Unemployment Persistence and the NAIRU: A Bayesian Approach *

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Abstract

This paper estimates the US and euro area NAIRU in a Bayesian framework. We set out a simple structural model explaining unemployment by demand and supply factors, which are treated as unobserved variables that have observable effects on measured unemployment, output and inflation. The model allows for unemployment persistence and a time-varying core inflation rate. The results show that although cyclical shocks are very persistent, most of the increase in European unemployment is driven by structural factors. The degree of persistence is lower in the US but demand shocks seem to be more important in explaining variation in unemployment.

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1 Introduction

The idea of a natural rate of unemployment has been pioneered by Friedman (1968) and Phelps (1968) who claim that unemployment is at its natural level when neither inflationary nor deflationary pressure emanates from the labour market. This is called the non-accelerating-inflation-rate of unemployment (NAIRU). The existence of a constant NAIRU has been questioned after the oil price shocks of the 1970s as unemployment remained high in Europe even after inflation had stabilised. More recently the NAIRU is assumed to be a function of labour market institutions (see e.g. Nickell et al., 2005) and real macroeconomic variables such as real interest rates (see e.g. Blanchard, 2003) or productivity growth (see e.g. Pissarides, 1990). As pointed out by Blanchard and Wolfers (2000) labour market institutions also interact with macroeconomic shocks. Due to labour market rigidities cyclical unemployment may translate into medium-run unemployment or even become permanent. The latter is known as hysteresis. These unemployment persistence effects can arise from insider-outsider effects in wage formation (see e.g. Blanchard and Summers, 1986) and/or depreciation of skills and search ineffectiveness of the unemployed (see e.g. Phelps, 1972).

As some of the factors driving the NAIRU are difficult to measure or even unobservable, one approach in the recent literature is to identify a time-varying NAIRU from observed variables using the Kalman filter and smoother. For example, Apel and Jansson (1999a,b) and Fabiani and Mestre (2004) estimate the NAIRU from a system combining an Okun relation, a Phillips curve and an equation for the unemployment gap with stochastic laws of motion for the NAIRU and potential output. However, this approach has a number of shortcomings. First, NAIRU estimates are obtained by specifying a reduced form model which essentially is a multivariate statistical decomposition of unemployment into trend and cycle. The model's parameters do not have a structural interpretation, though. Second, the use of a standard Phillips curve implies that unemployment is at its natural level when inflation has stabilised. As such, possible persistence or hysteresis effects due to labour market rigidities are neglected. In a recent paper Logeay and Tober (2006) combine the hysteresis approach with a time-varying NAIRU by allowing the latter to be affected by lagged unemployment. However this ad hoc modelling of hysteresis implies that next to cyclical shocks also shocks to the NAIRU induce hysteresis effects, i.e. as unemployment starts adjusting in response to a shift in the NAIRU the NAIRU itself will start shifting in response to the changes in lagged actual unemployment. As a result, a one percent increase in the NAIRU leads to a more than proportional increase in unemployment. There is no reason however why a shift in the NAIRU should induce hysteresis effects. Moreover, these models typically do not allow for permanent shifts in monetary policy as they assume a constant core inflation rate.[1] Third, calculating confidence bounds around the NAIRU point estimates in an unobserved component model is not a trivial task as filter and parameter uncertainty need to be combined. The dominant approach is to use simulation methods. The obtained confidence intervals are only approximations though.

Consistent with the literature, this paper models the NAIRU as an unobserved non-stationary process. We do not specify a reduced form equation for the deviation of unemployment from the NAIRU but derive an equation for this unemployment gap from a simple structural model including a Phillips curve, Okun's law and a demand equation. The specification of the Phillips curve allows for slow adjustment of unemployment towards the NAIRU by including elements of persistence. Furthermore we allow for time-varying core inflation. The model is estimated in a Bayesian framework for the euro area and the US using data for the period 1970Q1-2003Q4. The Bayesian approach has two main advantages over standard maximum likelihood estimation. First, it helps optimisation by down-weighting the likelihood function in regions where the parameters do not have a structural interpretation or are inconsistent with out-of-sample information. Second, the posterior distribution of the NAIRU can be calculated allowing for both parameter and filter uncertainty.

The paper is structured as follows. Section 2 presents the theoretical model. The estimation methodology is outlined in section 3. Section 4 presents the results. Section 5 concludes.

2 The model

2.1 Unemployment-inflation trade-off and the NAIRU

Consider a Phillips curve of the form

$$\Delta^2 p_t = -\theta_1 \left(u_t - u_t^*\right) - \theta_{11} \Delta u_t + \gamma(L) z_t, \tag{1}$$

[1] An exception is Domenech and Gomez (2006) who estimate simultaneously the NAIRU, core inflation, and the output gap.

where p_t is the log of prices, such that $\Delta^2 p_t$ is the change in inflation, u_t is the unemployment rate, u_t^* is the NAIRU, and z_t is a vector of cost-push shocks (e.g. import prices). The term $(u_t - u_t^*)$ represents a short-run unemployment-inflation trade-off, i.e. when u_t is below u_t^* inflation is rising, and vice versa. The size of this trade-off is determined by the parameter θ_1 which is a measure of nominal rigidities in wage and price setting due to for instance wage and price adjustment costs or staggered wage and price setting (see e.g. Layard et al., 2005, for a derivation of equation (1) from a standard wage and price setting schedule). Stronger rigidities imply a smaller value for θ_1, i.e. a flatter Phillips curve. The term Δu_t accounts for potential unemployment persistence effects. The inclusion of this term into the traditional Phillips curve is motivated from wage and price behaviour depending on the change, next to the level, of unemployment (see e.g. Layard et al., 2005; Franz, 2005). These change terms capture possible price effects of labour adjustment costs[2] and possible wage effects of insider-outsider behaviour and/or duration composition effects[3]. As a consequence, unemployment can deviate from its natural level even when inflation has stabilised. The impact of persistence effects becomes more clear if we rewrite equation (1) as

$$u_t = \frac{\theta_1}{\theta_1 + \theta_{11}} u_t^* + \frac{\theta_{11}}{\theta_1 + \theta_{11}} u_{t-1} - \frac{1}{\theta_1 + \theta_{11}} \Delta^2 p_t + \frac{\gamma(L)}{\theta_1 + \theta_{11}} z_t.$$

This specification shows that even when inflation is stable, unemployment can be far away from its natural rate due to persistence effects. The higher θ_{11} relative to θ_1 the more persistent unemployment is. The unemployment rate u_t^n which stabilises inflation (setting cost-push shocks to zero) is given by

$$u_t^n = \kappa u_t^* + (1 - \kappa) u_{t-1},$$

where $\kappa = \theta_1/(\theta_1 + \theta_{11})$. Layard et al. (2005) refer to this as the short-run NAIRU. It is a weighted average of u_t^* and u_{t-1}. We can distinguish three cases of interest. First, if $\kappa = 1$ unemployment is not affected by persistence effects, i.e. the short-run NAIRU u_t^n equals the long-run NAIRU u_t^*. Second, if $0 < \kappa < 1$ unemployment converges to u_t^* after a business cycle shock. However, the

[2] If these costs delay employment adjustment, marginal costs are higher in the short than in the long run (where employment is at its optimal level). The first difference of unemployment in the price setting schedule captures the effect of this short-run increase in marginal costs on prices.

