Image denoising using fractal and wavelet-based methods

K. U. Barthel[*] , H. L. Cycon[**],
Fachhochschule für Technik und Wirtschaft Berlin, University of Applied Sciences

D. Marpe[†]
Fraunhofer-Institut für Nachrichtentechnik HHI, Image Processing Department

ABSTRACT

There is a considerable amount of literature about image denoising using wavelet-based methods. Some new ideas where also reported using fractal methods. In this paper we propose a hybrid wavelet-fractal denoising method. Using a non-subsampled overcomplete wavelet transform we present the image as a collection of translation invariant copies in different frequency subbands. Within this multiple representation we do a fractal coding which tries to approximate a noise free image. The inverse wavelet transform of the fractal collage leads to the denoised image. Our results are comparable to some of the most efficient known denoising methods.

Keywords: image denoising, fractal image compression, fractal denoising, wavelet image denoising, image restoration

1. INTRODUCTION

Denoising i.e. restoration of electronically distorted images is an old but also still a relevant problem discussed in the literature [2, 3, 4, 5, 6, 7, 8]. There are many different cases of distortions. One of the most prevalent cases is distortion due to additive white Gaussian noise which can be caused by poor image acquisition or by transferring the image data in noisy communication channels. Early methods to restore the image used linear filtering or smoothing methods. These methods where simple and easy to apply but their effectiveness is limited since this often leads to blurred or smoothed out in high frequency regions.
All denoising methods use images artificially distorted with well defined white Gaussian noise to achieve objective test results. Note however that in real world images, to discriminate the distorting signal from the “true” image is an ill posed problem since it is not always well defined whether a pixel value belongs to the image or it is part of unwanted noise.

Newer and better approaches perform some thresholding in the wavelet domain of an image. The idea of wavelet thresholding relies on the assumption that the signal magnitudes dominate the magnitudes of the noise in a wavelet representation, so that wavelet coefficients can be set to zero if their magnitudes are less than a predetermined threshold. More recent developments focus on more sophisticated methods, like local or context-based thresholding in the wavelet domain [3, 4, 8]. Some methods are inspired by wavelet-based image compression methods [3, 4, 5, 6, 8]. A new approach to image denoising was proposed using fractal compression techniques for denoising. As fractal coding can be performed in the wavelet domain it is also possible to carry out the fractal denoising in the wavelet domain [9]. The background of all these methods is based on the idea that denoising is a special case of lossy image compression. In this sense lossy image compression can be understood as image restoration.

[[*] barthel | [**] hcycon]@fhtw-berlin.de ; phone +49 30 50192416 | +49 30 54699 363 ; http://www.fhtw-berlin.de
University of Applied Sciences (FHTW Berlin), Treskowallee 8, 10313 Berlin, Germany
[†] marpe@hhi.de ; phone +49 30 31002-619; fax +49 30 3927200; http://bs.hhi.de
Image Processing Department, Heinrich-Hertz-Institute (HHI), Einsteinufer 37, 10587 Berlin, Germany

In this paper we propose another way to combine fractal and wavelet-based methods inspired by [9]. Here we use a non-subsampled overcomplete wavelet representation of the image which combined with a modification of the conventional fractal coding approach. The major disadvantage of fractal image coders, their difficulty to encode finely structured image patterns will be used to for denoising images. Natural image structures possess similarities across resolution scales, which normally can be exploited for fractal image coding. Noisy structures however have no resemblance in other resolutions or other parts of the image and can therefore not be encoded using fractal coders. Encoding a noisy image with a fractal coder results in a good approximation of “natural” / self similar structures, whereas the noisy contents can not be described. Using conventional fractal coding schemes usually will lead to annoying blocking artifacts. This can be overcome if the fractal encoding is performed in the wavelet domain. This paper examines the ability to fractal denoise images using a non-subsampled overcomplete wavelet approach. Modifications to the standard fractal encoding scheme are described.
Our results are comparable or better than some than some of the most efficient known denoising methods.
The organization of the paper is as follows. In section 2 we describe the proposed algorithm and present the results in section 3.

