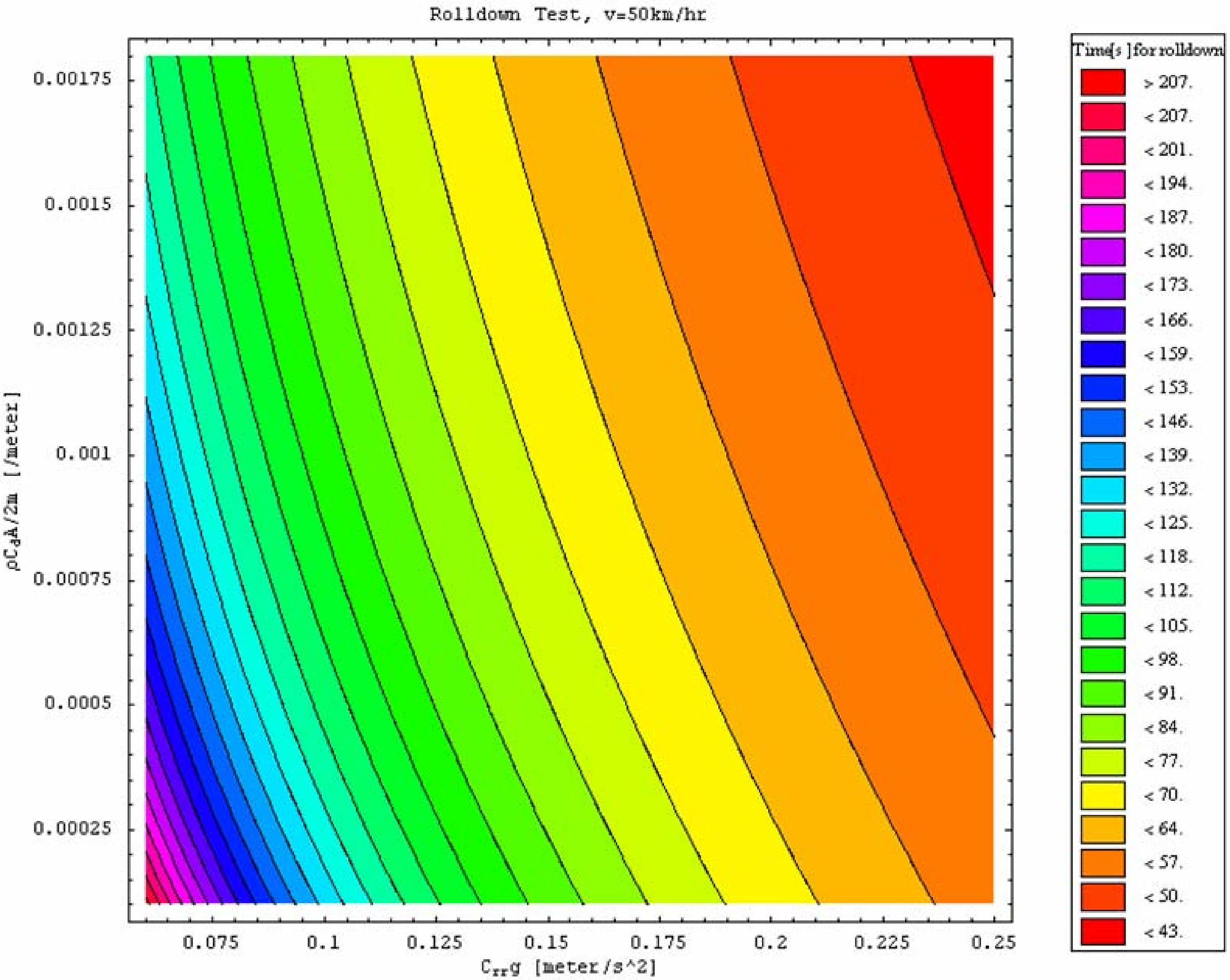
Rolldown Test, v=110 km/hr
0.00175
0.0015
0.00125
0.001
0.00075
0.0005
0.00025
ρC_dA/2m [/meter]
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
C_rrg [meter/s^2]
Time[s] for rolldown
> 245.
< 245.
< 237.
< 229.
< 221.
< 214.
< 206.
< 198.
< 191.
< 183.
< 175.
< 167.
< 160.
< 152.
< 144.
< 136.
< 129.
< 121.
< 113.
< 106.
< 98.
< 90.
< 82.
< 75.
< 67.
< 59.