[3] In the former, a transitory shock reduces the number of insiders and thus puts upward pressure on wages. This results in a positive effect of lagged unemployment which together with the standard negative effect of contemporaneous unemployment gives the change term of unemployment. In the latter, the duration of unemployment matters for aggregate wages as the long-term unemployed are less strong competitors for jobs and therefore put less pressure on wages than the short-run unemployed. The change term now captures the idea that wage pressure is lower when unemployment has recently risen as people that became recently unemployed are stronger competitors for jobs.

speed of adjustment depends on κ. In terms of policy, persistence means that once unemployment has risen it cannot be brought back at once without a permanent increase in inflation. But it can be reduced gradually without inflation rising. Third, $\kappa = 0$ means that cyclical shocks have a permanent impact on unemployment. The natural rate or long-run NAIRU u_t^* is not an attractor anymore since unemployment is only affected by its own history with no tendency to revert to an equilibrium. This extreme case, known as hysteresis, has been introduced by (Blanchard and Summers, 1986). More recently, the existence of pure hysteresis has been criticized as being too strong (see e.g. Blanchard, 2006), though. Given that pure hysteresis is at best doubtful, the focus of this paper is on measuring the degree of persistence in unemployment with hysteresis only being a limiting case of the model.

In the long run, unemployment is determined by long-run supply factors and equals u_t^*. In the short run, unemployment is determined by the interaction of aggregate supply, given by the Phillips curve in equation (1), and aggregate demand y_t^d which can be represented by

$$y_t^d = \frac{1}{\lambda_1}(m_t - p_t) + \frac{1}{\lambda_2}x_t, \tag{2}$$

where m_t is the nominal money stock and x_t captures all exogenous real factors driving demand, e.g. fiscal policy, relative import prices and world economic activity. This equation is simply the reduced form of an IS-LM system. Adding and subtracting $\overline{y}_t$ and taking first differences of both sides, to get rid of the level of prices, yields

$$\Delta y_t^d = \Delta \overline{y}_t + \frac{1}{\lambda_1}\left(\sigma_t^d - \Delta p_t\right) \tag{3}$$

where $\sigma_t^d = \Delta m_t - \frac{\lambda_1}{\lambda_2}\Delta x_t - \lambda_1 \Delta \overline{y}_t$ can be interpreted as a variable collecting demand factors, i.e. the growth rate of the money stock and real demand factors corrected for the growth rate of potential output. Note that as σ_t^d will be modelled as an unobserved variable (see section 2.2 below), specifying aggregate demand in this way does not imply that we are assuming (i) aggregate demand to be better characterised as depending on real balances rather than on real interest rates nor (ii) monetary policy to be better characterised by the growth of the money stock rather than an interest rate rule. The specification in equation (2) merely serves as a way to give an interpretation of what σ_t^d may be. As such, σ_t^d can also be interpreted as representing a nominal interest rate.

The link between aggregate demand and unemployment is given by Okun's Law

$$y_t^d - \overline{y}_t = -\omega(u_t - u_t^*), \quad (4)$$

where potential output $\overline{y}_t$ is defined as the level of output that corresponds to the equilibrium level of unemployment u_t^*.

2.2 Dynamics of unobserved variables

The model outlined in section 2.1 includes the observed endogenous variables y_t, u_t and Δp_t and the unobserved states $\overline{y}_t$, u_t^* and σ_t^d. The dynamics of these unobserved states are assumed to be given by

$$\overline{y}_{t+1} = \overline{y}_t + \psi_t + \eta_{1t} - \omega\eta_{3t}, \quad (5)$$

$$\psi_{t+1} = \psi_t + \eta_{2t}, \quad (6)$$

$$u_{t+1}^* = (1+\delta)u_t^* - \delta u_{t-1}^* + \eta_{3t}, \quad (7)$$

$$\sigma_{t+1}^d = \tau_t + \Delta\eta_{4t} + \eta_{5t}, \quad (8)$$

$$\tau_{t+1} = \tau_t + \eta_{6t}, \quad (9)$$

where the error terms η_{it} with $i = 1, ..., 6$ are mutually independent zero mean white noise processes representing structural shocks.

Following Harvey (1985) and Stock and Watson (1998), among others, equations (5)-(6) model potential output $\overline{y}_t$ as a random walk with drift, with the drift term ψ_t varying over time according to a random walk process. The time-variation in ψ_t allows for the possibility of permanent changes in the trend growth of real output, e.g. the productivity slowdown of the early 1970s. Potential output is further affected by structural unemployment u_t^* through the term $-\omega\eta_{3t}$. This negative relationship results from the definition of $\overline{y}_t$ as the level of output that corresponds to equilibrium unemployment, where from equation (4) $\frac{\partial\overline{y}}{\partial u^*} = -\omega$. Intuitively, it states that structural unemployment erodes the output potential of the economy.

Equation (7) specifies the natural rate of unemployment u_t^* as a non-stationary process, i.e. shifts in its underlying determinants are assumed to be permanent. As a pure random walk process would result in a non-smooth series that is hard to reconcile with the expected smooth evolution of the structural characteristics driving the natural rate, the AR(2) specification in equation (7) allows for a smooth evolution of u_t^* over time, i.e. the closer δ to one the smoother u_t^*.[4] If $\delta = 0$, u_t^* is a pure random walk process.

Equations (8)-(9) model the demand factor σ_t^d as the sum of three components: (i) an erratic component $\Delta\eta_{4t}$; (ii) a temporary component η_{5t}; and (iii) a level component τ_t driven by η_{6t}. The erratic component is included to capture temporary shifts in the level of demand, like e.g. a temporary increase in government spending. The temporary component captures permanent shifts in the level of demand. The level component captures permanent changes in monetary policy, i.e. a permanent change in the growth rate of the money stock m_t which, after correcting for trend growth, induces a permanent change in the level of inflation Δp_t. This is due to the fact that whenever demand differs from potential output, inflation has to adjust in order to bring demand and supply back in line. A permanent change in the growth rate of demand therefore implies a permanent change in the level of inflation. Thus τ_t is the (implicit) core inflation rate set by the central bank. As τ_t is modelled as a random walk, the model allows for a time-varying core inflation rate.

Note that not explicitly modelling the output gap and/or unemployment gap as a stationary process seems at odds with what is common in the literature (see e.g. Apel and Jansson, 1999a,b; Fabiani and Mestre, 2004). This literature directly specifies reduced form equations, though. In our model we specify structural equations and use these to derive a reduced form model (see 3.1 below). In this reduced form, the persistence in the output and unemployment gap is a function of the structural parameters in the model.

3 Estimation methodology

3.1 Reduced form and state space representation of the model

The reduced form for the observed endogenous variables y_t, u_t, and Δp_t, as a function of the unobserved states and the lagged observed endogenous variables, is obtained by solving equations

[4] Note that in order to induce this smoothness, the natural rate of unemployment is nowadays often modelled as an I(2) series, i.e. δ is set to one (see e.g. Orlandi and Pichelmann, 2000). We do not restrict δ to be equal to one in equation (7) as in this case u_t^* exhibits a (time-varying) drift, which would be hard to justify from an economic perspective.