1.1. Fractal image coding

Fractal image coding can be described as follows: The image to be encoded is partitioned into non-overlapping *range blocks* ***Y***. The task of the fractal coder is to find a larger block of the same image (a *domain block*) ***X'*** for every range block such that a transformation of the domain block is a good approximation of the range block (figure 1). The transformation consists of a *geometrical transformation* γ and a *luminance transformation* λ. The geometrical transformation performs a lowpass filtering and sub-sampling followed by a position shift. The luminance transformation scales the intensities and changes the mean of the downscaled domain block ***X***. The *collage* is the approximation that is obtained if all fractal transforms are applied to the original image. Fractal coding consist in finding a good collage that is very similar to the original image. Under the condition that these transformations are contractive, this set of transformations can iteratively be applied to any initial image which then will converge to the decoded image (the fractal *attractor*). Fractal encoding of images is lossy. Compression can be achieved if the set of transformations can be described more efficiently than the original pixel data. The error between the original image and the fractal collage will always be exceeded by the error of the decoded fractal attractor.

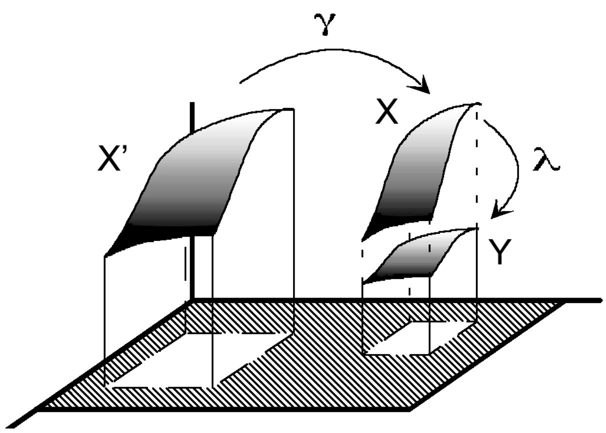


Fig. 1: A fractal approximation of a range block ***Y*** through a transformed domain block ***X'***

Smooth regions and edges are very self similar and can be coded efficiently by fractal coders. Irregular textures or noisy regions can not be approximated well, as they do not possess similarities across scales. This can be overcome by using a coding scheme with variable block sizes (e.g. quadtree partitioning) or hybrid coding approaches combining fractal coding with other coding techniques. Various optimizations of fractal coding schemes were performed. However as described in the next section, it turned out that fractal coding can be described as a mapping of coefficient-trees in a wavelet decomposition of an image. Fractal coding is nothing else but a wavelet coder with a very restricted mapping rule for coefficient (sub-)trees. This is one reason why other (non “fractal restricted”) wavelet coders outperformed pure fractal coding schemes [12].

1.2. Fractal coding in the wavelet domain

Under the partitioning constraint that every domain block is made up of an even number of range blocks conventional fractal coding can be described in the Haar-wavelet domain. The approximation of a range block through a contracted domain block in the spatial domain then can be described as the prediction (or extrapolation) of fine scale coefficients from coarse scale coefficients in the wavelet domain. The spatial contraction (lowpass average filtering and subsampling) corresponds to moving coefficients to the next higher frequency scale (figure 2). Now the decoding can be performed in a non-iterative way by consecutively extrapolating higher frequency coefficients from lower frequency coefficients. Fractal coding in the wavelet domain is not limited to the Haar-wavelet. The usage of smooth basis wavelets corresponds to fractal coding with overlapping range blocks in the spatial domain, thus avoiding blocking artefacts.

A detailed description of the analogy of fractal coding in the spatial and the wavelet domain can be found in [10]. The geometrical transformation consists in picking a wavelet coefficient-tree of a domain block ***X'*** and to eliminate the highest frequency coefficients. This corresponds to a lowpass filtering and subsampling. Then this reduced coefficient-tree ***X*** is mapped to the position of the coefficient-tree of the range block ***Y*** in the next higher frequency levels. The luminance transformation allows the values of the mapped coefficients to by multiplied by a scaling factor *a*. The mean of the range block to be changed or set by adjusting or setting one single (root) coefficient in the lowest frequency band.