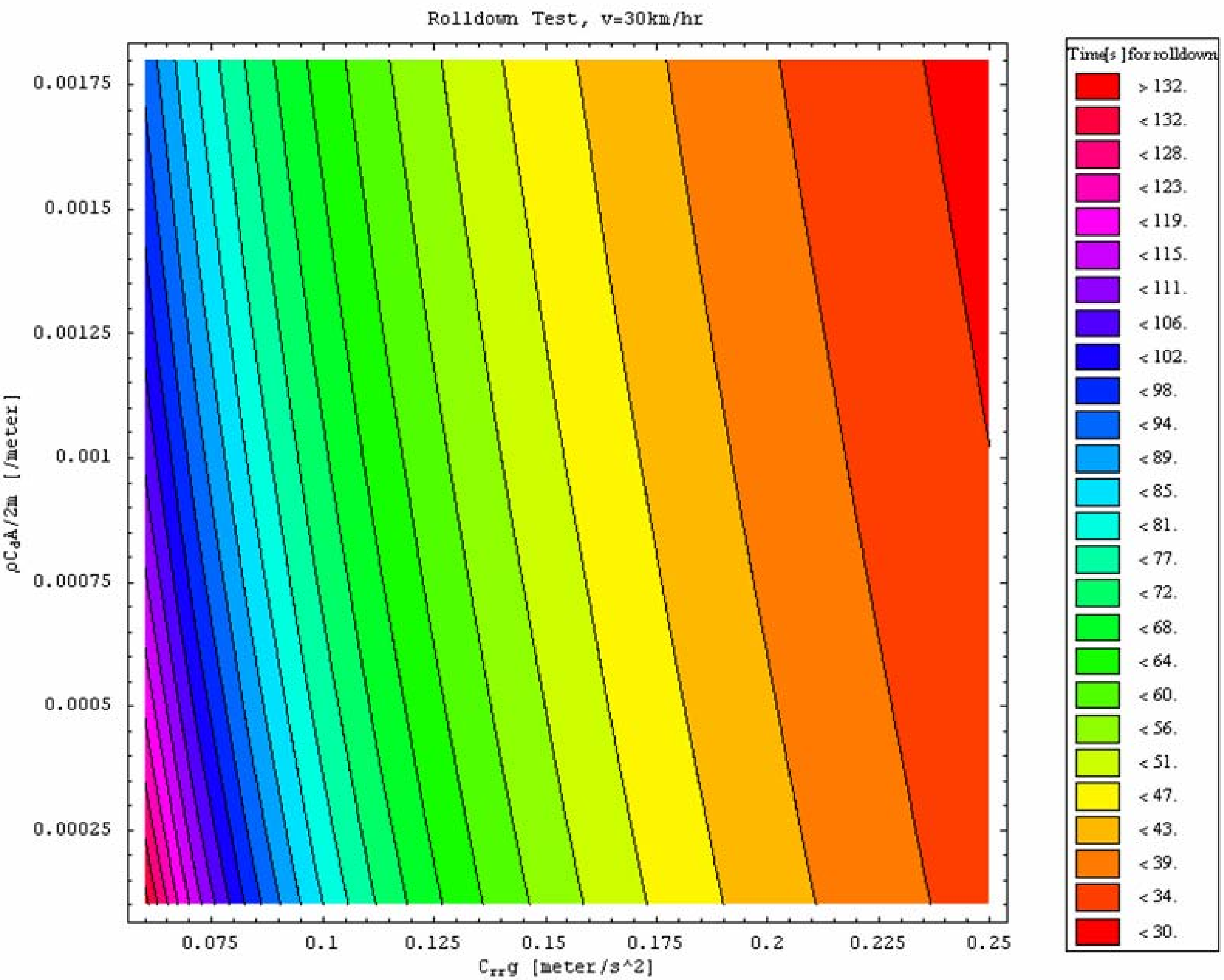
Rolldown Test, v=90 km/hr
0.00175
0.0015
0.00125
0.001
0.00075
0.0005
0.00025
$\rho C_d A/2m$ [/meter]
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
$C_{rr}g$ [meter/s^2]
Time[s] for rolldown
> 228.
< 228.
< 220.
< 213.
< 206.
< 199.
< 192.
< 185.
< 177.
< 170.
< 163.
< 156.
< 149.
< 142.
< 134.
< 127.
< 120.
< 113.
< 106.
< 99.
< 91.
< 84.
< 77.
< 70.
< 63.
< 56.