(1), (3) and (4) as

$$y_t^d = \overline{y}_t + \frac{\alpha}{\lambda_1}\left(\sigma_t^d + \lambda_1(y_{t-1}^d - \overline{y}_{t-1}) - \Delta p_{t-1} - \gamma(L) z_t\right) + \frac{\alpha\theta_{11}}{\lambda_1}\left(u_t^* - u_{t-1}\right) \tag{10}$$

$$u_t = u_t^* - \frac{\alpha}{\lambda_1 \omega}\left(\sigma_t^d + \lambda_1(y_{t-1}^d - \overline{y}_{t-1}) - \Delta p_{t-1} - \gamma(L) z_t\right) - \frac{\alpha\theta_{11}}{\lambda_1\omega}\left(u_t^* - u_{t-1}\right) \tag{11}$$

$$\Delta p_t = \sigma_t^d + \lambda_1(y_{t-1}^d - \overline{y}_{t-1}) - \alpha\left(\sigma_t^d + \lambda_1(y_{t-1}^d - \overline{y}_{t-1}) - \Delta p_{t-1} - \gamma(L) z_t\right) - \alpha\theta_{11}\left(u_t^* - u_{t-1}\right) \tag{12}$$

where $\alpha = \lambda_1\omega / (\theta_1 + \theta_{11} + \omega\lambda_1)$. This reduced form can be cast into a linear Gaussian state space model of the following general form[5]

$$y_t = Z\alpha_t + Ax_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0, H), \tag{13}$$

$$\alpha_{t+1} = T\alpha_t + R\eta_t, \qquad \eta_t \sim N(0, Q), \qquad t = 1, \ldots, n, \tag{14}$$

where y_t is a $p \times 1$ vector of p observed endogenous variables, modelled in the observation equation (13), x_t is a $k \times 1$ vector of k observed exogenous or predetermined variables and α_t is a $m \times 1$ vector of m unobserved states, modelled in the state equation (14). The vectors ε_t and η_t are assumed to hold mutually independent Gaussian error terms with the former representing measurement errors and the latter structural shocks. The exact specification of the vectors y_t, x_t and α_t and the matrices Z, A, T, R, H and Q that cast the model in equations (5)-(9) and (10)-(12) in the general state space representation in equations (13)-(14) is provided in Appendix A.1.

3.2 Parameter estimation: a Bayesian framework

For given parameter matrices Z, A, T, R, H, and Q, the unobserved state vector α_t can be identified from the observations $y_1, \ldots, y_n$ and $x_1, \ldots, x_n$ using the Kalman filter and smoother (see Appendix A.2 for technical details) . In practice these matrices generally depend on elements of an unknown parameter vector ψ. One possible approach is to derive the loglikelihood function for the model under study from the Kalman filter (see e.g. de Jong, 1991; Koopman and Durbin, 2000; Durbin and Koopman, 2001) and replace the unknown parameter vector ψ by its maximum likelihood (ML) estimate. This is not the approach pursued in this paper. The fairly large number of unknown parameters in combination with the large number of unobserved states makes the numerical optimisation of the sample loglikelihood function quite tedious. Therefore, we analyse the state space model from a Bayesian point of view, i.e. we use prior information to down-weight the likelihood function in regions of the parameter space that are inconsistent with out-of-sample

[5]See e.g. Durbin and Koopman (2001) for an extensive overview of state space models.

information and/or in which the structural model is not interpretable (Schorfheide, 2006). More formally, we treat ψ as a random parameter vector with a known prior density $p(\psi)$ and estimate the posterior densities $p(\psi \mid y, x)$ for the parameter vector ψ and $p(\widehat{\alpha}_t \mid y, x)$ for the smoothed state vector $\widehat{\alpha}_t$, where y and x denote the stacked vectors $(y_1', \ldots, y_n')'$ and $(x_1', \ldots, x_n')'$ respectively, by combining information contained in $p(\psi)$ and the sample data. This boils down to calculating the posterior mean $\overline{g}$

$$\overline{g} = E\left[g(\psi) \mid y, x\right] = \int g(\psi)\, p(\psi \mid y, x)\, d\psi \tag{15}$$

where g is a function which expresses the moments of the posterior densities $p(\psi \mid y, x)$ and $p(\widehat{\alpha}_t \mid y, x)$ in terms of the parameter vector ψ. In principle, the integral in equation (15) can be evaluated numerically by drawing a sample of n random draws of ψ, denoted $\psi^{(i)}$ with $i = 1, \ldots, n$, from $p(\psi \mid y, x)$ and then estimating $\overline{g}$ by the sample mean of $g(\psi)$. As $p(\psi \mid y, x)$ is not a density with known analytical properties, such a direct sampling method is not feasible, though. Therefore, we use importance sampling (see Appendix A.3 for technical details).

A second important advantage of the Bayesian framework over standard ML is that it straightforward to calculate the posterior densities of both the parameter vector ψ and the smoothed state vector $\widehat{\alpha}_t$ where the latter takes both parameter and filter uncertainty into account (see Appendix A.4 for technical details).

4 Estimation Results[6]

4.1 Data

We use quarterly data for the US and the euro area from 1970Q1 to 2003Q4. US data are taken from the OECD Economic Outlook and the International Monetary Fund (IMF) International Financial Statistics. Euro area data, which are aggregate series for 12 countries[7], are taken from the area-wide model of Fagan et al. (2005). The unemployment rate, u_t, is the quarterly unemployment rate. For inflation, Δp_t, we use the first difference of the log of the seasonally adjusted quarterly GDP deflator. Output, y_t^d, is the log of seasonally adjusted quarterly GDP in constant prices. As a measure for cost-push shocks in the Phillips curve we use the level and one lag of the second difference of log import prices.

[6]The GAUSS code to obtain the results presented in this section is available on the authors' webpage.

[7]Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain.

4.2 Prior distribution of the parameters

Prior information on the unknown parameter vector ψ is included in the analysis through the prior density $p(\psi)$. Detailed information on $p(\psi)$ can be found in Table 1. As stated above, the main motivation for setting these priors is to down-weight the likelihood function in regions of the parameter space that are inconsistent with out-of-sample information and/or in which the structural model is not interpretable. Previous estimates as well as economic theory give us an idea about the approximate value of the model's parameter. However using previous studies to set priors should be done with caution particularly if theses studies consider the same time period. We therefore use previous estimates only as a rough indication for the prior mean but chose the prior variance fairly loose. Okun's Law coefficient ω measures the percentage rise in the output gap when the unemployment gap falls by one percentage point. Okun (1970) stated that this relation is linear and ω is roughly three. We set the prior value for ω equal to 2.5 since more recent empirical studies found Okun's Law coefficient somewhat lower than 3 (see e.g Orlandi and Pichelmann, 2000) but leave a considerable amount of uncertainty around it. λ_1^{-1} links the monetary transmission mechanism to output. In Gerlach and Smets (1999) the long-run value of this parameter is found to be 1.11. However Gerlach and Smets link monetary policy to nominal interest rates whereas here it is treated as an unobserved state. The parameter is identified by the impact of inflation on the change in aggregate demand. The prior mean and variance for λ_1 are chosen so that the 5% and 95% percentiles of the prior distribution for λ_1^{-1} are 0.94 and 1.35 respectively, implying a roughly unit impact of monetary policy on aggregate demand. Setting priors on θ_1 and θ_{11} is more difficult as the vast majority of Phillips-curve estimates does not include a persistence measure θ_{11} and thus cannot be used here. Moreover, we do not want to make these priors too informative since measuring the degree of persistence is of particular interest in our analysis. The prior mean of both parameters are set to 0.5, implying $\kappa = 0.5$. As the model is not identified for $\theta_1 = 0$, i.e. the Phillips curve in equation (1) is horizontal implying that u_t^* is not identified, we restrict θ_1 to be positive and use a prior gamma distribution with a fairly loose 90% confidence interval. The parameter δ is included to allow for smoothness in u_t^*. We have chosen for a not too informative prior for δ. Priors on the state variances are set so that the resulting output gap matches with the commonly accepted timing of the business cycle with respect to shape and frequency of the output gap. Again, we leave a considerable amount of uncertainty around these prior variances.