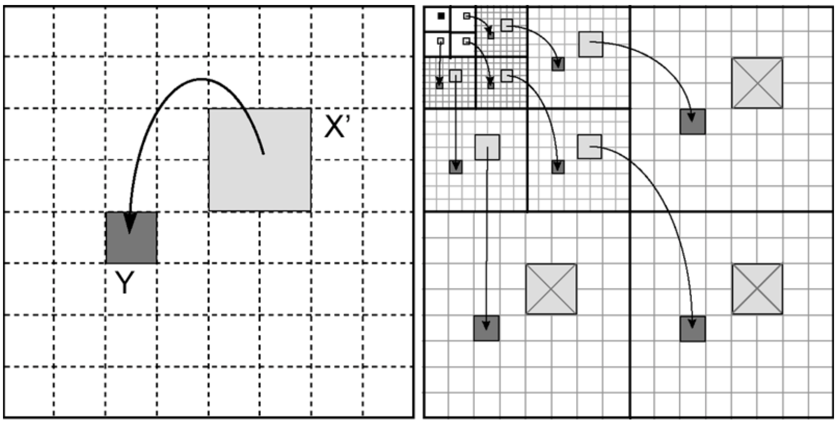


Fig. 2: Interpretation of fractal coding in the wavelet domain. The approximation of a range block ***Y*** through a spatially contracted domain block ***X'*** (left) can be done in the wavelet domain (right).

1.3. Wavelet based denoising schemes

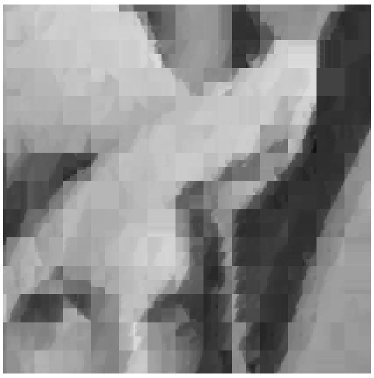
The idea of wavelet thresholding relies on the assumption that the signal magnitudes dominate the magnitudes of the noise in a wavelet representation, so that wavelet coefficients can be set to zero if their magnitudes are less than a predetermined threshold. Donoho and Johnstone [11] proposed *hard-* and *soft-thresholding* methods for denoising, where the former leaves the magnitudes of coefficients unchanged if they are larger than a given threshold, while the latter just shrinks them to zero by the threshold value.

However, the major problem with both methods and most of its variants is the choice of a suitable threshold value. Most signals show a spatially non-uniform energy distribution, which motivates the choice of a non-constant threshold. Since a given noisy signal may consist of some parts where the magnitudes of the signal are below the globally defined threshold and other parts where the noise magnitudes exceed that given threshold, methods relying on a globally defined threshold cut of parts of the signal, on the one hand, and leave some noise untouched, on the other hand. This observation led to the idea of a spatially adaptive threshold choice depending on the relationship of local energy (variance) of the observed signal and the noise variance.

Chang et al. [3, 4] were the first to propose this kind of spatially adaptive wavelet thresholding for image denoising. Their method of selecting a spatially adaptive threshold is based on a context model, which involves neighboring coefficients of the wavelet decomposition for the estimation of the local variance. The authors extended this idea by using a more elaborate context model and by iterating the context-based thresholding process in the denoised wavelet representation, which led to significantly improved results [8].