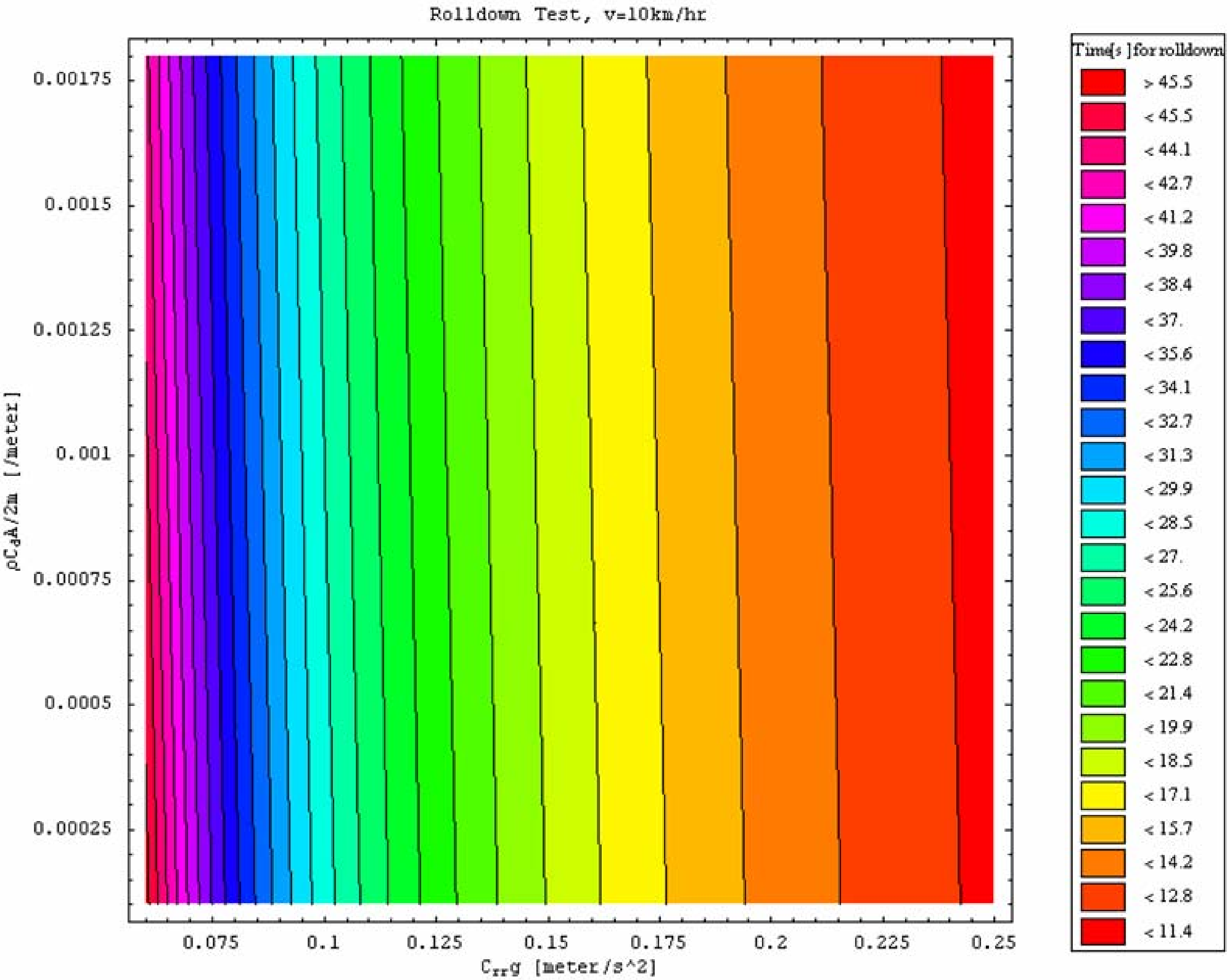


Rolldown Test, v= 70km/hr
0.00175
0.0015
0.00125
0.001
0.00075
0.0005
0.00025
ρC_dA/2m [/meter]
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
C_rr g [meter/s^2]
Time[s]for rolldown
> 206.
< 206.
< 200.
< 193.
< 187.
< 180.
< 174.
< 167.
< 161.
< 154.
< 148.
< 141.
< 135.
< 128.
< 122.
< 115.
< 109.
< 102.
< 96.
< 89.
< 83.
< 76.
< 70.
< 63.
< 57.
< 50.



Rolldown Test, v=50km/hr
0.00175
0.0015
0.00125
0.001
0.00075
0.0005
0.00025
ρC_dA/2m [/meter]
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
C_rr g [meter/s^2]
Time[s] for rolldown
> 207.
< 207.
< 201.
< 194.
< 187.
< 180.
< 173.
< 166.
< 159.
< 153.
< 146.
< 139.
< 132.
< 125.
< 118.
< 112.
< 105.
< 98.
< 91.
< 84.
< 77.
< 70.
< 64.
< 57.
< 50.
< 43.