Table 1: Prior Distribution

	Euro area		US	
Parameter	Mean	90% Interval	Mean	90% Interval
θ_1	0.50	[0.21 - 0.89]	0.50	[0.21 - 0.89]
θ_{11}	0.50	[-0.66 - 1.66]	0.50	[-0.66 - 1.66]
ω	2.50	[1.98 - 3.02]	2.50	[1.98 - 3.02]
λ_1	0.90	[0.74 - 1.06]	0.90	[0.74 - 1.06]
δ	0.50	[0.01 - 0.99]	0.50	[0.01 - 0.99]
γ_1	0.10	[-0.13 - 0.33]	0.10	[-0.13 - 0.33]
γ_2	0.10	[-0.13 - 0.33]	0.10	[-0.13 - 0.33]
$\sigma^2_{\varepsilon_1}$	0.05	[0.03 - 0.08]	0.05	[0.03 - 0.08]
$\sigma^2_{\varepsilon_2}$	0.05	[0.03 - 0.08]	0.05	[0.03 - 0.08]
$\sigma^2_{\varepsilon_3}$	0.05	[0.03 - 0.08]	0.05	[0.03 - 0.08]
$^{**}\sigma^2_{\eta_1}$	0.80	[0.49 - 1.17]	0.80	[0.49 - 1.17]
$^{**}\sigma^2_{\eta_2}$	0.09	[0.06 - 0.15]	0.30	[0.19 - 0.44]
$^{**}\sigma^2_{\eta_3}$	0.04	[0.02 - 0.05]	2.00	[1.24 - 2.91]
$\sigma^2_{\eta_4}$	0.10	[0.06 - 0.15]	0.15	[0.09 - 0.22]
$\sigma^2_{\eta_5}$	0.15	[0.09 - 0.22]	0.24	[0.15 - 0.35]
$^{*}\sigma^2_{\eta_6}$	0.70	[0.43 - 1.02]	0.80	[0.49 - 1.16]

The prior distribution is assumed to be Gaussian for all elements in ψ, except for θ_1 and the variance parameters which are assumed to be gamma distributed. $^{*}\sigma_{\eta_i} = \sigma^2_{\eta_i} * 10^{-2}$, $^{**}\sigma_{\eta_i} = \sigma^2_{\eta_i} * 10^{-4}$

4.3 Posterior distribution

Posterior distribution of the parameters

Table 2 presents the posterior mean and the 5% and 95% percentiles of the posterior distribution for the euro area and the US estimates. The importance of the change term of unemployment in the Phillips curve, as represented by θ_{11}, is found to be much higher in the euro area than in the US. The speed at which the short-run NAIRU u^n_t adjusts towards the natural rate u^*_t is measured by κ and given by $\frac{\theta_1}{\theta_1+\theta_{11}}$. In the euro area we find that $\kappa = 0.06$, implying that the adjustment is rather slow and thus unemployment is very persistent. The results for the US show that $\kappa = 0.33$ and therefore US unemployment is adjusting somewhat faster.[8] The finding of higher unemployment persistence in Europe than in the US is in line with previous findings. The posterior mean for θ_{11} in the euro area as well as the posterior mean of θ_1 in the US lie outside their 90% prior interval. Note that changing the prior mean or variance for these parameters does

[8]We obtain a posterior distribution for κ by calculating it in each of the importance samples. The 5% and 95% percentiles are $[0.03 - 0.10]$ for the euro area and $[0.18 - 0.60]$ for the US.

not affect their posterior distribution much.[9] This suggests that the data are rather informative with respect to the persistence measures in the Phillips curve. The estimates on ω, λ_1, γ_1 and γ_2 are consistent with the literature. Figures 3 and 4 in Appendix B show the prior together with the posterior distribution for all parameters.

Table 2: Posterior Distribution

	Euro area		US	
Parameter	Mean	90% Interval	Mean	90% Interval
θ_1	0.18	[0.09 - 0.29]	0.08	[0.04 - 0.12]
θ_{11}	2.60	[1.90 - 3.43]	0.16	[0.06 - 0.26]
ω	2.15	[1.99 - 2.30]	2.37	[2.21 - 2.55]
λ_1	0.97	[0.81 - 1.12]	1.01	[0.90 - 1.13]
δ	0.97	[0.96 - 0.98]	0.95	[0.91 - 0.99]
γ_1	0.10	[-0.13 - 0.33]	0.11	[-0.12 - 0.33]
γ_2	0.10	[-0.13 - 0.33]	0.11	[-0.12 - 0.34]
$\sigma^2_{\varepsilon_1}$	0.22	[0.18 - 0.26]	0.09	[0.06 - 0.14]
$\sigma^2_{\varepsilon_2}$	0.03	[0.02 - 0.04]	0.06	[0.04 - 0.08]
$\sigma^2_{\varepsilon_3}$	0.09	[0.05 - 0.12]	0.09	[0.07 - 0.10]
$^{**}\sigma^2_{\eta_1}$	0.74	[0.45 - 1.15]	0.72	[0.45 - 1.11]
$^{**}\sigma^2_{\eta_2}$	0.19	[0.13 - 0.27]	0.27	[0.17 - 0.42]
$^{**}\sigma^2_{\eta_3}$	0.07	[0.04 - 0.10]	3.16	[2.21 - 4.41]
$\sigma^2_{\eta_4}$	0.05	[0.03 - 0.08]	0.11	[0.07 - 0.16]
$\sigma^2_{\eta_5}$	0.13	[0.08 - 0.18]	0.38	[0.29 - 0.49]
$\sigma^2_{\eta_6}$	0.01	[0.01 - 0.01]	0.01	[0.01 - 0.01]

Note that the approximate covariance matrix $\hat{\Omega}$ is inflated with a factor 1.3. With $n = 10000$ for the initial importance function and all updates the probabilistic error bound for the importance sampling estimator $\overline{g}_n$ is well below 10% for all coefficients. The number of subsequent updates of the importance density is 9 for the euro area and 10 for the US (see Appendix A.3 for details). $^{**}\sigma_{\eta_i} = \sigma^2_{\eta_i} * 10^{-4}$.