1.4. Fractal denoising

Fractal denoising tries to use the fact that fractal coders can describe self similar structures across scales very well but do fail to approximate noisy structures. Consequently if a conventional fractal image coder is applied to a noisy image it will produce a noise reduction. The task of fractal denoising is to construct a fractal code for the noisy image such, that either the collage or the attractor is closer to the original noise-free image than the non encoded noisy image. Opposed to fractal compression no restrictions to the number or complexity of the transformations have to be made.
The fractal code for the image to be denoised has to be constructed in such a way that the original image parts have to be preserved (approximated as well as possible) whereas all noisy components should be discarded. In order to achieve this, a careful choice of fractal encoding parameters has to be made. Figure 3 shows the influence of the block size of a fractal coder if applied to a noisy image. If the range block sizes are chosen to be large, then all noisy components will be removed, however the quality of the original image will also be degraded. A smaller range block size will improve the image quality. If the range block size is too small however, all details from the original image can be approximated well but now also the noisy components will be approximated, which brings back the noisy components leading to a lower overall quality. This example demonstrates the importance of a proper choice of the fractal encoding parameters, that need to be adapted to the image content and the amount of noise. A simple approach is to use a quadtree partitioning scheme. If some decision criterion (like the approximation error) is exceeded for a range block, then this block is split into four smaller blocks. Figure 3 (lower line, middle) shows the coding result of a quadtree partitioning with improved denoising results. However also this result is still far from being acceptable. In addition it should be observed how the image quality is severely affected by blocking artifacts if a fractal coder operating in the spatial domain is used for denoising.



fractal attractor, bs = 16
PSNR = 24.3; RMSE = 15.5

fractal attractor, bs = 8
PSNR = 26.8; RMSE = 11.6

fractal attractor, bs = 4
PSNR = 25.7; RMSE = 13.3

noisy image

PSNR = 20.2, RMSE = 25

fractal attractor using a
quadtree partition bs = 16/8/4
PSNR = 27.3; RMSE = 11.0

original image

Fig. 3: Comparison of fractal encodings of a noisy image using variable range block sizes (bs)

Ghazel et. al. [9] proposed to estimate the fractal code of a noiseless image from a noisy image. If it is possible to determine this "noiseless code" then obviously the attractor will be very similar to the noise-less image. We will discuss our experiments and results concerning this idea in the next section.

2. PROPOSED ALGORITHM

2.1. Fractal denoising in the wavelet domain

Like fractal image coding also fractal image denoising can be performed in the wavelet domain. This is an effective approach to avoid blocking artifacts in the fractal approximation. However if typical octave band wavelet decompositions are used, only a limited set of domain blocks is available leading to reduced coding efficiency. This again can be overcome by employing a non-subsampling overcomplete wavelet decomposition of the image.

The usage of an overcomplete wavelet decomposition corresponds to the usage of a set of shifted images. If a fractal approximation is determined not only for one image but for all of these shifted versions, then an additional noise reduction gain is to be expected. If identical signals are superimposed by different statistical independent noise signals, then the addition of these noisy signals will lead to an attenuation of the noise as the wanted signals are correlated whereas the uncorrelated noisy signal attenuate each other.

Fractal denoising suffers from two problems: some parts of the original signal are not approximated well, whereas some noisy parts are approximated by the fractal coder although they are not part of the wanted signal. Under the assumption that these both problems do occur in different regions in the set of the shifted images, the inverse wavelet transform which corresponds to the superposition of the images (shifted back) will reduce the noise and improve the approximation quality of the reconstructed image.

To emphasize our approach two aspects shall be mentioned: Using an overcomplete wavelet decomposition even with a Haar-wavelet fractal coder no blocking artifacts will occur. In addition to our best knowledge the best denoising results are obtained using overcomplete wavelet decompositions, balancing the effects of uncertain thresholding of coefficients.

One further advantage of the wavelet domain is the fact, that range blocks need not be restricted to the wavelet-coefficient trees which correspond to the range blocks in the spatial domain. Our denoising scheme uses individual separate sub-trees in the three different frequency orientations (horizontal, vertical and diagonal direction).