Rolldown Test, v=30km/hr
0.00175
0.0015
0.00125
0.001
0.00075
0.0005
0.00025
ρC_dA/2m [/meter]
0.075
0.1
0.125
0.15
0.175
0.2
0.225
0.25
C_rr g [meter/s^2]
Time[s] for rolldown
> 132.
< 132.
< 128.
< 123.
< 119.
< 115.
< 111.
< 106.
< 102.
< 98.
< 94.
< 89.
< 85.
< 81.
< 77.
< 72.
< 68.
< 64.
< 60.
< 56.
< 51.
< 47.
< 43.
< 39.
< 34.
< 30.



As you can see the slower you start, the harder it is to find the aero drag component $\rho C_d A/2m$ since the curves are more and more vertical. And on the flip side it is no problem to find the rolling component $C_{rr}g$ since the aero component has almost no effect.

Tables constructed for the case of starting velocity=70km/hr (44mph).

```
a= 0.06 m/s^2
 Time= 270.   {b→0.00011 /m}
 Time= 254.   {b→0.00016}
 Time= 238.   {b→0.00022}
 Time= 222.   {b→0.00030}
 Time= 206.   {b→0.00040}
 Time= 190.   {b→0.00053}
 Time= 174.   {b→0.00070}
 Time= 158.   {b→0.00093}
 Time= 142.   {b→0.00124}
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Time= 126. {b→0.00170}
Time= 110. {b→0.00239}

a= 0.08
Time= 200. {b→0.00016}
Time= 189.5 {b→0.00022}
Time= 179. {b→0.00029}
Time= 168.5 {b→0.00038}
Time= 158. {b→0.00049}
Time= 147.5 {b→0.00062}
Time= 137. {b→0.00080}
Time= 126.5 {b→0.00102}
Time= 116. {b→0.00131}
Time= 105.5 {b→0.00170}
Time= 95. {b→0.00224}

a= 0.1
Time= 170. {b→0.00013}
Time= 161. {b→0.00019}
Time= 152. {b→0.00027}
Time= 143. {b→0.00037}
Time= 134. {b→0.00049}
Time= 125. {b→0.00064}
Time= 116. {b→0.00083}
Time= 107. {b→0.00108}
Time= 98. {b→0.00140}
Time= 89. {b→0.00184}
Time= 80. {b→0.00245}

a= 0.13
Time= 130. {b→0.00017}
Time= 124. {b→0.00025}
Time= 118. {b→0.00033}
Time= 112. {b→0.00044}
Time= 106. {b→0.00056}
Time= 100. {b→0.00071}
Time= 94. {b→0.00090}
Time= 88. {b→0.00113}
Time= 82. {b→0.00141}
Time= 76. {b→0.00178}
Time= 70. {b→0.00225}

a= 0.16
Time= 110. {b→0.00014}
Time= 105. {b→0.00022}
Time= 100. {b→0.00032}
Time= 95. {b→0.00043}
Time= 90. {b→0.00057}
Time= 85. {b→0.00073}
Time= 80. {b→0.00093}
Time= 75. {b→0.00118}
Time= 70. {b→0.00149}
Time= 65. {b→0.00188}
Time= 60. {b→0.00238}

a= 0.18
Time= 100. {b→0.00012}
Time= 95.5 {b→0.00020}
Time= 91. {b→0.00030}
Time= 86.5 {b→0.00042}
Time= 82. {b→0.00057}

```
 Time= 77.5    {b→0.00074}
 Time= 73.     {b→0.00095}
 Time= 68.5    {b→0.00120}
 Time= 64.     {b→0.00152}
 Time= 59.5    {b→0.00193}
 Time= 55.     {b→0.00245}

a= 0.2
 Time= 90.    {b→0.00013}
 Time= 86.    {b→0.00022}
 Time= 82.    {b→0.00033}
 Time= 78.    {b→0.00046}
 Time= 74.    {b→0.00062}
 Time= 70.    {b→0.00081}
 Time= 66.    {b→0.00103}
 Time= 62.    {b→0.00131}
 Time= 58.    {b→0.00166}
 Time= 54.    {b→0.00209}
 Time= 50.    {b→0.00264}
```

You now have your drag and aero coefficients a and b, where
a=Crr g and b=rho Cd A / 2m.   rho is the density of air, 1.2kg/m^3 at sea level, A is vehicle projected frontal area, m mass, g gravitational acceleration, 9.8m/s^2 at sea level.
The precision of a and b here will be something like the precision of your measurements but I haven't worked out the relation, maybe later.
Anyone with the heart to check the maths please do so, there may be mistakes in here.
Anyone who actually does the tests let me know how they worked out, what your values of a and b were.

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