Posterior distribution of the states

The mean and the 5% and 95% percentiles of the posterior distribution of u^*_t, $\overline{y}_t$ and τ_t together with u_t, y_t and Δp_t are plotted in Figures 1 and 2. The NAIRU for the euro area shows a clear upward trend from the beginning of the 1970s up to the middle of the 1990s while from that time on it is downward sloping. In contrast to earlier studies we find the euro area NAIRU to

[9] Overall, our experience from experimenting with alternative sets of priors and prior variances was that (i) most changes to the priors only have a minor impact on the posterior distributions while (ii) for some of the more sizeable changes either the estimation procedure failed to converge or the estimation results changed completely but were no longer economically interpretable. In that sense, we believe that our results are fairly insensitive to changing priors but we can, of course, not state that changing priors does not change the results in any circumstance. This is our explicit aim though as we use the Bayesian technique to down-weight the likelihood function in areas of the parameter space were the model is no longer interpretable.

be very smooth and substantially above the actual rate of unemployment in the 1970s. This may be explained by our persistence parameter. By neglecting persistence effects, i.e. restricting κ to be one, the short-run NAIRU and the long-run NAIRU are identical. With $\kappa = 0.06$ however, the short-run NAIRU follows the actual rate of unemployment more closely and therefore is more volatile than the long-run NAIRU. This shows that if the NAIRU is used as a measure for structural unemployment one need to take persistence effects into account and estimate the long-run rather than the short-run NAIRU. As potential output is defined as the level of output that corresponds to $u_t = u_t^*$, the slow adjustment of actual unemployment to the increasesd equilibrium level of unemployment in the 1970s and the early 1980s implies a persistent positive output gap. The estimated NAIRU for the US seems to be rather stable throughout the sample period with a decrease of 2% in the 1990s. Comparing the actual unemployment rate with the NAIRU shows that demand effects explain most of the unemployment variation in the US whereas the upward drift in euro area unemployment is supply side driven. In contrast to earlier work, but consistent with Fabiani and Mestre (2004), Figures 1 and 2 show that the NAIRU is measured fairly precise.

Both in the euro area and in the US, the core inflation rate τ_t decreased substantially over time. This again highlights the need of allowing for a time-varying equilibrium inflation rate. Note that potential output is very close to being a linear trend, i.e. there are no clear signs of a change in the drift term. Increasing the prior mean of the variance of the innovations to this drift term does not change the results.

Figure 1: euro area

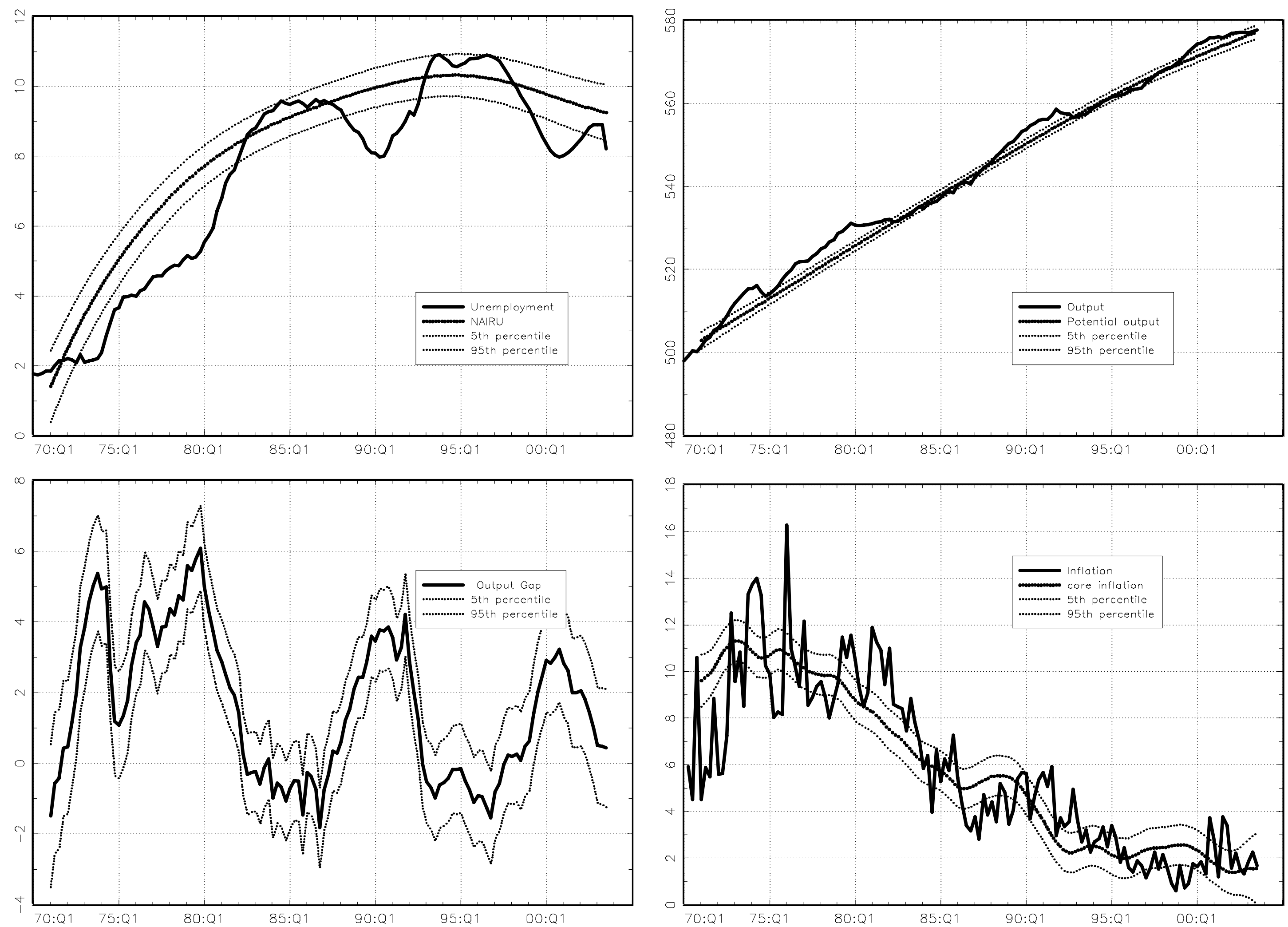
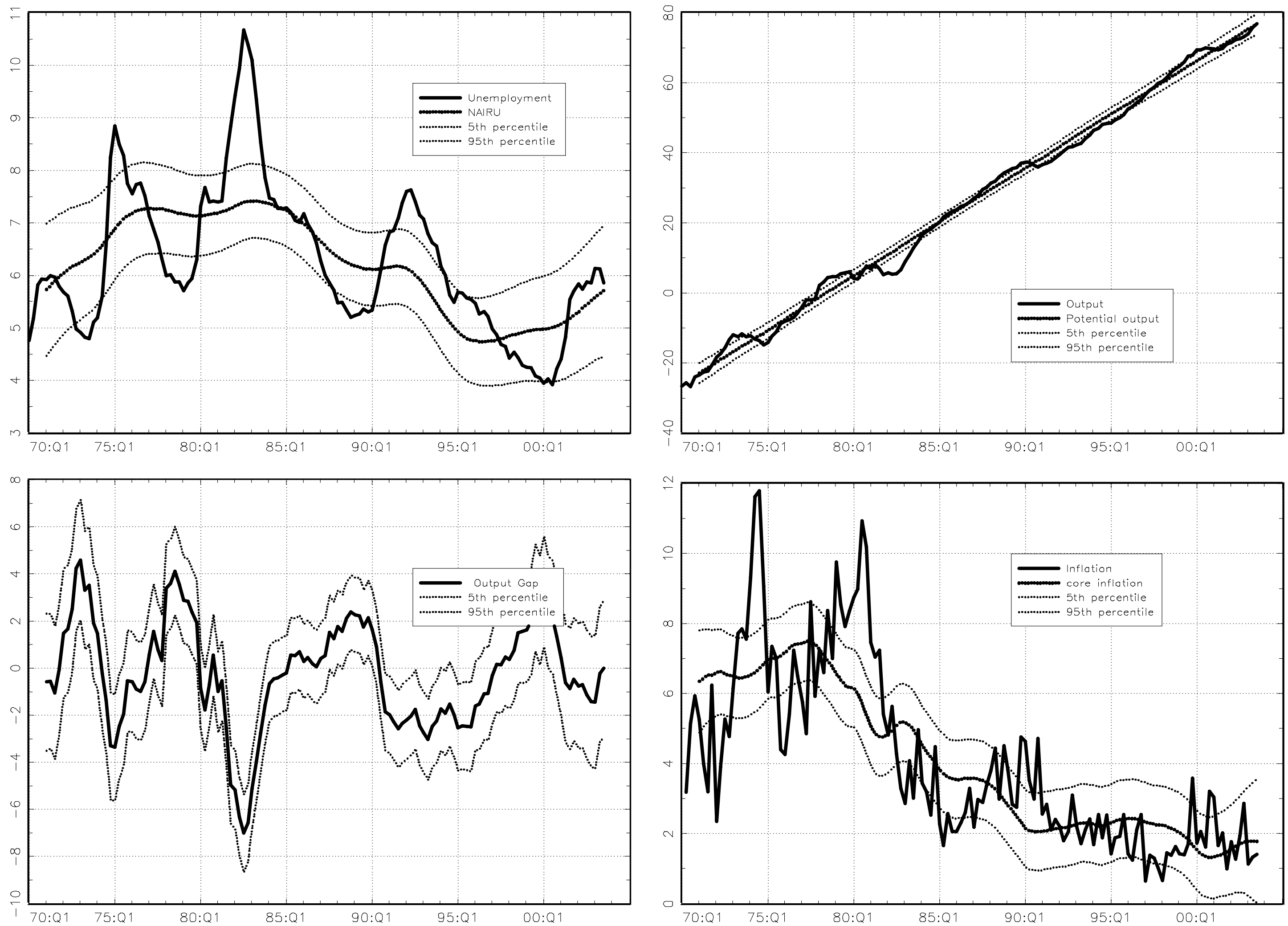