2.2. Estimation of noise-less fractal codes from a noisy image

The task of a fractal coder is to approximate each range block (or a wavelet sub-tree) Y_i by a scaled domain block X_j .

$$Y_i \approx a_{ij} \cdot X_j$$

In the noiseless case the optimal scaling factor a_{ij} is determined by

$$a_{ij} = \frac{E[X_j Y_i]}{E[X_j^2]},$$

where E[] denotes the expectation value. The best fractal approximation is to choose the domain block X_j leading to the smallest squared L^2 distance

$$\Delta_{ij}^2 = E[(Y_i - a_{ij} \cdot X_j)^2] = E[Y_i^2] + a_{ij}^2 E[X_j^2] - 2a_{ij} E[X_j Y_i]$$

In the presence of noise however, the domain and range vectors are distorted:

$$\hat{X} = X + N; \quad \hat{Y} = Y + N$$

If a scaling factor $\hat{a}$ is to be determined in the noisy image, the noise will affect the result.
(sub-indices i and j are omitted for simplicity)

$$\hat{a} = \frac{E[\hat{X}\hat{Y}]}{E[\hat{X}^2]}$$

Under the independence assumption between the noise and the image signal, we have:

$$E[\hat{X}\hat{Y}] = E[XY] \quad and \qquad E[\hat{X}^2] = E[X^2] + E[N^2]$$

Ghazel et. al. [9] propose to recalculate the scaling factor of the noise-free case from the noisy image data:

$$a = \hat{a}\left(1 + \frac{1}{\gamma}\right) \qquad \textit{with} \qquad \gamma = \frac{E[X^2]}{E[N^2]} \text{ (the SNR)}$$

Ghazel et. al. use this result to determine the collage error of the noise-less fractal code, which then can be computed using statistics from the noisy image. They claim that a significant improvement in fractal denoising can be achieved, if this modified scaling factor is used for the error criterion for the selection of the fractal code of the noise-less image:

$$\Delta_{ij}^2 = E[\hat{Y}_i^2] + E[N^2] + a_{ij}^2\left(E[\hat{X}_j^2] + E[N^2]\right) - 2a_{ij}E[\hat{X}_j\hat{Y}_i]^2$$

We tried to verify this result, but could not determine any gain in the denoising behavior of the fractal coder. This may be due to the fact, that we might have used a wrong estimate for γ.
However we think the calculation of the original scaling factor of the noise-less image from the noisy image has several problems: Δ^2_{ij} still is dependent on the noise-free image (which is not available) as γ depends on $E[X^2]$ which cannot easily be estimated or measured. $E[N^2]$ the variance of the noise can be determined for the entire image, however it will be different for individual domain and range blocks especially if these blocks are small. A third problem occurs if domain and range blocks share some wavelet coefficients, because now the noisy part is correlated. This overlap happens quite often for fractal encoding.
The result for the calculation of the noise-less scaling factor is rather unanticipated, as this means the scaling factor a can only become larger. However larger scaling factors advance the noise propagation to higher frequency bands.

We modified our fractal denoising scheme in a different way. First we also perform an estimation of the variance of the corrupting noise. The estimation of the “real noise variance” is performed by using the robust median estimator in the highest subband (highpass filtered in both directions) of each of the four different ‘branches’ of the whole quadtree of subbands as proposed by Donoho et al. [5, 11].

For the choice of the scaling factor a we distinguish two cases: If $E[\hat{Y}]$ ($\approx$ the variance of the range block) is much larger than the estimated global noise variance $\hat{\sigma}_N^2$; then the influence of the noise can be neglected. If $E[\hat{Y}]$ is smaller however, a large scaling factor could amplify the noise. We chose a modified scaling factor $\tilde{a}$ as

$$\tilde{a} = \begin{cases} \hat{a} & \textit{if } E[\hat{Y}^2] > K\hat{\sigma}_N^2 \\ \hat{a}\dfrac{E[\hat{Y}^2]}{K\hat{\sigma}_N^2} & \textit{if } E[\hat{Y}^2] \le K\hat{\sigma}_N^2 \end{cases}$$

Where K is a constant to be chosen, we found a value of K=3 to be a good choice. As collage error criterion for the selection of the fractal code we use

$$\Delta_{ij}^2 = E[\hat{Y}_i^2] + E[N^2] + \tilde{a}_{ij}^2\left(E[\hat{X}_j^2] + E[N^2]\right) - 2\tilde{a}_{ij}E[\hat{X}_j\hat{Y}_i]^2 ,$$

which is similar to the previous error selection criterion except for the modified scaling factor. Using this modification we observed a better denoising capability of the fractal coder.

2.3. Choice of parameters

We realized a fractal denoising scheme using a non-subsampled overcomplete wavelet representation with five wavelet decomposition levels for the domain blocks which corresponds to four decomposition levels for the range blocks. As wavelet filters we investigated the classical fractal Haar-, the Daubechies biorthogonal 9/7, and the Villasenor's biorthogonal 18/10-wavelet [1]. For the subsampled wavelet representation in all cases the 18/10-wavelet outperformed the Haar-wavelet by a large amount and gave constantly little better results than the 9/7-wavelet. This difference between the filters is reduced in the case of the non-subsampled overcomplete wavelet representation.
Range and domain sub-trees were treated independently in the different orientations. In the spatial domain the largest corresponding range block would have a size of 16 x 16 pixels. In the wavelet domain this domain block is represented by one DC-coefficient plus three individual sub-trees for each orientation, each containing 85 coefficients. The range sub-trees can be split using a quadtree partitioning scheme. Figure 4 (right) shows how a sub-tree is split into four smaller sub-trees. As splitting criterion we do not use the collage error, we found the variance of the range sub-tree to be a better splitting criterion. The maximum splitting depth can go down to single coefficients.

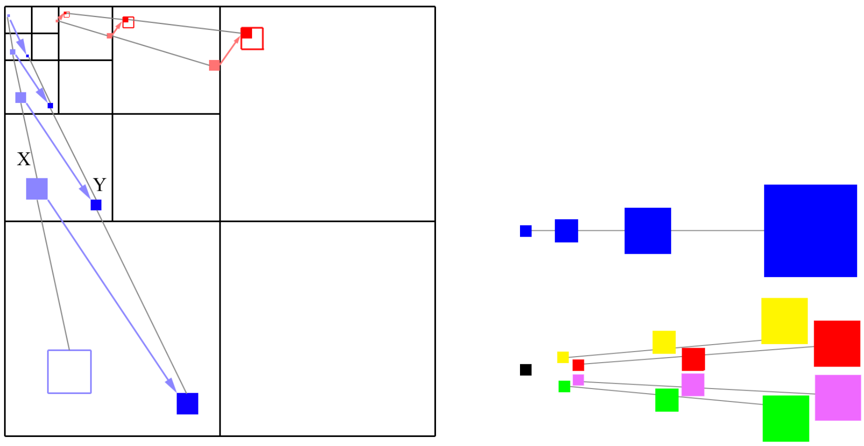


Fig. 4.: left: Approximation of a range wavelet sub-tree *Y* through a domain sub-tree *X* of the same spatial orientation, the upper mapping shows part of a sub-tree that has been split, right: splitting of one sub-tree to one coefficient plus four new smaller sub-trees.

As we use a non-subsampled overcomplete wavelet representation, fractal denoising has to performed for all branches of the wavelet quadtree. For a four level decomposition there are 256 images to be denoised. However this can be done very quickly as only a few suited domain sub-trees from a search region centered at the parent position of the range sub-tree are examined (Figure 5 right). In our current implementation the search for appropriate domain sub-trees is limited to the same wavelet tree of a particular shift.