Figure 2: US


Unemployment
NAIRU
5th percentile
95th percentile
Output
Potential output
5th percentile
95th percentile
Output Gap
5th percentile
95th percentile
Inflation
core inflation
5th percentile
95th percentile

5 Conclusion

This paper estimates the NAIRU for the US and the euro area. It differs from existing studies in that (i) we derive the NAIRU from a structural model which explains unemployment dynamics by demand and supply factors as well as by a persistence mechanism, (ii) inflation is allowed to have a time-varying mean, (iii) we estimate the model in a Bayesian framework which allows us to maintain the cross sectional restrictions of the model and also provides a posterior distribution for the NAIRU accounting for both filter and parameter uncertainty.

We found a fairly high degree of persistence in Europe while unemployment is much less persistent in the US. Nevertheless, the increase of euro area unemployment until the late 1980s is driven by supply side factors. Our results also indicate that neglecting persistence effects may lead to NAIRU estimates that differ considerably from structural unemployment, i.e. the long-run NAIRU. In contrast most of unemployment variation in the US since the beginning of the 1970s is driven by demand shocks. Further the uncertainty around the NAIRU estimates is found to be reasonable small.

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Appendices

Appendix A Technical details state space estimation

A.1 State space representation of the model in (10)-(9)

$$y_t = \begin{bmatrix} y_t^d & u_t & \Delta p_t \end{bmatrix}'; \; x_t = \begin{bmatrix} \Delta p_{t-1} & y_{t-1} & u_{t-1} & z_t & z_{t-1} \end{bmatrix}';$$

$$\alpha_t = \begin{bmatrix} \overline{y}_t & \overline{y}_{t-1} & \psi_t & u_t^* & \sigma_t^d & \sigma_{t-1}^d & \phi_t & \tau_t & u_{t-1}^* \end{bmatrix}';$$

$$A = \begin{bmatrix} -\frac{\alpha}{\lambda_1} & \alpha & -\frac{\alpha\theta_{11}}{\lambda_1} & -\frac{\alpha\gamma_1}{\lambda_1} & -\frac{\alpha\gamma_2}{\lambda_1} \\ \frac{\alpha}{\lambda_1\omega} & -\frac{\alpha}{\omega} & \frac{\alpha\theta_{11}}{\lambda_1\omega} & \frac{\alpha\gamma_1}{\lambda_1\omega} & \frac{\alpha\gamma_2}{\lambda_1\omega} \\ \alpha & \frac{\alpha(\theta_1+\theta_{11})}{\omega} & \alpha\theta_{11} & \alpha\gamma_1 & \alpha\gamma_2 \end{bmatrix};$$

$$Z = \begin{bmatrix} 1 & -\alpha & 0 & \frac{\alpha\theta_{11}}{\lambda_1} & \frac{\alpha}{\lambda_1} & -\frac{\alpha}{\lambda_1} & 0 & 0 & 0 \\ 0 & \frac{\alpha}{\omega} & 0 & \frac{\alpha(\theta_1+\theta_{11})}{\lambda_1\omega} & -\frac{\alpha}{\lambda_1\omega} & \frac{\alpha}{\lambda_1\omega} & 0 & 0 & 0 \\ 0 & -\frac{\alpha(\theta_1+\theta_{11})}{\omega} & 0 & -\alpha\theta_{11} & \frac{\alpha(\theta_1+\theta_{11})}{\lambda_1\omega} & -\frac{\alpha(\theta_1+\theta_{11})}{\lambda_1\omega} & 0 & 0 & 0 \end{bmatrix};$$

$$T = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1+\delta) & 0 & 0 & 0 & 0 & -\delta \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad R = \begin{bmatrix} 1 & 0 & 0 & \omega & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} & \varepsilon_{2t} & \varepsilon_{3t} \end{bmatrix}'; \; \eta_t = \begin{bmatrix} \eta_{1t} & \eta_{2t} & \eta_{3t} & \eta_{4t} & \eta_{5t} & \eta_{6t} \end{bmatrix}';$$

$$H = \begin{bmatrix} \sigma_{\varepsilon_1}^2 & 0 & 0 \\ 0 & \sigma_{\varepsilon_2}^2 & 0 \\ 0 & 0 & \sigma_{\varepsilon_3}^2 \end{bmatrix}; \; Q = \begin{bmatrix} \sigma_{\eta_1}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\eta_2}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\eta_3}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\eta_4}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\eta_5}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\eta_6}^2 \end{bmatrix}.$$