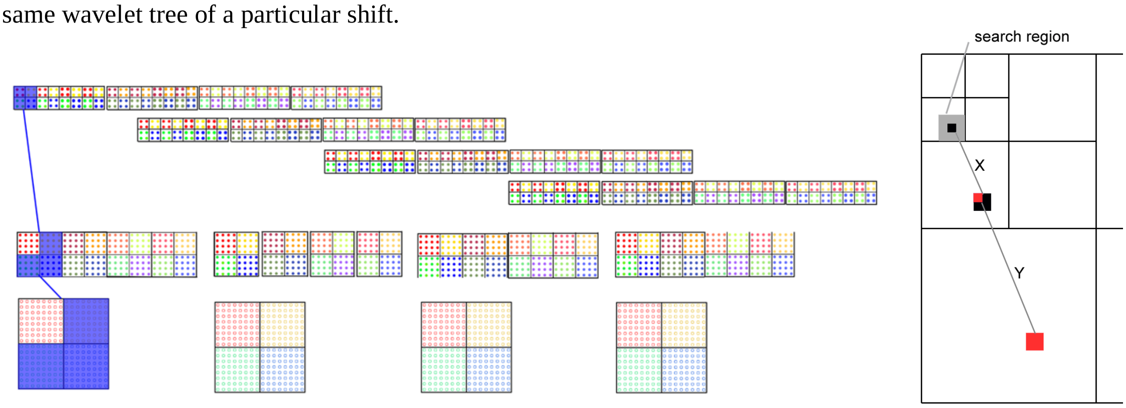


Fig. 5.: left: non-subsampled overcomplete wavelet representation, right: Approximation of a range wavelet sub-tree *Y* through a domain sub-tree *X* of the same spatial orientation, only domain sub-trees from a search region are used.

3. EXPERIMENTAL RESULTS

Table 1 compares our proposed fractal denoising scheme to the fractal wavelet denoising scheme proposed by Ghazel et. al. [9]. It should be observed, that the overcomplete non subsampled wavelet decomposition gives greatly improved denoising results, this at the cost of higher computation time.

Table 1: Comparisons of fractal denoising techniques (Lena image $\sigma_N = 25$, PSNR = 20.17 dB)

	Haar overcomplete	Haar subsampled	18/10 overcomplete	18/10 subsampled	Ghazel FW[9]
PSNR	30.76	28.08	**30.94**	29.13	29.80
RMSE	7.39	10.06	**7.23**	8.91	8.25

noisy image
$\sigma_N = 25$
PSNR = 20.17 dB

fractal denoised Haar-wavelet
(subsampled)
PSNR = 28.08 dB

fractal denoised 18/10-wavelet
(overcomplete)
PSNR = 30.94 dB

Fig. 6: Example images from table 1

Figure 7 compares the results of our fractal-wavelet denoising scheme with other sophisticated context-based denoising schemes in the wavelet domain [4, 8]. For the Lena image the results are comparable to the Chang denoising scheme, whereas for the Barbara image there is still an important gap between the results, which seems to come from the fact, that our splitting criterion does not detect the bad approximation of important high frequency coefficients. However it should be remarked that even our subsampled wavelet denoiser outperforms the classical Lee filter [2].