A.2 Diffuse initialisation and exact Kalman filter and smoother

In a linear Gaussian state space model, the distribution of α_t is entirely determined by the filtered state vector $a_t = E(\alpha_t \mid Y_t, X_t)$ and the filtered state variance matrix $P_t = Var(\alpha_t \mid Y_t, X_t)$, where $Y_t = \{y_1, \ldots, y_t\}$ and $X_t = \{x_1, \ldots, x_t\}$. The filter recursion can be initialised by the assumption that $\alpha_1 \sim N(a_1, P_1)$ where we assume

$$\alpha_1 = V\Gamma + R_0\eta_0, \qquad \eta_0 \sim N(0, Q_0), \qquad \Gamma \sim N(0, \kappa I_r), \tag{A-1}$$

where the $m \times r$ matrix V and the $m \times (m - r)$ matrix R_0 select the r elements of the state vector that are non-stationary and the $m - r$ elements that are stationary respectively. They are composed of columns of the identity matrix I_m and are defined so that, when taken together, their columns constitute all the columns of I_m and $V'R_0 = 0$. The unconditional variance matrix Q_0 of the stationary elements of the state vector is positive definite and can be computed from the model parameters. The $r \times 1$ vector Γ is a vector of unknown random quantities which, as we let $\kappa \to \infty$, is referred to as the diffuse vector. This leads to

$$\alpha_1 \sim N(0, P_1), \qquad P_1 = \kappa P_\infty + P_*, \tag{A-2}$$

where $P_\infty = VV'$ and $P_* = R_0Q_0R_0'$. The Kalman filter is modified to account for this diffuse initialisation implied by letting $\kappa \to \infty$ by using the exact initial Kalman filter introduced by Ansley and Kohn (1985) and further developed by Koopman (1997) and Koopman and Durbin (2003). Subsequently, the Kalman smoother algorithm is used to estimate the smoothed state vector $\widehat{a}_t = E(\alpha_t \mid Y_n, X_n)$ and the smoothed state variance matrix $\widehat{P}_t = Var(\alpha_t \mid Y_n, X_n)$, where $Y_n = \{y_1, \ldots, y_n\}$ and $X_n = \{x_1, \ldots, x_n\}$. In order to account for the diffuse initialisation of α_1, we use the exact initial state smoothing algorithm suggested by Koopman and Durbin (2003).

A.3 Computational aspects of importance sampling

The idea is to use an importance density $g(\psi \mid y, x)$ as a proxy for $p(\psi \mid y, x)$, where $g(\psi \mid y, x)$ should be chosen as a distribution that can be simulated directly and is as close to $p(\psi \mid y, x)$ as possible. By Bayes' theorem and after some manipulations, equation (15) can be rewritten as

$$\overline{g} = \frac{\int g(\psi)\, z^g(\psi, y, x)\, g(\psi \mid y, x)\, d\psi}{\int z^g(\psi, y, x)\, g(\psi \mid y, x)\, d\psi}, \tag{A-3}$$

with

$$z^{g}\left(\psi,y,x\right)=\frac{p\left(\psi\right)p\left(y\mid\psi\right)}{g\left(\psi\mid y,x\right)}. \tag{A-4}$$

Using a sample of n random draws $\psi^{(i)}$ from $g\left(\psi\mid y,x\right)$, an estimate $\overline{g}_n$ of $\overline{g}$ can then be obtained as

$$\bar{g}_n=\frac{\sum_{i=1}^{n}g\left(\psi^{(i)}\right)z^{g}\left(\psi^{(i)},y,x\right)}{\sum_{i=1}^{n}z^{g}\left(\psi^{(i)},y,x\right)}=\sum_{i=1}^{n}w_i g\left(\psi^{(i)}\right), \tag{A-5}$$

with w_i

$$w_i=\frac{z^{g}\left(\psi^{(i)},y,x\right)}{\sum_{i=1}^{n}z^{g}\left(\psi^{(i)},y,x\right)}. \tag{A-6}$$

The weighting function w_i reflects the importance of the sampled value $\psi^{(i)}$ relative to other sampled values. Geweke (1989) shows that if $g\left(\psi\mid y,x\right)$ is proportional to $p\left(\psi\mid y,x\right)$, and under a number of weak regularity conditions, $\overline{g}_n$ will be a consistent estimate of $\overline{g}$ for $n\to\infty$. As an importance density $g\left(\psi\mid y,x\right)$, we take a large sample normal approximation to $p\left(\psi\mid y,x\right)$, i.e.

$$g\left(\psi\mid y,x\right)=N\left(\widehat{\psi},\widehat{\Omega}\right) \tag{A-7}$$

where $\widehat{\psi}$ is the mode of $p\left(\psi\mid y,x\right)$ obtained from maximising

$$\log p\left(\psi\mid y,x\right)=\log p\left(y\mid\psi\right)+\log p\left(\psi\right)-\log p\left(y\right) \tag{A-8}$$

with respect to $\widehat{\psi}$ and where $\widehat{\Omega}$ denotes the covariance matrix of $\widehat{\psi}$. Note that $p\left(y\mid\psi\right)$ is given by the likelihood function derived from the Kalman filter and we do not need to calculate $p\left(y\right)$ as it does not depend on ψ.

As any numerical integration method delivers only an approximation to the integrals in equation $(A-3)$, we monitor the quality of the approximation by estimating the probabilistic error bound for the importance sampling estimator $\overline{g}_n$ ((Bauwens et al., 1999) chap. 3, eq. 3.34). This error bound represents a 95% confidence interval for the percentage deviation of $\overline{g}_n$ from $\overline{g}$. It should not exceed 10%.

Note that the normal approximation in equation $(A-7)$ selects $g\left(\psi\mid y,x\right)$ in order to match the location and covariance structure of $p\left(\psi\mid y,x\right)$ as good as possible. One problem is that the normality assumption might imply that $g\left(\psi\mid y,x\right)$ does not match the tail behaviour of $p\left(\psi\mid y,x\right)$. If $p\left(\psi\mid y,x\right)$ has thicker tails than $g\left(\psi\mid y,x\right)$, a draw $\psi^{(i)}$ from the tails of $g\left(\psi\mid y,x\right)$ can imply an explosion of $z^{g}\left(\psi^{(i)},y,x\right)$. This is due to a very small value for $g\left(\psi\mid y,x\right)$ being associated with a relatively large value for $p(\psi)\,p(y \mid \psi)$, as the latter is proportional to $p(\psi \mid y, x)$. Importance sampling is inaccurate in this case as this would lead to a weight w_i close to one, i.e. $\overline{g}_n$ is determined by a single draw $\psi^{(i)}$. This is signalled by instability of the weights and a probabilistic error bound that does not decrease in n. In order to help prevent explosion of the weights, we change the construction of the importance density in two respects (Bauwens et al., 1999, chap. 3). First, we inflate the approximate covariance matrix $\widehat{\Omega}$ by multiplying it by a factor 1.3. This reduces the probability that $p(\psi \mid y, x)$ has thicker tails than $g(\psi \mid y, x)$. Second, we use a sequential updating algorithm for the importance density. This algorithm starts from the importance density defined by $(A-7)$, with inflation of $\widehat{\Omega}$, estimates posterior moments for $p(\psi \mid y, x)$ and then defines a new importance density from these estimated moments. This improves the estimates for $\widehat{\psi}$ and $\widehat{\Omega}$. We continue updating the importance density until the weights stabilise. The number of importance samples n was chosen to make sure that the probabilistic error bound for the importance sampling estimator $\overline{g}_n$ does not exceed 10%.

A.4 Posterior distribution of parameter and states

An estimate $\widetilde{\psi}$ for the posterior mean $E[\psi \mid y, x]$ of the parameter vector ψ is obtained by setting $g\left(\psi^{(i)}\right) = \psi^{(i)}$ in equation $(A-5)$ and taking $\widetilde{\psi} = \overline{g}_n$. An estimate $\widetilde{\alpha}_t$ for the posterior mean $E[\widehat{\alpha}_t \mid y, x]$ of the smoothed state vector $\widehat{\alpha}_t$ is obtained by setting $g\left(\psi^{(i)}\right) = \widehat{\alpha}_t^{(i)}$ in equation $(A-5)$ and taking $\widetilde{\alpha}_t = \overline{g}_n$, where $\widehat{\alpha}_t^{(i)}$ is the smoothed state vector obtained from the Kalman smoother using the parameter vector $\psi^{(i)}$. In order to calculate the 5th and 95th percentiles of the posterior densities of both the parameter vector ψ and the smoothed state vector $\widehat{\alpha}_t$, let $F(\psi_j \mid y, x) = \Pr\left(\psi_j^{(i)} \leq \psi_j\right)$ with ψ_j denoting the j-th element in ψ. An estimate $\widetilde{F}(\psi_j \mid y, x)$ of $F(\psi_j \mid y, x)$ is obtained by setting $g\left(\psi^{(i)}\right) = I_j\left(\psi_j^{(i)}\right)$ in equation $(A-5)$ and taking $\widetilde{F}(\psi_j \mid y, x) = \overline{g}_n$, where $I_j\left(\psi_j^{(i)}\right)$ is an indicator function which equals one if $\psi_j^{(i)} \leq \psi_j$ and zero otherwise. An estimate $\widetilde{\psi}_j^{5\%}$ of the 5th percentile of the posterior density $p(\psi \mid y, x)$ is chosen such that $\widetilde{F}\left(\psi_j^{5\%} \mid y, x\right) = 0.05$. An estimate $\widetilde{\alpha}_{j,t}^{5\%}$ of the 5th percentile of the jth element of the posterior density $p(\widehat{\alpha}_t \mid y, x)$ is obtained by setting $g\left(\psi^{(i)}\right) = \widehat{\alpha}_{j,t}^{(i)} - 1.645\sqrt{\widehat{P}_{j,t}^{(i)}}$ in equation $(A-5)$ and taking $\widetilde{\alpha}_{j,t}^{5\%} = \overline{g}_n$, where $\widehat{\alpha}_{j,t}^{(i)}$ denotes the j-th element in $\widehat{\alpha}_t^{(i)}$ and $\widehat{P}_{j,t}^{(i)}$ is the (j, j)th element of the smoothed state variance matrix $\widehat{P}_t^{(i)}$ obtained using the parameter vector $\psi^{(i)}$. The 95th percentiles are constructed in a similar way. As such the posterior distribution of the smoothed state vector $\widehat{\alpha}$ take both parameter and filter uncertainty into account.

Appendix B Prior and Posterior distribution

Figure 3: Prior and Posterior distribution euro area

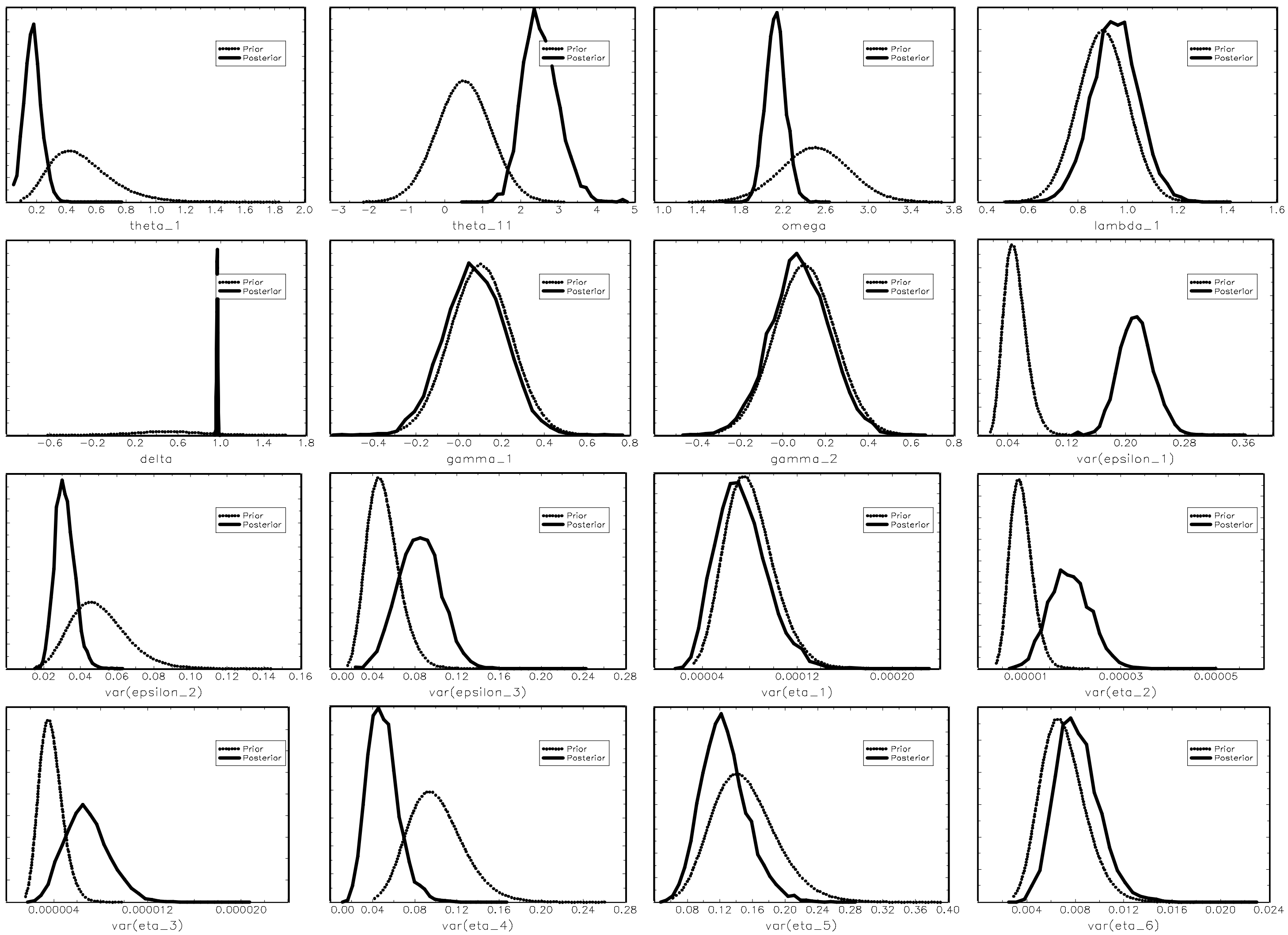

Figure 4: Prior and Posterior distribution US