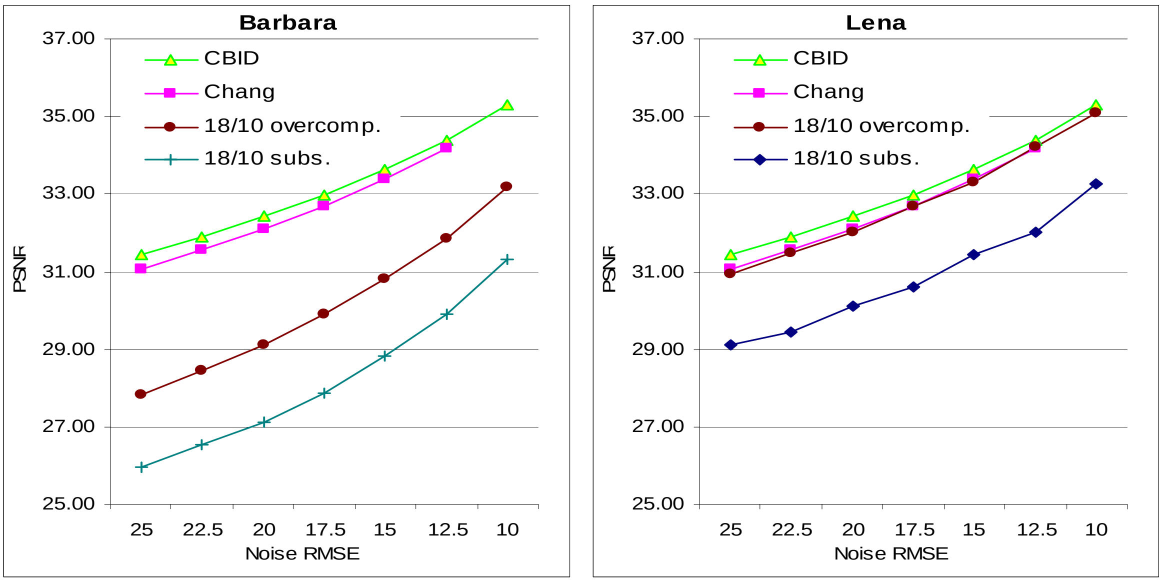


Fig. 7: Comparison with sophisticated non-fractal denoising schemes

4. CONCLUSIONS

We proposed a fractal denoising scheme operating in a non-subsampled overcomplete wavelet decomposition. Denoising results are significantly improved compared to a subsampled wavelet decomposition. For some images the denoising results are comparable to other state of the art wavelet denoisers. For other images there is still an important gap between the results. This is particular true for the Barbara image, which is related to the fact that better splitting criteria are needed in order to properly distinguish important signal components in the high frequency components.
Further research will investigate such techniques. In addition instead of a top-down approach, also a bottom-up partitioning scheme could be useful. Further improved approximation results are to be expected if domains from trees from shifted images are possible, which is not yet implemented in our current approach.

REFERENCES

1. J. D. Villasenor, B. Belzer and J. Liao, „Wavelet Filter Evaluation for Image Compression", IEEE Trans. Image Proc., August 1995.
2. J. S. Lee, "Digital image enhancement and noise filtering by use of local statistics", *IEEE PAMI*, vol. 2, no. 2, pp. 165-168, 1980.
3. S. G. Chang, B. Yu and M. Vetterli, "Spatially adaptive wavelet thresholding with context modeling for image denoising", *Proc. IEEE Int. Conf. on Image Processing*, Oct. 1998.
4. S. G. Chang, B. Yu and M. Vetterli, "Spatially adaptive wavelet thresholding with context modeling for image denoising", . *IEEE Trans. on Image Processing*, Vol. 9, No. 9, pp. 1522-1531, 2000. .
5. R. R. Coifman and D. L. Donoho, "Translation-invariant denoising" in *Wavelets and Statistics*,
6. X. Li and M. T. Orchard: "Spatially Adaptive Image Denoising under Overcomplete Expansions", *Proc. IEEE Int. Conf. on Image Processing*, Vancouver, 2000.
7. M. K. Mihcak, I. Kozintsev, and K. Ramchandran, "Spatially Adaptive Statistical Modeling of Wavelet Image Coefficients and Its Application to Denoising," *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, vol. 6, Mar. 1999, pp. 3253–3256.
8. D. Marpe, H. L. Cycon, G. Zander, K. U. Barthel, "Context-based Denoising of Images Using Iterative Wavelet Thresholding" *Proc. SPIE*, Vol. 4671, pp. 907-914, Jan. 2002.
9. M. Ghazel, G. H. Freeman, E. R. Vrscay, "Fractal-Wavelet Image denoising" *Proceedings of IEEE International Conference on Image Processing, (ICIP* 2002), I-836 – I-839
10. G. Davis, "A wavelet-based analysis fractal image compression, "IEEE Trans. Image Processing, vol. 7, pp. 141-154,1998.
11. D. L. Donoho and I. M. Johnstone, "Ideal Spatial Adaptation Via Wavelet Shrinkage ", Biometrica, vol.81, pp.425 –455,1994.
12. K. U. Barthel, S. Brandau, W. Hermesmeier, and G. Heising, "Zerotree Wavelet Coding Using Fractal Prediction", *Proceedings of IEEE International Conference on Image Processing (ICIP'97)*, Santa Barbara, CA, USA, Oct. 26-29, 1997